Indian Calendrical Calculations

Nachum Dershowitz[*]   Edward M. Reingold[†]

Abstract

We analyze various Indian calendars. We discuss the Indian day count, a generic solar calendar that generalizes various calendars including the mean Indian solar calendar, the true and astronomical Indian solar calendars, a generic lunisolar calendar that generalizes the Indian version, and the true and astronomical Indian lunisolar calendars. We also discuss aspects of the traditional Indian calculation of the time of sunrise and the determination of lunisolar holidays.

> The months of the Hindus are lunar, their years are solar;
> therefore their new year's day must in each solar year
> fall by so much earlier as the lunar year is shorter than the solar... .
> If this precession makes up one complete month,
> they act in the same way as the Jews,
> who make the year a leap year of thirteen months... ,
> and in a similar way to the heathen Arabs.
>
> —*Alberuni's India.*

1 Introduction

The world's many calendars are of three primary types: diurnal, solar, and lunar—see our *Calendrical Calculations: Millennium Edition* [5] (henceforth CCME). All three are represented among the many calendars of the Indian sub-continent.

- A *diurnal* calendar is a day count, either a simple count, like the Julian day number, or a complex, mixed-radix count, like the Mayan long count. The classical Indian day count (*ahargana*) is used for calendrical purposes.

- *Solar* calendars have a year length that corresponds to the solar year. All modern solar calendars add leap days at regular intervals to adjust the mean length of the calendar year to better approximate the true solar year. The solar (*saura*) calendar is more popular in northern India; a similar one is in use in Nepal.[1]

- A *lunar* (*cāndra*) calendar has as its primary component a mensual unit that corresponds to the lunar synodic month. It can be purely lunar, as in the twelve-month Islamic calendar year, or it can incorporate occasional leap months, as in the Hebrew lunisolar calendar. Several forms of lunisolar calendar are in use in India today; the Tibetan Phuglug calendar is somewhat similar.[2]

In general, a date is determined by the occurrence of a cyclical event (equinox, lunar conjunction, and so on) before some "critical" time of day, not necessarily during the same day. For a "mean" (*madhyama*) calendar, the event occurs at fixed intervals; for a "true" (*spaṣṭa*) calendar, the (approximate or precise) time of each occurrence of the event must be calculated. Various astronomical values were used by the Indian astronomers Āryabaṭa (circa 300 C.E.), Brahmagupta (circa 630 C.E.), the author of *Sūrya-Siddhānta* (circa 1000 C.E.), and others.

Indian month names are given in Table 1. Tamil names are different.

---

[*]School of Computer Science, Tel Aviv University, P.O. Box 39040, 69978 Ramat Aviv, Israel. Email: nachum@cs.tau.ac.il. This research was supported by the Israel Science Foundation (grant number 250/05).

[†]Department of Computer Science, Illinois Institute of Technology, 10 West 31st Street, Chicago, Illinois 60616-2987, USA. Email: reingold@iit.edu.

[1]We have to date been unable to ascertain the precise rules of the Nepalese solar calendar.

[2]We have to date been unable to ascertain the precise rules of the Tibetan lunisolar calendars.

	Vedic	Zodiacal Sign		Sanskrit	Month	Sanskrit
1	Madhu	Mesha	(Aries)	मेष	Chaitra	चैत्र
2	Mādhava	Vṛshabha	(Taurus)	वृषभ	Vaiśākha	वैशाख
3	Śukra	Mithuna	(Gemini)	मिथुन	Jyeshtha	ज्येष्ठ
4	Śuchi	Karka	(Cancer)	कर्क	Āshāḍha	आषाढ
5	Nabhas	Siṁha	(Leo)	सिंह	Śrāvaṇa	श्रावण
6	Nabhasya	Kanyā	(Virgo)	कन्या	Bhādrapada	भाद्रपद
7	Issa	Tulā	(Libra)	तुला	Āśvina	आश्विन
8	Ūrja	Vṛśchika	(Scorpio)	वृश्चिक	Kārtika	कार्तिक
9	Sahas	Dhanu	(Sagittarius)	धनु	Mārgaśīrsha	मार्गशीर्ष
10	Sahasya	Makara	(Capricorn)	मकर	Pausha	पौष
11	Tapas	Kumbha	(Aquarius)	कुम्भ	Māgha	माघ
12	Tapasya	Mīna	(Pisces)	मीन	Phālguna	फाल्गुन

Table 1: Indian month names.

Era	Current Year	Elapsed Year
Kali Yuga	+3102	+3101
Nepalese		+877
Kollam	+823	
Vikrama	+58	+57
Śaka	−77	−78
Bengal		−593
Caitanya		−1486

Table 2: Some eras, given as the offset from the Gregorian year.

We ignore many trivial differences between alternative calendars, like eras (year count). Some Indian calendars count "elapsed" years, beginning with year 0; others use "current", 1-based years. The offsets of some common eras from the Gregorian year are summarized in Table 2. There are also regional differences as to which is the first month of the year. Finally, calendars are local, in the sense that they depend on local times of sunrise, and the like.

We systematically apply the formulas for cyclic events in Chapter 1 of CCME to derive formulas for generic mean single-cycle and dual-cycle calendars. Solar calendars are based on the motion of the sun alone, so fit a single cycle pattern; lunisolar calendars, on the other hand, take both the solar and lunar cycles into account, so require double-cycle formulas. We apply these generic algorithms to the old Indian solar and lunisolar calendars, which are based on mean values (see Chapter 9 of CCME). We also use the code in CCME to compare the values obtained by the much more complicated true Indian calendars (Chapter 17 of CCME) with their modern astronomical counterparts. Unless noted otherwise, we center our astronomical calculations on the year 1000 C.E.

The next, brief section describes the Indian day count. Section 3 presents a generic solar calendar and shows how the mean Indian solar calendar fits the pattern. It is followed by a section that compares the later, true calendar with modern astronomical calculations. Similarly, Section 5 presents a generic lunisolar calendar and its application to the Indian version, and is followed by a section on the true and astronomical versions. Section 7 discusses aspects of the traditional calculation of the time of sunrise. Finally, Section 8 outlines the difficulty of computing the day of observance of holidays based on the lunisolar calendar.

Following the style of CCME, the algorithms in this paper are presented as mathematical function definitions in standard mathematical format. All calendar functions were automatically typeset directly from the Common Lisp functions listed in the appendix.

Date (Julian)		J.A.D.	Ahargana	R.D.
2 January 4713 B.C.E.	(noon)	1	$-588,464.5$	$-1,721,423.5$
18 February 3102 B.C.E.	(midnight)	588,465.5	0	$-1,132,959$
3 January 1 C.E.	(midnight)	1,721,425.5	1,132,960	1

Table 3: Day count correlations.

2 Diurnal Calendars

In most cases, a calendar date is a triple $\langle y, m, d \rangle$, where year y can be any positive or negative integer, and month m and day d are positive integers, possibly designated "leap". A day count is convenient as an intermediate device for conversions between calendars. One can count days from the first day of a calendar, normally $\langle 1, 1, 1 \rangle$, called the *epoch*. So $\langle 1, 1, 1 \rangle$, $\langle 1, 1, 2 \rangle$, and so on correspond to (elapsed) day count 0, 1, and so on. Days prior to the epoch have negative day counts and non-positive year numbers. Day counts can be 0-based or 1-based; that is, the epoch may be assigned 0 or 1. In CCME, we use the *Rata Die* (R.D.) count, day 1 of which is 1 January 1 (Gregorian).

The *ahargana* ("heap of days") is a 0-based day count of the Kali Yuga (K.Y.) era, used in Indian calendrical calculations. The K.Y. epoch, day 0 of the *ahargana* count, is Friday, 18 February 3102 B.C.E. (Julian). Its correlations with R.D. and with the midday-to-midday Julian Day Number (J.A.D., popular among astronomers) are given in Table 3. An earlier count, with much larger numbers, was used by Āryabata. We use the onset of the Kali Yuga, R.D. $-1,143,959$, for our **hindu-epoch**.

