Closures

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Formal Semantics of Functional Programs

• Compile into typed lambda calculus
• Small step operational semantics
  – Environment $\times$ Expression $\Rightarrow$ Environment $\times$ Expression
Essential OCcaml sublanguage

e ::= c
| x
| (e₁, ..., eₙ)
| e₁ e₂
| fun x -> e
| let x = e₁ in e₂
| match e₀ with pᵢ -> eᵢ
Evaluation of Expression

• Expressions evaluate to values in a dynamic environment
  \[ \text{env} :: e \rightarrow v \]

• Evaluation is meaningless if expression does not type check

• Values are a syntactic subset of expressions:
  \[
  v ::= c \mid (v_1, \ldots, v_n) \\
  \mid \text{fun } x \rightarrow e
  \]
Dealing with Functions as Values

• Anonymous functions `fun x-> e` are values
  – `env :: (fun x -> e) --> (fun x -> e)`
Evaluating “let expressions”

- To evaluate \textbf{let } x = e_1 \textbf{ in } e_2 \textbf{ in environment } env:
  1. \textbf{Evaluate} the binding expression \( e_1 \) to a value \( v_1 \) in environment \( env \)
     \[
     env :: e_1 \rightarrow v
     \]
  2. \textbf{Extend} the environment to bind \( x \) to \( v_1 \)
     \[
     env' = env [x \mapsto v_1]
     \]
     (newer bindings temporarily \textit{shadow} older bindings)
  3. \textbf{Evaluate} the body expression \( e_2 \) to a value \( v_2 \) in environment \( env' \)
     \[
     env' :: e_2 \rightarrow v_2
     \]
  4. \textbf{Return} \( v_2 \)
Evaluating Function Application take 1

• To evaluate \( e_1 e_2 \) in environment \( \text{env} \)
  1. Evaluate \( e_2 \) to a value \( v_2 \) in environment \( \text{env} \)
     \[ \text{env} :: e_2 \rightarrow v_2 \]
     Note: right to left order, like tuples, which matters in the presence of side effects
  2. Evaluate \( e_1 \) to a value \( v_1 \) in environment \( \text{env} \)
     \[ \text{env} :: e_1 \rightarrow v_1 \]
     Note that \( v_1 \) must be a function value \( \text{fun} \ x \rightarrow e \)
  3. Extend environment to bind formal parameter \( x \) to actual value \( v_2 \)
     \[ \text{env}' = \text{env} [x \mapsto v_2] \]
  4. Evaluate body \( e \) to a value \( v \) in environment \( \text{env}' \)
     \[ \text{env}' :: e \rightarrow v \]
  5. Return \( v \)
Evaluating Function Application take 1

if env :: e2 --> v2
and env :: e1 --> (fun x -> e)
and env[x↦v2] :: e --> v
then env :: e1 e2 --> v
Evaluating Function Application Simple Example

```let f = fun x -> x in f 0```

**1. Evaluate** binding expression `fun x->x` to a value in empty environment `env_0`

**2. Extend** environment to bind `f` to `fun x->x`

```
env_1 = env_0[f ↦ fun x -> x ] = [f ↦ fun x -> x]
```

**3. Evaluate** let-body expression `f 0` in environment `env_1`

```
env_1 :: f 0 --> v1
```

1. Evaluate `0` to a value `0` in environment `env_1`

2. Evaluate `f` to `fun x -> x`

3. Extend environment to bind formal parameter `x` to actual value `0`

```
env_2 = env_1[x ↞ 0] = [f ↞ ..., x ↞ 0]
```

4. Evaluate the function body `x` in environment `env_2`

```
env_2 :: x--> 0
```

**4. Return** `0`
Hard Example

```
let x = 1 in
  let f = fun y -> x in
  let x = 2 in
    f 0
```

1. What is the result of the expression?
2. What does OCaml say?
3. What do you say?
Hard Example Ocaml

let x = 1 in
  let f = fun y -> x in
    let x = 2 in
      f 0
warning 26: x unused variable
:- int 1
Hard Example “C”

