

Compilation Summary Class

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Advanced Topics

(if interested please send email)

- Just in time compilation
- Compiler correctness

Topics

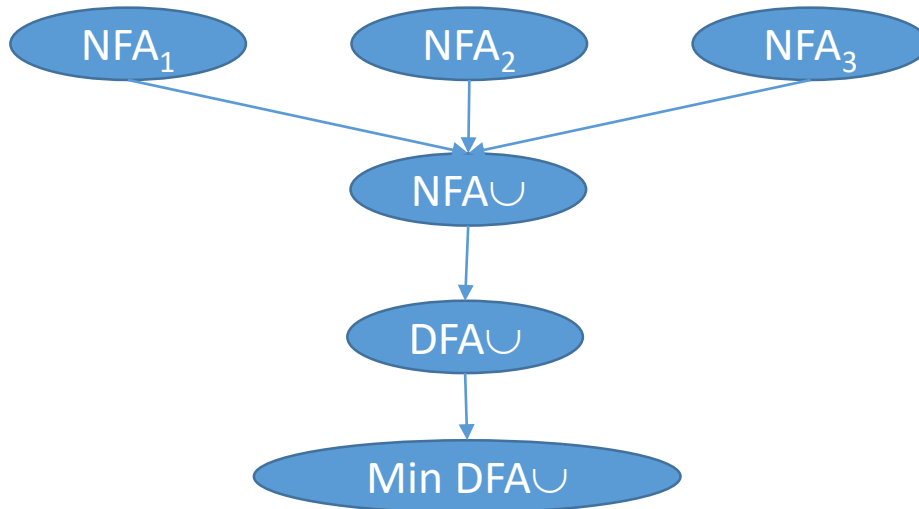
- Lexical Analysis
- Syntax Analysis
 - LL(1)
 - SLR(1)
- Semantic Analysis
- Intermediate Code Generation
- Register Allocation
- Machine code generation
- Assembler/Linker/Loader
- Memory allocation and Garbage Collection

Lexical Analysis (Scanning)

- input
 - program text (file)
- output
 - sequence of tokens
- Read input file
- Identify language keywords and standard identifiers
- Handle include files and macros
- Count line numbers
- Remove whitespaces
- Report illegal symbols
- [Produce symbol table]

Generating Lexical Analysis

Token definition (Regular Expression)	Java Code for match
if	return(IF);
[a-z][a-z0-9]*	return(ID);
[0-9]*	return(NUM);



Maximal Match Scanner using DFA

```
Token nextToken()
{
  lastFinal = 0;
  currentState = 1 ;
  inputPositionAtLastFinal = input;
  currentPosition = input;
  while (not(isDead(currentState))) {
    nextState = edges[currentState][*currentPosition];
    if (isFinal(nextState)) {
      lastFinal = nextState ;
      inputPositionAtLastFinal = currentPosition; }
    currentState = nextState;
    advance currentPosition;
  }
  input = inputPositionAtLastFinal ;
  return action[lastFinal];
}
```

Summary Lexical Analysis

- Based on regular expressions and automata theory
- Excellent open source tools LEX/FLEX/JLEX/...
- Some tricky implementation tricks
 - Input buffering
 - Maintain state in program counter

LL(1) Syntax Analysis

- Work for a restricted class of programs
- Identifies the left most derivation or syntax error
- The simplest implementation includes one (recursive) function for non-terminal
- The function for non-terminal A reads one token and **uniquely decides** on which of the productions of A to apply
 - $A \rightarrow \alpha$
 - $A \rightarrow \beta$
- A rule $A \rightarrow \alpha$ is applied for tokens $t \in \text{select}(A \rightarrow \alpha)$

Nullable Nonterminals

- A non-terminal is **nullable** if it can derive the empty word
- $A \rightarrow^* \varepsilon$
- Computed iteratively
- Extended to right hand side of derivation

```
while changes do {  
    for every rule of the form  $A \rightarrow X_1 X_2 \dots X_n$   
        where all  $X_i$  is nullable make  $X$  nullable  
}
```

The set of terminals in which a grammar symbols may begin

- $\text{First}(X) = \{ t \mid X \rightarrow^* t \alpha \}$
- Computed iteratively
- Extended to right hand sides of rules

Computing First Iteratively

For each token t , $\text{First}(t) := \{t\}$

For each non-terminal $\langle A \rangle$, $\text{First}(\langle A \rangle) = \{\}$

while changes occur do

 if there exists a non-terminal $\langle A \rangle$ and

 a rule $\langle A \rangle \rightarrow V_1 V_2 \dots V_n \alpha$ and

V_1, V_2, \dots, V_{n-1} are nullable

 a token $t \in \text{First}(V_n)$

 add t to $\text{First}(\langle A \rangle)$

The set of terminals $\text{first}(\alpha)$
for a string of grammar symbols α

- Tokens in which α **may** begin
- $\text{First}(\alpha) = \{ t \mid \alpha \rightarrow^* \beta \}$
- Computed iteratively

Computing First Iteratively

For the empty string, $\text{first}(\alpha) := \{\}$ for every rule $A \rightarrow \alpha$

while changes occur do

if there exists a non-terminal $\langle A \rangle$ and

a rule $\langle A \rangle \rightarrow V_1 V_2 \dots V_n \alpha$ and

V_1, V_2, \dots, V_{n-1} are nullable

a token $t \in \text{First}(V_n)$

add t to $\text{First}(V_1 V_2 \dots V_n \alpha)$

The set of terminals which may follow a non-terminal

- For a non-terminal $\langle A \rangle$ define
 - $\text{Follow}(\langle A \rangle) = \{ t \mid \exists \alpha, \beta: \langle S \rangle \rightarrow^* \alpha \langle A \rangle t \beta \}$
- For a rule $\langle A \rangle \rightarrow \alpha$
 - If α is nullable then
$$\text{select}(\langle A \rangle \rightarrow \alpha) = \text{First}(\alpha) \cup \text{Follow}(\langle A \rangle)$$
 - Otherwise $\text{select}(\langle A \rangle \rightarrow \alpha) = \text{First}(\alpha)$
- The grammar is LL(1) if for every two grammar rules $\langle A \rangle \rightarrow \alpha$ and $\langle A \rangle \rightarrow \beta$
 - $\text{Select}(A \rightarrow \alpha) \cap \text{Select}(A \rightarrow \beta) = \emptyset$

Computing Follow Iteratively

For each non-terminal A , $\text{Follow}(A) := \{\}$

$\text{Follow}(S) = \{\$ \}$

while changes occur do

 for each rule $V \rightarrow \alpha A \beta$

 for each token $t \in \text{First}(\beta)$ and

 add t to $\text{Follow}(A)$

 if β is nullable

 for each token $t \in \text{Follow}(V)$

 add t to $\text{Follow}(A)$

Predictive Parser

```
<S> → id := <E>
<S> → if (<E>) <S> else <S>
<E> → id <EP> | (<E>)
<EP> → ε | + <E> <EP>
```

```
def parse_S():
    if id(input):
        match(input, id)
        match(input, assign)
        parse_E()
    elif if_tok(input):
        match(input, if_tok)
        match(input, lp)
        parse_E()
        match(input, rp)
        parse_S()
        match(input, else_tok)
        parse_S()
    else:
        syntax_error()
```

```
def parse_E():
    if id(input):
        match(input, id)
        parse_EP()
    elif lp(input):
        match(input, lp)
        parse_E()
        match(input, rp)
    else:
        syntax_error()
```

```
def parse_EP():
    if plus(input):
        match(input, plus)
        parse_E()
    elif
        rp(input) or
        else_tok(input) or
        eof(input):
        return // ε
    else:
        syntax_error()
```


1: $\langle S \rangle \rightarrow \mathbf{id} := \langle E \rangle$
 2: $\langle S \rangle \rightarrow \mathbf{if} (\langle E \rangle) \langle S \rangle \mathbf{else} \langle S \rangle$
 3: $\langle E \rangle \rightarrow \mathbf{id} \langle EP \rangle$
 4: $\langle E \rangle \rightarrow (\langle E \rangle)$
 5: $\langle EP \rangle \rightarrow \varepsilon$
 6: $\langle EP \rangle \rightarrow \mathbf{+} \langle E \rangle \langle EP \rangle$

First

S	E	EP
id, if	id, (+

Follow

S	E	EP
\$, else	\$,), else	\$,), else

Select

rule	α	First(α)	Select(α)
1	$\mathbf{id} := \langle E \rangle$	id	id
2	$\mathbf{if} (\langle E \rangle) \langle S \rangle \mathbf{else} \langle S \rangle$	if	if
3	$\mathbf{id} \langle EP \rangle$	<u>id</u>	id
4	$(\langle E \rangle)$	((
5	ε		\$,), else

Predictive Parser

```
<S> → id := <E>  
<S> → if (<E>) <S> else <S>  
<E> → id <EP> | (<E>)  
<EP> → ε | + <E> <EP>
```

```
def parse_S():  
    if id(input):  
        match(input, id)  
        match(input, assign)  
        parse_E()  
    elif if_tok(input):  
        match(input, if_tok)  
        match(input, lp)  
        parse_E()  
        match(input, rp)  
        parse_S()  
        match(input, else_tok)  
        parse_S()  
    else:  
        syntax_error()
```

```
def parse_E():  
    if id(input):  
        match(input, id)  
        parse_EP()  
    elif lp(input):  
        match(input, lp)  
        parse_E()  
        match(input, rp)  
    else:  
        syntax_error()
```

```
def parse_EP():  
    if plus(input):  
        match(input, plus)  
        parse_E()  
        parse_EP()  
    elif  
        rp(input) or  
        else_tok(input) or  
        eof(input):  
        return // ε  
    else:  
        syntax_error()
```

Bottom-Up Syntax Analysis

- Input
 - A context free grammar
 - A stream of tokens
- Output
 - A syntax tree or error
- Method
 - Construct parse tree in a bottom-up manner
 - Find the rightmost derivation in (reversed order)
 - For every potential right hand side and token decide when a production is found
 - Report an error as soon as the input is not a prefix of valid program

