

## Program analysis

Mooly Sagiv

<http://www.cs.tau.ac.il/~msagiv/courses/wcc08.html>

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## Outline

- What is (static) program analysis
- Example
- Undecidability
- An Iterative Algorithm
- Properties of the algorithm
- The theory of Abstract Interpretation

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## Abstract Interpretation Static analysis

- Automatically identify program properties
  - No user provided loop invariants
- Sound but incomplete methods
  - But can be rather precise
- Non-standard interpretation of the program operational semantics
- Applications
  - Compiler optimization
  - Code quality tools
    - Identify potential bugs
    - Prove the absence of runtime errors
    - Partial correctness

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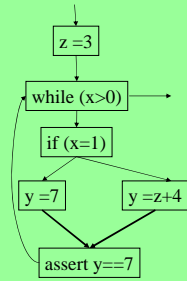
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## Control Flow Graph(CFG)

```

z = 3
while (x>0) {
  if (x = 1)
    y = 7;
  else
    y = z + 4;
  assert y == 7
}
    
```




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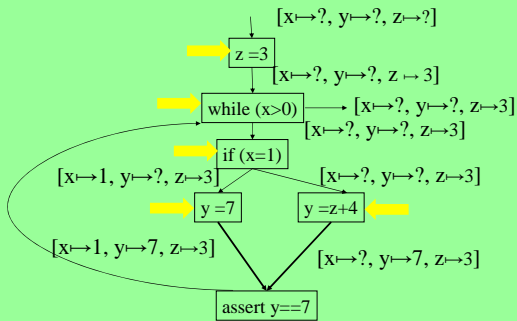
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## Constant Propagation




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## Memory Leakage

List reverse(Element \*head)

```

{
  List rev, n;
  rev = NULL;
  while (head != NULL) {
    n = head->next;
    head->next = rev;
    head = n;
    rev = head;
  }
  return rev;
}
    
```

*potential leakage of address pointed to by head*

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## Memory Leakage

```
Element* reverse(Element *head)
{
    Element *rev, *n;
    rev = NULL;
    while (head != NULL) {
        n = head → next;
        head → next = rev;
        rev = head; ↻ No memory leaks
        head = n;
    }
    return rev; }

```

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## A Simple Example

```
void foo(char *s )
{
    while ( *s != ' ' )
        s++;
    *s = 0;
}

```

Potential buffer overrun:  
 $\text{offset}(s) \geq \text{alloc}(\text{base}(s))$

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## A Simple Example

```
void foo(char *s) @require string(s)
{
    while ( *s != ' ' && *s != 0 )
        s++;
    *s = 0;
}

```

⚡ No buffer overruns

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## Example Static Analysis Problem

- Find variables which are **live** at a given program location
- Used before set on some execution paths from the current program point

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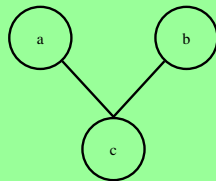
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## A Simple Example

```
/* c */
L0:  a := 0
/* ac */
L1:  b := a + 1
/* bc */
     c := c + b
/* bc */
     a := b * 2
/* ac */
     if c < N goto L1
/* c */
     return c
```



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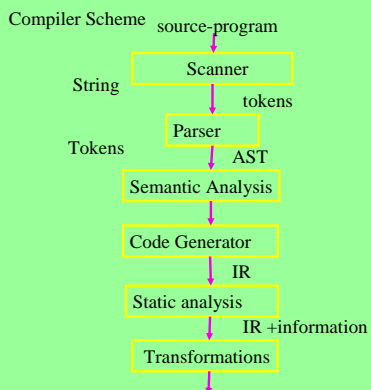
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## Undecidability issues

- It is impossible to compute exact static information
- Finding if a program point is reachable
- Difficulty of interesting data properties

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## Undecidability

- A variable is **live** at a given point in the program
  - if its current value **is used** after this point prior to a definition in **some execution path**
- It is undecidable if a variable is live at a given program location

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## Proof Sketch

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L:  $x := y$

Is y live at L?

