Rewrite-Based Deduction: Expansion and Contraction

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At the heart of automated deduction lies an inference engine that repeatedly expands the database of formulæ by applying rules of inference (forwards or backwards). Unlike classical proof theory, the database is not only expanded during deduction, but also contracted as a result of (typically ad-hoc) deletion of formulæ deemed "redundant" (e.g. tautology, subsumption, and demodulation tests).

Research in rewriting has contributed in various ways, including the suggestion of refinements of existing inference calculi. On the conceptual level, notions from rewriting have led to a formalization of contraction steps in theorem provers. A (possibly goal-oriented) complexity measure for proofs can be used to characterize redundant expansions that can be glossed over, as well as contractions to eliminate newly introduced redundancies. This approach provides a methodology for completeness proofs in the presence of contraction. Every proof based on the current formula set that requires a non-redundant inference must contain a subproof that can be replaced—after some expansion step—to obtain a strictly smaller proof. Then contraction does not harm completeness if it never forces the complexity of a proof to increase.

Improved versions of completion (with oriented instances of equations, contextual simplification, and critical pair criteria), resolution (with ordering strategies, subsumption and simplification), Horn-clause deduction (with ordering-based strategies and simplification by decreasing instances), and inductive proofs (with rippling) can be phrased within this framework. The choice of complexity measure dictates the precise contractions that are permissible.