Constructing LR(0) parsing table

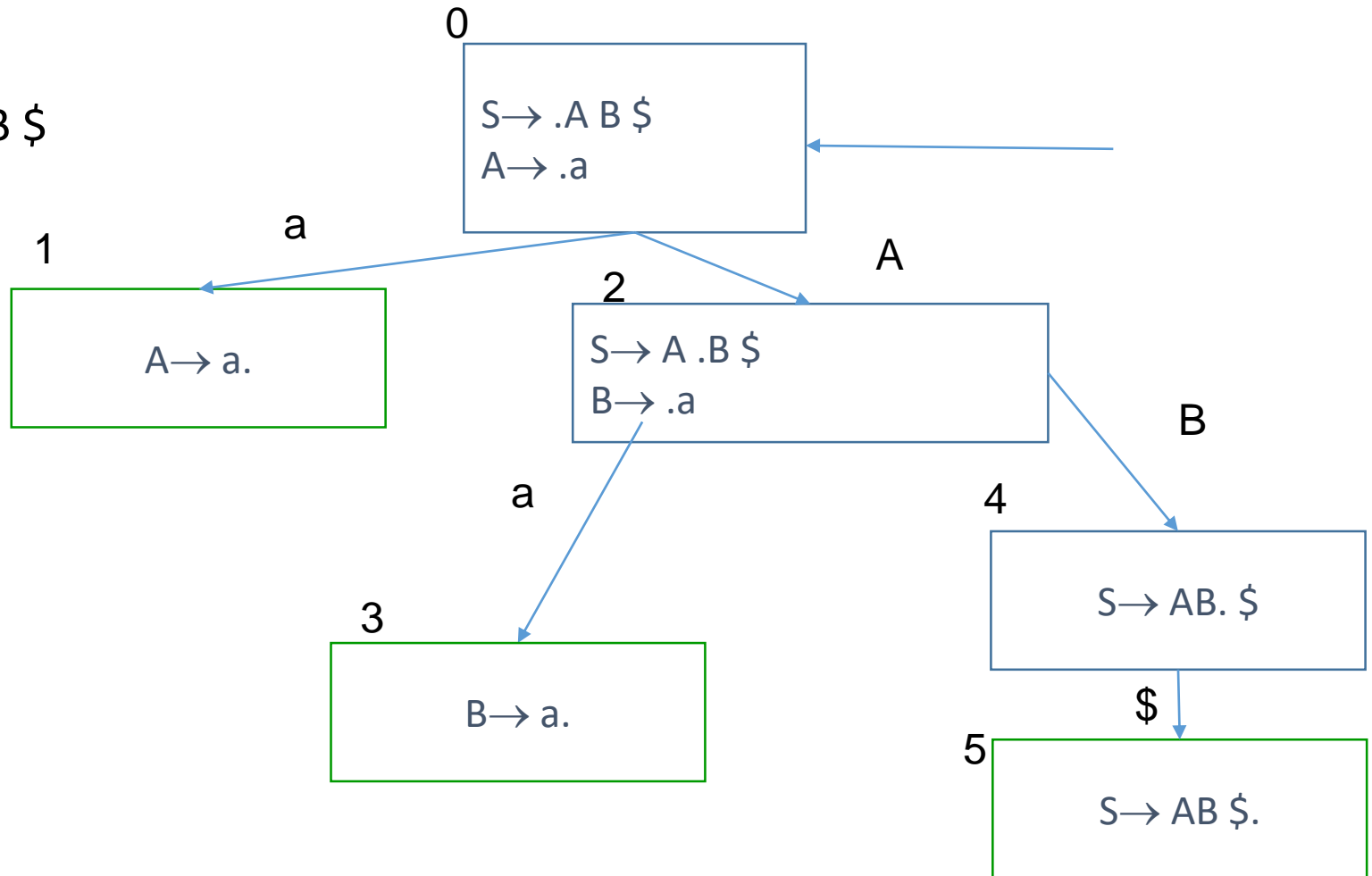
- Add a production $S' \rightarrow S\$$
- Construct a deterministic finite automaton accepting “valid stack symbols”
 - States are set of items $A \rightarrow \alpha \bullet \beta$
 - The initial state includes $S' \rightarrow \bullet S$ and its epsilon closure
 - All the states are accepting (excluding the sink which not shown)
 - There is a transition labeled by X between states include $A \rightarrow \alpha \bullet X \beta$ and the epsilon closure of $A \rightarrow \alpha X \bullet \beta$
- Fill the parsing table
 - $A \rightarrow \alpha \bullet \Rightarrow$ **reduce** $A \rightarrow \alpha$ on all terminals
 - $A \rightarrow \alpha \bullet t \beta \Rightarrow$ **shift** the appropriate state on t
 - $A \rightarrow \alpha \bullet X \beta \Rightarrow$ **goto** the appropriate state on X (happens after reduce on the remaining stack)

Constructing SLR(1) parsing table

- Add a production $S' \rightarrow S\$$
- Construct a deterministic finite automaton accepting “valid stack symbols”
 - States are set of items $A \rightarrow \alpha \bullet \beta$
 - The initial state includes $S' \rightarrow \bullet S$ and its epsilon closure
 - All the states are accepting (excluding the sink which not shown)
 - There is a transition labeled by X between states include $A \rightarrow \alpha \bullet X \beta$ and the epsilon closure of $A \rightarrow \alpha X \bullet \beta$
- Fill the parsing table
 - $A \rightarrow \alpha \bullet \Rightarrow$ **reduce** $A \rightarrow \alpha$ on terminals in $\text{Follow}(A)$
 - $A \rightarrow \alpha \bullet t \beta \Rightarrow$ **shift** the appropriate state on t
 - $A \rightarrow \alpha \bullet X \beta \Rightarrow$ **goto** the appropriate state on X (happens after reduce on the remaining stack)

A Trivial Example

- $S \rightarrow A B \$$
- $A \rightarrow a$
- $B \rightarrow a$



LR(0) Control Table Trivial Example

state	terminal			nonterminal		
	a	\$	other	S	A	B
0 S → .A B \$ A → .a	shift 1	err	err		2	
1 A → a.	reduce A → a					
2 S → A .B \$ B → .a	shift 3	err	err			4
3 B → a.	reduce B → a					
4 S → AB. \$	err	shift 5	err			
5 S → AB \$.	accept					

SLR(1) Control Table Trivial Example

state	terminal			nonterminal		
	a	\$	other	S	A	B
0 S → .A B \$ A → .a	shift 1	err	err		2	
1 A → a.	reduce A → a	err	err			
2 S → A .B \$ B → .a	shift 3	err	err			4
3 B → a.	err	reduce B → a	err			
4 S → AB. \$	err	shift 5	err			
5 S → AB \$.	accept					

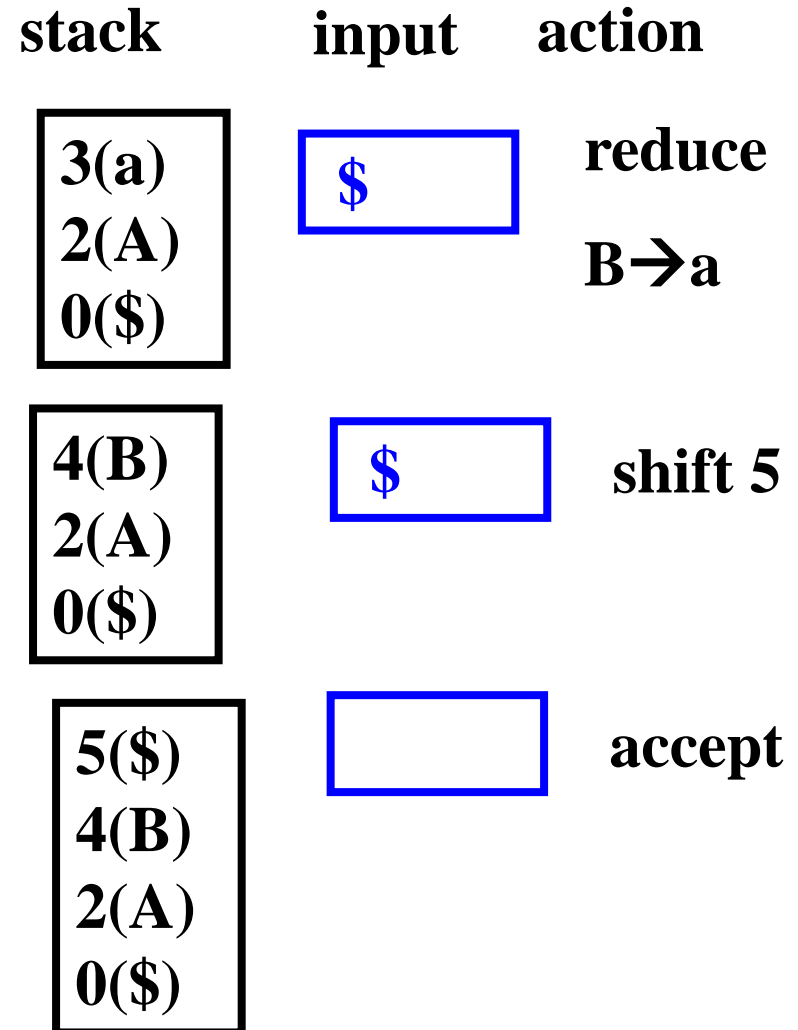
Parsing aa

state	terminal			nonterminal		
	a	\$	other	S	A	B
0	s1	err	err		2	
1	reduce $A \rightarrow a$					
2	s3	err	err			4
3	reduce $B \rightarrow a$					
4	err	s5	err			
5	accept					

stack	input	action
0(\$)	aa \$	shift 1
1(a)	a \$	reduce
0(\$)		$A \rightarrow a$
0(\$)	a \$	A
2(A)	a \$	shift 3
0(\$)		

Parsing aa (Cont)

state	terminal			Nonterminal		
	a	\$	other	S	A	B
0	s1	err	err		2	
1	reduce $A \rightarrow a$					
2	s3	err	err			4
3	reduce $B \rightarrow a$					
4	err	s5	err			
5	accept					



