

Closures

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Formal Semantics of Functional Programs

- Small step operational semantics
 - Environment \times Expression \Rightarrow Environment \times Expression
- Compile into typed lambda calculus

Essential OCaml sublanguage

```
e ::= c  
| (op)  
| x  
| (e1, ..., en)  
| C e  
| e1 e2  
| fun x -> e  
| let x = e1 in e2  
| match e0 with pi -> ei
```

Evaluation of Expression

- Expressions evaluate to values in a dynamic environment
 - $env :: e \rightarrow v$
- Evaluation is meaningless if expression does not **type check**
- Values are a *syntactic subset of expressions*:

```
v ::= c | (op) | (v1, ..., vn)  
    | C v  
    | fun x -> e
```

Dealing with Functions as Values

- Anonymous functions `fun x-> e` are values
 - `env :: (fun x -> e) --> (fun x -> e)`

Evaluating “let expressions”

- To evaluate **let $x = e_1$ in e_2** in environment **env**:
 1. **Evaluate** the binding expression e_1 to a value v_1 in environment **env**
 $\text{env} :: e_1 \rightarrow v$
 2. **Extend** the environment to bind x to v_1
 $\text{env}' = \text{env} [x \mapsto v_1]$
(newer bindings temporarily *shadow older bindings*)
 3. **Evaluate** the body expression e_2 to a value v_2 in environment **env'**
 $\text{env}' :: e_2 \rightarrow v_2$
 4. **Return** v_2

Compiling “let expressions” into Lambda Calculus

let $v = e_1$ in e_2

Evaluating Function Application take 1

- To **evaluate** $e_1 e_2$ in environment env
 1. **Evaluate** e_2 to a value v_2 in environment env
 $env :: e_2 \rightarrow v_2$
Note: right to left order, like tuples, which matters in the presence of side effects
 2. **Evaluate** e_1 to a value v_1 in environment env
 $env :: e_1 \rightarrow v_1$
Note that v_1 must be a function value $fun\ x \rightarrow e$
 3. **Extend** environment to bind formal parameter x to actual value v_2
 $env' = env [x \mapsto v_2]$
 4. **Evaluate** body e to a value v in environment env'
 $env' :: e \rightarrow v$
 5. **Return** v

Evaluating Function Application take 1

if $\text{env} :: e_2 \rightarrow v_2$
and $\text{env} :: e_1 \rightarrow (\text{fun } x \rightarrow e)$
and $\text{env}[x \mapsto v_2] :: e \rightarrow v$
then $\text{env} :: e_1 \ e_2 \rightarrow v$

Evaluating Function Application Simple Example

let f = fun x -> x in f 0

$env_0 = []$

1. **Evaluate** binding expression **fun x->x** to a value in empty environment env_0

2. **Extend** environment to bind **f** to **fun x->x**

$env_1 = env_0[f \mapsto \text{fun } x \rightarrow x] = [f \mapsto \text{fun } x \rightarrow x]$

3. **Evaluate** let-body expression **f 0** in environment env_1
 $env_1 :: f \ 0 \rightarrow v_1$

1. Evaluate **0** to a value 0 in environment env_1

2. Evaluate **f** to **fun x -> x**

3. Extend environment to bind formal parameter **x** to actual value 0

$env_2 = env_1[x \mapsto 0] = [f \mapsto \dots, x \mapsto 0]$

4. Evaluate the function body **x** in environment env_2
 $env_2 :: x \rightarrow 0$

$env_2 :: x \rightarrow 0$

4. **Return 0**

Hard Example

```
let x = 1 in
  let f = fun y -> x in
    let x = 2 in
      f 0
```

1. What is the result of the expression?
2. What does OCaml say?
3. What do you say?

Hard Example Ocaml

```
let x = 1 in
  let f = fun y -> x in
    let x = 2 in
      f 0
warning 26: x unused variable
:- int 1
```

Why different answers?

- Two different rules for variable scope
 - Rule of dynamic scope (lisp)
 - Rule of lexical (static) scope (Ocaml, Javascript, Scheme, ...)

