Calendars*

Nachum Dershowitz† Edward M. Reingold‡

May 2011

An algorithm that gives a name to each day. These day names are generally distinct, but not always. For example, the day of the week is a calendar, in a trivial sense, with infinitely many days having the same day name. The design of calendars has been driven by human needs varying over millennia, so the study of calendars is intricately bound to history, astronomy, and religion. Ancient calendars were based on simple observations of various phenomena such as the waxing and waning of the moon, the change in seasons, or the movement of heavenly bodies. Because observational methods suffer from the vagaries of weather and chance, modern calendars tend to be based solely on arithmetical rules, distanced from their motivation in nature.

Most calendars divide a year into an integral number of months and divide months into an integral number of days. However, these astronomical periods—day, month, and year—are incommensurate, so exactly how one coordinates these time periods and the accuracy with which they approximate their astronomical values are what differentiate one calendar from another.

Dozens of calendars are still in use, in addition to the almost universal use of the Gregorian calendar. Many religious holidays and national events are determined by dates on these non-Gregorian calendars. Numerous tables, algorithms, and rules of thumb for conversion of dates between calendars have been published.

Solar calendars—including the Egyptian, Julian, Coptic, Ethiopic, Gregorian and Persian—are based on the yearly solar cycle, whereas lunar calendars, such as the Islamic, and lunisolar calendars, such as the Hebrew, Hindu, and Chinese, take the monthly lunar cycle as their basic building block. Most solar calendars are divided into months, but these months are divorced from the lunar events; they are sometimes related to the movement of the sun through the twelve signs of the zodiac.

Almost every calendar incorporates a notion of "leap" year to correct the cumulative error caused by approximating a year by an integral number of days and months. Solar calendars add a day every few years to keep up with the astronomical year.

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* *McGraw-Hill Encyclopedia of Science & Technology*, to appear.

† School of Computer Science, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel.

‡ Department of Computer Science, Illinois Institute of Technology, 10 West 31st Street, Chicago, Illinois 60616-2987, USA.

Diurnal Calendars

The simplest naming convention would be to assign an integer to each day; fixing day 1 would determine the whole calendar. The Babylonians had such a day count, as did the Maya and the Hindus. Such *diurnal* calendars are used by astronomers who use *julian day numbers* to specify dates. The "Julian period," introduced in 1583 by Joseph Justus Scaliger was originally a counting of *years* in a cycle of 7980 years, starting from 4713 B.C.E.; nineteenth century astronomers adapted the system into a strict counting of *days* backward and forward from

$$\begin{aligned} \text{JD } 0 &= \text{Noon on Monday, January 1, 4713 B.C.E.}^1 \text{ (Julian)} \\ &= \text{Noon on Monday, November 24, } -4713 \text{ (Gregorian).} \end{aligned}$$

A fractional part of a julian day gives the fraction of a day beyond noon.

Computer scientists often use diurnal calendars as an intermediate device for converting from one calendar to another.

Solar Calendars

The *Gregorian calendar*, now in common use throughout the world, is based on twelve-month year that closely approximates the earth's solar cycle. This calendar was designed by a commission assembled by Pope Gregory XIII in the sixteenth century; the main author of the new system was the Naples astronomer Aloysius Lilius. This calendar is based on a 365-day common year divided into twelve months of lengths 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, and 31 days, and a 366-day leap years, the extra day being added to make the second month 29 days long. A year is a leap year if it is divisible by 4 and is not a century year (multiple of 100) or if it is divisible by 400. For example, 1900 is not a leap year; 2000 is. The Gregorian calendar differs from its predecessor, the old-style or *Julian calendar*, only in that the Julian calendar did not include the century rule for leap years—all century years were leap years.

A simpler solar calendar with a fixed year length of 365 days and no leap year rule was in use in Egypt for millennia prior to the adoption of the Julian calendar in the third century C.E., and was also used in Babylon and Persia. It served as the canonical calendar for astronomers until the sixteenth century. Each month had thirty days, except for the last five days of the year, called *epagomenæ*. Though this approximation to the tropical year results in a noticeable shift in the date of onset of the seasons, computationally such a calendar is extremely simple, and it persists in the Armenian calendar.

The Julian calendar was instituted on January 1, 709 *ab urbe condita* (from the traditional founding of Rome) by Julius Cæsar, with the help of Alexandrian astronomer Sosigenes. The counting of years according to the Christian era was instituted by the Roman monk Dionysius Exiguus in the sixth century, but only

[1] Before the common era; or, B.C.

became commonplace a few centuries later; Dionysius erred by a few years in his determination of the year of the birth of Jesus. Dionysius did not invent the notion of "B.C."—his system started at 1. Year 1 B.C.E.(B.C.) is the year before 1 C.E. in the system introduced and popularized by the Venerable Bede around 731; the concept of zero as a number was then unknown in Europe. In this system, the number of years between the same day in years x B.C.E. and y C.E.is $x+y?1$ (not $x+y$). The use of a year 0 preceding year 1 on the Gregorian calendar is due to Jacques Cassini in 1740.