The Rightmost Derivation in Reverse Order

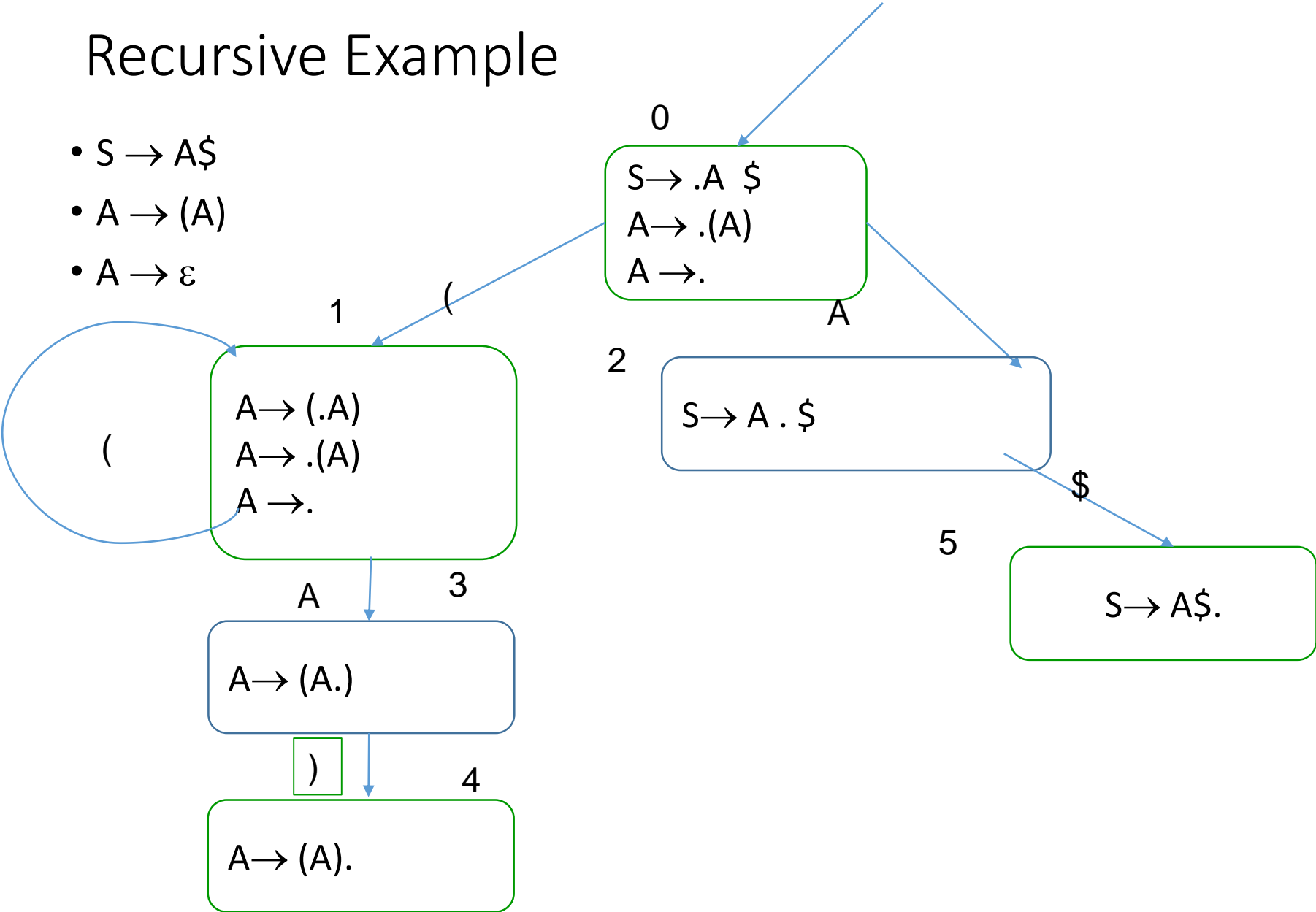
1. $S \rightarrow BA\$$

2. $B \rightarrow a$

3. $A \rightarrow a$

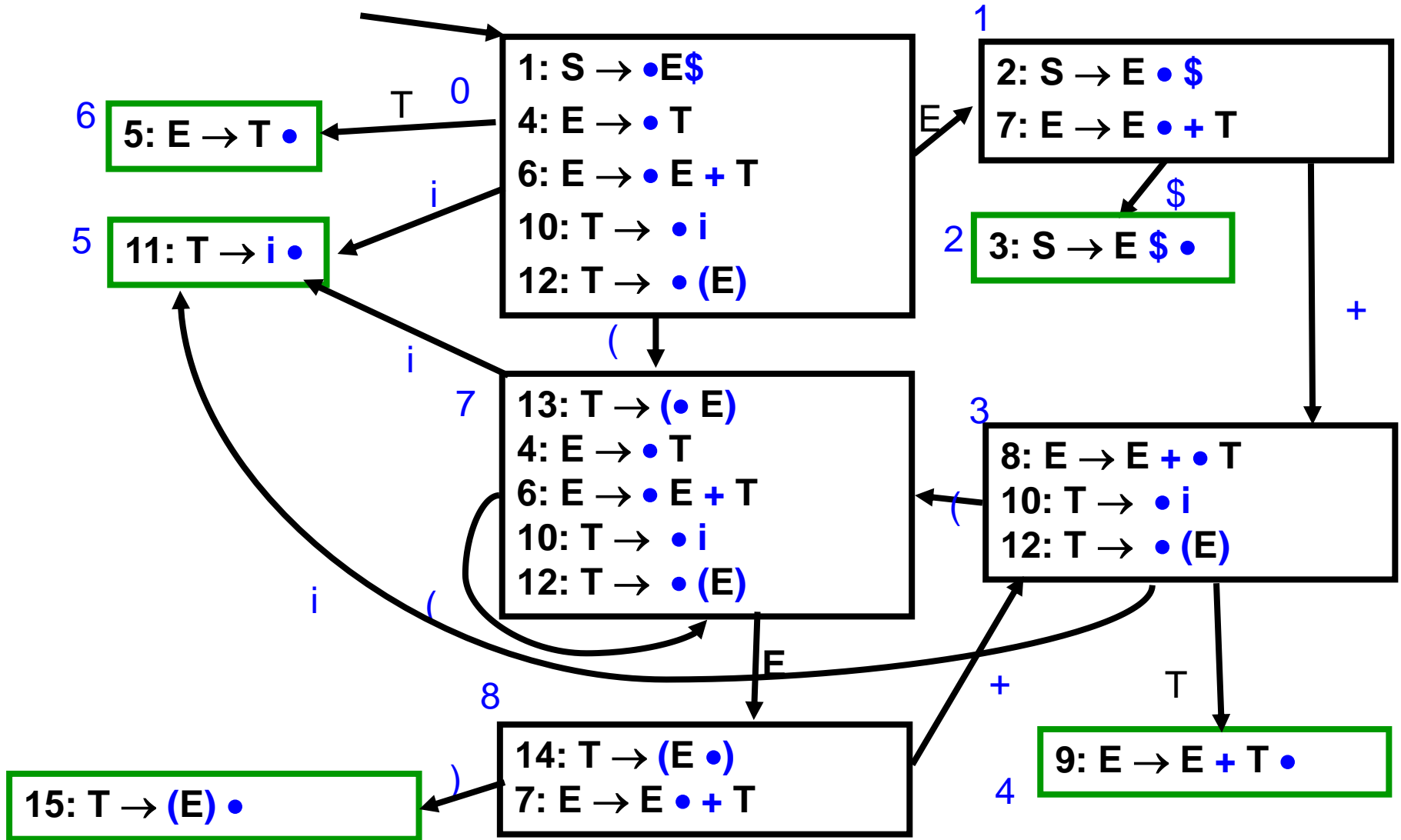
Recursive Example

- $S \rightarrow A\$$
- $A \rightarrow (A)$
- $A \rightarrow \epsilon$



Control Table Recursive Example

state	terminal				nonterminal	
	()	\$	other	S	A
0 S → .A\$ A → .(A) A → .	shift 1	err	err	err		2
1 A → (.A) A → .(A) A → .	shift 1	reduce A →				
2 S → A . \$	shift 5	err				
3 A → (A.)	err	shift 4				
4 A → (A).	err	err	shift 5			
5 S → A\$.	accept					



9

Follow

S	E	T
	+,)	+,)

Example Control Table

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E + T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

Example Control Table

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E + T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

0(\$)

input

i + i \$

shift 5

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

5 (i)
0 (\$)

input

+ i \$

reduce $T \rightarrow i$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

6 (T)
0 (\$)

input

+ i \$

reduce $E \rightarrow T$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

1(E)
0 (\$)

input

+ i \$

shift 3

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

3 (+)
1(E)
0 (\$)

input

i \$

shift 5

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

5 (i)

3 (+)

1(E)

0(\$)

input

\$

reduce $T \rightarrow i$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E + T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

4 (T)
3 (+)
1 (E)
0 (\$)

input

\$

reduce $E \rightarrow E + T$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

input

1 (E)
0 (\$)

\$

shift 2

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

2 (\$)
1 (E)
0 (\$)

input

accept

The Rightmost Derivation in Reverse Order

1. $S \rightarrow E\$$

2. $E \rightarrow E + T$

3. $T \rightarrow I$

4. $E \rightarrow T$

5. $T \rightarrow i$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

0(\$)

input

((i) \$

shift 7

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

7(i)
0(\$)

input

(i) \$

shift 7

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

7 ()
7 ()
0 (\$)

input

i) \$

shift 5

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

5 (i)
7 (()
7 (()
0 (\$)

input

) \$

reduce $T \rightarrow i$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

6 (T)
7 (()
7 (()
0 (\$)

input

) \$

reduce $E \rightarrow T$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

8 (E)
7 (()
7 (()
0 (\$)

input

) \$

shift 9

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce E → E+T						
5	reduce T → i						
6	reduce E → T						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce T → (E)						

stack

9 ()
8 (E)
7 ()
7()
0(\$)

input

\$

reduce T → (E)

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

6 (T)
7()
0(\$)

input

\$

reduce $E \rightarrow T$

	i	+	()	\$	E	T
0	s5	err	s7	err	err	1	6
1	err	s3	err	err	s2		
2	acc						
3	s5	err	s7	err	err		4
4	reduce $E \rightarrow E+T$						
5	reduce $T \rightarrow i$						
6	reduce $E \rightarrow T$						
7	s5	err	s7	err	err	8	6
8	err	s3	err	s9	err		
9	reduce $T \rightarrow (E)$						

stack

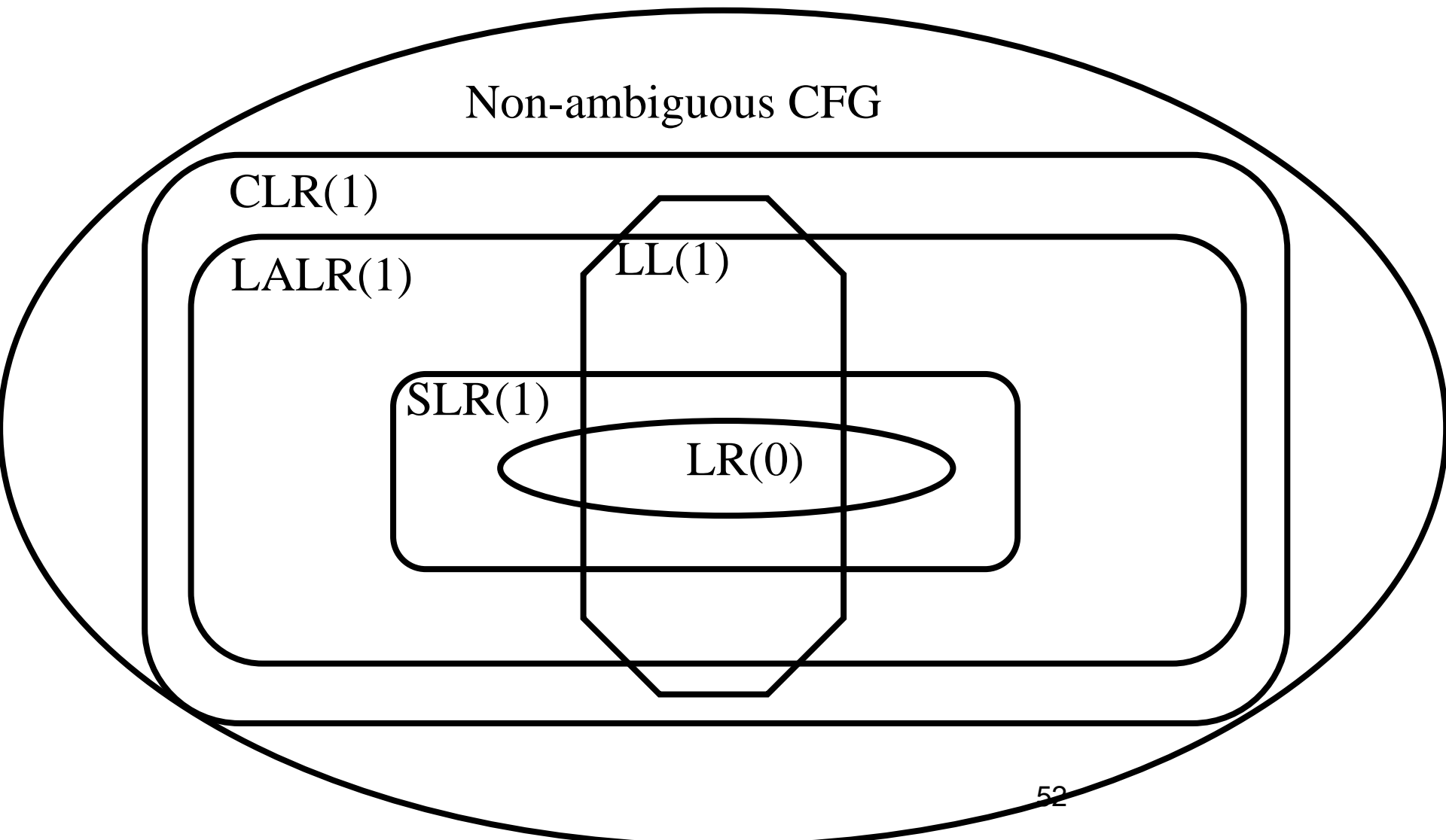
8 (E)
7(())
0(\$)

input

\$

err

Grammar Hierarchy



Interesting Non LR(1) Grammars

- Ambiguous
 - Arithmetic expressions
 - Dangling-else
- Common derived prefix
 - $A \rightarrow B_1 a b \mid B_2 a c$
 - $B_1 \rightarrow \varepsilon$
 - $B_2 \rightarrow \varepsilon$
- Optional non-terminals
 - $st \rightarrow \text{OptLab Ass}$
 - $\text{OptLab} \rightarrow \text{id} : \mid \varepsilon$
 - $\text{Ass} \rightarrow \text{id} := \text{Exp}$

