Mathematical analysis of some termination principles

Andreas Weiermann
Institut für Mathematische Logik und Grundlagenforschung
der Westfälischen Wilhelms-Universität Münster
Einsteinstr. 62, D-48149 Münster, Germany

Abstract

In this survey talk we start with recalling some celebrated combinatorial independence results. These include Friedman's miniaturization of the well-foundedness of ε_0, Friedman's miniaturization of Kruskal's theorem (KT) and Kirby's and Paris' results on Goodstein sequences and Hydra battles. We then give a mathematical classification of these principles in terms of growth rate conditions using recursion theory and methods from analytic combinatorics, a field which is familiar in computer science from the average case analysis of algorithms.

We relate Otter's tree constant 2.95576... to Friedman's miniaturization of KT and obtain the following refinement [to appear in JSL] of a result by Loebl and Matoušek. Let α be Otter's tree constant and $c := \frac{1}{\log_2(\alpha)}$. Then for any rational number $r > c$ the following assertion is true but independent of first order Peano arithmetic: For any natural number K there exists a natural number M so large that for any sequence $T_0, \ldots, T_M$ of finite trees satisfying that the number of nodes in T_i is bounded by $K + r \cdot \log_2(i+1)$ for $i = 0, \ldots, M$ we find indices i and j less than or equal to M such that $i < j$ and T_i is homeomorphically embeddable into T_j.

This result is rather sharp since for $r < c$ the corresponding assertion can be proved within primitive recursive arithmetic.

Friedman's miniaturization of the well-foundedness for ε_0 and iterated ω-powers ω_h is classified similarly with an application of the saddle point method. Using a result from random walk theory we give a classification of Friedman's miniaturization of KT for binary trees. Using related techniques we also obtain complete classification results for Hydra battles and Goodstein sequences. These investigations lead to new Goodstein style sequences for which termination is independent. We further characterize the derivation lengths for the finitary Ackermann functions using Brigham's theorem from analytic number theory (joint work with I. Lepper).