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## Conservative (Sound)

- The compiler need not generate the optimal code
- Can use more registers (“spill code”) than necessary
- Find an upper approximation of the live variables
- Err on the safe side
- A superset of edges in the interference graph
- Not too many superfluous live variables

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## Conservative(Sound) Software Quality Tools

- Can never miss an error
- But may produce false alarms
  - Warning on non existing errors

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## Data Flow Values

- Order data flow values
  - $a \sqsubseteq b \Leftrightarrow a$  “is more precise than”  $b$
  - In live variables
    - $a \sqsubseteq b \Leftrightarrow a \subseteq b$
  - In constant propagation
    - $a \sqsubseteq b \Leftrightarrow a$  includes more constants than  $b$
- Compute the least solution
- Merge control flow paths optimistically
  - $a \sqcup b$
  - In live variables
    - $a \sqcup b = a \cup b$

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## Transfer Functions

- Program statements operate on data flow values conservatively

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## Transfer Functions (Constant Propagation)

- Program statements operate on data flow values conservatively
- If  $a=3$  and  $b=7$  before  
“ $z = a + b;$ ”
  - then  $a=3$ ,  $b =7$ , and  $z =10$  after
- If  $a=?$  and  $b=7$  before  
“ $z = a + b;$ ”
  - then  $a=?$ ,  $b =7$ , and  $z =?$  After
- For  $x = \text{exp}$ 
  - $\text{CpOut} = \text{CpIn} [x \mapsto [[\text{exp}]](\text{CpIn})]$

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## Transfer Functions LiveVariables

- If  $a$  and  $c$  are potentially live after  
“ $a = b *2$ ”
  - then  $b$  and  $c$  are potentially live before
- For “ $x = \text{exp};$ ”
  - $\text{LiveIn} = \text{Livout} - \{x\} \cup \text{arg}(\text{exp})$

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## Iterative computation of conservative static information

- Construct a control flow graph(CFG)
- Optimistically start with the best value at every node
- “Interpret” every statement in a conservative way
- Forward/Backward traversal of CFG
- Stop when no changes occur

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## Pseudo Code (forward)

```

forward(G(V, E): CFG, start: CFG node, initial: value){
  // initialization
  value[start]:= initial
  for each v ∈ V - {start} do value[v] := ⊥
  // iteration
  WL = V
  while WL != {} do
    select and remove a node v ∈ WL
    for each u ∈ V such that (v, u) ∈ E do
      value[u] := value[u] ∪ f(v, u)(value[v])
    if value[u] was changed WL := WL ∪ {u}
  
```

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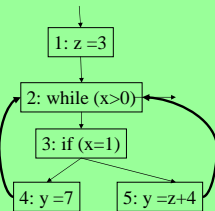
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## Constant Propagation



N	Val	WL
	v[1]=[x→?, y→?, z→?]	{1, 2, 3, 4, 5}
1	v[2]=[x→?, y→?, z→3]	{2, 3, 4, 5}
2	v[3]=[x→?, y→?, z→3]	{3, 4, 5}
3	v[4]=[x→?, y→?, z→3] v[5]=[x→?, y→?, z→3]	{4, 5}
4		{5}
5		{}

Only values before CFG are shown

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## Pseudo Code (backward)

```
backward(G(V, E); CFG, exit: CFG node, initial: value){
  // initialization
  value[exit]:= initial
  for each v ∈ V - {exit} do value[v] := ⊥
  // iteration
  WL = V
  while WL != {} do
    select and remove a node v ∈ WL
    for each u ∈ V such that (u, v) ∈ E do
      value[u] := value[u] ⊔ f(v, u)(value[v])
      if value[u] was changed WL := WL ∪ {u}
```

