Problems in Rewriting III*

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1 Introduction

We presented lists of open problems in the theory of rewriting in the proceedings of the previous two conferences [36; 37]. We continue with that tradition this year. We give references to solutions to eleven problems from the previous lists, report on progress on several others, provide a few reformulations of old problems, and include ten new problems.

2 Old Problems

Some progress has been made on previously listed problems. For convenience, we repeat the problems about which we are able to report progress.

Problem 4. One of the outstanding open problems in typed lambda calculi is the following: Given a term in ordinary untyped lambda calculus, is it decidable whether it can be typed in the second-order $\lambda 2$ calculus? See [11; 48].

This question has been solved in the negative. In [105] J.B. Wells proves that given a closed, type-free lambda term, the question whether it is typable in second-order $\lambda 2$ calculus, is undecidable. Moreover, given a closed type-free lambda term M and a type σ, then it is also undecidable in second-order $\lambda 2$ calculus whether M has type σ.

Problem 6 (A. Middeldorp [73]). If R and S are two term-rewriting systems with disjoint vocabularies, such that for each of R and S any two convertible normal forms must be identical, then their union $R \cup S$ also enjoys this property [73]. Accordingly, we say that unicity of normal forms (UN) is a "modular" property of term-rewriting systems. "Unicity of normal

* The first author was supported in part by the National Science Foundation under Grants CCR-90-07195 and CCR-90-24271; the second author was partially supported by the ESPRIT working groups COMPASS and CCL; the third author's work was partially supported by ESPRIT BRA project 6454: Confer.

forms with respect to reduction" ($UN^{\rightarrow}$) is the weaker property that any two normal forms of the same term must be identical. For non-left-linear systems, this property is not modular. The question remains: Is $UN^{\rightarrow}$ a modular property of left-linear term-rewriting systems?

A positive solution is given in [70].

Problem 23 (E. A. Cichon [23]). The following system [35], based on the "Battle of Hydra and Hercules" in [60], is terminating, but not provably so in Peano Arithmetic:

$$\begin{aligned}
h(z, e(x)) &\rightarrow h(c(z), d(z, x)) \\
d(z, g(0,0)) &\rightarrow e(0) \\
d(z, g(x,y)) &\rightarrow g(e(x), d(z,y)) \\
d(c(z), g(g(x,y),0)) &\rightarrow g(d(c(z), g(x,y)), d(z, g(x,y))) \\
g(e(x), e(y)) &\rightarrow e(g(x,y))
\end{aligned}$$

Transfinite (ϵ_0-) induction is required for a proof of termination. Must any termination *ordering* have the Howard ordinal as its order type, as conjectured in [23]?

If the notion of termination ordering is formalized by using ordinal notations with variables, then a termination proof using such orderings yields a slow growing bound on the lengths of derivations. If the order type is less than the Howard-Bachmann ordinal then, by Girard's Hierarchy Theorem, the derivation lengths are provably total in Peano Arithmetic. Hence a termination proof for this particular rewrite system for the Hydra game cannot be given by such an ordering [A. Weiermann, personal communication].

Problem 24. The existential fragment of the first-order theory of the "recursive path ordering" (with multiset and lexicographic "status") is decidable when the precedence on function symbols is total [25; 57], but is undecidable for arbitrary formulas. Is the existential fragment decidable for partial precedences? The Σ_4 ($\exists^*\forall^*\exists^*\forall^*$) fragment is undecidable, in general [101]. The positive existential fragment for the empty precedence (that is, for homeomorphic tree embedding) is decidable [13]. One might also ask whether the first-order theory of *total* recursive path orderings is decidable. Related results include the following: The existential fragment of the subterm ordering is decidable, but its Σ_3 ($\exists^*\forall^*\exists^*$) fragment is not [102]. The first-order theory of encompassment (the instance-of-subterm relation) is decidable [19]. Once we're at it, we might as well ask what the complexity of the satisfiability test for the existential fragment is—in the total case.

Though the first-order theory of encompassment is decidable [19], the first-order (Σ_2) theory of the recursive (lexicographic status) path ordering, assuming certain simple conditions on the precedence, is not [27].

Rephrased Problem 25 (R. Treinen [100]). Consider a finite set of function symbols containing at least one AC (associative-commutative) function symbol. Let T be the corresponding set of terms (modulo the AC properties). It is known from [101] that the first-order theory (Σ_3 fragment) of T is undecidable when F contains at least a non-constant symbol (besides the AC symbol). When F only contains an AC symbol and constants, the theory reduces to Presburger's arithmetic and is hence decidable. On the other hand the Σ_1 fragment of T is always decidable [26]. The decidability of the Σ_2 fragment of the theory of T remains open. Even more, the solvability of the following important particular case is open: given $t, t_1, \ldots, t_n \in T(F, X)$, is there an instance of t which is not an instance of $t_1, \ldots, t_n$ modulo the AC axioms? This is known as *complement problems* modulo AC.

