Types and Type Inference

Mooly Sagiv

Slides by Kathleen Fisher and John Mitchell

Reading: "Concepts in Programming Languages",

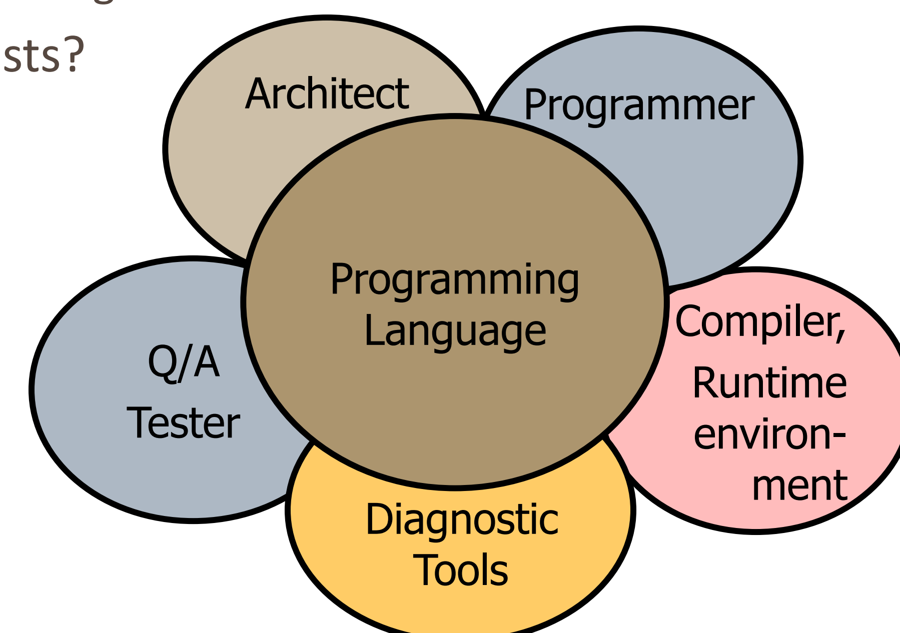
Revised Chapter 6 - handout on the course homepage

Outline

- General discussion of types
  - What is a type?
  - Compile-time versus run-time checking
  - Conservative program analysis
- Type inference
  - Discuss algorithm and examples
  - Illustrative example of static analysis algorithm
- Polymorphism
  - Uniform versus non-uniform implementations

Language Goals and Trade-offs

- Thoughts to keep in mind
  - What features are convenient for programmer?
  - What other features do they prevent?
  - What are design tradeoffs?
    - Easy to write but harder to read?
    - Easy to write but poorer error messages?
  - What are the implementation costs?



What is a type?

- A type is a collection of computable values that share some structural property.

Examples

```
Integer
String
Int → Bool
(Int → Int) → Bool
[a] → a
[a]  × a → [a]
```

Non-examples

```
{3, True, \x->x}
Even integers
{f:Int → Int | x>3 =>
      f(x) > x *(x+1)}
```

Distinction between sets of values that are types and sets that are not types is *language dependent*

Advantages of Types

- Program organization and documentation
  - Separate types for separate concepts
    - Represent concepts from problem domain
  - Document intended use of declared identifiers
    - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true – "Bill"
- Support optimization
  - Example: short integers require fewer bits
  - Access components of structures by known offset

What is a type error?

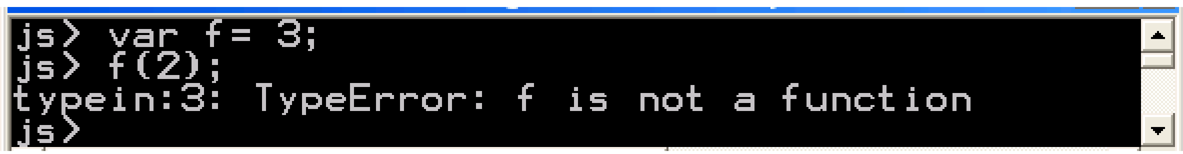
- Whatever the compiler/interpreter says it is?
- Something to do with bad bit sequences?
  - Floating point representation has specific form
  - An integer may not be a valid float
- Something about programmer intent and use?
  - A type error occurs when a value is used in a way that is inconsistent with its definition
    - Example: declare as character, use as integer

Type errors are language dependent

- Array out of bounds access
  - C/C++: runtime errors
  - Haskell/Java: dynamic type errors
- Null pointer dereference
  - C/C++: run-time errors
  - Haskell/ML: pointers are hidden inside datatypes
    - Null pointer dereferences would be incorrect use of these datatypes, therefore static type errors

Compile-time vs Run-time Checking

- JavaScript and Lisp use run-time type checking
  - f(x)   Make sure f is a function before calling f



- Haskell and Java use compile-time type checking
  - f(x)   Must have f :: A $\rightarrow$ B and x :: A
- Basic tradeoff
  - Both kinds of checking prevent type errors
  - Run-time checking slows down execution
  - Compile-time checking restricts program flexibility
    - JavaScript array: elements can have different types
    - Haskell list: all elements must have same type
  - Which gives better programmer diagnostics?

Expressiveness

- In JavaScript, we can write a function like

```
function f(x) { return x < 10 ? x : x(); }
```

Some uses will produce type error, some will not

- Static typing always conservative

```
    if  (complicated-boolean-expression)
                then  f(5);
                else  f(15);
```

Type Safety

- Type safe programming languages protect its own abstractions
- Type safe programs cannot go wrong
- No run-time errors
- But exceptions are fine
- The small step semantics cannot get stuck
- Type safety is proven at language design time

Relative Type-Safety of Languages

- Not safe: BCPL family, including C and C++
  - Casts, unions, pointer arithmetic
- Almost safe: Algol family, Pascal, Ada
  - Dangling pointers
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p
    - Hard to make languages with explicit deallocation of memory fully type-safe
- Safe: Lisp, Smalltalk, ML, Haskell, Java, JavaScript
  - Dynamically typed: Lisp, Smalltalk, JavaScript
  - Statically typed: ML, Haskell, Java

If code accesses data, it is handled with the type associated with the creation and previous manipulation of that data

Type Checking vs Type Inference

- Standard type checking:

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2; };
```

  - Examine body of each function
  - Use declared types to check agreement
- Type inference:

~~int~~ f(~~int~~ x) { return x+1; };
~~int~~ g(~~int~~ y) { return f(y+1)*2; };

  - Examine code without type information
  - Infer the most general types that could have been declared

ML and Haskell are *designed* to make type inference feasible

Why study type inference?

- Types and type checking
  - Improved steadily since Algol 60
    - Eliminated sources of unsoundness
    - Become substantially more expressive
  - Important for modularity, reliability and compilation
- Type inference
  - Reduces syntactic overhead of expressive types
  - Guaranteed to produce most general type
  - Widely regarded as important language innovation
  - Illustrative example of a flow-insensitive static analysis algorithm

History

- Original type inference algorithm
  - Invented by Haskell Curry and Robert Feys for the simply typed lambda calculus in 1958
- In 1969, Hindley
  - extended the algorithm to a richer language and proved it always produced the most general type
- In 1978, Milner
  - independently developed equivalent algorithm, called algorithm W, during his work designing ML
- In 1982, Damas proved the algorithm was complete.
  - Currently used in many languages: ML, Ada, Haskell, C# 3.0, F#, Visual Basic .Net 9.0. Have been plans for Fortress, Perl 6, C++0x,…

uHaskell

- Subset of Haskell to explain type inference.
  - Haskell and ML both have overloading
  - Will not cover type inference with overloading

```
<decl> ::= [<name> <pat>  = <exp>]
<pat>  ::=  Id  | (<pat>, <pat>) | <pat> :
<pat> | []
<exp>  ::= Int | Bool | [] | Id | (<exp>)
           | <exp> <op> <exp>
           | <exp> <exp>  | (<exp>, <exp>)
           | if <exp> then <exp> else <exp>
```

Type Inference: Basic Idea

- Example

```
f x = 2 + x
> f :: Int -> Int
```

- What is the type of f?

  + has type: $Int \rightarrow Int \rightarrow Int$

  2 has type: Int

  Since we are applying + to x we need x :: Int

  Therefore f x = 2 + x has type $Int \rightarrow Int$

Type Inference: Basic Idea

- Another Example