3 Mean Solar Calendars

The modern Indian solar calendar is based on a close approximation to the *true* times of the sun's entrance into the signs of the sidereal zodiac; the Hindu names of the zodiac are given in Table 1. Traditional calendarists employ medieval epicyclic theory (see Section 17.1 of CCME); others use a modern ephemeris. Before about 1100 C.E., however, Hindu calendars were usually based on *average* times. The basic structure of the calendar is similar for both the mean (*madhyama*) and true (*spaṣṭa*) calendars. The Gregorian, Julian and Coptic calendars are other examples of mean solar calendars; the Persian and French Revolutionary are two examples of true solar calendars. In this and the following section, we examine these two solar calendar schemes.

The Indian mean solar calendar, though only of historical interest, has a uniform and mathematically pleasing structure. (See [3] for connections between leap-year structures and other mathematical tasks.) Using the astronomical constants of the *Ārya-Siddhānta*, yields 149 leap years of 366 days, which are distributed evenly in a cycle of 576 years. Similarly, 30-day and 31-day months alternate in a perfectly even-handed manner.

3.1 Single-Cycle Calendars

The mean solar calendar is an instance of a general single-cycle calendar scheme. Consider a calendar with mean year length of Y days, where Y is a positive real number. If Y is not a whole number, then there will be common years of length $\lfloor Y \rfloor$ and leap years of length $\lceil Y \rceil$, with a leap-year frequency of $Y \bmod 1$.

To convert between R.D. dates and single-cycle dates, we apply formulas (1.65) and (1.68) from Section 1.12 of CCME. Suppose that a year is divided into months of length as close to equal as possible. For a standard 12-month year, the average length would be $M = Y/12$. Some months, then, should be $\lfloor M \rfloor$ days long, and the rest $\lceil M \rceil$ days. A day is declared New Year if the solar event occurs *before* some critical moment. In other words, if t_n is the critical moment of day n, then n is New Year if and only if the event occurs during the interval $[t_n - 1, t_n)$. (The beginnings of new months may be handled similarly or may be determined by simpler schemes, depending on the calendar; we discuss this below.)

Suppose that the sun was at the critical longitude at the critical time t_{-1} of day -1, the day before the epoch, so that day -1 just missed being New Year. Finally, assume that a leap day, when there is one, is added at year's end. The number of elapsed days from the calendar's epoch $\langle 0, 0, 0 \rangle$ until a date $\langle y, m, d \rangle$ (all three components are for now

3

0-based) is simply:

$$\lfloor yY \rfloor + \lfloor mM \rfloor + d \tag{1}$$

with inverse

$$
\begin{aligned}
y &= \lceil (n+1)/Y \rceil - 1 \\
n' &= n - \lfloor yY \rfloor \\
m &= \lceil (n'+1)/M \rceil - 1 \\
d &= n' - \lfloor mM \rfloor
\end{aligned}
\tag{2}
$$

If the rule is that the event may occur up to *and including* the critical moment, then n is New Year if and only if the event occurs during the interval $(t_n - 1, t_n]$. Accordingly, we need to change some ceilings and floors in the above formulas. Supposing that the event transpired exactly at that critical moment t_0 of the epoch, the elapsed-day calculation becomes

$$\lceil yY \rceil + \lceil mM \rceil + d \tag{3}$$

The inverse function, assuming $Y \geq M \geq 1$, converting a day count n into a 0-based date, $\langle y, m, d \rangle$, is

$$
\begin{aligned}
y &= \lfloor n/Y \rfloor \\
n' &= n \bmod Y \\
m &= \lfloor n'/M \rfloor \\
d &= \lfloor n' \bmod M \rfloor
\end{aligned}
\tag{4}
$$

The above four formulas assume that months are determined in the same way as are years, from a specified average value M, and, therefore, follow the same pattern every year (except for the leap day in the final month of leap years). There is an alternative version of mean solar calendars in which month lengths vary, and are determined by the mean position of the sun each month. In this case, we combine the calculation of the number of days in the elapsed years and those of the elapsed months, giving

$$\lfloor yY + mM \rfloor + d \tag{5}$$

or

$$\lceil yY + mM \rceil + d \tag{6}$$

depending on whether the "before" or "not after" version is required. The inverses for these two variable-month versions are

$$
\begin{aligned}
y &= \lceil (n+1)/Y \rceil - 1 \\
m' &= \lceil (n+1)/M \rceil - 1 \\
m &= m' \bmod 12 \\
d &= n - \lfloor m'M \rfloor
\end{aligned}
\tag{7}
$$

or

$$
\begin{aligned}
y &= \lfloor n/Y \rfloor \\
m &= \lfloor n/M \rfloor \bmod 12 \\
d &= \lfloor n \bmod M \rfloor
\end{aligned}
\tag{8}
$$

respectively.

3.2 Generic Single-Cycle Calendars

The critical event for a calendar sometimes occurs exactly at the calendar's epoch (K.Y., in the Indian case). However, often an additional complication is introduced, wherein the relevant critical event occurred some fraction of a day before the critical time for the epoch. Furthermore, the cyclical month pattern may have its own starting point. Accordingly, for the fixed-month calendar, we are given the following constants:

1. the calendar epoch, **single-cycle-epoch**, an integer;

4

2. the offset of the first critical event, **delta-year**, a number in the range $[0,1)$;

3. the average year length, **average-year-length**, of at least 1 day;

4. the average month length, **average-month-length**, at least 1 day long, but no longer than an average year; and

5. the offset for the first month, **delta-month**, also in the range $[0,1)$.

The critical yearly event for the epochal year occurred **delta-year** days after the earliest possible moment, which is $1 -$ **delta-year** days before the critical time; the critical monthly event for the first month of the calendar occurred **delta-month** days after its earliest possible time.

To convert a single-cycle 1-based date to an R.D. date, we add to the epoch the days before the *year*, the days before *month* in *year*, and the days before *day* in *month*, taking the initial offsets into account:

fixed-from-single-cycle (9)

$$\left(\boxed{\; \boxed{year} \;\big|\; \boxed{month} \;\big|\; \boxed{day} \;} \right) \quad \overset{\text{def}}{=}$$

single-cycle-epoch $+ \;\lfloor(year - 1) \times$ **average-year-length** $+$ **delta-year**$\rfloor\; +$
$\lfloor(month - 1) \times$ **average-month-length** $+$ **delta-month**$\rfloor + day - 1$

In the other direction, we compute the single-cycle date from an R.D. date by determining the year from the start of the mean year using (1.68), the month from (1.68) applied to the month parameters, and the day by taking the remainder:

single-cycle-from-fixed(*date*) $\quad \overset{\text{def}}{=}$ (10)

$$\boxed{\; \boxed{year} \;\big|\; \boxed{month} \;\big|\; \boxed{day} \;}$$

where

$$days \quad = \quad date - \textbf{single-cycle-epoch}$$

$$year \quad = \quad \left\lceil \frac{days + 1 - \textbf{delta-year}}{\textbf{average-year-length}} \right\rceil$$

$$n \quad = \quad days - \lfloor \textbf{delta-year} + (year - 1) \times \textbf{average-year-length} \rfloor$$

$$month \quad = \quad \left\lceil \frac{n + 1 - \textbf{delta-month}}{\textbf{average-month-length}} \right\rceil$$

$$day \quad = \quad n + 1 - \lfloor \textbf{delta-month} + (month - 1) \times \textbf{average-month-length} \rfloor$$

The Coptic calendar, with **average-year-length** of $365\frac{1}{4}$ days, is also a single-cycle calendar, but we need to use a fictitious **average-month-length** of 30 to accommodate its twelve 30-month days, which are followed by an extra "month" of epagomenē lasting 5–6 days. Also, **single-cycle-epoch** $=$ R.D. $103,605$, **delta-year** $= \frac{1}{4}$, and **delta-month** $= 0$. Compare Table 1.3 and Chapter 4 of CCME.

The Julian (old style) calendar, on the other hand, even though it has the same year length as the Coptic, does not fit our scheme, on account of its irregular month lengths. See Chapter 3 of CCME.

It should be stressed that for these functions to operate correctly for rational parameters, precise arithmetic is incumbent. Otherwise, four years, say, of average length $365\frac{1}{4}$ might not add up to an integral number of days, wreaking havoc to functions using floors, ceilings, and modular arithmetic.