Several special cases have been solved [40; 67], and in unpublished work in progress.

Problem 35. Huet's proof [47] of the "completeness" of completion is predicated on the assumption that the ordering supplied to completion does not change during the process. Assume that at step i of completion, the ordering used is able to order the current rewriting relation $\rightarrow_{R_i}$, but not necessarily $\rightarrow_{R_k}$ for $k < i$ (since old rules may have been deleted by completion). Is there an example showing that completion is then incomplete (the persisting rules are not confluent)?

The answer is yes, even when completion terminates with finitely many rules [93].

Problem 37 (U. Reddy, F. Bronsard). In [17] a rewriting-like mechanism for clausal reasoning called "contextual deduction" was proposed. It specializes "ordered resolution" by using pattern matching in place of unification, only instantiating clauses to match existing clauses. Does contextual deduction always terminate? (In [17] it was taken to be obvious, but that is not clear; see also [79].) It was shown in [17] that the mechanism is complete for refuting ground clauses using a theory that contains all its "strong-ordered" resolvents. Is there a notion of "complete theory" (like containing all strong-ordered resolvents not provable by contextual refutation) for which contextual deduction is complete for refutation of ground clauses?

Contextual deduction as defined in [17] does not terminate. Bronsard and Reddy have gone on to solve this [18] by using a more restricted, decidable mechanism. A completeness proof, incorporating equational inference with complete systems, is given in [16].

Problem 38 (J. Siekmann). Is satisfiability of equations in the theory of distributivity (unification modulo modulo one right- and one left-distributivity axiom) decidable? (With just one of these, the problem had already been solved in [97].) A partial positive solution is given in [29], based on a striking result on the structure of certain proofs modulo distributivity. Although many more cases are described in [28; 30], the general case remains open.

This theory is decidable [95; 94].

Problem 43. Design a framework for combining constraint solving algorithms. Some particular cases have been attacked: In [4] it was shown how decision procedures for solvability of unification problems can be combined. In [5] a similar technique is applied to (unquantified) systems of equations and disequations. In [90] the combination of unification algorithms is extended to the case where alphabets share constants. In related work [12], unification is performed in the combination of an equational theory and membership constraints.

Some progress is in [91].

Problem 44 (H. Comon). "Syntactic" theories enjoy the property that a (semi) unification algorithm can be derived from the axioms [53; 61]. This algorithm terminates for some particular cases (for instance, if all variable occurrences in the axioms are at depth at most one, and cycles have no solution) but does not in general. For the case of associativity and commutativity (AC), with a seven-axiom syntactic presentation, the derivation tree obtained by the non-deterministic application of the syntactic unification rules (*Decompose, Mutate, Merge, Coalesce, Check*, Delete*) in [53] can be pruned so as to become finite in most cases. The basic idea is that one unification problem (up to renaming) must appear infinitely times on every infinite branch of the tree (since there are finitely many axioms in the syntactic presentation). Hence, it should be possible to prune or freeze every infinite branch from some point on. The problem is to design such pruning rules so as to compute a finite derivation tree (hence, a finite complete set of unifiers) for every finitary unification problem of a syntactic equational theory.

The core of this problem has been solved [14].

Problem 46 (D. Kapur). Ground reducibility of extended rewrite systems, modulo congruences like associativity and commutativity (AC), is undecidable [59]. For left-linear AC systems, on the other hand, it is decidable [55]. What can be said more generally about restrictions on extended rewriting that give decidability? This problem is related to number 2.

Progress has been made in [63], where it is proven that ground reducibility remains undecidable when a single non-constant function symbol is associative.

Problem 50. Combinations of typed λ-calculi with term-rewriting systems have been studied extensively in the past few years [7; 15; 38; 39]. The strongest termination result allows first-order rules as well as higher-order rules defined by a generalization of primitive recursion. Suppose all rules for functional constant F follow the schema:

$$F(\bar{l}[\bar{X}],\bar{Y}) \rightarrow v[F(\bar{r}_1[\bar{X}],\bar{Y}),\ldots,F(\bar{r}_m[\bar{X}],\bar{Y}),\bar{Y})]$$