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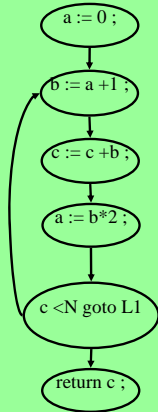
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```
/* c */
L0:  a := 0
/* ac */
L1:  b := a + 1
/* bc */
     c := c + b
/* bc */
     a := b * 2
/* ac */
     if c < N goto L1
/* c */
     return c
```




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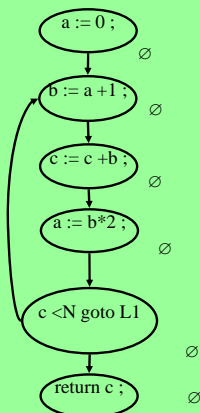
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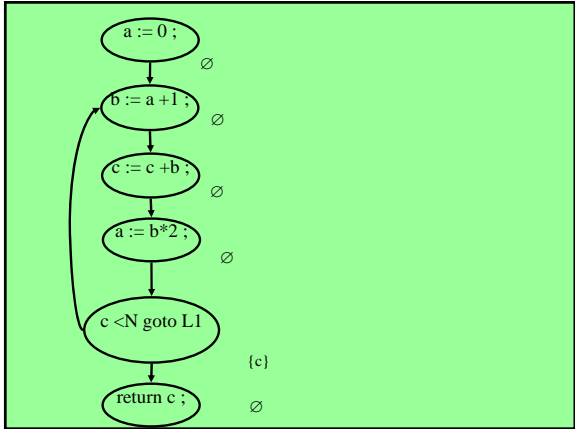
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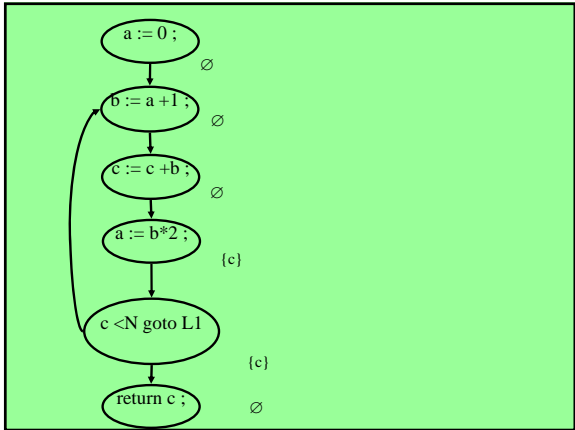
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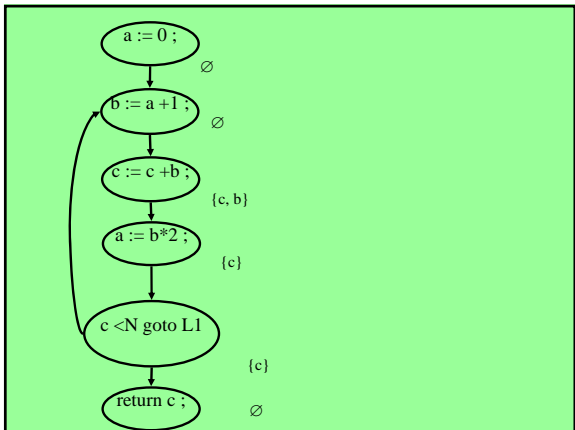
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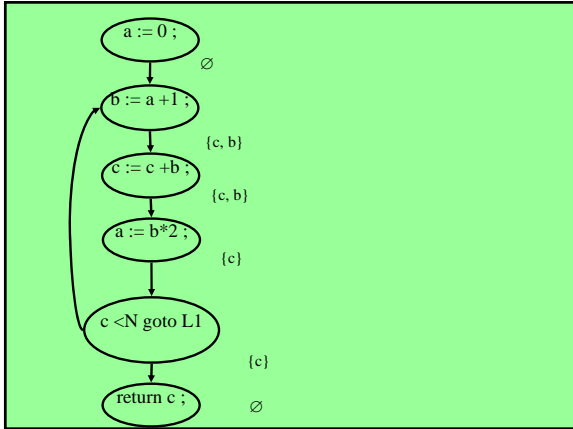
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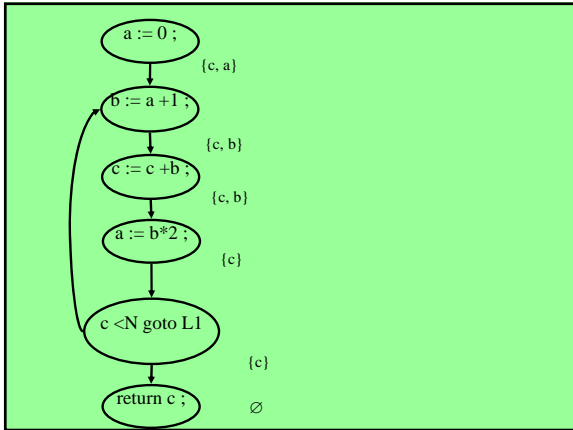
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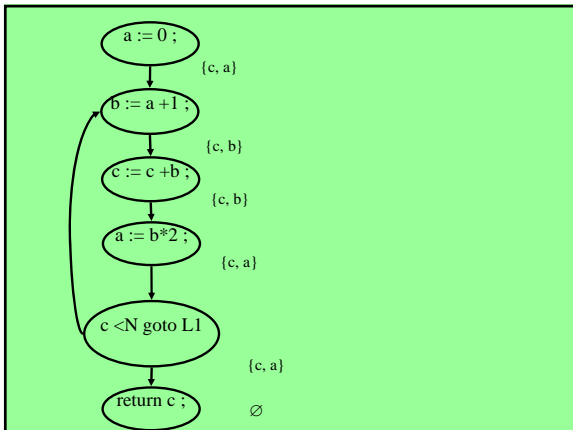
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## Summary Iterative Procedure

- Analyze one procedure at a time
  - More precise solutions exit
- Construct a control flow graph for the procedure
- Initializes the values at every node to the most optimistic value
- Iterate until convergence

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## Abstract Interpretation

- The mathematical foundations of program analysis
- Established by Cousot and Cousot 1979
- Relates static and runtime values

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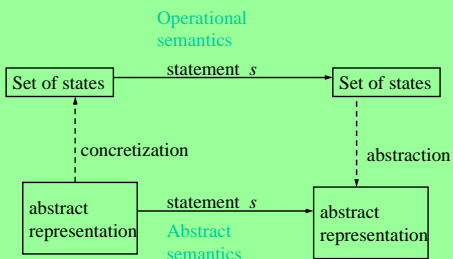
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## Abstract (Conservative) interpretation



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## Example rule of signs

- Safely identify the sign of variables at every program location
- Abstract representation {P, N, ?}
- Abstract (conservative) semantics of \*

*#	P	N	?
P	P	N	?
N	N	P	?
?	?	?	?

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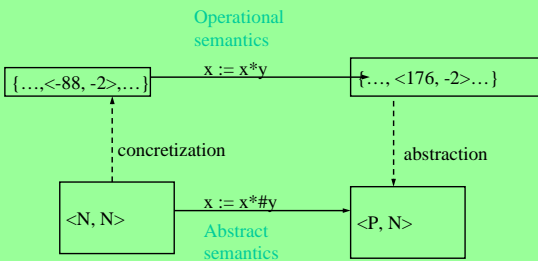
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## Abstract (conservative) interpretation




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## Example rule of signs

- Safely identify the sign of variables at every program location
- Abstract representation {P, N, ?}
- $\alpha(C) =$  if all elements in C are positive then return P else if all elements in C are negative then return N else return ?
- $\gamma(a) =$  if (a==P) then return {0, 1, 2, ... } else if (a==N) return {-1, -2, -3, ... } else return Z

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## Example Constant Propagation

- Abstract representation
  - set of integer values and an extra value “?” denoting variables not known to be constants
- Conservative interpretation of +

+#	?	0	1	2
?	?	?	?	?
0	?	0	1	2
1	?	1	2	3
2	?	2	3	4

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## Example Program