```
f (g, h) = g (h(0))
> f :: (a -> b, Int -> a) -> b
```

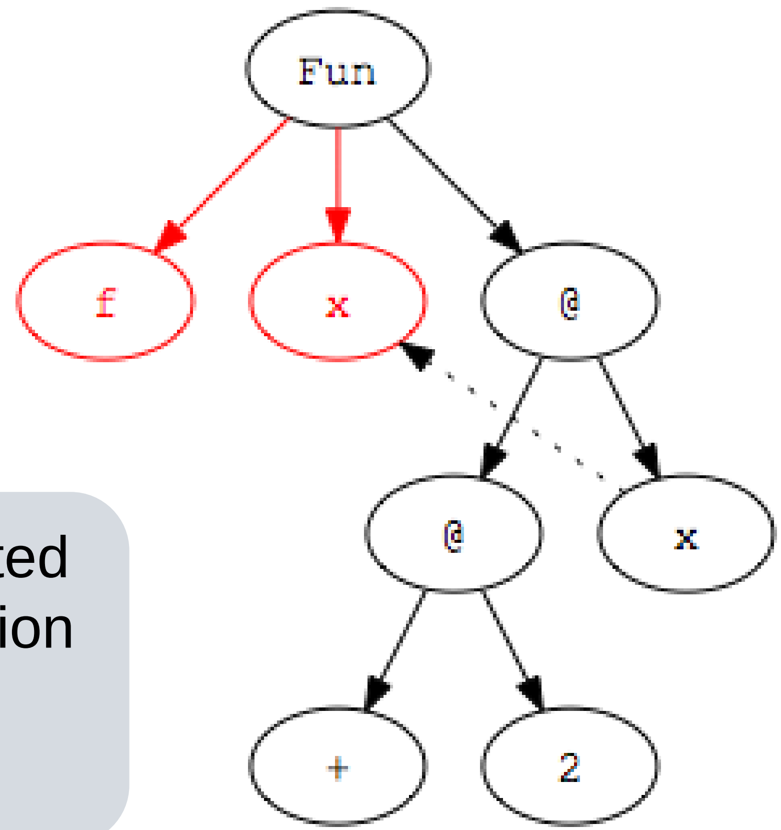
Imperative Example

```
x := b[z]
a [b[y]] := x
```

Step 1: Parse Program

- Parse program text to construct parse tree

```
f x = 2 + x
```

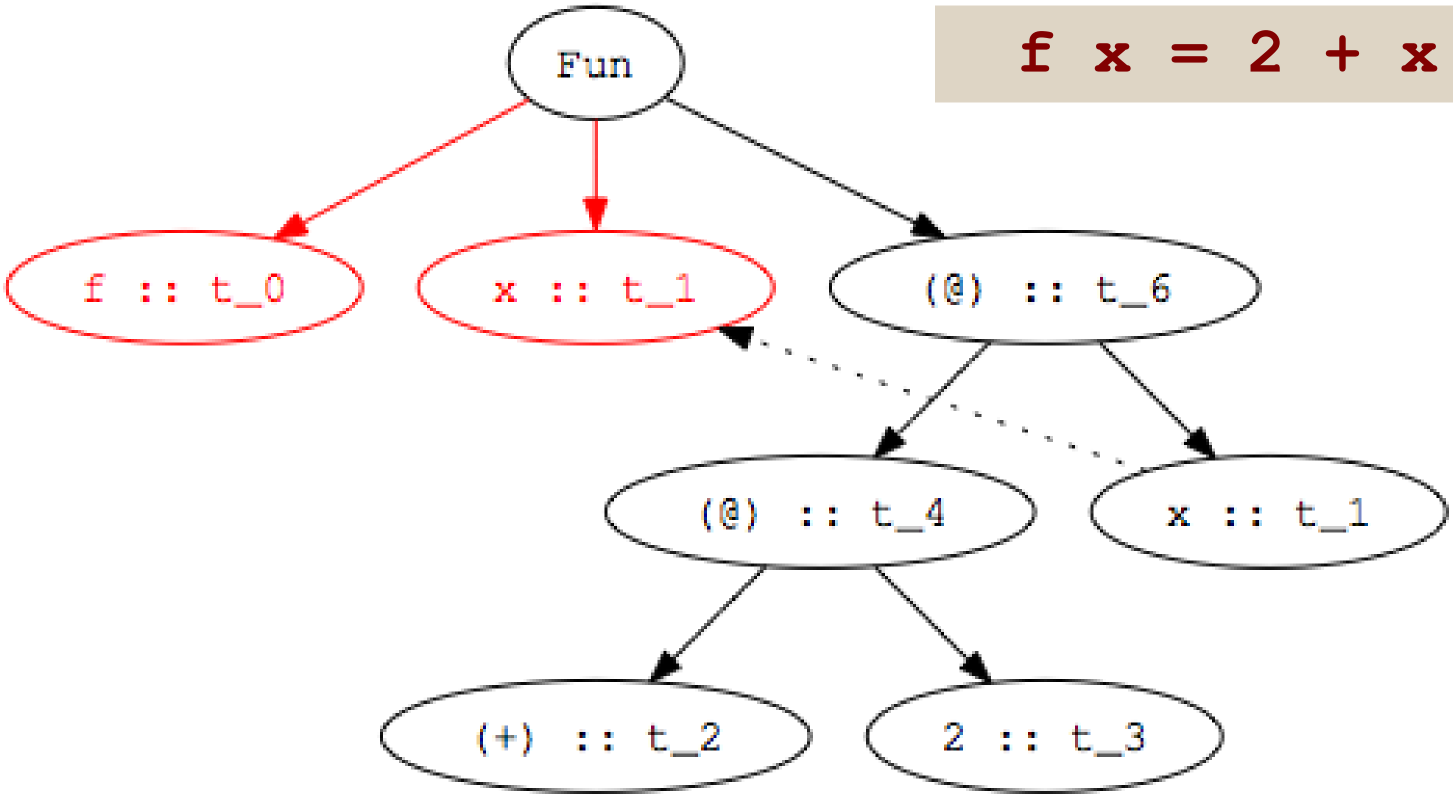


Infix operators are converted to Curied function application during parsing:

2 + x ➔ (+) 2 x

Step 2: Assign type variables to nodes

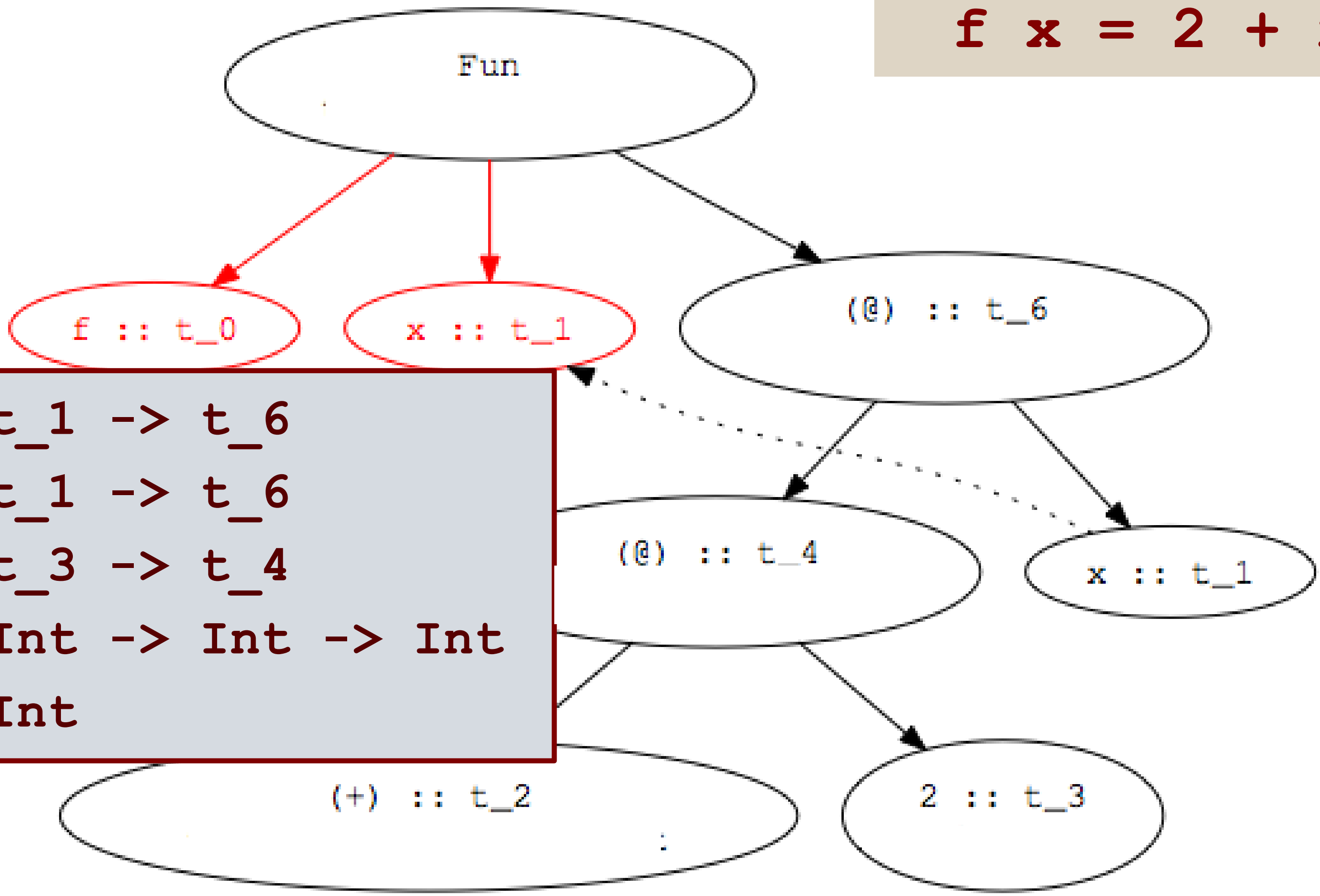
`f x = 2 + x`



Variables are given same type as binding occurrence

Step 3: Add Constraints