$St \rightarrow \text{id: Ass} \mid \text{Ass}$

Interesting Non LR(1) Grammars

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 - Arithmetic expressions
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 - $\text{Ass} \rightarrow \text{id} := \text{Exp}$

Summary

- LR is a powerful technique
- Generates efficient parsers
- Generation tools exist LALR(1)
 - Bison, yacc, CUP, ply
- But some grammars need to be tuned
 - Shift/Reduce conflicts
 - Reduce/Reduce conflicts
 - Efficiency of the generated parser
- There exist more general methods
 - GLR
 - Arbitrary grammars in n^3
 - Early parsers
 - CYK algorithms

Code Generation

- Instruction Selection
- Register allocation
 - Caller save vs. Calle Save registers
 - Local vs. Global Register Allocation

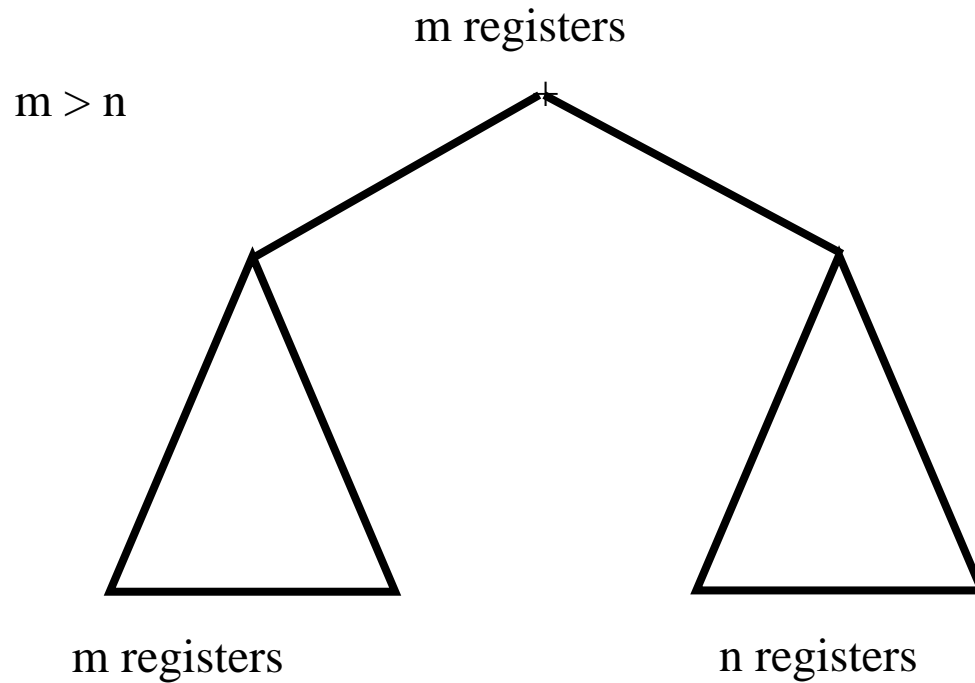
Two Phase Solution

Dynamic Programming

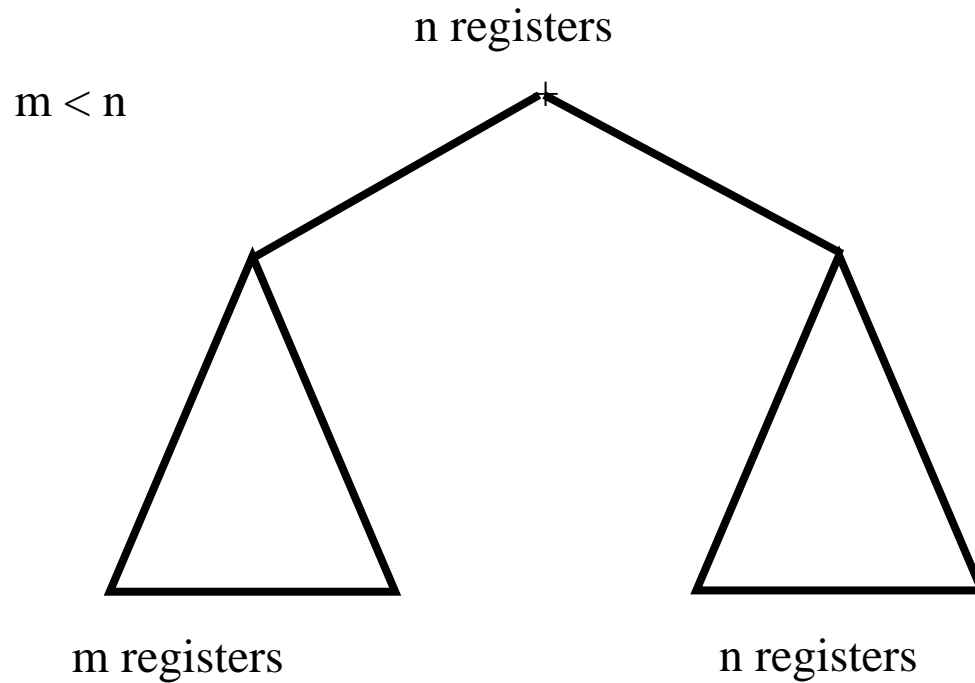
Sethi & Ullman

- Bottom-up (labeling)
 - Compute for every subtree
 - The minimal number of registers needed
 - Weight
- Top-Down
 - Generate the code using labeling by preferring “heavier” subtrees (larger labeling)
 - Can integrate spilling

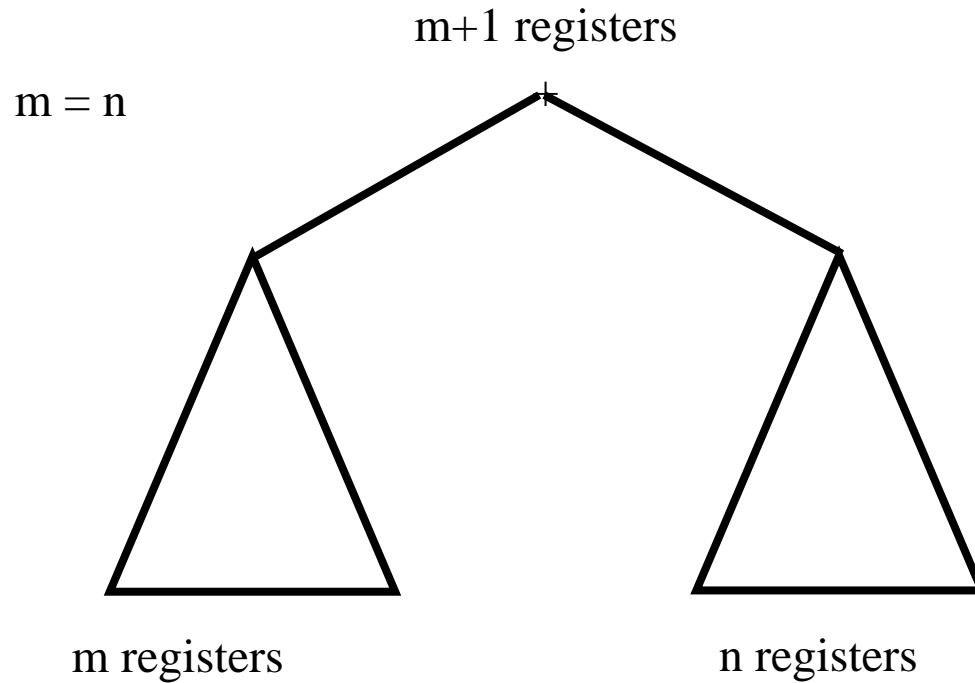
The Labeling Principle



The Labeling Principle



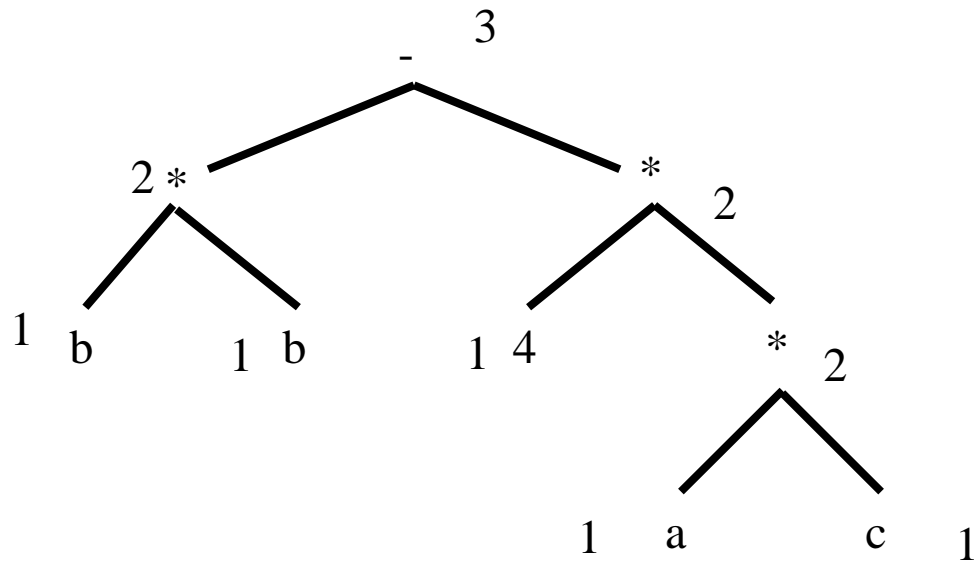
The Labeling Principle



The Labeling Algorithm

```
weight(Node: expression): integer {  
  switch node: {  
    case number(n: integer): return 1;  
    case localVariable(v: symbol) return 1;  
    case e1: Node + e2: Node {  
      let lw: integer = weight(e1);  
      let rw: integer = weight(e2);  
      if (lw < rw) return rw ;  
      else if (lw > rw) return lw;  
      else return lw + 1 ;  
    }  
  }  
  ...  
}
```

Labeling the example (weight)



The need for global register allocation

```
int foo() {  
    int x = 1 ;  
    x = x + 1 ;  
    x = x + 1 ;  
    ...  
    printf(“%d”, x);  
}
```

```
foo():  
    push    rbp  
    mov     rbp, rsp  
    sub     rsp, 16  
    mov     DWORD PTR [rbp-  
4], 1  
    add     DWORD PTR [rbp-  
4], 1  
    add     DWORD PTR [rbp-  
4], 1  
    mov     eax, DWORD PTR [r  
bp-4]  
    mov     esi, eax  
    mov     edi, OFFSET FLAT:.L  
C1  
    mov     eax, 0  
    call   printf  
    nop  
    leave  
    ret
```

```
foo():  
    push    rbp  
    mov     rbp, rsp  
    sub     rsp, 16  
    mov     eax, 1  
    add     eax, 1  
    add     eax, 1  
    mov     esi, eax  
    mov     edi, OFFSET FLAT:.L  
C1  
    mov     eax, 0  
    call   printf  
    nop  
    leave  
    ret
```