```c
{  
    int x = 1
    {  
      int f(int y)
      {  
        return x ;
      }
    }
    {  
      int x = 2;
      printf("%d", f(0)) ;
    }
}
```
Why different answers?

- Two different rules for variable scope
  - Rule of dynamic scope (lisp)
  - Rule of lexical (static) scope (Ocaml, Javascript, Scheme, ...)

Dynamic Scope

• **Rule of dynamic scope:** The body of a function is evaluated in the current dynamic environment at the time the function is called, not the old dynamic environment that existed at the time the function was defined.

• Use latest binding of \( x \)

• Thus return 2
Lexical Scope

- **Rule of lexical scope**: The body of a function is evaluated in the old dynamic environment that existed at the time the function was defined, not the current environment when the function is called.
- Causes OCaml to use earlier binding of \( x \).
- Thus return 1.
Scope

- **Rule of dynamic scope**: The body of a function is evaluated in the current dynamic environment at the time the function is called, not the old dynamic environment that existed at the time the function was defined.
- **Rule of lexical scope**: The body of a function is evaluated in the old dynamic environment that existed at the time the function was defined, not the current environment when the function is called.

In both, environment is extended to map formal parameter to actual value.

Why would you want one vs. the other?
Lexical vs. dynamic scope

• Consensus after decades of programming language design is that **lexical scope is the right choice**
• Dynamic scope is convenient in some situations
• Some languages use it as the norm (e.g., Emacs LISP, LaTeX)
• Some languages have special ways to do it (e.g., Perl, Racket)
• But most languages just don’t have it
Why Lexical Scope (1)

- Programmer can freely change names of local variables

\[
\begin{align*}
(* 1 *) & \text{ let } x = 1 \\
(* 2 *) & \text{ let } f y = \\
& \quad \text{ let } x = y + 1 \text{ in } \\
& \quad \text{ fun } z \to x+y+z \\
(* 3 *) & \text{ let } x = 3 \\
(* 4 *) & \text{ let } w = (f 4) 6
\end{align*}
\]

\[
\begin{align*}
(* 1 *) & \text{ let } x = 0 \\
(* 2 *) & \text{ let } f y = \\
& \quad \text{ let } q = y + 1 \text{ in } \\
& \quad \text{ fun } z \to q+y+z \\
(* 3 *) & \text{ let } x = 3 \\
(* 4 *) & \text{ let } w = (f 4) 6
\end{align*}
\]
Why Lexical Scope (2)

- Type checker can prevent run-time errors

```
(* 1 *) let x = 1
(* 2 *) let f y =
    let x = y + 1 in
    fun z -> x+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

```
(* 1 *) let x = 0
(* 2 *) let f y =
    let x = y + 1 in
    fun z -> x+y+z
(* 3 *) let x = “hi”
(* 4 *) let w = (f 4) 6
```
Exception Handling

- Resembles dynamic scope:
- `raise e` transfers control to the “most recent” exception handler
- like how dynamic scope uses “most recent” binding of variable
Where is an exception caught?

- Dynamic scoping of handlers
  - Throw to most recent catch on run-time stack

- Dynamic scoping is not an accident
  - User knows how to handler error
  - Author of library function does not
Implementing time travel (lexical)

Q  How can functions be evaluated in old environments?
A  The language implementation keeps them around as necessary

A function value is really a data structure that has two parts:
1. The code
2. The environment that was current when the function was defined
   1. Gives meaning to all the *free variables of the function body*
      – Like a “pair”
      • But you cannot access the pieces, or directly write one down in the language syntax
      • All you can do is call it
      – This data structure is called a *function closure*

A function application:
– evaluates the code part of the closure
– in the environment part of the closure extended to bind the function argument
Hard Example Revisited

[1] let x = 1 in
[2] let f = fun y -> x in
[3] let x = 2 in
[4] let z = f 0 in z

With lexical scope:
• Line 2 creates a closure and binds f to it:
  – Code: fun y -> x
  – Environment: [x↦1]
• Line 4 calls that closure with 0 as argument
  – In function body, y maps to 0 and x maps to 1
• So z is bound to 1
Another Example

[1] let x = 1 in
[2] let f y = x + y in
[3] let x = 3 in
[4] let y = 4 in
[5] let z = f (x + y) in z

With lexical scope:
1. Creates a closure and binds f to it:
   – Code: `fun y -> x + y`
   – Environment: `[x↦1]`
2. Line 5 env = `[x↦3, y↦4]`
3. Line 5 calls that closure with `x+y=7` as argument
   – In function body, `x` maps to `1`
   • So `z` is bound to `8`
Another Example

[1] let x = 1 in
[2]  let f y = x + y in
[3]   let x = 3 in
[4]    let y = 4 in
[5]     let z = f (x + y) in z

With dynamic scope:
1. Line 5 env = [x \mapsto 3, y \mapsto 4]
2. Line 5 calls that closure with \texttt{x+y=7} as argument
   - In function body, \texttt{x} maps to 3, so \texttt{x+y} maps to 10
   - Note that argument \texttt{y} shadows \texttt{y} from line 4
   • So \texttt{z} is bound to 10
Closure Notation

\[
<<\text{code, environment}>>
\]

\[
<<\text{fun y -> x+y, [x\rightarrow 1]>}>\]

With lexical scoping, well-typed programs are guaranteed never to have any variables in the code body other than function argument and variables bound by closure environment.
Evaluating Function Application take 2