```
f x = 2 + x
```




```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_2 = t_3 -> t_4
t_2 = Int -> Int -> Int
t_3 = Int
```

Step 4: Solve Constraints

```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_2 = t_3 -> t_4
t_2 = Int -> Int -> Int
t_3 = Int
```

```
t_3 -> t_4 = Int -> (Int -> Int)
```

```
t_3 = Int
t_4 = Int -> Int
```

```
t_0 = t_1 -> t_6
t_4 = t_1 -> t_6
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

```
t_1 -> t_6 = Int -> Int
```

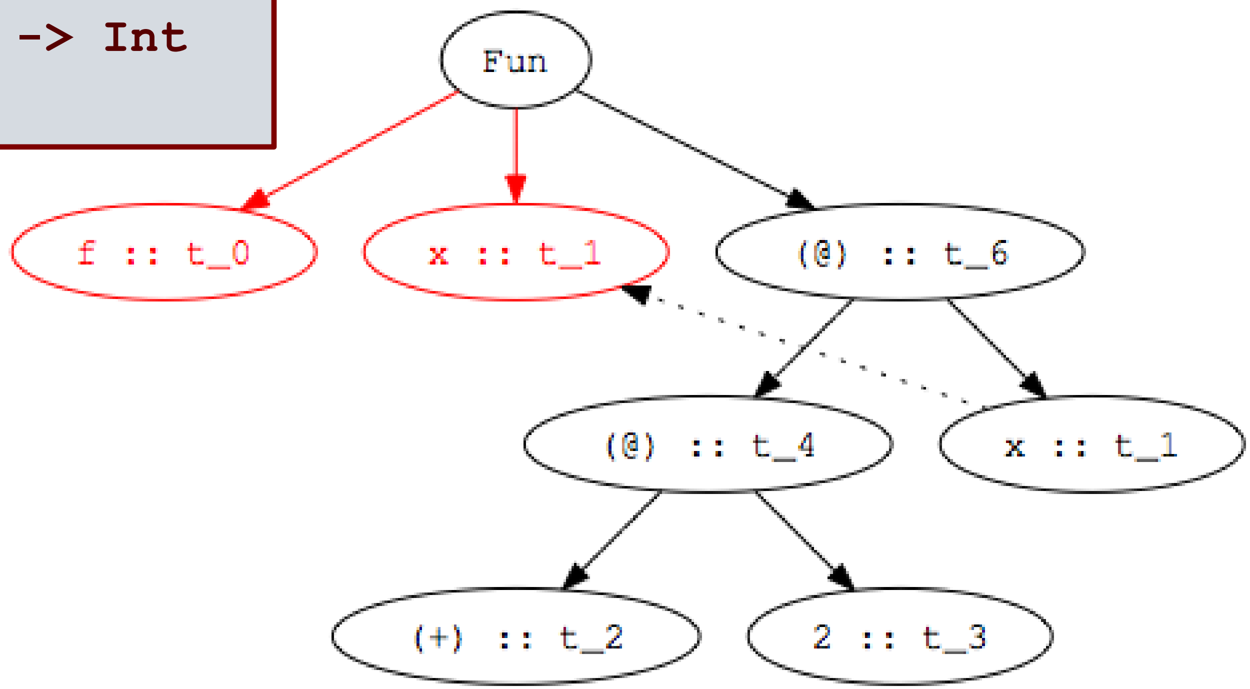
```
t_1 = Int
t_6 = Int
```

```
t_0 = Int -> Int
t_1 = Int
t_6 = Int
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

Step 5:
Determine type of declaration

```
t_0 = Int -> Int
t_1 = Int
t_6 = Int -> Int
t_4 = Int -> Int
t_2 = Int -> Int -> Int
t_3 = Int
```

```
f x = 2 + x
> f :: Int -> Int
```



Unification

- Given two type terms t_1, t_2
- Compute the most general unifier of t_1 and t_2
  - A mapping m from type variables to typed terms such that
    - $t_1 \{m\} == t_2 \{m\}$
    - Every other unifier is a refinement of m
- Example

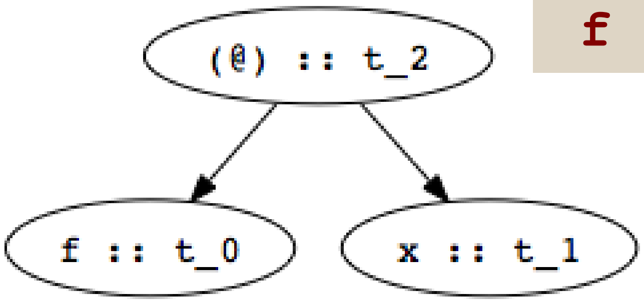
  mgu(**`t_3 -> t_4, Int -> (Int -> Int)=`**

  `[t_3 ↦ Int, t_4 ↦Int -> Int]=`

Type Inference Algorithm

- Parse program to build parse tree
- Assign type variables to nodes in tree
- Generate constraints:
  - From environment: literals (2), built-in operators (+), known functions (tail)
  - From form of parse tree: e.g., application and abstraction nodes
- Solve constraints using *unification*
- Determine types of top-level declarations

Constraints from Application Nodes

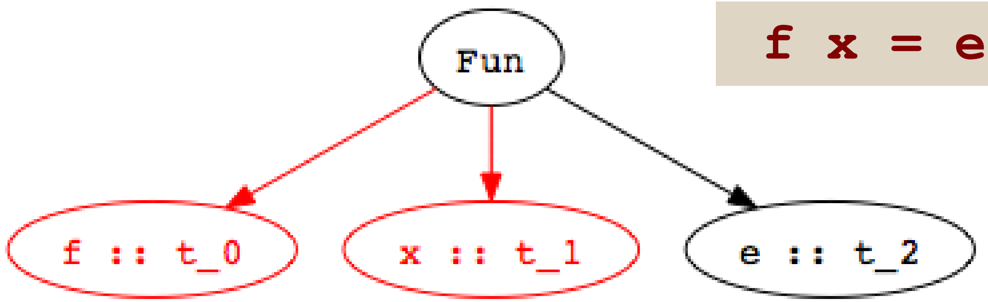