For the alternate version, where the critical event may occur at the critical time, **delta-year** is the fraction of the day *before* the critical moment of the epoch at which the event occurred. We have, instead:

alt-fixed-from-single-cycle (11)

$$\left(\ \boxed{\begin{array}{c|c|c} \textit{year} & \textit{month} & \textit{day} \end{array}}\ \right) \quad \overset{\mathrm{def}}{=}$$

single-cycle-epoch $+\ \lceil (\textit{year} - 1) \times$ **average-year-length** $-$ **delta-year** $\rceil\ +$
$\lceil (\textit{month} - 1) \times$ **average-month-length** $-$ **delta-month** $\rceil + \textit{day} - 1$

alt-single-cycle-from-fixed $(\textit{date}) \quad \overset{\mathrm{def}}{=}$ (12)

$$\boxed{\begin{array}{c|c|c} \textit{year} & \textit{month} & \textit{day} \end{array}}$$

where

$$\textit{days} \quad = \quad \textit{date} - \textbf{single-cycle-epoch} + \textbf{delta-year}$$

$$\textit{year} \quad = \quad \left\lfloor \frac{\textit{days}}{\textbf{average-year-length}} \right\rfloor + 1$$

$$n \quad = \quad (\textit{days} \bmod \textbf{average-year-length}) + \textbf{delta-month}$$

$$\textit{month} \quad = \quad \left\lfloor \frac{n}{\textbf{average-month-length}} \right\rfloor + 1$$

$$\textit{day} \quad = \quad \lfloor n \bmod \textbf{average-month-length} \rfloor + 1$$

This version, too, works for the Coptic calendar, but with **delta-year** $= \frac{1}{2}$.

Now for the variable-month version. As before, we have the epoch of the calendar **single-cycle-epoch**, the average year length **average-year-length**, and the initial offset, **delta-year**. However, instead of the fixed-month structure given by **average-month-length** and **delta-month**, we simply specify the number of months in the year, **months-per-year**. To convert between R.D. dates and dates on this single-cycle mean calendar, we again apply formulas (1.65) and (1.68) from Section 1.12 of CCME, but with minor variations.

In this case, to convert a single-cycle date to an R.D. date, we add the days before the mean *month* in *year*, and the days before *day* in *month*:

var-fixed-from-single-cycle (13)

$$\left(\ \boxed{\begin{array}{c|c|c} \textit{year} & \textit{month} & \textit{day} \end{array}}\ \right) \quad \overset{\mathrm{def}}{=}$$

single-cycle-epoch $+$
$\lfloor (\textit{year} - 1) \times$ **average-year-length** $+$ **delta-year** $+ (\textit{month} - 1) \times \textit{average-month-length} \rfloor$
$+\ \textit{day} - 1$

where

$$\textit{average-month-length} \quad = \quad \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

In the other direction, we compute the single-cycle date from an R.D. date by determining the year from the start of the mean year using (1.68), the month from (1.68) applied to the month parameters, and the day by subtraction:

var-single-cycle-from-fixed $(\textit{date}) \quad \overset{\mathrm{def}}{=}$ (14)

6

$$\boxed{\; year \;|\; month \;|\; day \;}$$

where

$$days \quad = \quad date - \textbf{single-cycle-epoch}$$

$$offset \quad = \quad days + 1 - \textbf{delta-year}$$

$$year \quad = \quad \left\lceil \frac{offset}{\textbf{average-year-length}} \right\rceil$$

$$average\text{-}month\text{-}length \; = \; \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

$$month \quad = \quad 1 + \left(\left(\left\lceil \frac{offset}{average\text{-}month\text{-}length} \right\rceil - 1 \right) \bmod \textbf{months-per-year} \right)$$

$$day \quad = \quad days + 1 - \left\lfloor \textbf{delta-year} + \left(\left\lceil \frac{offset}{average\text{-}month\text{-}length} \right\rceil - 1 \right) \times average\text{-}month\text{-}length \right\rfloor$$

3.3 Indian Mean Solar Calendar

Unlike other solar calendars, especially the universally-used Gregorian, the Indian calendars are based on the sidereal (*nakshatra*) year. The old Hindu mean (*madhyama*) solar calendar is an example of the second version of our generic solar calendar, using an estimate of the length of the sidereal year and mean sunrise as the critical time.

However, we need the fourth version of the formulas, with the determining event occurring before or at the critical time:

alt-var-fixed-from-single-cycle (15)

$$\left(\boxed{\; year \;|\; month \;|\; day \;} \right) \quad \overset{\text{def}}{=}$$

$$\textbf{single-cycle-epoch} +$$
$$\left\lceil (year - 1) \times \textbf{average-year-length} - \textbf{delta-year} + (month - 1) \times average\text{-}month\text{-}length \right\rceil$$
$$+ \, day - 1$$

where

$$average\text{-}month\text{-}length \; = \; \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

alt-var-single-cycle-from-fixed(*date*) $\quad \overset{\text{def}}{=}$ (16)

$$\boxed{\; year \;|\; month \;|\; day \;}$$

where

$$days \quad = \quad date - \textbf{single-cycle-epoch} + \textbf{delta-year}$$

$$average\text{-}month\text{-}length \; = \; \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

$$year = \left\lfloor \frac{days}{\textbf{average-year-length}} \right\rfloor + 1$$

$$month = 1 + \left(\left\lfloor \frac{days}{average\text{-}month\text{-}length} \right\rfloor \bmod \textbf{months-per-year} \right)$$

$$day = \lfloor days \bmod average\text{-}month\text{-}length \rfloor + 1$$

Following the *First Ārya Siddhānta* regarding year length, the constants we would need are

$$\textbf{single-cycle-epoch} \quad \overset{\text{def}}{=} \quad \textbf{hindu-epoch} \tag{17}$$

$$\textbf{average-year-length} \quad \overset{\text{def}}{=} \quad 365\frac{149}{576} \tag{18}$$

$$\textbf{delta-year} \quad \overset{\text{def}}{=} \quad \frac{1}{4} \tag{19}$$

$$\textbf{months-per-year} \quad \overset{\text{def}}{=} \quad 12 \tag{20}$$

The above algorithms give a 1-based year number. The necessary changes for versions of the Indian calendar that use elapsed years (including those in CCME) are trivial.

4 True Solar Calendars

One may say that a solar calendar is astronomical if the start of its years is determined by the actual time of a solar event. The event is usually an equinox or solstice, so we presume it is the moment the true solar longitude attains some given value, named **critical-longitude** below, and can assume that the true and mean times of the event differ by at most 5 days.

The astronomical Persian calendar (Chapter 13 of CCME) uses a critical longitude of 0° for the spring equinox and apparent noon in Tehran as its critical moment. The defunct French Revolutionary calendar (Chapter 15 of CCME) used a critical longitude of 180° for the autumnal equinox and apparent midnight in Paris as its critical moment.[3]

4.1 Generic Solar Calendars

Fixed-month versions of the true calendar usually have idiosyncratic month lengths. This is true of both the Persian and Bahá'í calendars; see Chapters 13–14 of CCME. So we restrict ourselves to the determination of New Year. First, we define a function to determine the true longitude at the critical time of any given day, where the critical time is determined by some function **critical-time**:

$$\textbf{true-longitude}\,(date) \quad \overset{\text{def}}{=} \tag{21}$$

$$\textbf{solar-longitude}\,(\textbf{critical-time}\,(date))$$

Since solar longitude increases at different paces during different seasons, we search for the first day the sun attains the **critical-longitude** beginning five days prior to the mean time:

$$\textbf{solar-new-year-on-or-after}\,(date) \quad \overset{\text{def}}{=} \tag{22}$$

$$\underset{d \geq start}{\text{MIN}} \left\{ \begin{array}{l} \textbf{critical-longitude} \leq \textbf{true-longitude}\,(d) \\ \leq \textbf{critical-longitude} + 2 \end{array} \right\}$$

---

[3]A generic version of such calendars were given in our paper [2] as Lisp macros.

where

$$\lambda \quad = \quad \textbf{true-longitude}\,(\textit{date})$$

$$\textit{start} \quad = \quad \textit{date} - 5 + \left\lfloor \textbf{average-year-length} \times \tfrac{1}{360} \right. $$
$$\left. \times \Big(\big(\textbf{critical-longitude} - \lambda \big) \mod 360 \Big) \right\rfloor$$

The initial estimate is based on the current solar longitude λ, with an average daily increase of $360°/Y$.