Dynamic Scope

- **Rule of dynamic scope:** The body of a function is evaluated in the current dynamic environment at the time the function is called, not the old dynamic environment that existed at the time the function was defined
- Use latest binding of **x**
- Thus return 2

Lexical Scope

- **Rule of lexical scope:** The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called
- Causes OCaml to use earlier binding of **x**
- Thus return 1

Scope

- **Rule of dynamic scope:** The body of a function is evaluated in the current dynamic environment at the time the function is called, not the old dynamic environment that existed at the time the function was defined
- **Rule of lexical scope:** The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called
- *In both, environment is extended to map formal parameter to actual value*
- Why would you want one vs. the other?

Implementing time travel

Q How can functions be evaluated in old environments?

A The language implementation keeps them around as necessary

A function value is really a data structure that has two parts:

1. The code
2. The environment that was current when the function was defined
 1. Gives meaning to all the *free variables of the function body*
 - Like a “pair”
 - But you cannot access the pieces, or directly write one down in the language syntax
 - All you can do is call it
 - This data structure is called a ***function closure***

A function application:

- evaluates the code part of the closure
- in the environment part of the closure extended to bind the function argument

Hard Example Revisited

```
[1] let x = 1 in
[2] let f = fun y -> x in
[3]   let x = 2 in
[4]     let z = f 0 in z
```

With lexical scope:

- Line 2 creates a closure and binds **f** to it:
 - Code: **fun y -> x**
 - Environment: **[x↔1]**
- Line 4 calls that closure with **0** as argument
 - In function body, **y** maps to **0** and **x** maps to **1**
- So **z** is bound to **1**

Another Example

```
[1] let x = 1 in
[2] let f y = x + y in
[3]   let x = 3 in
[4]     let y = 4 in
[5]       let z = f (x + y) in z
```

With lexical scope:

1. Creates a closure and binds **f** to it:
 - Code: **fun y -> x + y**
 - Environment: **[x ↦ 1]**
2. Line 5 env = [x ↦ 3, y ↦ 4]
3. Line 5 calls that closure with **x+y=7** as argument
 - In function body, **x** maps to **1**
 - So **z** is bound to **8**

Another Example

```
[1] let x = 1 in
[2] let f y = x + y in
[3]   let x = 3 in
[4]     let y = 4 in
[5]       let z = f (x + y) in z
```

With dynamic scope:

1. Line 5 env = [x ↦ 3, y ↦ 4]
2. Line 5 calls that closure with **x+y=7** as argument
 - In function body, **x** maps to **3**, so **x+y** maps to **10**
 - Note that argument **y** shadows **y** from line 4
 - So **z** is bound to **10**

Closure Notation

<<code, environment>>

<<fun y -> x+y, [x↦1]>>

With lexical scoping, well-typed programs are guaranteed never to have any variables in the code body other than function argument and variables bound by closure environment

Evaluating Function Application take 2

- To **evaluate** $e_1 e_2$ in environment env
 1. **Evaluate** e_2 to a value v_2 in environment env
 $env :: e_2 \rightarrow v_2$
Note: right to left order, like tuples, which matters in the presence of side effects
 2. **Evaluate** e_1 to a value v_1 in environment env
 $env :: e_1 \rightarrow v_1$
Note that v_1 must be a closure with function value $fun\ x \rightarrow e$ and environment env'
 3. **Extend** environment to bind formal parameter x to actual value v_2
 $env'' = env' [x \mapsto v_2]$
 4. **Evaluate** body e to a value v in environment env''
 $env'' :: e \rightarrow v$
 5. **Return** v

Evaluating Function Application take 2

```
if env :: e2 --> v2
and env :: e1 -->
  <<fun x -> e, env'>>
and env'[x ↦ v2] :: e --> v
then env :: e1 e2 --> v
```

Evaluating Anonymous Function Application take 2

Anonymous functions **fun x -> e** are closures
env :: (fun x -> e) -->
<<fun x -> e, env>>

Why are Closure useful?

- Hides states in an elegant way
- Useful for
 - Implementing objects
 - Web programming
 - Operated system programming
 - Emulating control flow
 - ...