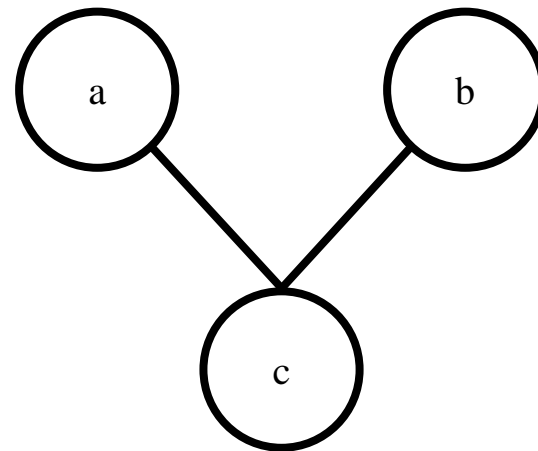
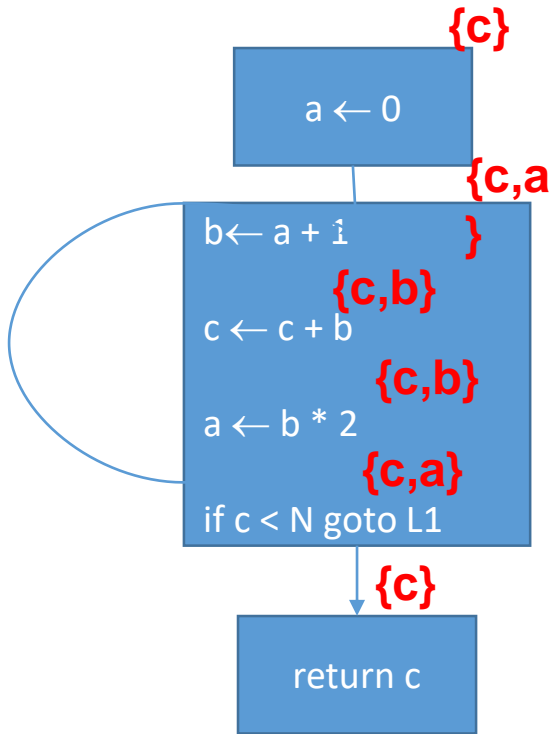

Caller-Save and Callee-Save Registers

- **callee-save-registers** (MIPS 16-23, X86 r12-15, rbp, rsp)
 - Saved by the callee when modified
 - Values are automatically **preserved** across calls
- **caller-save-registers**
 - Saved by the caller when needed
 - Values are not automatically preserved
- Usually the architecture defines caller-save and callee-save registers
 - Separate compilation
 - Interoperability between code produced by different compilers/languages
- But compilers can decide when to use caller/callee registers

X86lite Registers: 16 64-bit registers

register	usage	Callee save
rax	general purpose accumulator	N
rbx	base register, pointer to data	N
rcx	counter register for strings & loops	N
rdx	data register for I/O	N
rsi	pointer register, string source register	N
rdi	pointer register, string destination register	N
rbp	base pointer, points to the stack frame	Y
rsp	stack pointer, points to the top of the stack	Y
r08-r11	General purpose registers	N
r12-15	General purpose registers	Y

Using Liveness Information



A Complete Example (Andrew Appel) <https://www.cs.princeton.edu/~appel/>

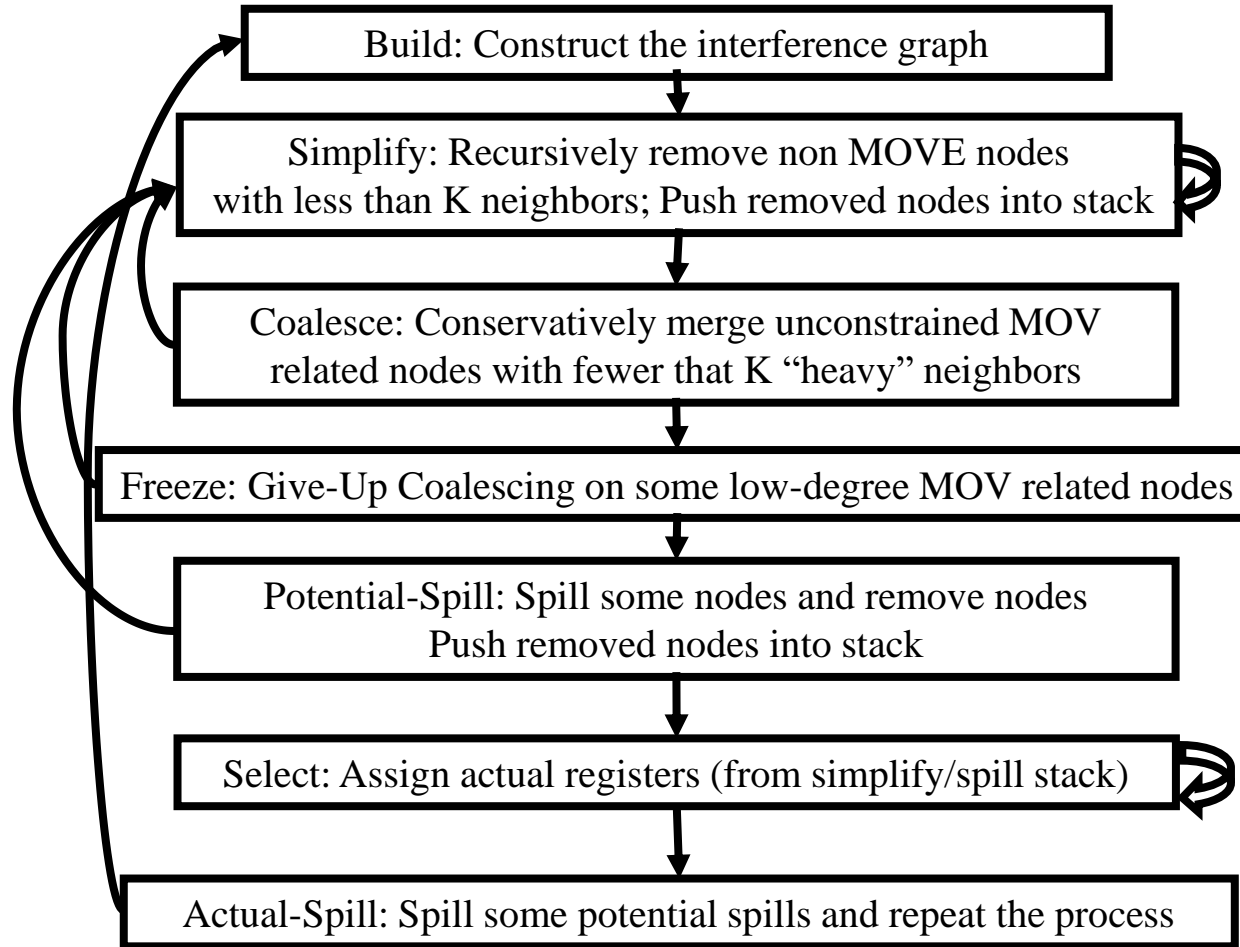
enter:

```
c := r3    r1, r2  caller save
           r3      callee-save
a := r1
b := r2
d := 0
e := a
```

loop:

```
d := d+b
e := e-1
if e>0 goto loop
r1 := d
r3 := c
return /* r1,r3 */
```

Graph Coloring with Coalescing



A Complete Example

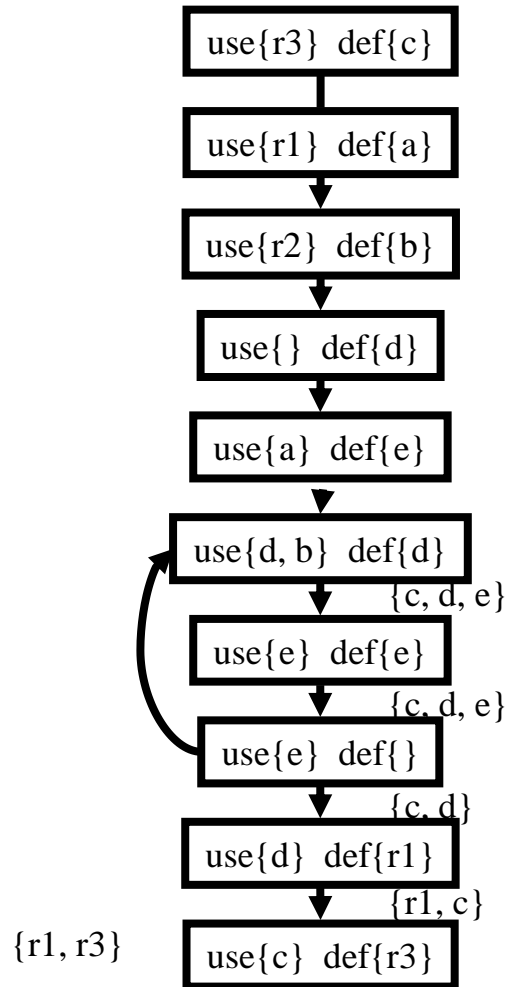
enter:

$c := r3$ **r1, r2** caller save
 $a := r1$ **r3** callee-save
 $b := r2$
 $d := 0$
 $e := a$

loop:

$d := d+b$
 $e := e-1$
 if $e > 0$ goto loop
 $r1 := d$
 $r3 := c$

return /* **r1,r3** */



A Complete Example

enter:

`c := r3`

`a := r1`

`b := r2`

`d := 0`

`e := a`

loop:

`d := d+b`

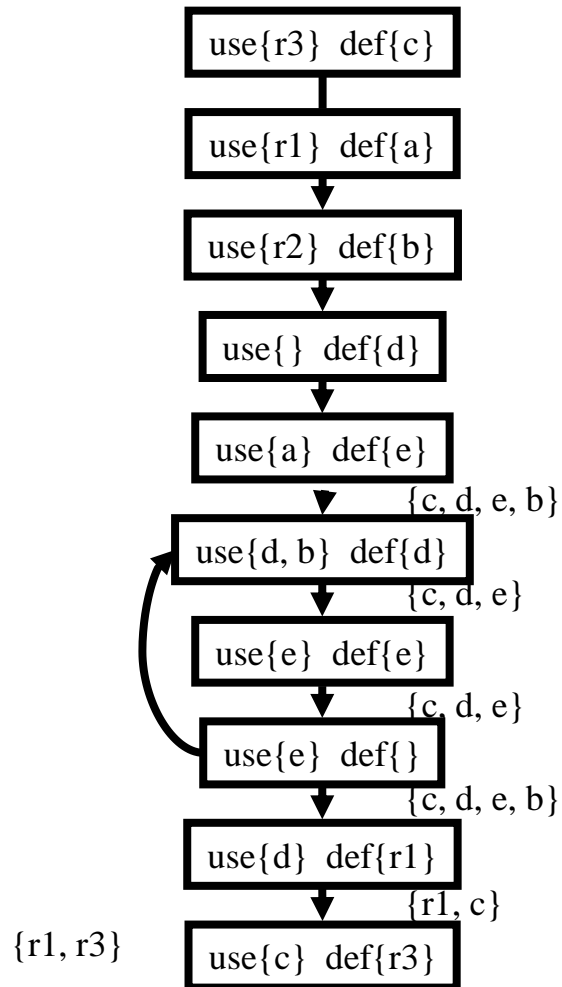
`e := e-1`

`if e>0 goto loop`

`r1 := d`

`r3 := c`

`return /* r1,r3 */`



A Complete Example

enter:

c := r3

a := r1

b := r2

d := 0

e := a

loop:

d := d+b

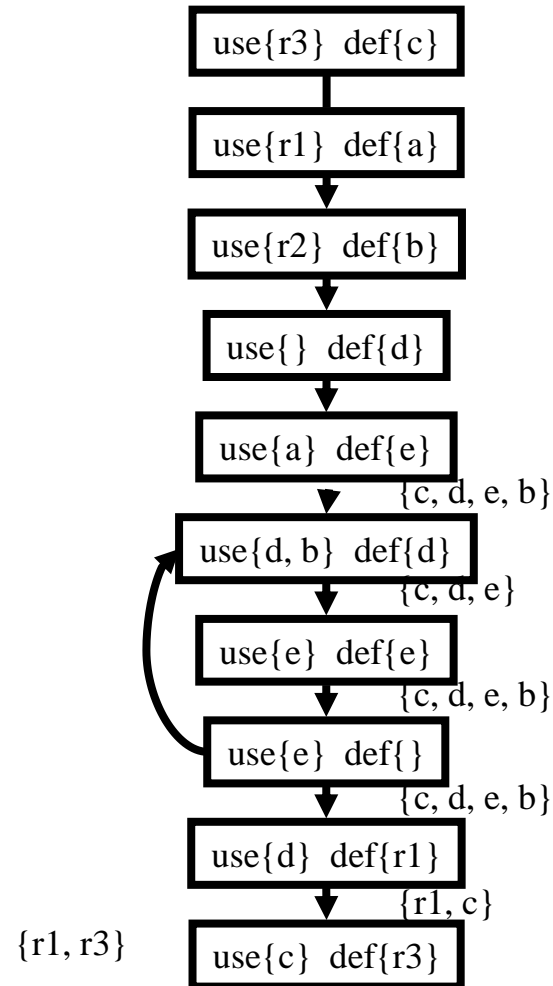
e := e-1

if e>0 goto loop

r1 := d

r3 := c

return /* r1,r3 */



A Complete Example

enter:

`c := r3`

`a := r1`

`b := r2`

`d := 0`

`e := a`

loop:

`d := d+b`

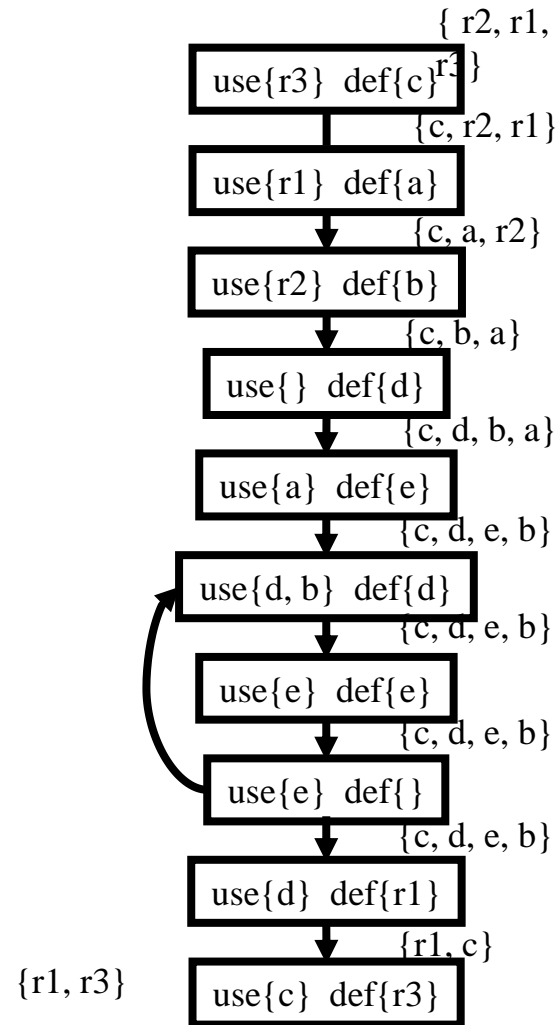
`e := e-1`

`if e>0 goto loop`

`r1 := d`

`r3 := c`

`return /* r1,r3 */`



Live Variables Results

enter:

c := r3

a := r1

b := r2

d := 0

e := a

loop:

d := d+b

e := e-1

if e>0 goto loop

r1 := d

r3 := c

return /* r1,r3 */

enter: /* r2, r1, r3 */

c := r3 /* c, r2, r1 */

a := r1 /* a, c, r2 */

b := r2 /* a, c, b */

d := 0 /* a, c, b, d */

e := a /* e, c, b, d */

loop:

d := d+b /* e, c, b, d */

e := e-1 /* e, c, b, d */

if e>0 goto loop /* c, d */

r1 := d /* r1, c */

r3 := c /* r1, r3 */

return /* r1, r3 */

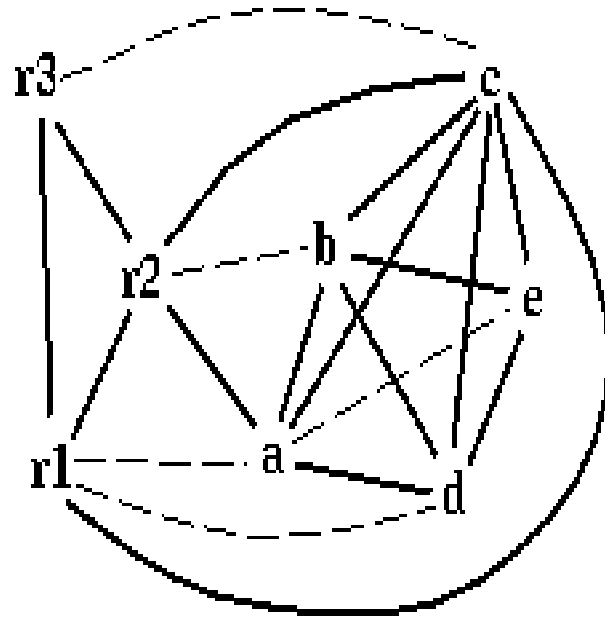
```

enter      /* r2, r1, r3 */
c := r3   /* c, r2, r1 */
a := r1   /* a, c, r2 */
b := r2   /* a, c, b */
d := 0    /* a, c, b, d */
e := a    /* e, c, b, d */

loop:
d := d+b  /* e, c, b, d */
e := e-1  /* e, c, b, d */
if e>0 goto loop /* c, d */
r1 := d   /* r1, c */
r3 := c  /* r1, r3 */

return /* r1, r3 */

```



$$\text{spill priority} = (\text{uo} + 10 \text{ ui})/\text{deg}$$

```

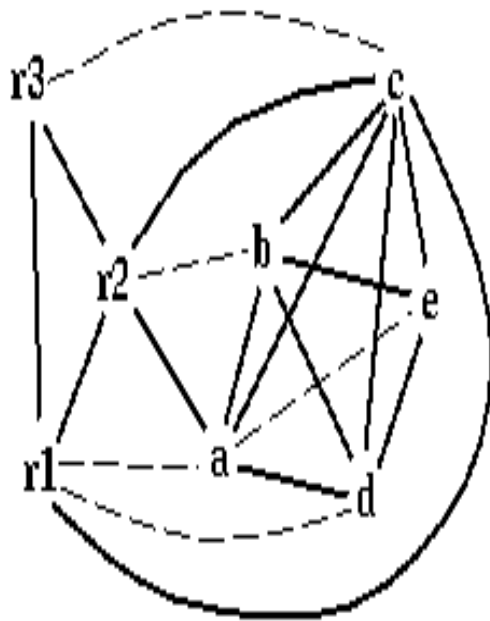
enter:          /* r2, r1, r3 */
c := r3 /* c, r2, r1 */
a := r1 /* a, c, r2 */
b := r2 /* a, c, b */
d := 0 /* a, c, b, d */
e := a /* e, c, b, d */

loop:
d := d+b /* e, c, b, d */
e := e-1 /* e, c, b, d */
if e>0 goto loop /* c, d */
r1 := d /* r1, c */
r3 := c /* r1, r3 */
return /* r1, r3 */

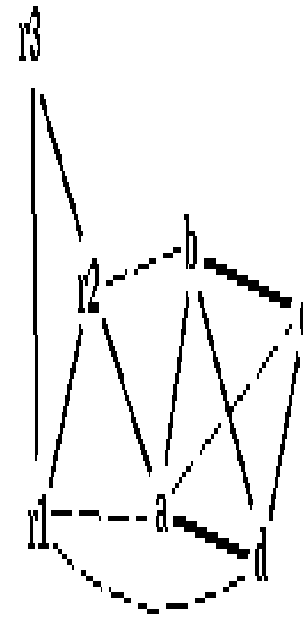
```

	use+ def outside loop	use+ def within loop	deg	spill priority
a	2	0	4	0.5
b	1	1	4	2.75
c	2	0	6	0.33
d	2	2	4	5.5
e	1	3	3	10.3