- To evaluate $e_1 e_2$ in environment $env$
  1. Evaluate $e_2$ to a value $v_2$ in environment $env$
     $$env :: e_2 \rightarrow v_2$$
     Note: right to left order, like tuples, which matters in the presence of side effects
  2. Evaluate $e_1$ to a value $v_1$ in environment $env$
     $$env :: e_1 \rightarrow v_1$$
     Note that $v_1$ must be a closure with function value $\text{fun } x \rightarrow e$ and environment $env'$
  3. Extend environment to bind formal parameter $x$ to actual value $v_2$
     $$env'' = env' [x\rightarrow v_2]$$
  4. Evaluate body $e$ to a value $v$ in environment $env''$
     $$env'' :: e \rightarrow v$$
  5. Return $v$
if \( \text{env} :: e_2 \rightarrow v_2 \)
and \( \text{env} :: e_1 \rightarrow \langle\langle \text{fun} \ x \rightarrow e, \ \text{env}'\rangle\rangle \)
and \( \text{env}'[x\mapsto v_2] :: e \rightarrow v \)
then \( \text{env} :: e_1 \ e_2 \rightarrow v \)
Evaluating Anonymous Function
Application take 2

Anonymous functions `fun x-> e` are closures
`env :: (fun x -> e) -->
<<fun x -> e, env>>`
Why are Closure useful?

• Hides states in an elegant way
• Useful for
  – Implementing objects
  – Web programming
  – Operated system programming
  – Emulating control flow
  – ...

Simple Example

```plaintext
let startAt x =
    let incrementBy y = x + y
    in incrementBy
val startAt : int -> int -> int = <fun>

let closure1 = startAt 3
val closure1 : int -> int = <fun>

let closure2 = startAt 5
val closure2 : int -> int = <fun>

closure1 7
:- int =10

closure2 9
:- int =14
```
Another Example

let derivative f dx =
    fun x -> f (x + dx) - f x / dx
val derivative : (int -> int) -> int -> int -> int -> int = <fun>
Implementation Notes

• Duration of closure can be long
  – Usually implemented with garbage collection

• It is possible to support lexical scopes without closure (using stack) if one of the following is forbidden:
  – Nested scopes (C, Java)
  – Returning a function (Algol, Pascal)
Essential OCaml sublanguage

e ::= c
    | x
    | e_1 e_2
    | fun x -> e
    | let x = e_1 in e_2
    | match e_0 with p_i -> e_i
Essential OCcaml sublanguage+rec

\[
e ::= c \\
| x \\
| (e_1, \ldots, e_n) \\
| e_1 e_2 \\
| \text{fun } x \rightarrow e \\
| \text{let } x = e_1 \text{ in } e_2 \\
| \text{match } e_0 \text{ with } p_i \rightarrow e_i \\
| \text{let rec } f \ x = e_1 \text{ in } e_2
\]
let rec Evaluation

• How to handle
  let rec f x = e₁ in e₂
let rec Evaluation

• To evaluate let rec f x = e₁ in e₂ in environment env
  – don’t evaluate the binding expression e₁
  1. Extend the environment to bind f to a recursive closure
     env’ = env [f ↦ <<<f, fun x -> e₁, env>>]
  2. Evaluate the body expression e₂ to a value v₂ in
     environment env’
     env’ :: e₂ --> v₂
  3. Return v₂
Closure in OCaml

• *Closure conversion is an important phase of compiling many functional languages*
• Expands on ideas we’ve seen here
• Many optimizations possible
• Especially, better handling of recursive functions
Closures in Java

- Nested classes can simulate closures
- Used everywhere for Swing GUI!
- [http://docs.oracle.com/javase/tutorial/uiswing/events/generalrules.html#innerClasses](http://docs.oracle.com/javase/tutorial/uiswing/events/generalrules.html#innerClasses)
- Java 8 adds higher-order functions and closures
- Can even think of OCaml closures as resembling Java objects:
  - closure has a single method, the code part, that can be Invoked
  - closure has many fields, the environment part, that can be accessed
Closures in C

• In C, a function pointer is just a code pointer, period, No environment
• To simulate closures, a common idiom:
  – Define function pointers to take an extra, explicit environment argument
  – But without generics, no good choice for type of list elements or the environment
• Use void* and various type casts...
• From Linux kernel:
  – http://lxr.free-electrons.com/source/include/linux/kthread.h#L13
Summary

• Lexical scoping is natural
• Permit general programming style
  – Works well with higher order functions
• Well understood
• Implemented with closures
  – But requires long lived objects
• Integrated into many programming languages
• Some surprises (javascript)
Summary (Ocaml)

- Functional programs provide concise coding
- Compiled code compares with C code
- Successfully used in some commercial applications
  - F#, ERLANG, Jane Street
- Ideas used in imperative programs
- Good conceptual tool
- Less popular than imperative programs