```
f x
```

```
t_0 = t_1 -> t_2
```

- Function application (apply f to x)
  - Type of f (t_0 in figure) must be domain $\rightarrow$ range
  - Domain of f must be type of argument x (t_1 in fig)
  - Range of f must be result of application (t_2 in fig)
  - Constraint: t_0 = t_1 -> t_2

Constraints from Abstractions



`f x = e`

`t_0 = t_1 -> t_2`

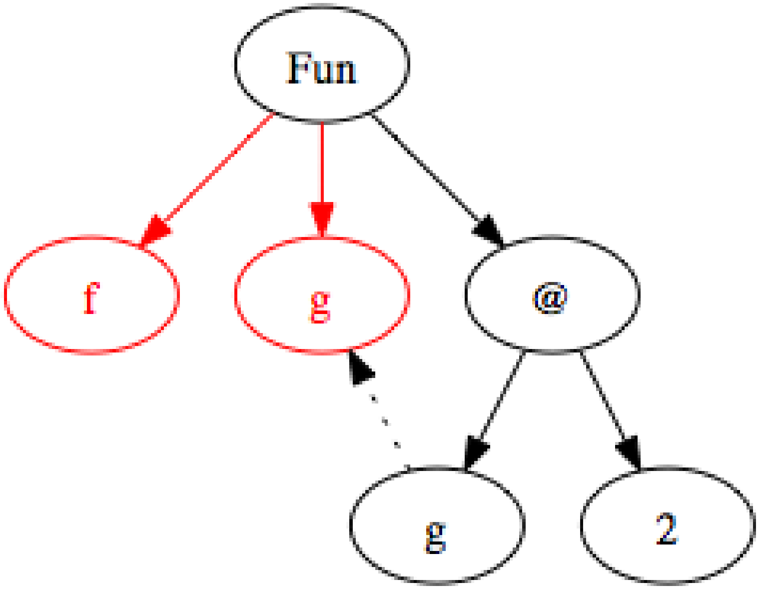
- Function declaration:
  - Type of f (t_0 in figure) must domain $\rightarrow$ range
  - Domain is type of abstracted variable x (t_1 in fig)
  - Range is type of function body e (t_2 in fig)
  - Constraint: t_0 = t_1 -> t_2

Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 1:
  Build Parse Tree

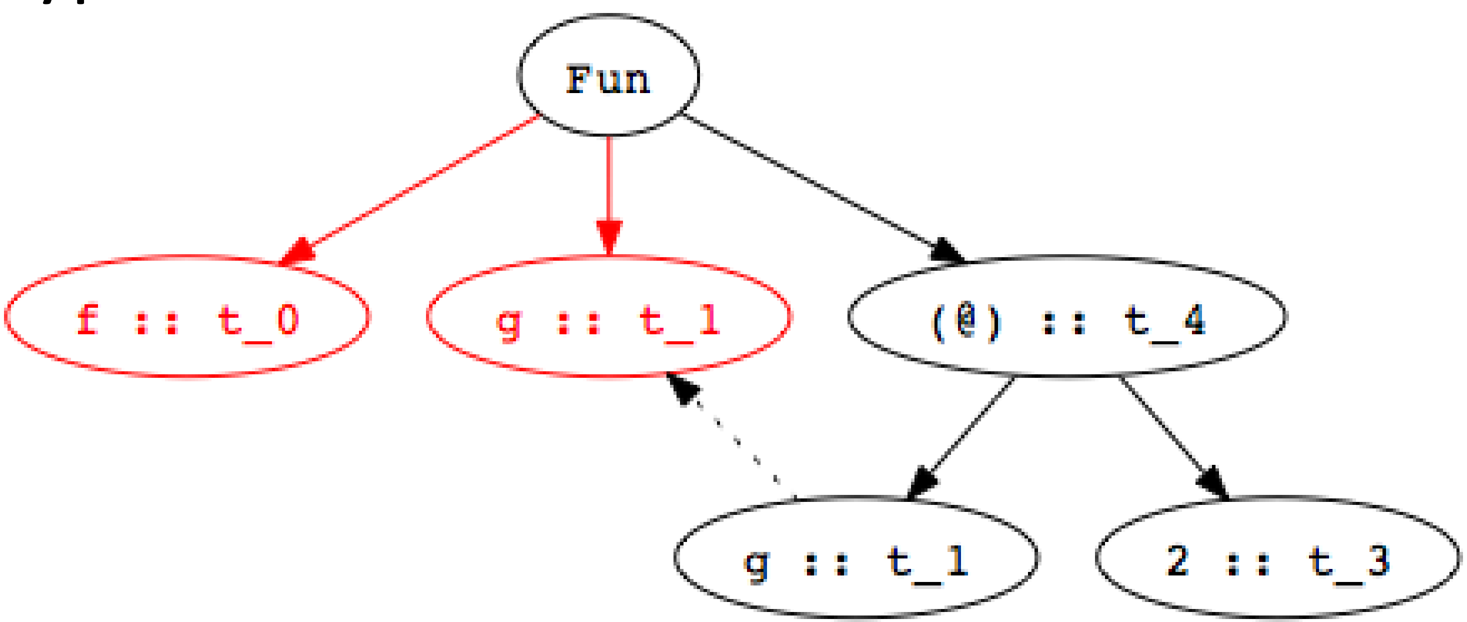


Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 2:
  Assign type variables



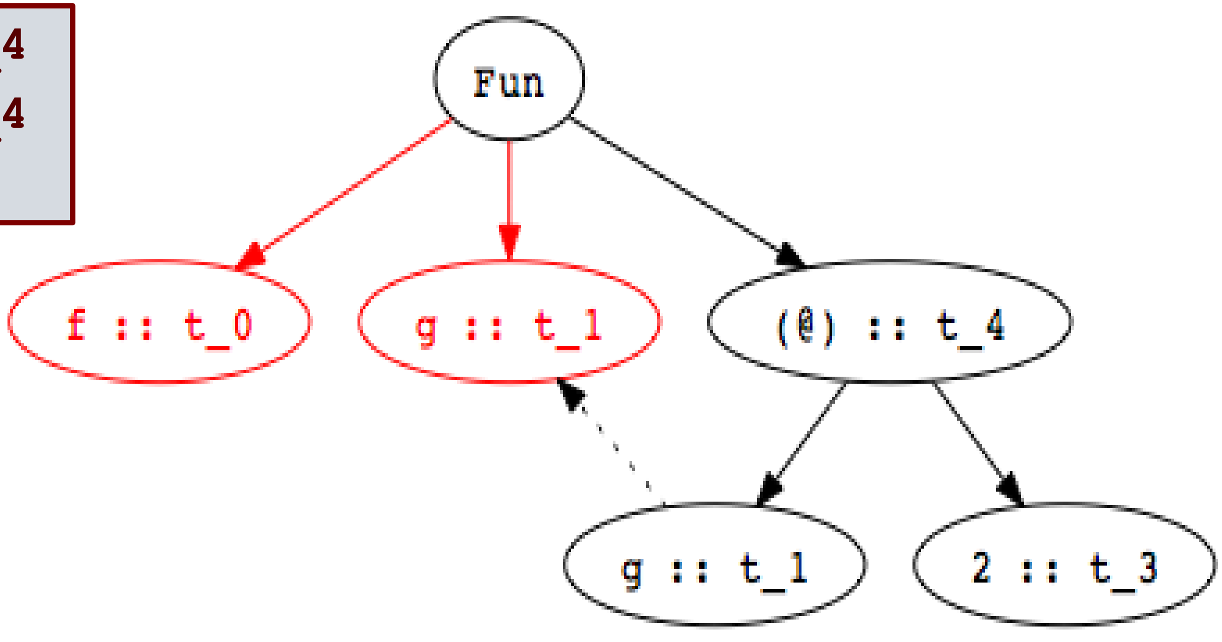
Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 3:
  Generate constraints

```
t_0 = t_1 -> t_4
t_1 = t_3 -> t_4
t_3 = Int
```



Inferring Polymorphic Types

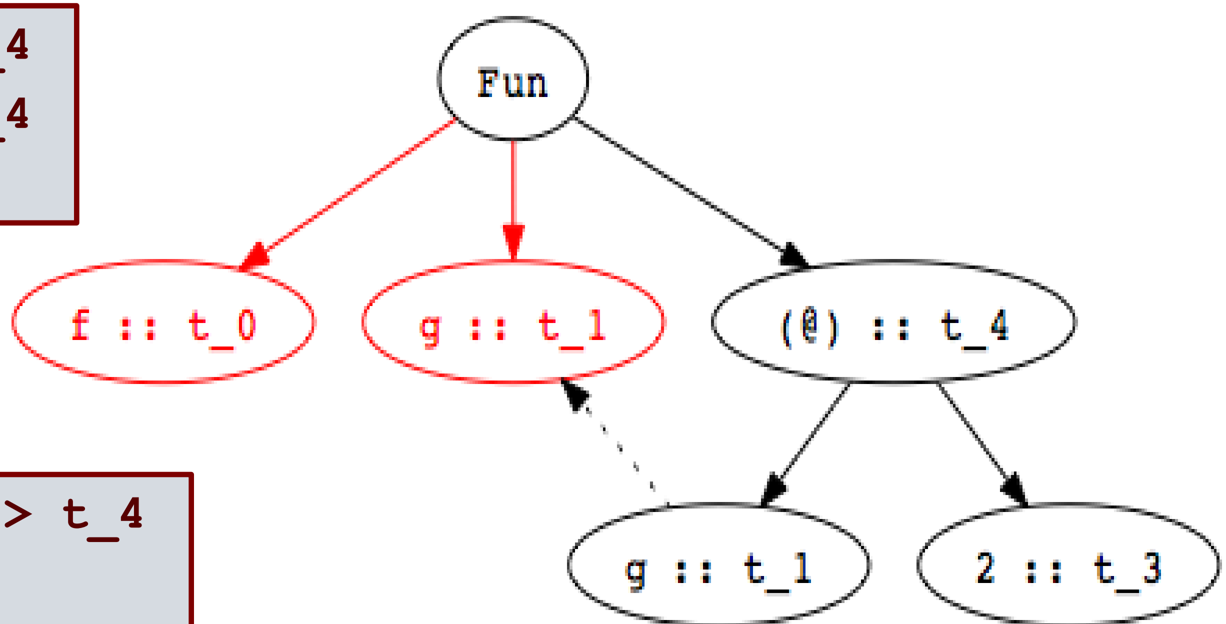
- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 4:
  Solve constraints

