Part II — Solutions

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Question 1 Find a confluent and terminating rewrite system for the following equational theory:

$$\begin{aligned} \epsilon * x &= x \\ x * \epsilon &= x \\ (x : y) * z &= x : (y * z) \\ x * (y * z) &= (x * y) * z \end{aligned}$$

Answer:

$$\begin{aligned} \epsilon * x &\rightarrow x \\ x * \epsilon &\rightarrow x \\ (x : y) * z &\rightarrow x : (y * z) \\ (x * y) * z &\rightarrow x * (y * z) \end{aligned}$$

Question 2 Find a confluent and terminating rewrite system for the above theory *plus* the following equations:

$$\begin{aligned} r(\epsilon) &= \epsilon \\ r(x : \epsilon) &= x : \epsilon \\ r(x * y) &= r(y) * r(x) \end{aligned}$$

Answer: Add

$$\begin{aligned} r(\epsilon) &\rightarrow \epsilon \\ r(x : y) &\rightarrow r(y) * (x : \epsilon) \\ r(x * y) &\rightarrow r(y) * r(x) \end{aligned}$$

Question 3 What happens if you add the rule

$$r(x) * y \quad \rightarrow \quad t(x, y)$$

to the system you obtained for the previous question?

Answer: Add to (1)

$$\begin{aligned} r(x) &\rightarrow t(x, \epsilon) \\ t(\epsilon, y) &\rightarrow y \\ t(x : y, Z) &\rightarrow t(y, x : z) \\ t(x, y) * z &\rightarrow t(x, y * z) \\ t(x * y, \epsilon) &\rightarrow t(y, t(x, \epsilon)) \end{aligned}$$

Question 4 Find an ordered rewrite system for the following equational theory:

$$\begin{aligned} x + 0 &= x \\ x + s(y) &= s(x + y) \\ x + y &= y + x \\ d(x) &= x + x \end{aligned}$$

using a recursive path ordering with precedence $+ > d > s > 0$.

Answer:

$$\begin{aligned} x + 0 &\rightarrow x \\ x + s(y) &\rightarrow s(x + y) \\ d(0) &\rightarrow 0 \\ d(s(x)) &\rightarrow s(s(d(x))) \\ x + y &\leftrightarrow y + x \end{aligned}$$

Question 5 Show that there is a computable ordering and computable strategy such that, given any finite set of equations E and valid identity $s = t$, completion will prove that $E \vdash s = t$ by generating enough equational consequences of E to reduce both sides of the identity to the same term, without backtracking and undoing any simplification (contracting inference) step.