Simple Example

```
let startAt x =  
  let incrementBy y = x + y  
  in incrementBy  
val startAt : int -> int -> int = <fun>
```

```
let closure1 = startAt 3  
val closure1 : int -> int = <fun>
```

```
let closure2 = startAt 5  
val closure2 : int -> int = <fun>
```

```
closure1 7  
:- int =10
```

```
closure2 9  
:- int =14
```

Another Example

```
let derivative f dx =
```

```
  fun x -> f (x + dx) - f x / dx
```

```
val derivative : (int -> int) -> int -> int -> int = <fun>
```

Implementation Notes

- Duration of closure can be long
 - Usually implemented with garbage collection
- It is possible to support lexical scopes without closure (using stack) if one of the following is forbidden:
 - Nested scopes (C, Java)
 - Returning a function (Algol, Pascal)

Lexical vs. dynamic scope

- Consensus after decades of programming language design is that **lexical scope is the right choice**
- Dynamic scope is convenient in some situations
- Some languages use it as the norm (e.g., Emacs LISP, LaTeX)
- Some languages have special ways to do it (e.g., Perl, Racket)
- But most languages just don't have it

Why Lexical Scope (1)

- Programmer can freely change names of local variables

```
(* 1 *) let x = 1
(* 2 *) let f y =
    let x = y + 1 in
    fun z -> x+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

```
(* 1 *) let x = 0
(* 2 *) let f y =
    let q = y + 1 in
    fun z -> q+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

Why Lexical Scope (2)

- Type checker can prevent run-time errors

```
(* 1 *) let x = 1
(* 2 *) let f y =
    let x = y + 1 in
    fun z -> x+y+z
(* 3 *) let x = 3
(* 4 *) let w = (f 4) 6
```

```
(* 1 *) let x = 0
(* 2 *) let f y =
    let x = y + 1 in
    fun z -> x+y+z
(* 3 *) let x = "hi"
(* 4 *) let w = (f 4) 6
```

Exception Handling

- Resembles dynamic scope:
- **raise e** transfers control to the “most recent” exception handler
- like how dynamic scope uses “most recent” binding of variable

Where is an exception caught?

- Dynamic scoping of handlers
 - Throw to most recent catch on run-time stack
- Dynamic scoping is not an accident
 - User knows how to handler error
 - Author of library function does not

Essential OCaml sublanguage

```
e ::= c  
| (op)  
| x  
| (e1, ..., en)  
| C e  
| e1 e2  
| fun x -> e  
| let x = e1 in e2  
| match e0 with pi -> ei
```

Essential OCaml sublanguage+rec

```
e ::= c  
| (op)  
| x  
| (e1, ..., en)  
| C e  
| e1 e2  
| fun x -> e  
| let x = e1 in e2  
| match e0 with pi -> ei  
| let rec f x = e1 in e2
```

let rec Evaluation

- How to handle
let rec f x = e₁ in e₂

let rec Evaluation

- To evaluate **let rec f x = e₁ in e₂** in environment **env**
 - *don't evaluate the binding expression e₁*
 1. **Extend** the environment to bind f to a *recursive closure*
env' = env [f ↦ <<f, fun x -> e₁, env>>]
 2. **Evaluate** the body expression e₂ to a value v₂ in environment **env'**
env' :: e₂ --> v₂
 3. **Return v₂**

Closure in OCaml

- *Closure conversion is an important phase of compiling many functional languages*
- Expands on ideas we've seen here
- Many optimizations possible
- Especially, better handling of recursive functions

Closures in Java

- Nested classes can simulate closures
- Used everywhere for Swing GUI!
- <http://docs.oracle.com/javase/tutorial/uiswing/events/generalrules.html#innerClasses>
- Java 8 adds higher-order functions and closures
- Can even think of OCaml closures as resembling Java objects:
 - closure has a single method, the code part, that can be invoked
 - closure has many fields, the environment part, that can be accessed

Closures in C

- In C, a *function pointer is just a code pointer, period*, No environment
- To simulate closures, a common **idiom**:
 - Define function pointers to take an extra, explicit environment argument
 - But without generics, no good choice for type of list elements or the environment
- Use **void*** and various type casts...
- From Linux kernel:
 - <http://lxr.free-electrons.com/source/include/linux/kthread.h#L13>

Summary

- Lexical scoping is natural
- Permit general programming style
 - Works well with higher order functions
- Well understood
- Implemented with closures
 - But requires long lived objects
- Integrated into many programming languages
- Some surprises (javascript)

Summary (Ocaml)

- Functional programs provide concise coding
- Compiled code compares with C code
- Successfully used in some commercial applications
 - F#, ERLANG, Jane Street
- Ideas used in imperative programs
- Good conceptual tool
- Less popular than imperative programs