```
t_0 = t_1 -> t_4
t_1 = t_3 -> t_4
t_3 = Int
```

```
t_0 = (Int -> t_4) -> t_4
t_1 =  Int -> t_4
t_3 =  Int
```



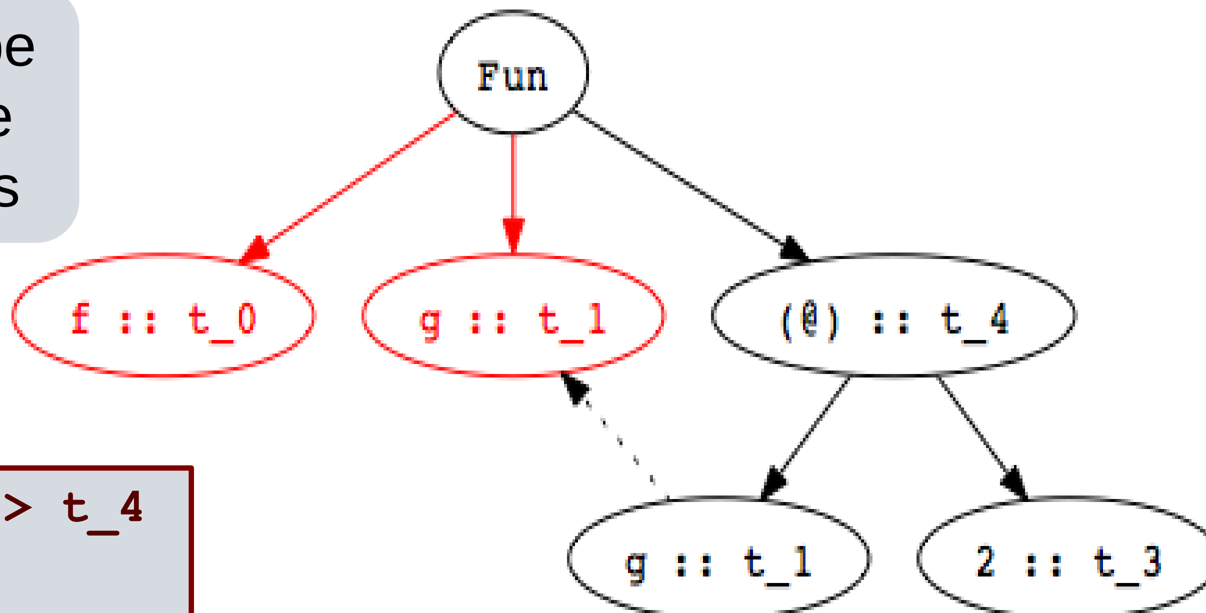
Inferring Polymorphic Types

- Example:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Step 5:
  Determine type of top-level declaration

Unconstrained type variables become polymorphic types




```
t_0 = (Int -> t_4) -> t_4
t_1 =  Int -> t_4
t_3 =  Int
```

Using Polymorphic Functions

- Function:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Possible applications:

```
add x = 2 + x
> add :: Int -> Int

f add
> 4 :: Int
```

```
isEven x = mod (x, 2) == 0
> isEven:: Int -> Bool

f isEven
> True :: Bool
```

Recognizing Type Errors

- Function:

```
f g = g 2
> f :: (Int -> t_4) -> t_4
```

- Incorrect use

```
not x = if x then True else False
> not :: Bool -> Bool
f not
> Error: operator and operand don't agree
  operator domain: Int -> a
  operand:         Bool -> Bool
```

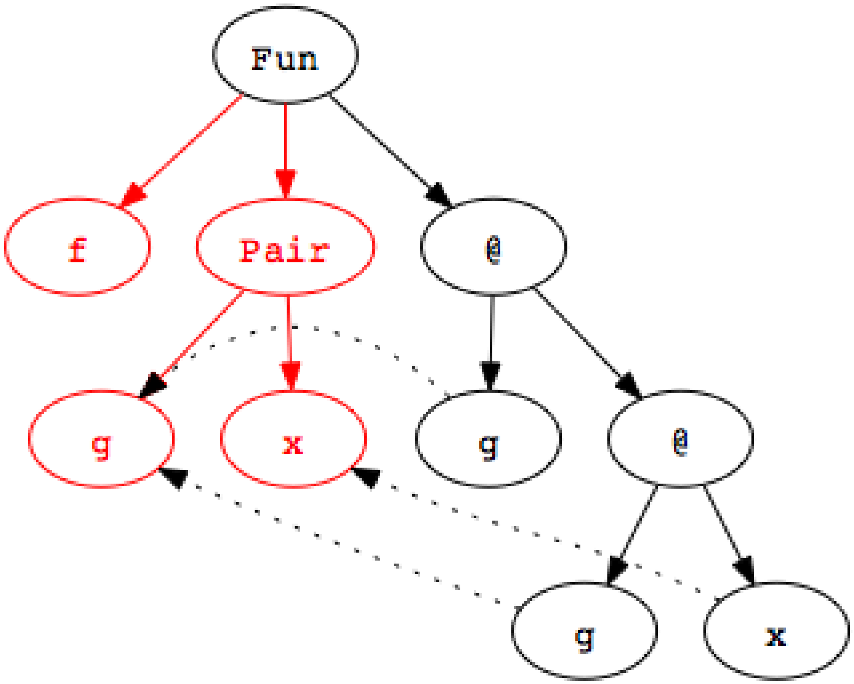
- Type error:
  cannot unify Bool $\rightarrow$ Bool and Int $\rightarrow$ t

Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 1:
  Build Parse Tree

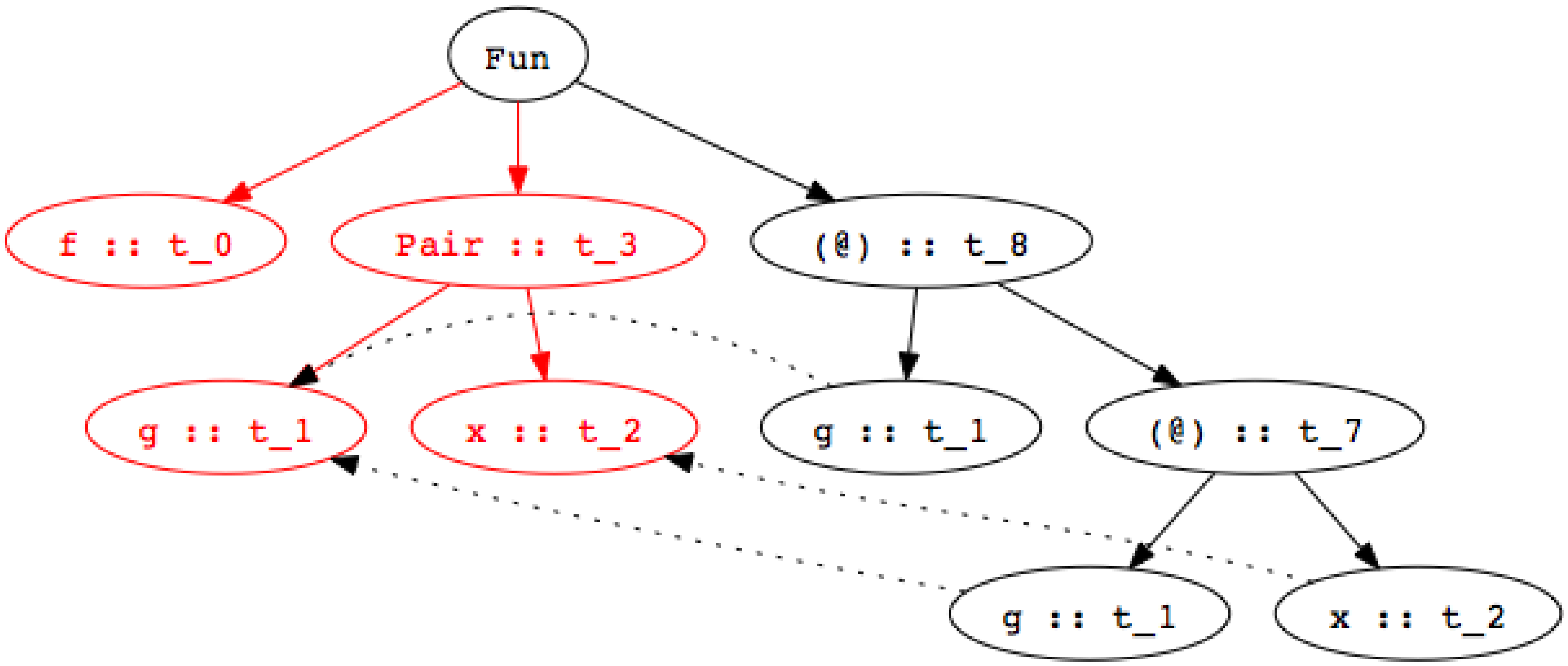


Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 2:
  Assign type variables



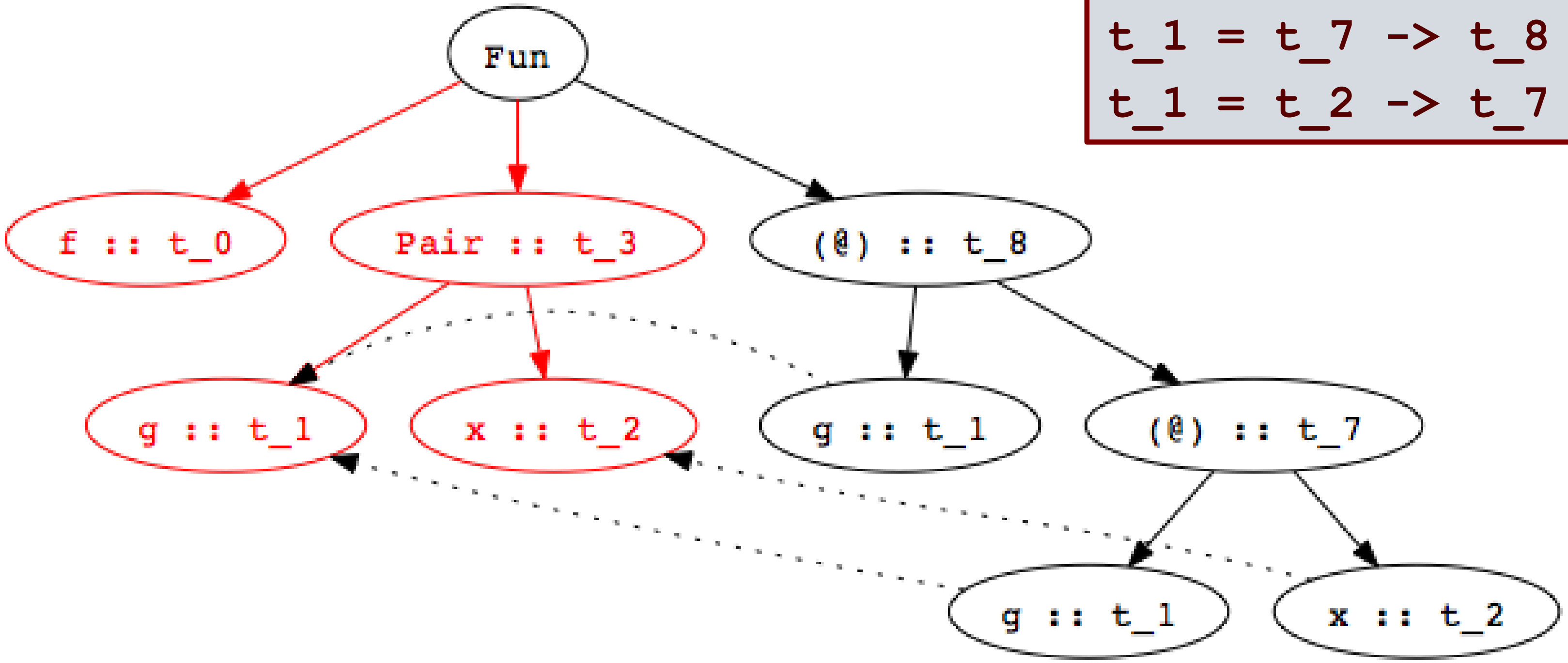
Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 3:
  Generate constraints

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```



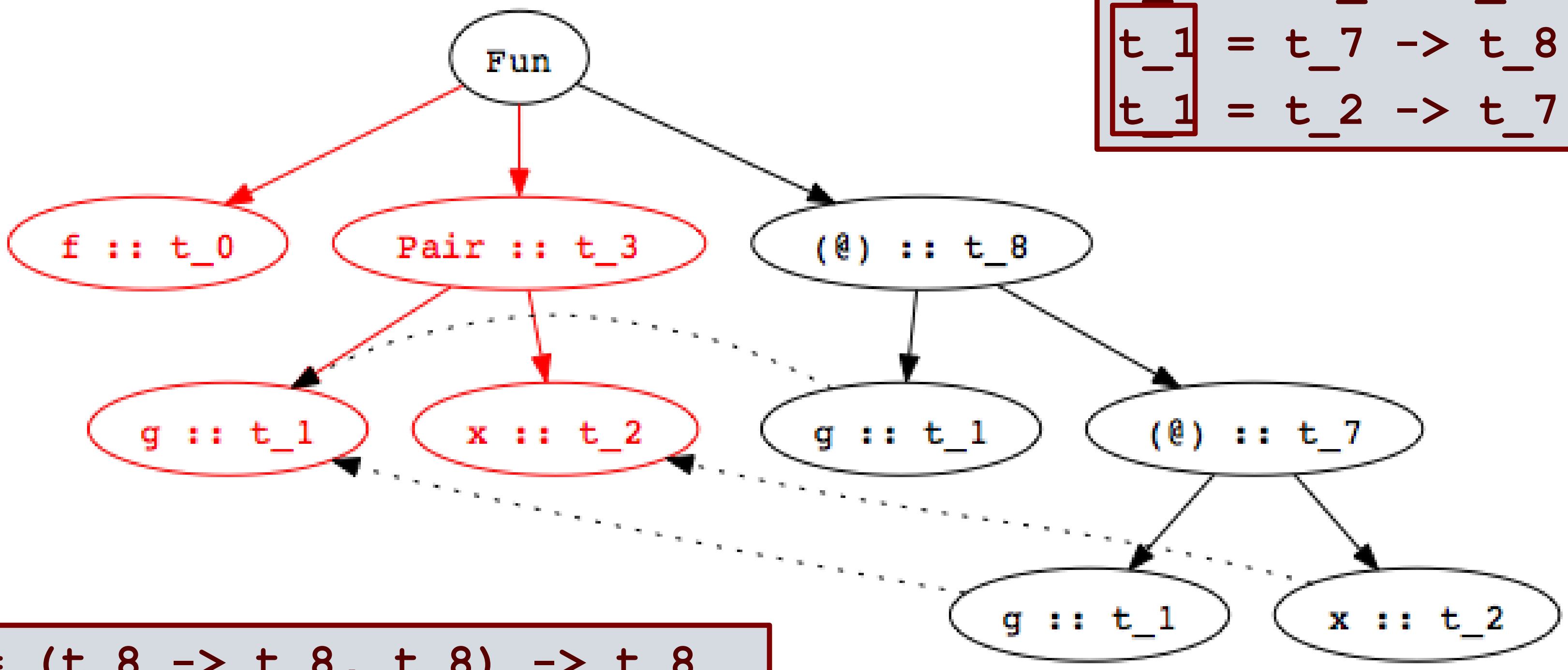
Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 4:
  Solve constraints

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```