Solar New Year (*Sowramana Ugadi*) in a given Gregorian year is then computed as follows:

$$\textbf{hindu-solar-new-year}\,(\textit{g-year}) \quad \overset{\text{def}}{=} \tag{23}$$

$$\textbf{solar-new-year-on-or-after}$$
$$\Big(\textbf{gregorian-from-fixed} \Big(\boxed{\textit{g-year} \quad \textbf{january} \quad 1} \Big) \Big)$$

which uses the R.D. to Gregorian conversion function **gregorian-from-fixed** of CCME.

For a variable-month version of the true calendar, such as the Indian solar calendar and its relatives, the start of *each* month is also determined by the true solar longitude:

$$\textbf{solar-from-fixed}\,(\textit{date}) \quad \overset{\text{def}}{=} \tag{24}$$

$$\boxed{\textit{year} \quad m+1 \quad \textit{date} - \textit{begin} + 1}$$

where

$$\lambda \quad = \quad \textbf{true-longitude}\,(\textit{date})$$

$$m \quad = \quad \left\lfloor \frac{\lambda}{30°} \right\rfloor$$

$$\textit{year} \quad = \quad \text{round}$$
$$\left(\frac{\textbf{critical-time}\,(\textit{date}) - \textbf{solar-epoch}}{\textbf{average-year-length}} - \frac{\lambda}{360°} \right)$$

$$\textit{approx} \quad = \quad \textit{date} - 3 - (\lfloor \lambda \rfloor \mod 30°)$$

$$\textit{begin} \quad = \quad \underset{i \geq \textit{approx}}{\text{MIN}} \left\{ m = \left\lfloor \frac{\textbf{true-longitude}\,(i)}{30°} \right\rfloor \right\}$$

This function can be inverted using the methods of CCME (Section 17.2).

4.2 True Indian Solar Calendar

For the Indian solar calendar, we need to use the Indian sidereal longitude function (**hindu-solar-longitude** in CCME) in place of **solar-longitude** (in the **true-longitude** function). The length of the sidereal year according to the *Sūrya-Siddhānta* is

$$\textbf{average-year-length} \quad = \quad 365\frac{279457}{1080000}$$

See Table 4. The year begins when the sun returns to sidereal longitude $0°$. There are various critical times of day that are used to determine exactly which day is New Year.

Source	Length	
First Ārya-Siddhānta	365.258681	$365^d6^h12^m30^s$
Brahma-Siddhānta	365.258438	$365^d6^h12^m\ 9^s$
Original Sūrya-Siddhānta	365.258750	$365^d6^h12^m36^s$
Present Sūrya-Siddhānta	365.258756	$365^d6^h12^m36.56^s$
Modern Value (for 1000 C.E.)	365.256362	$365^d6^h\ 9^m\ 8.44^s$

Table 4: Sidereal year values.

- According to the Orissa rule (followed also in Punjab and Haryana), sunrise is used. In other words, the solar month is determined by the stellar position of the sun the following morning:

$$\textbf{orissa-critical}\,(date) \quad \overset{\text{def}}{=} \quad \textbf{hindu-sunrise}\,(date+1) \tag{25}$$

where **hindu-sunrise** is sunrise according to Indian rule or practice (Eq. 17.35 in CCME). See Section 7 for details.

- According to the Tamil rule, sunset of the current day is used:

$$\textbf{tamil-critical}\,(date) \quad \overset{\text{def}}{=} \quad \textbf{hindu-sunset}\,(date) \tag{26}$$

where **hindu-sunset** is sunset according to Indian rule or practice.

- According to the Malayali (Kerala) rule, 1:12 p.m. (seasonal time) on the current day is used:

$$\textbf{malayali-critical}\,(date) \quad \overset{\text{def}}{=} \tag{27}$$

$$\textbf{hindu-sunrise}\,(date) + \tfrac{3}{5} \times (\textbf{hindu-sunset}\,(date) - \textbf{hindu-sunrise}\,(date))$$

Kerala also uses a different critical longitude.

- According to some calendars from Madras, apparent midnight at the end of the day is used:

$$\textbf{madras-critical}\,(date) \quad \overset{\text{def}}{=} \tag{28}$$

$$\textbf{hindu-sunset}\,(date) + \tfrac{1}{2} \times (\textbf{hindu-sunrise}\,(date+1) - \textbf{hindu-sunset}\,(date))$$

- According to the Bengal rule (also in Assam and Tripura), midnight at the start of the civil day is usually used, unless the zodiac sign changes between 11:36p.m. and 12:24a.m. (temporal time), in which case various special rules apply, depending on the month and on the day of the week.

See [8, page 12] and [1, page 282]. The function **critical-time** should be set to one of these.

4.3 Indian Astronomical Solar Calendar

For an astronomical Indian solar calendar, we need to substitute an astronomical calculation of *sidereal* longitude for **solar-longitude** in **true-longitude**. We also should use astronomical geometric sunrise (and/or sunset) for **hindu-sunrise** (and **hindu-sunset**) in **critical-time**; see Section 7.

The difference between the equinoctal and sidereal longitude (the *ayanamsha*) changes with time, as a direct consequence of precession of the equinoxes. It is uncertain what the zero point of Indian sidereal longitude is, but it is customary to say that the two measurements coincided circa 285 C.E., the so-called "Lahiri *ayanamsha*". Others (for
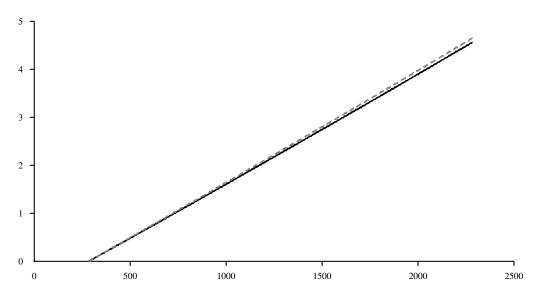
10

Figure 1: Increasing difference, for 285–2285 C.E., between siddhāntic and astronomical sidereal longitudes in degrees (solid black) and days (dashed gray), assuming coincidence in 285.
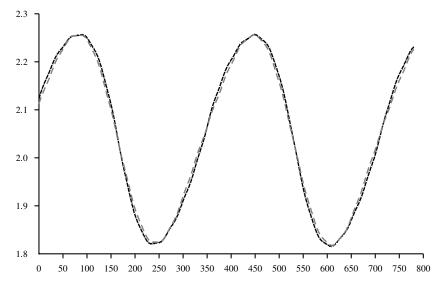

Figure 2: Difference (1000–1001 C.E.) between siddhāntic and astronomical sidereal solar longitude (solid black line). Dotted white line (atop the black line) uses mathematical sine function, rather than an interpolated tabular sine; dashed gray line uses fixed epicycle; they are virtually indistinguishable.

example [10, Section 18]) suggest the two measurements coincided around 560 C.E. Either way, the over-estimate of the length of the mean sidereal year used by the siddhantas leads to a growing discrepancy in the calculation of solar longitude; see Table 4. (The length of the sidereal year is increasing by about 10^{-4} seconds per year.)

The Indian Vernal equinox, when the sun returns to sidereal longitude 0°, is called Mesha saṃkrānti. Solar New Year, the day of Mesha saṃkrānti, as computed by **hindu-solar-new-year** with traditional year lengths, is nowadays about 4 days later than that which would be obtained by astronomical calculation (assuming the Lahiri value).

