Problem #47 (Solved !)

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> *Summary:* Prove a Parallel Moves Lemma *for reductions of infinite length.*

For reductions of transfinite length, a version of the Parallel Moves Lemma can be proved if one considers only "strongly converging" infinite reductions in the sense of [KKSd91]. However, if one wants to consider converging reductions, as in [DKP91], then it is not difficult to construct a counterexample, not to the infinite Parallel Moves Lemma itself, but to the method of proof (cf. [KKSd90]). An infinite Parallel Moves Lemma might involve a different notion of "descendant".

Remark

[Sim04] shows that it is not possible to obtain a Parallel Moves Lemma for (Cauchy-)convergent infinite reductions which relies on a notion of *residual* maintaining some of the basic properties of residuals known from the finite case. His counterexamples, however, are somewhat particular in that the right-hand sides of the rewrite rule are not normalized. The question remains whether it is possible to salvage a Parallel Moves Lemma for (Cauchy-)convergent reductions for restricted classes of rewrite systems.

Bibliography

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[KKSd90]   J. R. Kennaway, Jan Willem Klop, M. R. Sleep, and F. J. de Vries. Transfinite reductions in orthogonal term rewriting systems. Technical Report CS-R9041, CWI, Amsterdam, 1990.

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