Declarative and Logic Programming

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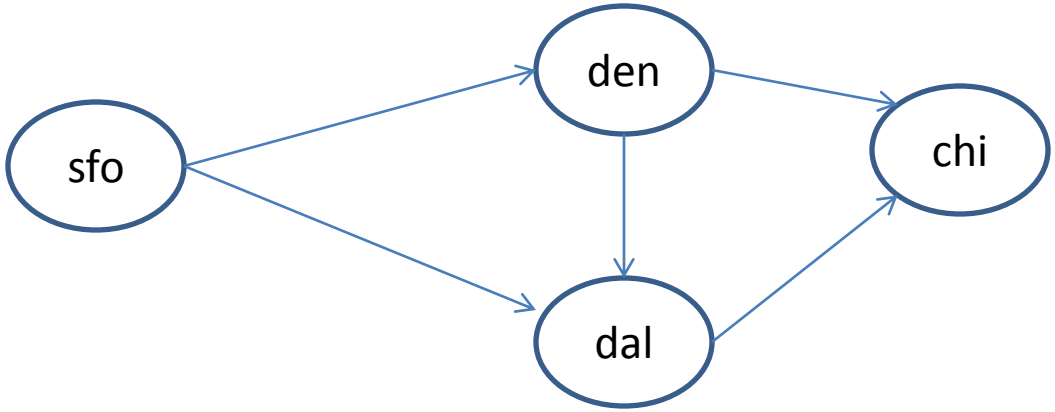
Adapted from Peter Hawkins
Jeff Ullman (Stanford)

Why declarative programming

- Turing complete languages (C, C++, Java, Javascript, Haskel, …) are powerful
  - But require a lot of efforts to do simple things
  - Especially for non-expert programmers
  - Error-prone
  - Hard to be optimize
- Declarative programming provides an alternative
  - Restrict expressive power
  - High level abstractions to perform common tasks
  - Interpreter/Compiler guarantees efficiency
  - Useful to demonstrate some of the concepts in the course

Flight database

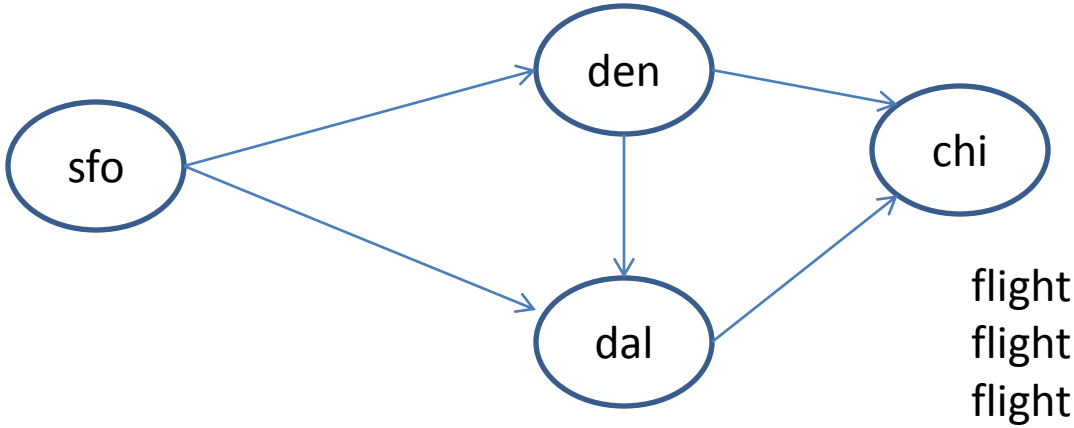


flight	source	dest
	sfo	den
	sfo	dal
	den	chi
	den	dal
	dal	chi

Where can we fly from Denver?

SQL: SELECT dest FROM flight WHERE source="den";

Flight database SQL vs. Datalog



flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

Where can we fly from Denver?

SQL: SELECT dest FROM flight WHERE source="den“;

Datalog: flight(den, X).

X= chi;