To calculate sidereal longitude, we use the algorithm for precession in [7, pages 136–137]:

$$\textbf{precession}(t) \stackrel{\text{def}}{=} (p + P - arg) \bmod 360 \tag{29}$$

where

$$c \quad = \quad \textbf{julian-centuries}(t)$$

$$\eta \quad = \quad \left(47.0029'' \times c + (-0.03302)'' \times c^2 + (6 \times 10^{-5})'' \times c^3 \right) \bmod 360$$

$$P \quad = \quad \left(174.876384° + (-869.8089)'' \times c + 0.03536'' \times c^2 \right) \bmod 360$$

$$p \quad = \quad \left(5029.0966'' \times c + 1.11113'' \times c^2 + (6 \times 10^{-6})'' \times c^3 \right) \bmod 360$$

$$A \quad = \quad \cos\eta \times \sin P$$

$$B \quad = \quad \cos P$$

$$arg \quad = \quad \textbf{arctan}_2(A, B)$$

where **julian-centuries** measures the (dynamical) time before/after noon on 1 January 2000, and the second argument of **arctan**$_2$ determines the quadrant:

$$\textbf{arctan}_2(a, b) \stackrel{\text{def}}{=} \left\{ \begin{array}{ll} \alpha & \textbf{if } b \geq 0 \\ \alpha + 180° & \textbf{otherwise} \end{array} \right\} \bmod 360 \tag{30}$$

where

$$x \quad = \quad \frac{a}{b}$$

$$\alpha \quad = \quad \arctan x$$

We need the value of this function when the *ayanamsha* was 0.

$$\textbf{sidereal-start} \stackrel{\text{def}}{=} \tag{31}$$

$$\textbf{precession}$$

$$\left(\textbf{universal-from-local} \right.$$

$$\left. (\textbf{mesha-samkranti}(285 \text{ C.E.}), \textbf{hindu-locale}) \right)$$

where **mesha-samkranti** gives the local time of the (sidereal) equinox, Ujjain is our **hindu-locale**, and **universal-from-local** is one of the time conversion functions of CCME. Then

$$\textbf{sidereal-solar-longitude}(t) \stackrel{\text{def}}{=} \tag{32}$$

$$\left(\textbf{solar-longitude}(t) - \textbf{precession}(t) + \textbf{sidereal-start} \right) \bmod 360$$

12

That done, we can compare the astronomical calendar with the approximations used in the true Indian calendar.

The cumulative effect over the centuries of the difference in length of the sidereal year on the time of Mesha saṃkrānti, and on the sidereal longitude at that time, is shown in Figure 1. In 1000 C.E., it stood at about $1°37'$.

The average difference between the calculated sidereal longitude and the astronomical values was $2°3'$ during 1000–1002 C.E. In addition, Figure 2 shows a periodic discrepancy of up to $\pm 12'$ between the siddhāntic estimate of solar longitude and the true values. The figure also suggests that neither the interpolated stepped sine function used in traditional astronomy, nor the fluctuating epicycle of Indian theory (using the smallest [best] size, instead), make a noticeable difference for the sun. In other words, the tabular sine and arcsine functions (see Table 17.2 in CCME) are precise enough for the purpose, while the theory of changing epicycle (see Figure 17.1 in CCME) is unnecessary for the sun.

The difference in longitude for a given moment t, Ujjain local time, is calculated as

$$\textbf{hindu-solar-longitude}(t) - \textbf{sidereal-solar-longitude}(\textbf{universal-from-local}(t, \textbf{hindu-locale}))$$

5   Lunisolar Calendars

The lunisolar calendar type is represented by the Chinese, Hebrew, and Easter calendars today, as well as some of the Indian and other Asian cultures, and was historically very popular. The basic idea is that months follow the lunar cycle, with leap months added every 2–3 years, so that the average year length matches the sun's apparent celestial revolution. Indian lunisolar calendars can be further subdivided into those whose months begin with each new moon (the *amānta* scheme) and those that go from full-moon to full-moon (*pūrṇimānta*).

The Hebrew and Easter calendars follow a fixed leap-year cycle, as did the old Hindu mean lunisolar calendar; the Chinese and modern Hindu calendars determine each month and year according to the true positions of the sun and moon. Unlike the Hebrew lunisolar calendar, with its 19-year cycle of 7 leap years, Indian intercalated months, in the mean scheme, do not follow a short cyclical pattern. Rather, in the *Ārya-Siddhānta* version, there are $66,389$ leap years in every $180,000$-year cycle. The Hebrew, Easter, and mean Indian leap-year rules all distribute leap years evenly over the length of the cycle.

Lunisolar calendars can also come in the same two flavors, fixed- and variable-month patterns. The Indian mean lunisolar calendar has variable months, like its solar sister; the Hebrew calendar has a more-or-less fixed scheme (see Chapter 7 of CCME for details).

In the fixed-month scheme, one fixed month (usually the last) of the 13 months of a leap year is considered the leap month, so one can just number them consecutively. This is not true of the Indian calendar, in which any month can be leap, if it starts and ends within the same (sidereal) zodiacal sign.

Unlike other calendars, a day on the mean Indian calendar can be *omitted* any time in a lunar month, since the day number is determined by the phase of the mean moon. Here we concentrate on the leap year structure; see CCME for other details.

Let Y and M be the lengths of the mean solar year and lunar month in days, respectively, where $Y \geq M \geq 1$ are positive reals. If Y is not a multiple of M, then there will be common years of $\lfloor Y/M \rfloor$ months and leap years of length $\lceil Y/M \rceil$ months. Then a year has Y/M months on the average, with a leap-year frequency of $(Y \bmod M)/M$.

The basic idea of the dual-cycle calendar is to first aggregate days into months and then months into years. Elapsed months are counted in the same way as years are on the single-cycle calendar, using an average length of M instead of Y. Then, years are built from multiple units of months, rather than days, again in a similar fashion to a single-cycle calendar.

For the Indian mean lunar calendar, according to the *Ārya Siddhānta*, we would use the values

$$Y = 365\tfrac{149}{576}$$
$$M = 29\tfrac{2362563}{4452778}$$

and sunrise as the critical time of day. The Hebrew calendar also follows a dual-cycle pattern, with

$$M = 29\tfrac{13753}{25920}$$
$$Y = \tfrac{285}{19}M$$

and noon as critical moment, but exceptions can lead to a difference of up to 3 days.

To convert from a 0-based lunisolar date (y, m, d) to a day count:

$$\lceil (\lfloor yY/M \rfloor + m)M \rceil + d$$

In the other direction, we have

$$
\begin{aligned}
m' &= \lfloor n/M \rfloor \\
y &= \lceil (m'+1)M/Y \rceil - 1 \\
m &= m' - \lfloor yY/M \rfloor \\
d &= n - \lceil m'M \rceil
\end{aligned}
$$

When the leap month is not fixed and any month can be leap, as in the Indian calendar, we would use an extra, Boolean component for dates, $\langle y, m, \ell, d \rangle$, and need to determine which month is leap. On the Chinese calendar, the first lunar month in a leap year (counting from month 11 to month 11) during which the sun does not change its zodiacal sign is deemed leap. In the Indian scheme, the rule is similar: Any month in which the sidereal sign does not change is leap.

As was the case for the solar calendars, there are variants corresponding to whether the critical events may also occur at the critical moments.

6 True Lunisolar Calendar

The general form of the determination of New Year on a lunisolar calendar is as follows:

1. Find the moment s when the sun reaches its critical longitude.

2. Find the moment p when the moon attains its critical phase before (or after) s.

3. Choose the day d before (or after) p satisfying additional criteria.

   Some examples include:

- The Nicæan rule for Easter is the first Sunday after the first full moon on or after the day of the Spring equinox. See Chapter 8 of CCME.

- The classical rule for the first month (*Nisan*) of the Hebrew year was that it start on the eve of the first observable crescent moon no more than a fortnight before the Spring equinox. See Section 12.9 of CCME.

- The eleventh month of the Chinese calendar almost always begins with the new moon on or before the day of the winter solstice (270°). Chinese New Year is almost always the new moon on or after the day the sun reaches 300°. See Chapter 16 of CCME.

- Indian Lunar New Year is the (sunrise-to-sunrise) day of the last new moon before the sun reaches the fixed First Point in Aries (0° sidereal). See Chapter 17 of CCME.