```
t_0 = (t_8 -> t_8, t_8) -> t_8
```

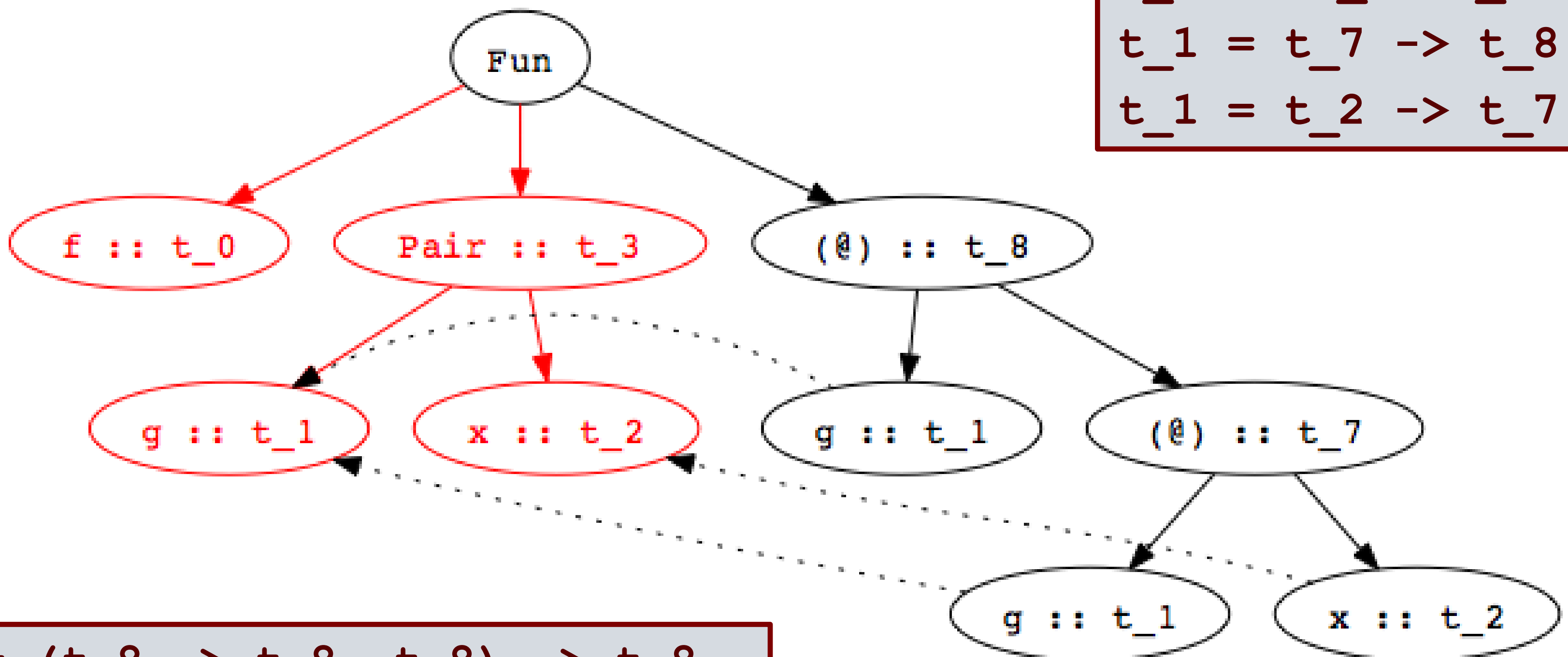
Another Example

- Example:

```
f (g,x) = g (g x)
> f :: (t_8 -> t_8, t_8) -> t_8
```

- Step 5:
  Determine type of f

```
t_0 = t_3 -> t_8
t_3 = (t_1, t_2)
t_1 = t_7 -> t_8
t_1 = t_2 -> t_7
```




```
t_0 = (t_8 -> t_8, t_8) -> t_8
```

Polymorphic Datatypes

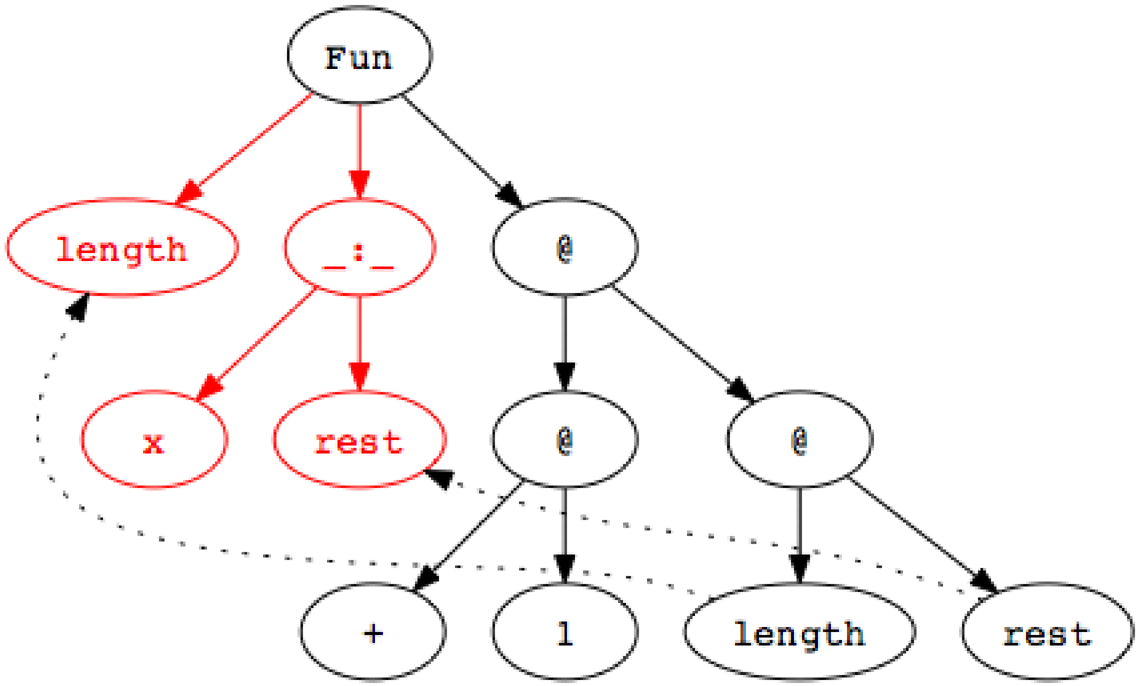
- Functions may have multiple clauses

```
length [] = 0
length (x:rest) = 1 + (length rest)
```

- Type inference
  - Infer separate type for each clause
  - Combine by adding constraint that all clauses must have the same type
  - Recursive calls: function has same type as its definition

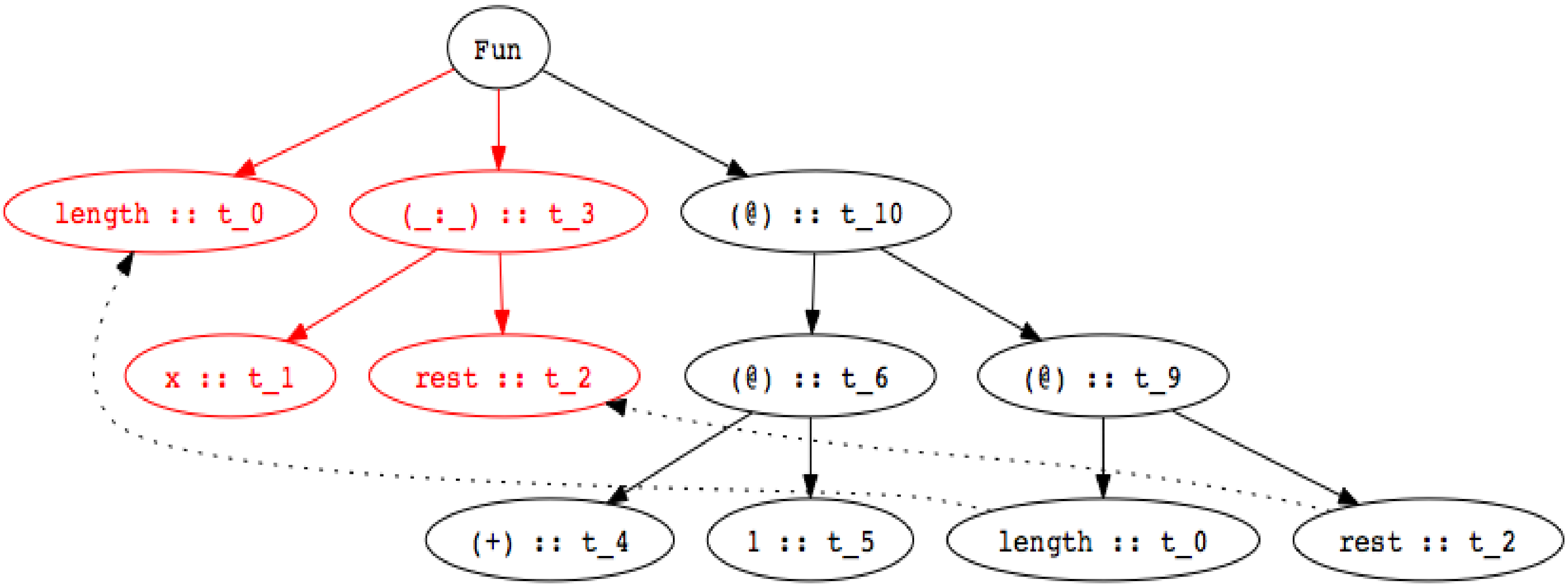
Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 1: Build Parse Tree



Type Inference with Datatypes

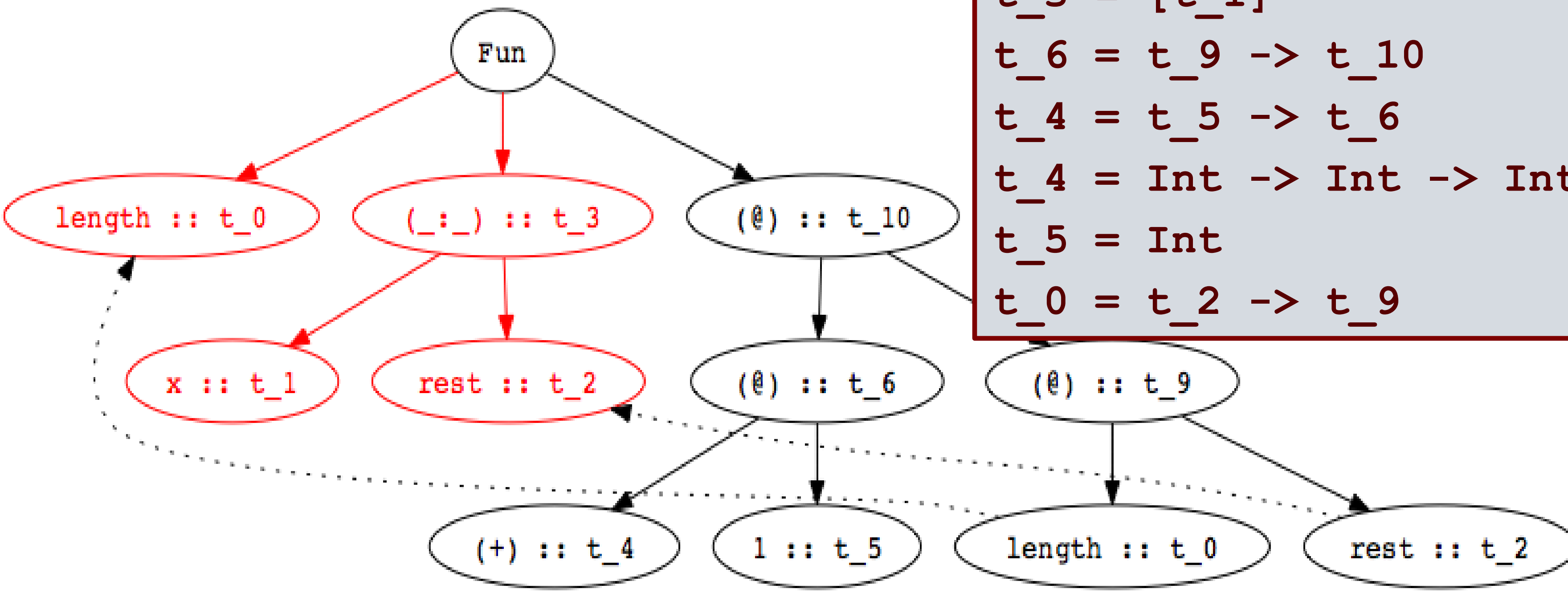
- Example: `length (x:rest) = 1 + (length rest)`
- Step 2: Assign type variables



Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 3: Generate constraints

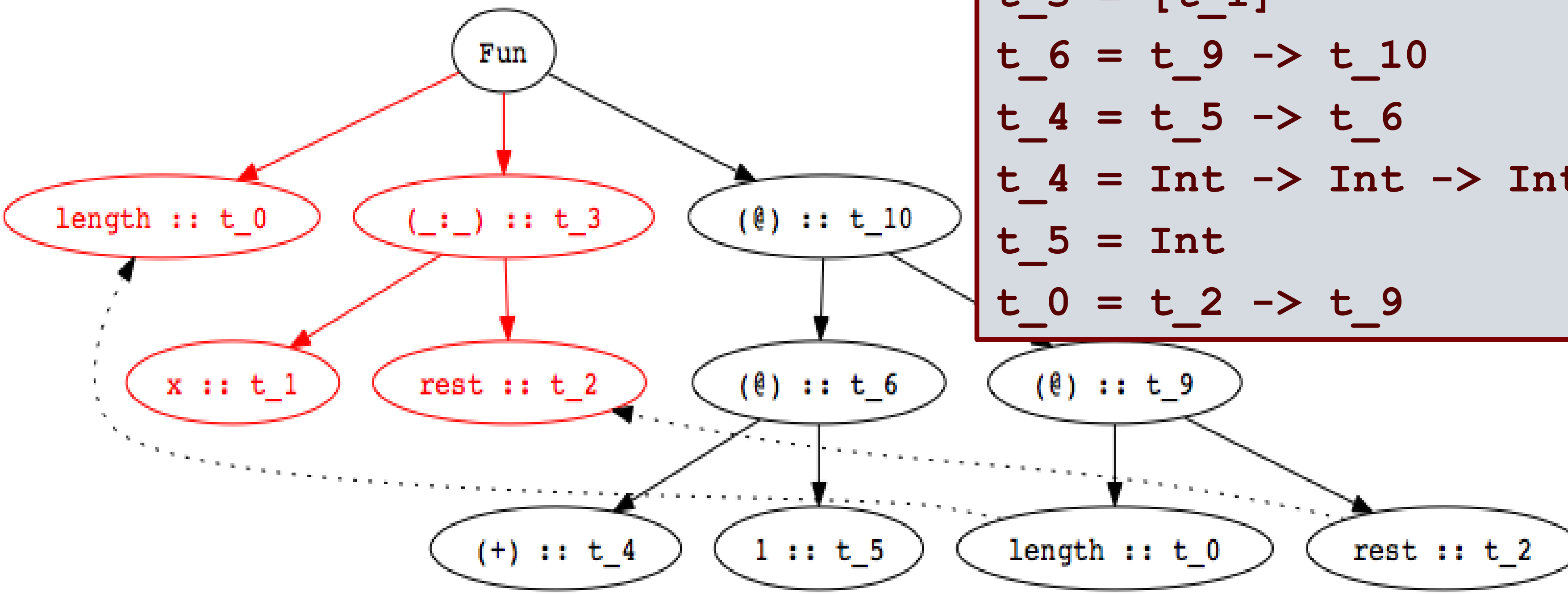
```
t_0 = t_3 -> t_10
t_3 = t_2
t_3 = [t_1]
t_6 = t_9 -> t_10
t_4 = t_5 -> t_6
t_4 = Int -> Int -> Int
t_5 = Int
t_0 = t_2 -> t_9
```



Type Inference with Datatypes

- Example: `length (x:rest) = 1 + (length rest)`
- Step 3: Solve Constraints