where the (not necessarily disjoint) variables in $\bar{X}$ and $\bar{Y}$ are of arbitrary order, each of $\bar{l},\bar{r}_1,\ldots,\bar{r}_m$ is in $\mathcal{T}(\mathcal{F},\{\bar{X}\})$, $v[\bar{z},\bar{Y}]$ is in $\mathcal{T}(\mathcal{F},\{\bar{Y},\bar{z}\})$, for new variables $\bar{z}$ of appropriate types, and $\bar{r}_1, \ldots, \bar{r}_m$ are each less than $\bar{l}$ in the multiset extension of the strict subterm ordering. If $\mathcal{T}(\mathcal{F}, \mathcal{X})$ is the term-algebra which includes only *previously* defined functional constants—forbidding the use of mutually recursive functional constants—termination is ensured [56]. Does termination also hold when there are mutually recursive definitions? Does this also hold when the subterm assumption is unfulfilled? (In [56] an alternative schema is proposed, with the subterm assumption weakened at the price of having only first-order variables in $\bar{X}$.) Questions of confluence of combinations of typed λ-calculi and higher-order systems also merit investigation. These results have been extended to combinations with more expressive type systems [9; 8].

An extension to the Calculus of Constructions has been reported in [10]. One can also allow the use of lexicographic and other "statuses" for the higher-order constants when comparing the subterms of F in left and right hand sides [Jouannaud and Okada, unpublished]. Finally, this can also be done when the rewrite rules follow from the induction schema in the initial algebra of the constructors [106].

Rephrased Problem 51 (H. Comon, M. Dauchet). Given an arbitrary finite term rewriting system R, is the first order theory of one-step rewriting ($\rightarrow_R$) decidable? Decidability would imply the decidability of the first-order theory of encompassment (that is, being an instance of a subterm) [19], as well as several known decidability results in rewriting. (It is well known that the theory of $\rightarrow_R^*$ is in general undecidable.)

Problem 56 (V. van Oostrom). An abstract reduction system is "decreasing Church-Rosser", if there exists a labelling of the reduction relation by a well-founded set of labels, such that all local divergences can be completed to form a "decreasing diagram" (see [84] for precise definitions). Does the Church-Rosser property imply decreasing Church-Rosser? That is, is it always possible to localize the Church-Rosser property? This is known to be the case for (weakly) normalizing and finite systems.

It is now known to hold for countable systems [68],[85, Cor. 2.3.30].

Rephrased Problem 57 (F. Baader [3]). Does there exist a semigroup theory (without constants in the equations) for which there is a reduced canonical term-rewriting system (with the right-hand side and subterms of the left in normal form) that is not length decreasing?

Problem 58 (M. Oyamaguchi). Is any "strongly" non-overlapping right-linear term-rewriting system confluent? ("Strong" in the sense that left-hand sides are non-overlapping even when the occurrences of variables have been renamed apart [21].) On the one hand, strongly non-overlapping systems need not be confluent [46]; on the other hand, strongly non-overlapping right-ground systems are [88].

A partial positive solution is given in [83; 99], namely, any strongly non-overlapping right-linear term-rewriting system is confluent if it satisfies the condition that for any rewrite rule, no variables occurring more than once in the left-hand-side occur in the right-hand-side.

Problem 60 (H. Zantema). Let R be a many-sorted term-rewriting system and R' the one-sorted system consisting of the same rules, but in which all operation symbols are considered to be of the same sort. Any rewrite in R is also a rewrite in R'. The converse does not hold, since terms and rewrite steps in R' are allowed that are not well-typed in R. In [108] it was shown that termination of R is in general not equivalent to termination of R', but it is if R does not contain both collapsing and duplicating rules. Are termination of R and of R' equivalent in the case where all variables occurring in R are of the same sort? If this statement holds, it would follow that simulating operation symbols of arity n greater than 2 by $n-1$ binary symbols in a straightforward way does not affect termination behavior.

A positive solution has recently been claimed [M. Marchiori, personal communication].

Problem 61 (T. Nipkow, M. Takahashi). For higher-order rewrite formats as given by combinatory reduction systems [62] and higher-order rewrite systems [80; 96], confluence has been proved in the restricted case of orthogonal systems. Can confluence be extended to such systems when they are weakly orthogonal (all critical pairs are trivial)? When critical pairs arise only at the root, confluence is known to hold.

Weakly orthogonal higher-order rewriting systems are confluent. This has been shown both via the Tait-Martin-Löf method and via finite developments [86, Sec. 3].

Problem 62 (V. van Oostrom). Let R and S be two left-linear, confluent combinatory reduction systems with the same alphabet. Suppose the rules of R do not overlap the rules of S. Is $R \cup S$ confluent? This is true for the restricted case when R is a term-rewriting system (an easy generalization of a result by F. Müller [77]), or if neither system has critical pairs. (The restriction to the same alphabet is essential, since confluence is in general not preserved under the addition of function symbols, not even for left-linear systems.)