   Using the functions provided in CCME:

1. The moment s can be found with **solar-longitude-after** (Section 12.4).

2. Finding the moment p can be accomplished with **lunar-phase-before** or **lunar-phase-after** (Section 12.5).

3. Choosing the day is facilitated by **kday-on-or-after** and its siblings (Section 1.10).

For example, the winter-solstice-to-winter-solstice period is called a *suì* (岁) on the Chinese calendar. Hence, the start of the Chinese month in which a *suì* begins, that is, the month containing the winter solstice (almost always the eleventh month, but on occasion a leap eleventh month) is determined by

$$\textbf{sui-month-start}\,(date) \quad \stackrel{\text{def}}{=} \tag{33}$$

$$\lfloor \textbf{standard-from-universal}$$
$$(\textit{moon}, \textbf{chinese-location}\,(\textit{date})) \rfloor$$

where

$$\textit{sun} \quad = \quad \textbf{universal-from-standard}$$
$$\left(\, \lfloor \textbf{standard-from-universal} \right.$$
$$\left(\, \textbf{solar-longitude-after}\,(\textit{date}, 270°)\,, \textbf{chinese-location}\,(\textit{date})\,\right)\rfloor\,,$$
$$\textbf{chinese-location}\,(\textit{date})\,\big)$$

$$\textit{moon} \quad = \quad \textbf{lunar-phase-before}\,(\textit{sun}+1, 0°)$$

For the Indian calendars, the functions should use sidereal longitudes and can be traditional or astronomical, as desired. Using the astronomical code of CCME, we can define

$$\textbf{sidereal-solar-longitude-after}\,(t, \phi) \quad \overset{\text{def}}{=} \tag{34}$$

$$\underset{x \in [l:u]}{\overset{u-l<\varepsilon}{\text{MIN}}} \left\{ \big(\big(\textbf{sidereal-solar-longitude}\,(x) - \phi \big) \bmod 360 \big) < 180° \right\}$$

where

$$\varepsilon \quad = \quad 10^{-5}$$

$$\textit{rate} \quad = \quad \frac{\textbf{average-year-length}}{360°}$$

$$\tau \quad = \quad t + \textit{rate} \times \big(\big(\phi - \textbf{sidereal-solar-longitude}(t) \big) \bmod 360 \big)$$

$$l \quad = \quad \max\{t, \tau - 5\}$$

$$u \quad = \quad \tau + 5$$

The function MIN performs a bisection search in $[l, u]$ with accuracy ε.

For the traditional Hindu calendar, we would use **hindu-solar-longitude-after** (Chapter 17 of CCME) instead of **sidereal-solar-longitude-after** and use the following in place of **lunar-phase-before**:

$$\textbf{hindu-lunar-phase-before}\,(t, \phi) \quad \overset{\text{def}}{=} \tag{35}$$

$$\underset{x \in [l:u]}{\overset{u-l<\varepsilon}{\text{MIN}}} \left\{ \begin{array}{l} ((\textbf{hindu-lunar-phase}\,(x) - \phi) \bmod 360) \\ < 180° \end{array} \right\}$$

where

$$\varepsilon \quad = \quad 2^{-17}$$

$$\tau \quad = \quad t - \textbf{hindu-synodic-month} \times \tfrac{1}{360}$$
$$\times ((\textbf{hindu-lunar-phase}\,(t) - \phi) \bmod 360)$$

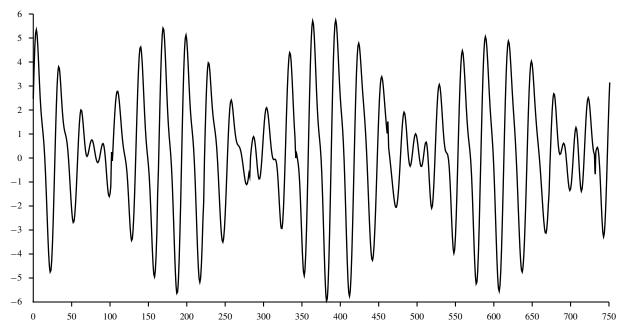$$l \quad = \quad \tau - 2$$

$$u \quad = \quad \min\{t, \tau + 2\}$$

Figure 3: Tithi time differences, 1000–1001 C.E. (in hours).

Then we can use the following to compute the start of Indian lunisolar month m:

$$\textbf{hindu-lunar-month-on-or-after}\,(\textit{date},m) \quad \stackrel{\text{def}}{=} \tag{36}$$

$$\begin{cases} \textit{date} & \textbf{if } \textit{moon} \leq \textbf{hindu-sunrise}\,(\textit{date}) \\ \textit{date}+1 & \textbf{otherwise} \end{cases}$$

where

$$\lambda \quad = \quad (m-1) \times 30°$$

$$\textit{sun} \quad = \quad \textbf{hindu-solar-longitude-after}\,(\textit{date},\lambda)$$

$$\textit{moon} \quad = \quad \textbf{hindu-lunar-phase-before}\,(\textit{sun},0°)$$

$$\textit{date} \quad = \quad \lfloor \textit{moon} \rfloor$$

This code does not check whether m may be expunged.

The time of the *tithis* (lunar "days", corresponding to 30ths of the lunar phase cycle) differs an average of less than 13 minutes between the traditional and astronomical calculations (again in 1000 C.E.). See Figure 3.

Figure 4 shows the same (nil) impact of sine and epicycle (using the biggest epicycle) on the calculation of lunar sidereal longitude as we found in the solar case. Moreover, the 16th century correction (*bija*) of Gaṇṇesa Daivajna for the length of the anomalistic months (from $488,203$ revolutions of the apogee in a *yuga* to $488,199$) also is of no consequence (in the 16th century as well as in the 11th). The difference between the calculated longitude and astronomical values was $1°56' \pm 3°25'$.

In the true version of the Indian lunisolar calendar, months, called *kshaya*, may also be expunged, when two zodiacal sign transitions occur in one lunar month. Thus, even a 12-month year can have a leap month (as was the case in 1963–4), and a leap year can even have two (as in 1982–3). See our *Calendrical Tabulations* [6].

There are several competing conventions as to the placement and naming of leap months and excision of suppressed months; see [8, page 26].
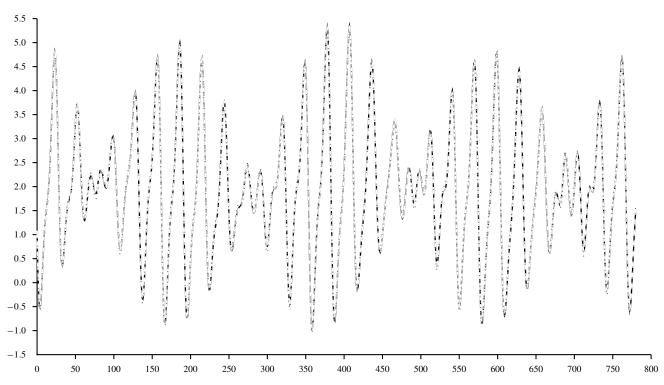
16

Figure 4: Difference (1000–1001 C.E.) between siddhāntic and astronomical sidereal lunar longitude (solid black line). Dashed light gray line (atop the black line) uses the mathematical sine function, rather than an interpolated tabular sine; white dashed line (largely overlaying the solid black line) uses a fixed epicycle; the gray dotted line is sans *bija* correction; they are virtually indistinguishable.
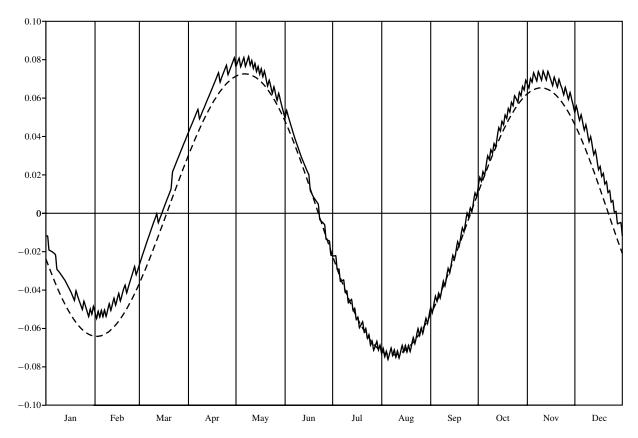
Figure 5: Sunrise, Hindu and astronomical, 1000 C.E. Difference in sunrise times is shown as a solid line; the difference of just the equation of time calculation is shown as a dashed line.