```
x = 5;  
y = 7;  
if (getc())  
    y = x + 2;  
z = x + y;
```

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## Example Program (2)

```
if (getc())  
    x = 3 ; y = 2;  
else  
    x = 2; y = 3;  
z = x + y;
```

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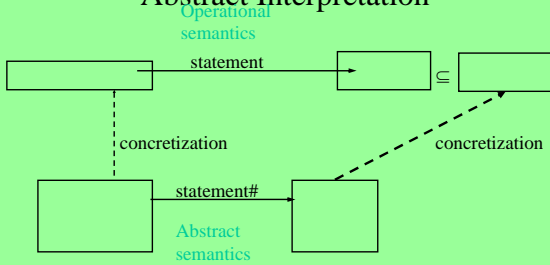
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## Local Soundness of Abstract Interpretation



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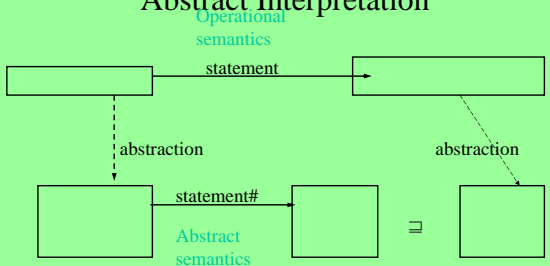
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## Local Soundness of Abstract Interpretation



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## Some Success Stories Software Quality Tools

- The prefix Tool identified interesting bugs in Windows
- The Microsoft SLAM tool checks correctness of device driver
  - Driver correctness rules
- Polyspace checks ANSI C conformance
  - Flags potential runtime errors

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## Summary

- Program analysis provides non-trivial insights on the runtime executions of the program
  - Degenerate case – types (flow insensitive)
- Mathematically justified
  - Operational semantics
  - Abstract interpretation (lattice theory)
- Employed in compilers
- Will be employed in software quality tools

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