1700 FORESTS

NACHUM DERSHOWITZ

Arvoles lloran por lluvia
y montañas por aire
Ansí lloran mis ojos
por ti querido amante

Ladino folk song

The number of ordered forests (sequences of non-trivial ordered rooted trees) with a total of n edges is

$$F_n = \frac{1}{2}\binom{2n}{n} = \binom{2n-1}{n}$$

Every tree in the forest must have at least one edge, or else there would be infinitely many forests for every n.

To see this: Ordered trees with n edges are in bijection with Dyck paths of length $2n$. So a forest, which is a sequence of trees, corresponds to a sequence of Dyck paths. Every (Grand-Dyck) lattice path returning to the baseline (the diagonal) after $2n$ steps can be interpreted as a sequence of Dyck paths, one per (non-trivial) tree, delineated by the points at which the path crosses the baseline. There are $\binom{2n}{n}$ such paths (since they must have n $\nearrow$-steps and n $\searrow$-steps). But a path and its mirror image correspond to the same forest. The equation follows.

This is sequence A001700 in Sloane's OEIS:

n	0	1	2	3	4	5	6	7	8	$\cdots$
F_{n+1}	1	3	10	35	126	462	1716	6435	24310	$\cdots$

That is, A001700$(n) = F_{n+1}$ counts the number of ordered forests with $n+1$ edges (and no trivial trees). Contrast the Catalan numbers, A000108(n), which count ordered forests with n *nodes* and allow for trivial trees.

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