7 Sunrise

Generally, Indian calendarists advocate the use of geometric sunrise for calendrical determinations:[4]

$$\textbf{hindu-sunrise}\,(date) \quad \overset{\text{def}}{=} \tag{37}$$

$$\textbf{dawn}\,(date, \textbf{hindu-locale}, 0°)$$

Lahiri, however, suggests a depression angle of $47'$ (including $31'$ for refraction); astronomers use $50'$.

As is well-known, the original siddhāntic calculation for sunrise uses a simple approximation for the equation of time. Figure 6 compares the two versions. Using an accurate equation of time, but otherwise following the siddhāntic method for sunrise, agrees closely with geometric sunrise. See Figure 5.

8 Holidays

Many of the holidays in India depend on the local lunisolar calendar. Table 5 lists some of the more popular holidays. (For a comprehensive list in English, see [9].) There is a very wide regional variance in timing and duration of holidays.

In general, holidays do not occur in leap months or on leap days. If a month is skipped, as happens intermittently (with gaps of 19 to 141 years between occurrences), then the "lost" holidays are moved to the next nonth depending, again, on regional conventions. In many places, rather than skip a whole month, two half months are skipped and their holidays are moved backward or forward, depending on which lost half-month they are meant to occur in.

---

[4]Pal Singh Purewal [personal communication, April 29, 2002]: "Most Indian almanac editors give and advocate the use of centre of solar disk for sunrise without refraction."
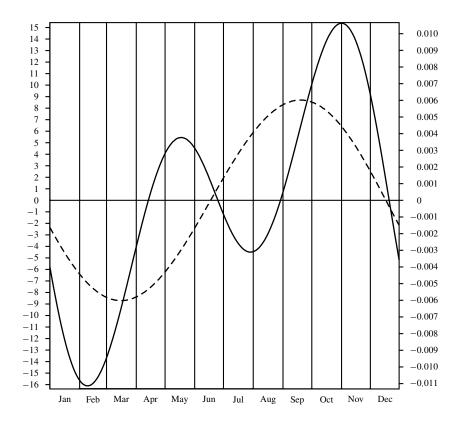
Figure 6: The equation of time in 1000 C.E. The astronomical version is shown as a solid line; the Hindu version is shown as a dashed line. The left vertical axis is marked in minutes and the right vertical axis is marked in fractions of a day.

Holiday	Lunar Date(s)
New Year (Chandramana Ugadi)	Chaitra 1
Birthday of Rāma	Chaitra 9
Birthday of Krishna (Janmāshṭamī)	Śrāvaṇa 23
Ganēśa Chaturthī	Bhādrapada 3 or 4
Dashara (Nava Rathri), last 3 days	Āśvina 8–10
Diwali, last day	Kārtika 1
Birthday of Vishnu (Ekadashi)	Mārgaśīrsha 11
Night of Śiva	Māgha 28 or 29
Holi	Phālguna 15

Table 5: Some Hindu lunisolar holidays.

The precise day of observance of a lunisolar event usually depends on the time of day (sunrise, noon, midnight, etc.) at which the moon reaches a critical phase (*tithi*). According to [4], for example, Gaṇēśa Chaturthī is celebrated on the day in which *tithi* (lunar day) 4 is current in whole or in part during the midday period 10:48a.m. – 1:12p.m. (temporal time). If that lunar day is current during that time frame on two consecutive days, or if it misses that time frame on both days, then it is celebrated on the former day.[5]

Appendix—The Lisp Code

This appendix contains Common Lisp source code for the algorithms presented in the preceding sections. CCME should be consulted for undefined functions and macros.

```
(defun fixed-from-single-cycle (s-date)
  ;; TYPE single-cycle-date -> fixed-date
  (let* ((year (standard-year s-date))
         (month (standard-month s-date))
         (day (standard-day s-date)))
    (+ single-cycle-epoch
       (floor (+ (* (1- year) average-year-length)
                 delta-year))
       (floor (+ (* (1- month) average-month-length)
                 delta-month))
       day -1)))
```

```
(defun single-cycle-from-fixed (date)
  ;; TYPE fixed-date -> single-cycle-date
  (let* ((days (- date single-cycle-epoch))
         (year (ceiling (- days -1 delta-year)
                        average-year-length))
         (n (- days (floor (+ delta-year
                              (* (1- year) average-year-length)))))
         (month (ceiling (- n -1 delta-month) average-month-length))
         (day (- n -1 (floor (+ delta-month
                                (* (1- month)
                                   average-month-length))))))
    (hindu-solar-date year month day)))
```

```
(defun alt-fixed-from-single-cycle (s-date)
  ;; TYPE single-cycle-date -> fixed-date
  (let* ((year (standard-year s-date))
         (month (standard-month s-date))
         (day (standard-day s-date)))
    (+ single-cycle-epoch
       (ceiling (- (* (1- year) average-year-length)
                   delta-year))
       (ceiling (- (* (1- month) average-month-length)
                   delta-month))
       day -1)))
```

```
(defun alt-single-cycle-from-fixed (date)
  ;; TYPE fixed-date -> single-cycle-date
  (let* ((days (+ (- date single-cycle-epoch) delta-year))
         (year (1+ (quotient days average-year-length)))
         (n (+ (mod days average-year-length) delta-month))
         (month (1+ (quotient n average-month-length)))
         (day (1+ (floor (mod n average-month-length)))))
    (hindu-solar-date year month day)))
```