The answer is yes [86, Thm. 3.13].

Problem 63 (M. Oyamaguchi).
Is confluence of right-ground term-rewriting systems decidable? Compare [87; 33; 34; 88].

Related is [76].

Problem 65 (D. Cohen, P. Watson [24]). An interesting system for doing arithmetic by rewriting was presented in [24]. Unfortunately, its termination has not been proved.

Termination of a related system is proved in [103].

Problem 68 (H. Comon). Consider the existential fragment of the theory defined by a binary predicate symbol $\subseteq$, a finite set of function symbols $f_1, \ldots, f_n$, the function symbols $\cap, \cup, \neg$, and the projection symbols $f_{i,j}^{-1}$ for $j \leq arity(f_i)$. Variables are interpreted as subsets of the Herbrand Universe. With the obvious interpretation of these symbols, is satisfiability of such formulæ decidable? Special cases have been solved in [44; 2; 6; 42].

This has been solved positively [43; 20; 1].

3 New Problems

Problem 78 (P. Lescanne). There are confluent calculi of explicit substitutions, but these do not preserve termination (strong normalization) [31; 72], and there are calculi that are not confluent on open terms, but which do preserve termination [65]. Is there a calculus of explicit substitution that is both confluent and preserves termination?

Problem 79 (M. Ogawa). Does a system that is nonoverlapping under unification with infinite terms (unification without "occur-check" [71]) have unique normal forms? This conjecture was originally proposed in [81] with an incomplete proof, as an extension of the result on strongly nonoverlapping systems [62; 21]. Related results appear in [88; 99; 69], but the original conjecture is still open. This is related to Problem 2. This problem is also related with modularity of confluence of systems sharing constructors, see [82].

Problem 80 (H. Comon). *Strong sequentiality* is a property of rewrite systems introduced in [49] (see [51]), which ensures the existence of optimal reduction strategies. Is strong sequentiality decidable for arbitrary rewrite systems? What is the complexity of strong sequentiality in the linear case? in the orthogonal case? Decidability results for particular rewrite systems are given in [52; 98; 58], among others.

Problem 81 (A. Weiermann). If the termination of a finite rewrite system over a finite signature can be proved using a simplification ordering, then the derivation lengths are bounded by a Hardy function of ordinal level less than the small Veblen number $\phi_{\Omega^\omega}0$. (See [104].) Is it possible to lower this bound by replacing the Hardy function by a slow growing function? That is, is it possible to bound the derivation lengths by a multiply recursive function?

Problem 82 (J. Zhang). Is there a convergent extended rewrite system for ternary boolean algebra, for which the following permutative equations hold:

$$f(x,y,z) = f(x,z,y) = f(y,x,z) = f(y,z,x) = f(z,x,y) = f(z,y,x)$$
$$f(f(x,y,z),u,x) = f(x,y,f(z,u,x))$$

See [107; 110; 22; 66].

Problem 83. A collection of rewrite orderings operating on disjoint signatures can be extended to an ordering operating on the union of the signatures, while still preserving part of the properties [92]. Such constructions can be used for proving modular termination properties of rewrite systems. Do they extend to the case where one of the starting orderings is given by $\beta\eta$ reductions on typed lambda terms?

Problem 84. Unification of patterns (à la [75]) modulo associativity and commutativity has been shown decidable [89]. Does it extend to equational theories whose axioms have the same set of variables on left and right hand side?

Problem 85 (M. Rusinowitch). Ordered paramodulation is known to be complete for simplification orderings that are total on ground terms [45]. Other theorem proving strategies are similarly restricted. How can these restrictions be relaxed?

Problem 86 (H. Zantema). When there exists a monotonic well-ordering ("monotonic" means that replacing a subterm with a smaller one decreases the whole term) of ground terms that shows termination of a rewrite system, the system is called "totally terminating." The union of two totally terminating rewrite systems which do not share any symbols is totally terminating if at least one of them does not contain a rule that has more occurrences of some variable on the right than on the left [41]. What if variables are duplicated?

Problem 87 (H. Zantema). Termination of string-rewriting systems is known to be undecidable [49]. Termination of a single term-rewriting rule was proved undecidable in [32; 64]. It is also undecidable whether there exists a simplification ordering that proves termination of a single term rewriting rule [74] (cf. [54]). Is it decidable whether a single term rewrite rule can be proved terminating by a monotonic ordering that is total on ground terms? (With more rules it is not [109].)

4 Coda

Please send any contributions by electronic or ordinary mail to any of us. We hope to continue periodically publicizing new problems and solutions to old ones.

Acknowledgements

We thank all the individuals who contributed questions, updates and solutions. Many thanks to Stefan Blom for substantial help producing this list.

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