Since every fourth year on the Julian calendar was a leap year, a cycle of 4 years contained $4\times365+1 = 1461$ days, giving an average length of year of 365.25 days. This is somewhat more than the mean length of the tropical year, and over the centuries the calendar slipped with respect to the seasons. By the sixteenth century, the date of the spring equinox had shifted from around March 21 to around March 11. If this error were not corrected, eventually Easter, whose date depends on that equinox, would migrate through the whole calendar year. Pope Gregory XIII instituted only a minor change in the calendar—century years not divisible by 400 would no longer be leap years. (He also improved the rules for Easter.) Thus, three out of four century years are common years, giving a cycle of 400 years containing $400\times365+97 = 146{,}097$ days and an average year length of $146097/400 = 365.2425$ days. He also corrected the accumulated 10-day shift in the calendar by proclaiming that Thursday, October 4, 1582 C.E., the last date in the old-style (Julian) calendar, would be followed by Friday, October 15, 1582, the first day of the new-style (Gregorian) calendar. Catholic countries followed his rule, however, Protestant countries resisted. Great Britain and her colonies (including the United States until 1752); Russia held out until 1918, after the Bolshevik Revolution. Different parts of what is now the United States changed over at different dates; Alaska, for example, changed only when it was purchased by the U.S. in 1867. Turkey did not switch to the Gregorian calendar until 1927.

An alternative to arithmetic rules to determine whether a year has 365 or 366 days is to fix the date of the new year according to some astronomical event. The Persian astronomical calendar, for example, uses the vernal (spring) equinox; the short-lived French Revolutionary calendar used the autumnal equinox.

Lunar Calendars

The Islamic calendar is a example of a strictly lunar calendar, with no intercalation of months (unlike lunisolar calendars). Its independence of the solar cycle means that its months do not occur in fixed seasons, but migrate through the solar year. Virtually all Muslims follow an observation-based calendar in which new moons (and hence months) are proclaimed by officials when seen. The calendar is computed, by the majority of the Muslim world, starting at sunset of Thursday, July 15, 622 C.E. (Julian), the year of Mohammed's *hijra*. The introduction of the calendar is often attributed to the Caliph 'Umar, in 639 C.E., but there is evidence that it was in use before his succession.

Islamic astronomers developed an arithmetic approximation that is used for estimation. In it, there are twelve months, which contain, alternately, 30 or 29 days; the twelfth month contains 29 days in an ordinary year and contains 30 days in a leap year—the 2nd, 5th, 7th, 10th, 13th, 16th, 18th, 21st, 24th, 26th, and 29th years of a 30-year cycle. This gives an average month of $29.5305555\cdots$ days and an average year of $354\frac{11}{30}$ days.

Lunisolar Calendars

Lunisolar calendars invariably alternate twelve- and thirteen-month years. The so-called *Metonic cycle* is based on the observation that 19 solar years contain almost exactly 235 lunar months. This correspondence, named after the Athenian astronomer Meton and known much earlier to ancient Babylonian and Chinese astronomers, makes a relatively simple and accurate fixed solar/lunar calendar feasible. The $235 = 12 \times 12 + 7 \times 13$ months in the cycle are divided into twelve years of twelve months and seven leap years of thirteen months. The seven leap years are evenly distributed within the nineteen year cycle, with gaps of one or two years between them. The Metonic cycle was used as the basis of lunisolar calendars in Mesopotamia from the fourth century B.C.E., and later in the Seleucid empire. It is (currently) accurate to within 6.5 minutes a year and is still employed in the Hebrew calendar (instituted in 359 C.E.) and for the ecclesiastical calculation of Easter (in both the Gregorian version and the Dionysian formulation of the earlier Nicæan rule). The Hebrew calendar has an average year length of 365.2468 days and month length of $29\frac{13753}{25920} \approx 29.530594$ days; the Gregorian calculation of Easter results in a mean month length of $29\frac{37405943}{70499183} \approx 29.530587$ days. Both these calendars may introduce a shift in the start of the year (of a day or two) for technical purposes. (The Hebrew new year, Rosh HaShanah, for example, is not allowed to fall on the alternate days Sunday, Wednesday and Friday; the calculated Easter full moon is not allowed to fall twice on the same Gregorian date within any one 19-year cycle.) The result is that the Hebrew calendar repeats only after 689,472 years and the ecclesiastical after 5,700,000.

The seven-out-of-nineteen rule of the Metonic cycle is only one of many possibilities. Some of the older Hindu lunisolar calendars evenly distribute 66,389 leap months over 180,000 years.

Alternatively, a lunisolar calendar can follow astronomically determined patterns, as in the Chinese and Hindu, in which each month begins with a new moon, and a thirteenth month is added whenever there are thirteen new moons within a solar year. The Chinese calendar has undergone numerous reforms, but since 1645 C.E. it has used astronomical calculations to fix the start of years and months. The Hindu lunisolar calendar is distinguished by the occasional occurrence of expunged months and days. The classic version is based on the algorithm of the *Surya Siddhanta* (circa 1000 C.E.), which employs Ptolemaic methods.

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