```
t_0 = t_3 -> t_10
t_3 = t_2
t_3 = [t_1]
t_6 = t_9 -> t_10
t_4 = t_5 -> t_6
t_4 = Int -> Int -> Int
t_5 = Int
t_0 = t_2 -> t_9
```




```
t_0 = [t_1] -> Int
```

Multiple Clauses

- Function with multiple clauses

```
append ([],r) = r
append (x:xs, r) = x : append (xs, r)
```

- Infer type of each clause
  - First clause:

```
> append :: ([t_1], t_2) -> t_2
```

  - Second clause:

```
> append :: ([t_3], t_4) -> [t_3]
```

- Combine by equating types of two clauses

```
> append :: ([t_1], [t_1]) -> [t_1]
```

Most General Type

- Type inference produces the *most general type*

```
map (f, []   ) = []
map (f, x:xs) = f x : map (f, xs)
> map :: (t_1 -> t_2, [t_1]) -> [t_2]
```

- Functions may have many less general types

```
> map :: (t_1  -> Int, [t_1])  -> [Int]
> map :: (Bool -> t_2, [Bool]) -> [t_2]
> map :: (Char -> Int, [Char]) -> [Int]
```

- Less general types are all instances of most general type, also called the *principal type*

Type Inference Algorithm

- When Hindley/Milner type inference algorithm was developed, its complexity was unknown
- In 1989, Kanellakis, Mairson, and Mitchell proved that the problem was exponential-time complete
- Usually linear in practice though…
  - Running time is exponential in the depth of polymorphic declarations

Information from Type Inference

- Consider this function…

```
reverse [] = []
reverse (x:xs) = reverse xs
```

… and its most general type:

```
> reverse :: [t_1] -> [t_2]
```

- What does this type mean?

Reversing a list should not change its type, so there must be an error in the definition of reverse!

Type Inference: Key Points

- Type inference computes the types of expressions
  - Does not require type declarations for variables
  - Finds the most general type by solving constraints
  - Leads to polymorphism
- Sometimes better error detection than type checking
  - Type may indicate a programming error even if no type error
- Some costs
  - More difficult to identify program line that causes error
  - Natural implementation requires uniform representation sizes
  - Complications regarding assignment took years to work out
- Idea can be applied to other program properties
  - Discover properties of program using same kind of analysis

Haskell Type Inference

- Haskell uses type classes
  - supports user-defined overloading, so the inference algorithm is more complicated
- ML restricts the language
  - to ensure that no annotations are required
- Haskell provides additional features
  - like polymorphic recursion for which types cannot be inferred and so the user must provide annotations

Parametric Polymorphism: Haskell vs C++

- Haskell polymorphic function
  - Declarations (generally) require no type information
  - Type inference uses type variables to type expressions
  - Type inference substitutes for type variables as needed to instantiate polymorphic code
- C++ function template
  - Programmer must declare the argument and result types of functions
  - Programmers must use explicit type parameters to express polymorphism
  - Function application: type checker does instantiation

Example: Swap Two Values

- Haskell

```
swap :: (IORef a, IORef a) -> IO ()
swap (x,y) = do {
  val_x <- readIORef x; val_y <- readIORef y;
  writeIORef y val_x;   writeIORef x val_y;
  return () }
```

- C++

```
template <typename T>
void swap(T& x, T& y){
      T tmp = x;  x=y;  y=tmp;
}
```

Declarations both swap two values polymorphically, but they are compiled very differently

Implementation

- Haskell
  - **`swap`** is compiled into one function
  - Typechecker determines how function can be used
- C++
  - **`swap`** is compiled differently for each instance (details beyond scope of this course …)
- Why the difference?
  - Haskell ref cell is passed by pointer. The local **`x`** is a pointer to value on heap, so its size is constant
  - C++ arguments passed by reference (pointer), but local **`x`** is on the stack, so its size depends on the type

Polymorphism vs Overloading

- Parametric polymorphism
  - Single algorithm may be given many types
  - Type variable may be replaced by any type
  - if **`f::t→t`** then **`f::Int→Int`**, **`f::Bool→Bool`**, ...
- Overloading
  - A single symbol may refer to more than one algorithm
  - Each algorithm may have different type
  - Choice of algorithm determined by type context
  - Types of symbol may be arbitrarily different
  - In ML, **`+`** has types **`int*int→int`**, **`real*real→real`**, no others

Varieties of Polymorphism

- Parametric polymorphism A single piece of code is typed generically
  - Imperative or first-class polymorphism
  - ML-style or let-polymorphism
- Ad-hoc polymorphism The same expression exhibit different behaviors when viewed in different types
  - Overloading
  - Multi-method dispatch
  - Intentional polymorphism
- Subtype polymorphism A single term may have many types using the rule of subsumption allowing to selectively forget information

Summary

- Types are important in modern languages
  - Program organization and documentation
  - Prevent program errors
  - Provide important information to compiler
- Type inference
  - Determine best type for an expression, based on known information about symbols in the expression
- Polymorphism
  - Single algorithm (function) can have many types