Spill C



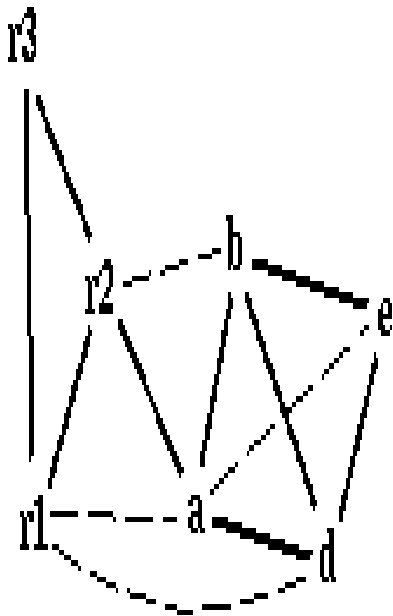
stack



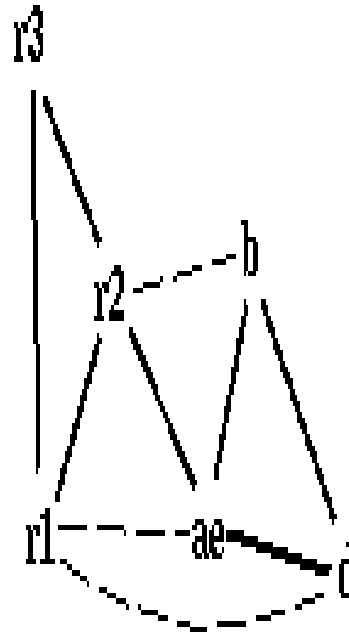
stack



Coalescing $a+e$



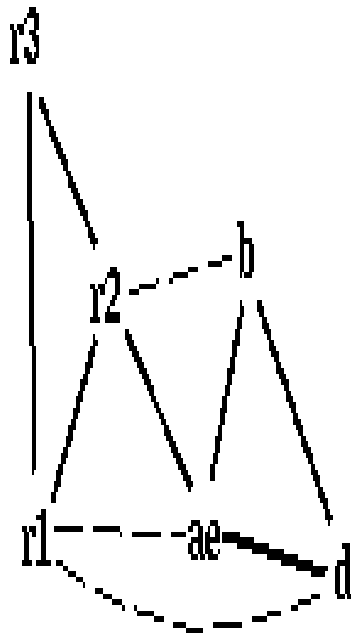
stack



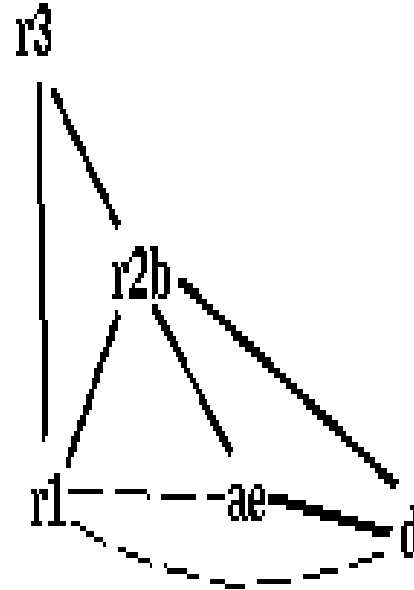
stack



Coalescing $b+r_2$



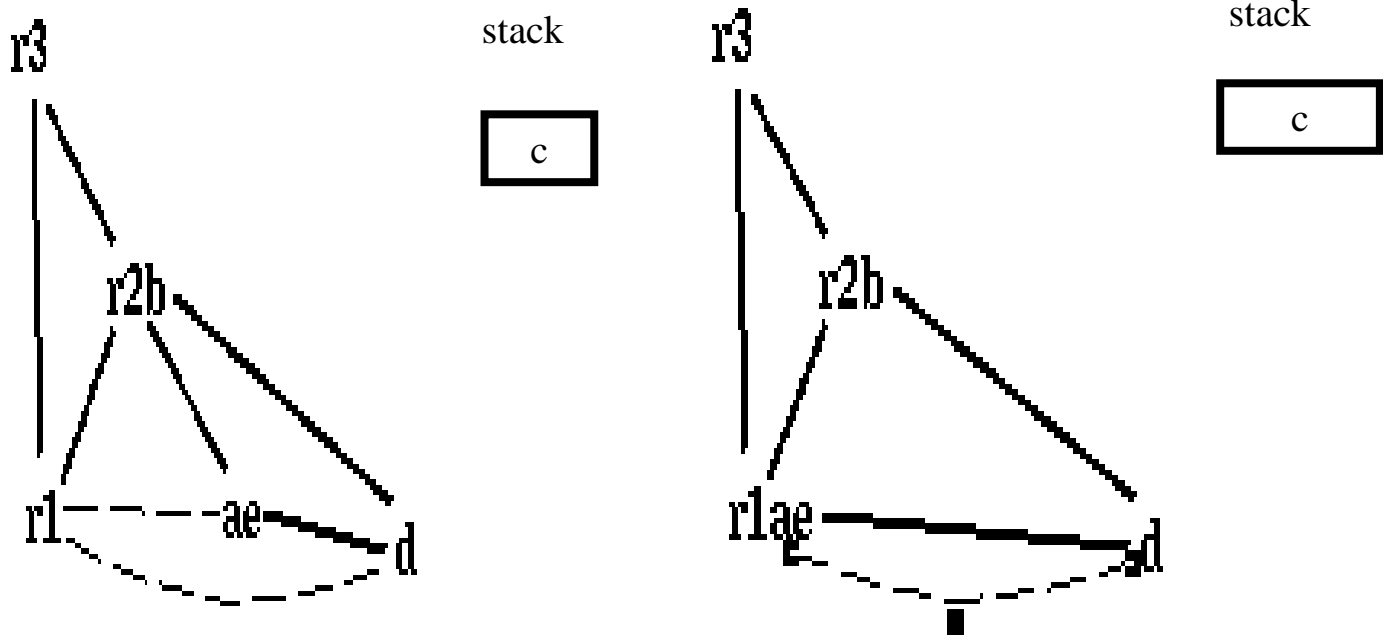
stack



stack

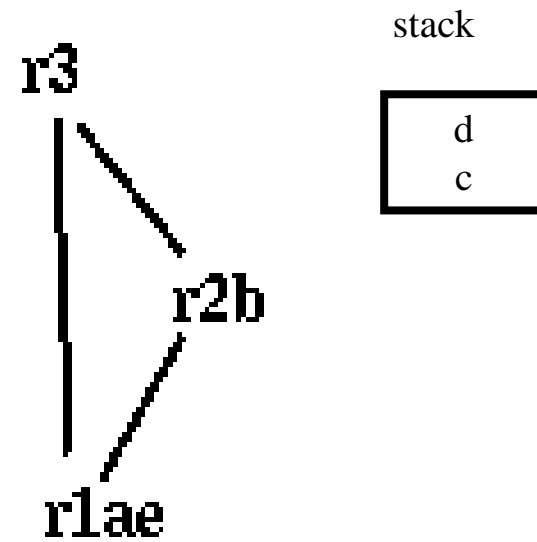
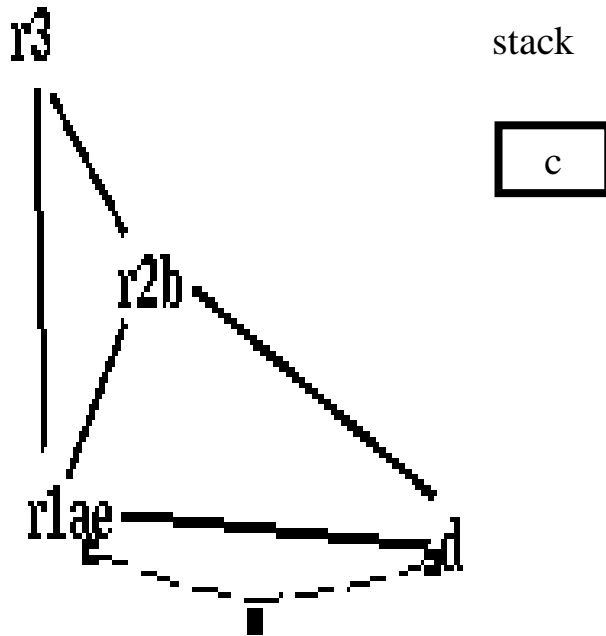


Coalescing $ae+r1$

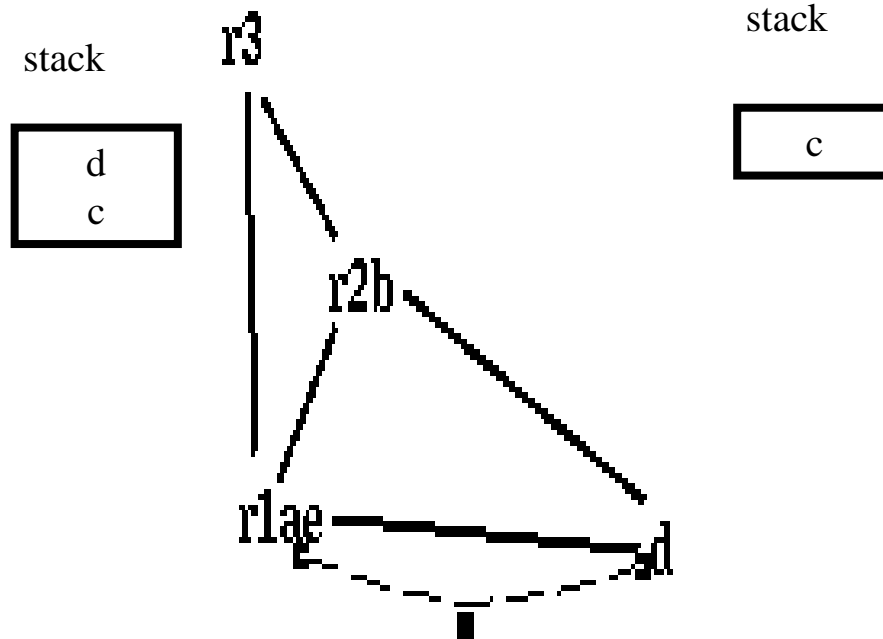
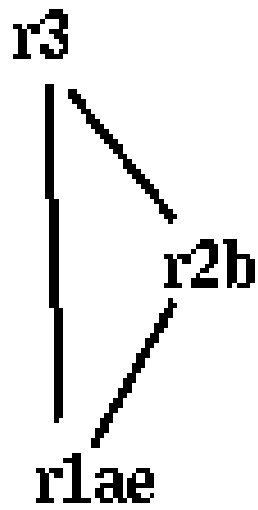


$r1ae$ and d are constrained

Simplifying d

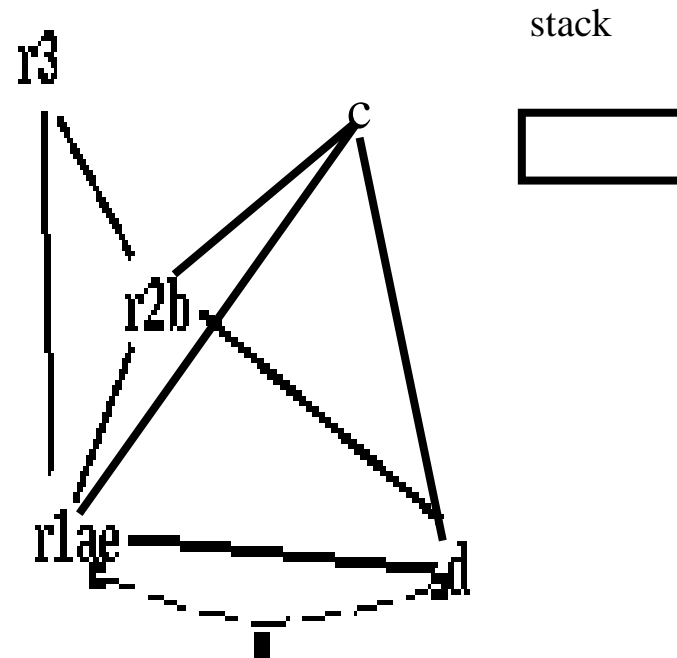
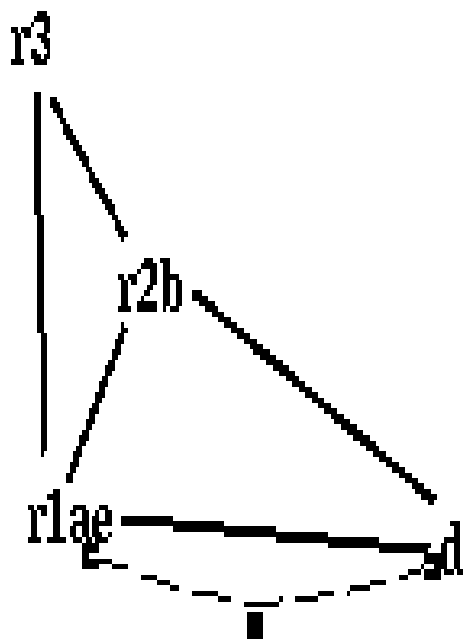


Pop *d*



d is assigned to **r3**

Pop c



actual spill!

<pre> enter: /* r2, r1, r3 */ c := r3 /* c, r2, r1 */ a := r1 /* a, c, r2 */ b := r2 /* a, c, b */ d := 0 /* a, c, b, d */ e := a /* e, c, b, d */ loop: d := d+b /* e, c, b, d */ e := e-1 /* e, c, b, d */ if e>0 goto loop /* c, d */ r1 := d /* r1, c */ r3 := c /* r1, r3 */ return /* r1,r3 */ </pre>	<pre> enter: /* r2, r1, r3 */ c1 := r3 /* c1, r2, r1 */ M[c_loc] := c1 /* r2 */ a := r1 /* a, r2 */ b := r2 /* a, b */ d := 0 /* a, b, d */ e := a /* e, b, d */ loop: d := d+b /* e, b, d */ e := e-1 /* e, b, d */ if e>0 goto loop /* d */ r1 := d /* r1 */ c2 := M[c_loc] /* r1, c2 */ r3 := c2 /* r1, r3 */ return /* r1,r3 */ </pre>
---	--