---

[5]Precise details for the individual holidays are difficult to come by.

```
1   (defun var-fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date))
6            (average-month-length (/ average-year-length
7                                     months-per-year)))
8       (+ single-cycle-epoch
9          (floor (+ (* (1- year) average-year-length)
10                   delta-year
11                   (* (1- month) average-month-length)))
12         day -1)))

1   (defun var-single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (- date single-cycle-epoch))
4            (offset (- days -1 delta-year))
5            (year (ceiling offset average-year-length))
6            (average-month-length (/ average-year-length
7                                     months-per-year))
8            (month (+ 1 (mod (1- (ceiling offset average-month-length))
9                             months-per-year)))
10           (day (- days -1
11                   (floor (+ delta-year
12                             (* (1- (ceiling offset average-month-length))
13                                average-month-length))))))
14      (hindu-solar-date year month day)))

1   (defun alt-var-fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date))
6            (average-month-length (/ average-year-length
7                                     months-per-year)))
8       (+ single-cycle-epoch
9          (ceiling (+ (* (1- year) average-year-length)
10                     (- delta-year)
11                     (* (1- month) average-month-length)))
12         day -1)))

1   (defun alt-var-single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (+ (- date single-cycle-epoch) delta-year))
4            (average-month-length (/ average-year-length
5                                     months-per-year))
6            (year (1+ (quotient days average-year-length)))
7            (month (+ 1 (mod (quotient days average-month-length)
8                             months-per-year)))
9            (day (1+ (floor (mod days average-month-length)))))
10      (hindu-solar-date year month day)))

1   (defun true-longitude (date)
2     ;; TYPE moment -> longitude
3     (solar-longitude (critical-time date)))

1   (defun solar-new-year-on-or-after (date)
2     ;; TYPE fixed-date -> fixed-date
3     ;; Fixed date of solar new year on or after fixed date.
```

```
4     (let* ((lambda (true-longitude date))
5           (start
6            (+ date -5
7               (floor (* average-year-length 1/360
8                         (mod (- critical-longitude lambda) 360))))))
9       (next d start
10               (<= critical-longitude
11                   (true-longitude d)
12                   (+ critical-longitude 2)))))))
```

```
1     (defun hindu-solar-new-year (g-year)
2       ;; TYPE gregorian-year -> fixed-date
3       ;; Fixed date of Hindu solar New Year in Gregorian year.
4        (solar-new-year-on-or-after
5          (gregorian-from-fixed
6            (gregorian-date g-year january 1))))
```

```
1     (defun solar-from-fixed (date)
2       ;; TYPE fixed-date -> solar-date
3       ;; Solar date equivalent to fixed date.
4       (let* ((lambda (true-longitude date))
5             (m (quotient lambda (deg 30)))
6             (year (round (- (/ (- (critical-time date) solar-epoch)
7                                average-year-length)
8                             (/ lambda (deg 360)))))
9             (approx ; 3 days before start of mean month.
10              (- date 3
11                 (mod (floor lambda) (deg 30))))
12             (begin ; Search forward for beginning...
13              (next i approx ; ... of month.
14                    (= m (quotient (true-longitude i)
15                                   (deg 30))))))
16         (hindu-solar-date year (1+ m)  (- date begin -1))))
```

```
1     (defun orissa-critical (date)
2       ;; TYPE fixed-date -> moment
3       ;; Universal time of critical moment on or after date
4       ;; according to the Orissa rule
5       (hindu-sunrise (1+ date)))
```

```
1     (defun tamil-critical (date)
2       ;; TYPE fixed-date -> moment
3       ;; Universal time of critical moment on or after date
4       ;; according to the Tamil rule
5       (hindu-sunset date))
```

```
1     (defun malayali-critical (date)
2       ;; TYPE fixed-date -> moment
3       ;; Universal time of critical moment on or after date
4       ;; according to the Malayali rule
5       (+ (hindu-sunrise date)
6          (* 3/5 (- (hindu-sunset date) (hindu-sunrise date)))))
```

```
1     (defun madras-critical (date)
2       ;; TYPE fixed-date -> moment
3       ;; Universal time of critical moment on or after date
4       ;; according to the Madras rule
5       (+ (hindu-sunset date)
6          (* 1/2 (- (hindu-sunrise (1+ date)) (hindu-sunset date)))))
```

```
1   (defun precession (tee)
2     ;; Meeus, page 136-137
3     ;; Using 0,0 as J2000 coordinates
4     (let* ((c (julian-centuries tee))
5           (eta (mod (+ (* (seconds 47.0029l0) c)
6                        (* (seconds -0.03302l0) (expt c 2))
7                        (* (seconds 0.00006l0) (expt c 3)))
8                     360))
9           (cap-P (mod (+ (deg 174.876384l0)
10                         (* (seconds -869.8089l0) c)
11                         (* (seconds 0.03536l0) (expt c 2)))
12                      360))
13          (p (mod (+ (* (seconds 5029.0966l0) c)
14                     (* (seconds 1.11113l0) (expt c 2))
15                     (* (seconds 0.000006l0) (expt c 3)))
16                  360))
17          (cap-A (* (cosine-degrees eta) (sin-degrees cap-P)))
18          (cap-B (cosine-degrees cap-P))
19          (arg (arctan2 cap-A cap-B)))
20       (mod (- (+ p cap-P) arg) 360)))

1   (defun arctan2 (A B)
2     (let* ((x (/ A B))
3           (alpha (radians-to-degrees (atan x))))
4       (mod (if (>= B 0)
5               alpha
6             (+ alpha (deg 180l0)))
7            360)))

1   (defconstant sidereal-start
2     (precession (universal-from-local
3                 (mesha-samkranti (ce 285))
4                 hindu-locale)))

1   (defun sidereal-solar-longitude (tee)
2     ;; TYPE moment -> angle
3     ;; Sidereal solar longitude at moment tee
4     (mod (+ (solar-longitude tee)
5            (- (precession tee))
6             sidereal-start)
7          360))

1   (defun sui-month-start (date)
2     ;; TYPE fixed-date -> fixed-date
3     ;; Fixed date of start of Chinese month containing solstice
4     ;; occurring on or after date.
5     (let* ((sun (universal-from-standard
6                 (floor (standard-from-universal
7                        (solar-longitude-after date (deg 270))
8                        (chinese-location date)))
9                 (chinese-location date)))
10          (moon (lunar-phase-before (1+ sun) (deg 0))))
11      (floor (standard-from-universal moon (chinese-location date)))))

1   (defun sidereal-solar-longitude-after (tee phi)
2     ;; TYPE (moment season) -> moment
3     ;; Moment UT of the first time at or after tee
4     ;; when the sidereal solar longitude will be phi degrees.
5     (let* ((varepsilon 1d-5) ; Accuracy of solar-longitude.
```

```
  6          (rate ; Mean days for 1 degree change.
  7           (/ average-year-length (deg 360)))
  8          (tau ; Estimate (within 5 days).
  9           (+ tee
 10             (* rate
 11                (mod (- phi (sidereal-solar-longitude tee)) 360))))
 12          (l (max tee (- tau 5))) ; At or after tee.
 13          (u (+ tau 5)))
 14    (binary-search ; Bisection search.
 15     l l
 16     u u
 17     x (< (mod (- (sidereal-solar-longitude x) phi) 360)
 18          (deg 180l0))
 19     (< (- u l) varepsilon))))
```

```
  1  (defun hindu-lunar-phase-before (tee phi)
  2    ;; TYPE (moment phase) -> moment
  3    ;; Moment UT of the last time at or before tee
  4    ;; when the Hindu lunar-phase was phi degrees.
  5    (let* ((varepsilon (expt 2 -17)) ; Accuracy.
  6           (tau ; Estimate.
  7            (- tee
  8              (* hindu-synodic-month 1/360
  9                 (mod (- (hindu-lunar-phase tee) phi) 360))))
 10           (l (- tau 2))
 11           (u (min tee (+ tau 2)))) ; At or before tee.
 12    (binary-search ; Bisection search.
 13     l l
 14     u u
 15     x (< (mod (- (hindu-lunar-phase x) phi) 360)
 16          (deg 180l0))
 17     (< (- u l) varepsilon))))
```

```
  1  (defun hindu-lunar-month-on-or-after (date m)
  2    ;; TYPE fixed-date -> fixed-date
  3    ;; Fixed date of first lunar moon on or after fixed date.
  4    (let* ((lambda (* (1- m) (deg 30)))
  5           (sun (hindu-solar-longitude-after date lambda))
  6           (moon (hindu-lunar-phase-before sun (deg 0)))
  7           (date (floor moon)))
  8    (if (<= moon (hindu-sunrise date))
  9        date (1+ date))))
```

```
  1  (defun hindu-sunrise (date)
  2    ;; TYPE fixed-date -> moment
  3    ;; Geometrical sunrise at Hindu locale on date.
  4    (dawn date hindu-locale (deg 0)))
```

Acknowledgment

References

[1] S. K. Chatterjee, *Indian Calendric System*, Ministry of Information and Broadcasting, India, 1998.

[2] Nachum Dershowitz and Edward M. Reingold, "Implementing Solar Astronomical Calendars", in: *Birashkname*, M. Akrami, ed., Shahid Beheshti University, Tehran, Iran, pages 477-487, 1998.

[3] Mitchell A. Harris and Edward M. Reingold, "Line Drawing and Leap Years", *ACM Computing Surveys* **36**, pages 68–80, 2004.

[4] Franz Kielhorn, "Festal Days of the Hindu Lunar Calendar," *The Indian Antiquary*, volume XXVI, pp. 177–187, 1897.

[5] Edward M. Reingold and Nachum Dershowitz, *Calendrical Calculations: The Millennium Edition,* 3rd printing, Cambridge University Press, 2001.

[6] Edward M. Reingold and Nachum Dershowitz, *Calendrical Tabulations,* Cambridge University Press, 2002.

[7] Jean Meeus, *Astronomical Algorithms,* 2nd. ed., Willmann-Bell, Inc., Richmond, VA, 1998.

[8] Robert Sewell and Sankara B. Dikshit, *The Indian Calendar–with tables for the conversion of Hindu and Muhammadan into* A.D. *dates, and vice versâ*, Motilal Banarsidass Publ., Delhi, 1995. Originally published in 1896.

[9] Muriel M. Underhill, *The Hindu Religious Year,* Association Press, Calcutta, 1921.

[10] Usha-Shashi, "Hindu Astrological Calculations (According to Modern Methods)", Sagar Publ., New Delhi, 1978.

[11] Walther E. van Wijk, "On Hindu Chronology, I–V", *Acta Orientalia,* 1922–1927.