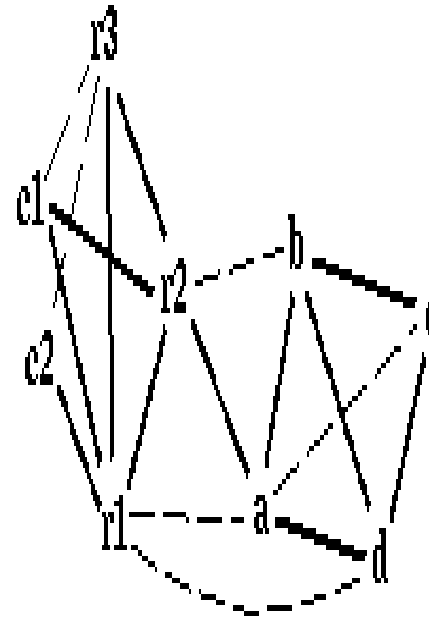
```

enter:      /* r2, r1, r3 */
            c1 := r3 /* c1, r2, r1 */
            M[c_loc] := c1 /* r2 */
            a := r1 /* a, r2 */
            b := r2 /* a, b */
            d := 0 /* a, b, d */
            e := a /* e, b, d */

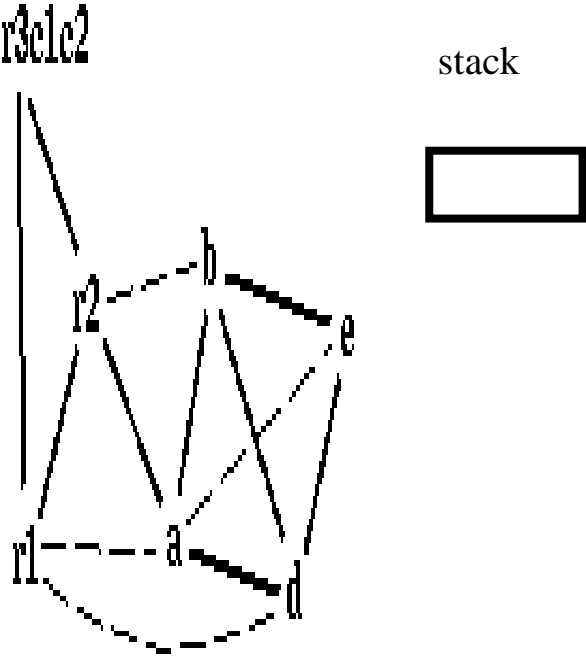
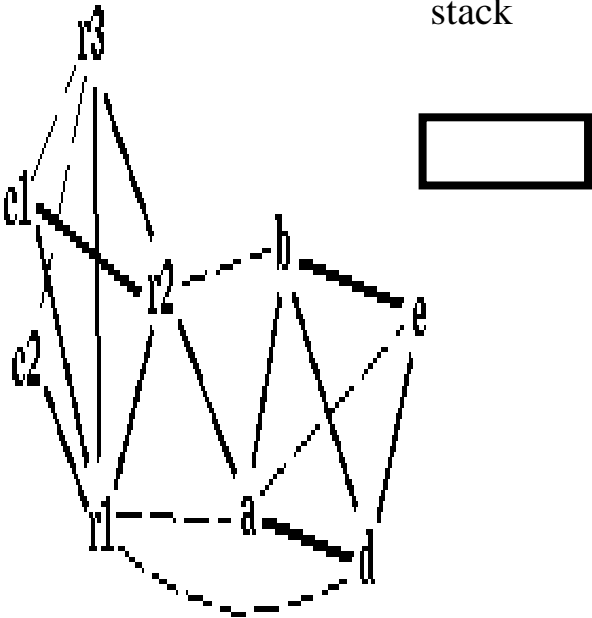
loop:
            d := d+b /* e, b, d */
            e := e-1 /* e, b, d */
            if e>0 goto loop /* d */
            r1 := d /* r1 */
            c2 := M[c_loc] /* r1, c2 */
            r3 := c2 /* r1, r3 */

return /* r1, r3 */

```

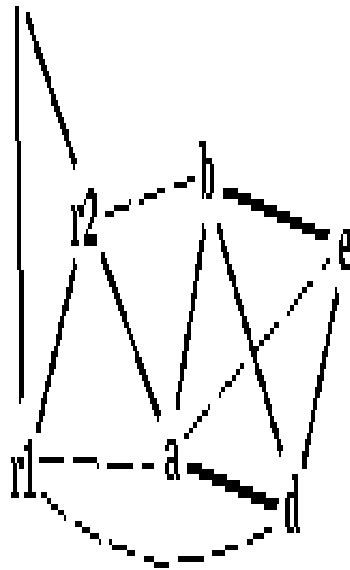


Coalescing $c1+r3$; $c2+c1r3$



Coalescing a+e; b+r2

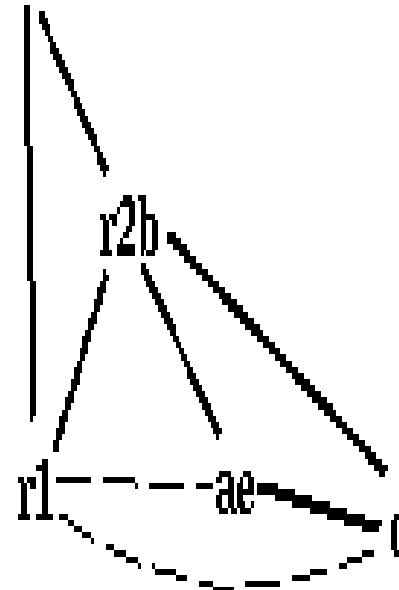
r3c1c2



stack



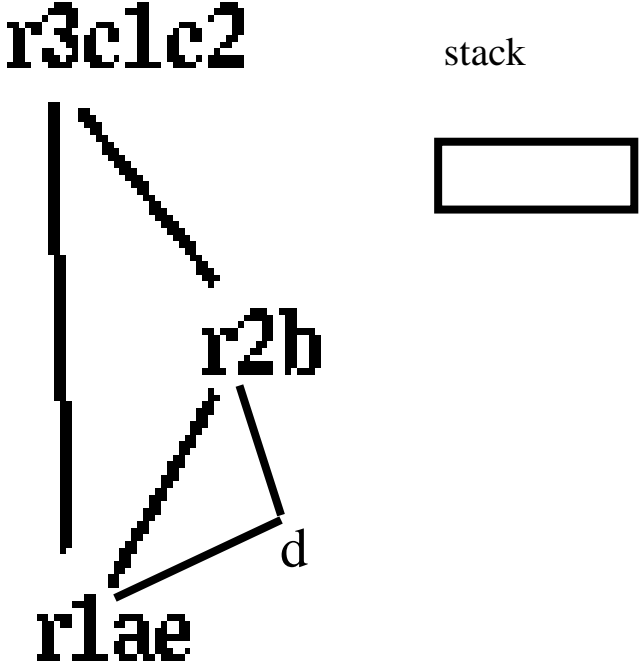
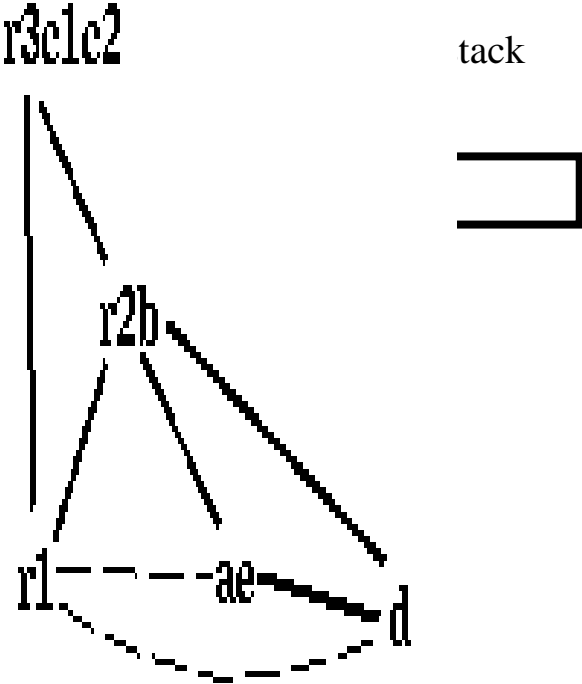
r3c1c2



stack



Coalescing ae+r1

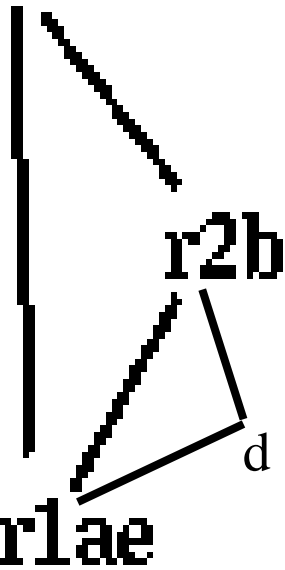


r1ae and d are constrained

Simplify d

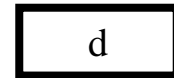
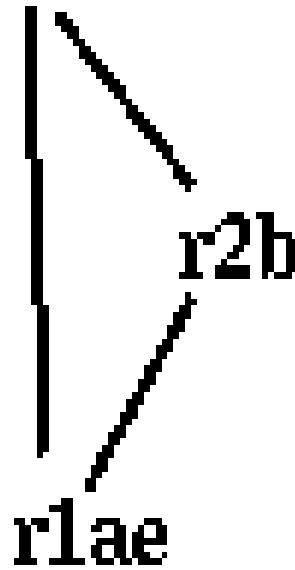
r3c1c2

stack

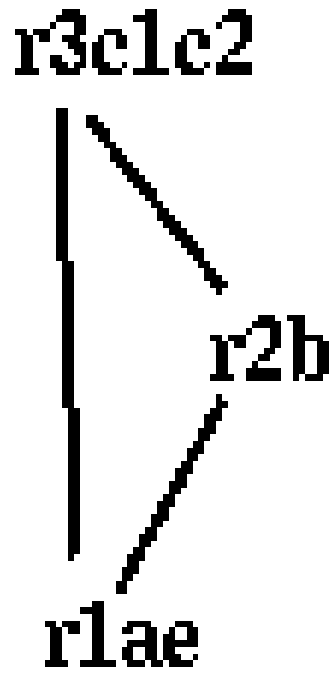


r3c1c2

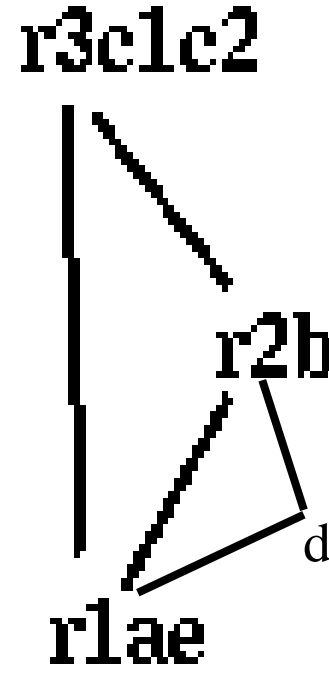
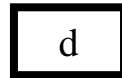
stack



Pop d



stack



stack



a
b
c1
c2
d
e

r1
r2
r3
r3
r3
r1

enter:

c1 := r3

M[c_loc] := c1

a := r1

b := r2

d := 0

e := a

a	r1
b	r2
c1	r3
c2	r3
d	r3
e	r1

loop:

d := d+b

e := e-1

if e>0 goto loop

r1 := d

c2 := M[c_loc]

r3 := c2

return /* r1,r3 */

enter:

r3 := r3

M[c_loc] := r3

r1 := r1

r2 := r2

r3 := 0

r1 := r1

loop:

r3 := r3+r2

r1 := r1-1

if r1>0 goto

loop

r1 := r3

r3 := M[c_loc]

r3 := r3

return /* r1,r3 */

```

enter:
    r3 := r3
    M[c_loc] := r3
    r1 := r1
    r2 := r2
    r3 := 0
    r1 := r1
loop:
    r3 := r3+r2
    r1 := r1-1
    if r1>0 goto
loop
    r1 := r3
    r3 := M[c_loc]
    r3 := r3
return /* r1,r3 */

```

```

enter:
    M[c_loc] := r3
    r3 := 0
loop:
    r3 := r3+r2
    r1 := r1-1
    if r1>0 goto
loop
    r1 := r3
    r3 := M[c_loc]
return /* r1,r3 */

```

Garbage Collection Techniques

- Reference counting
- Mark and sweep
- Copying
- Generational
- Incremental
- [Parallel]

Assembler/Linker/Loader

- Assembler convert symbolic instructions into binary format
 - Resolve local labels and generate relocation tables
- Linker merge several files into a single executable
 - Relocate instructions
 - Resolve external calls
- Loader build a runtime state from executable

Course Summary

- Many useful techniques
- Some implementation details
- Become a better programmer