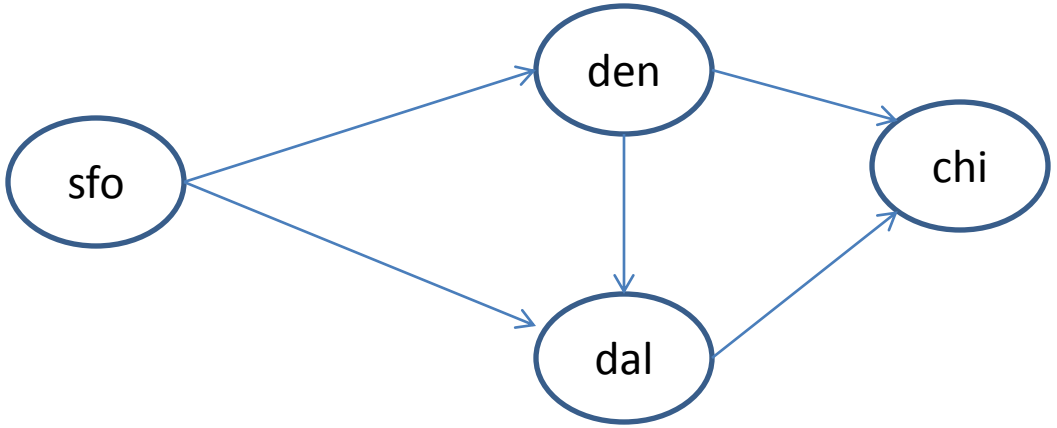
X= dal;

no

X is a logical variable

Instantiated with the satisfying assignments

Flight database conjunctions



flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

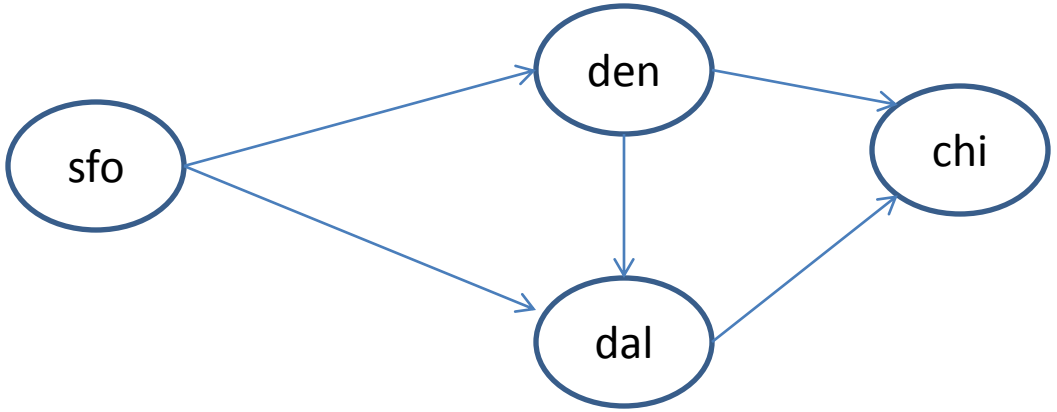
Logical conjunction: "," let us combine multiple conditions into one query

Which intermediate airport can I go between San Francisco and Chicago?

Datalog: flight(sfo, X), flight(X, chi).

X = den;
X = dal;
no

Recursive queries Datalog



flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

reachable(X, Y) :- flight(X, Y).
reachable(X, Y) :- reachable(X, Z), flight(Z, Y).

Where can we reach from San Francisco?

reachable(sfo, X).

X=den;

X=dal;

X=chi;

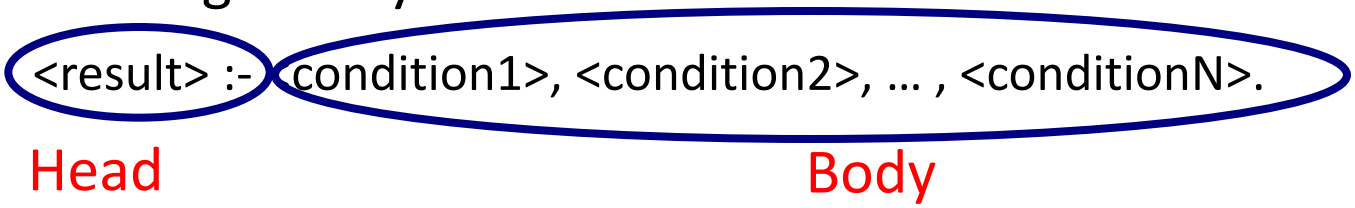
no

Why Not Just Use SQL?

1. Recursion is much easier to express in Datalog
2. Rules express things that go on in both FROM and WHERE clauses, and let us state some general principles

Syntax of Datalog

Datalog rule syntax:



- Body consists of one or more conditions (input tables)
- Head is an output table
  - Recursive rules: result of head in rule body

Terminology and Convention

reachable(S,D) :- flight(S,Z), reachable(Z,D).

- An ***atom*** is a ***predicate***, or relation name with ***arguments***
- Convention: Variables begin with a capital, predicates begin with lower-case
- The **head** is an atom; the **body** is the AND of one or more atoms
- *Extensional database predicates* (**EDB**) – source tables
- *Intensional database predicates* (**IDB**) – derived tables

Datalog Programs

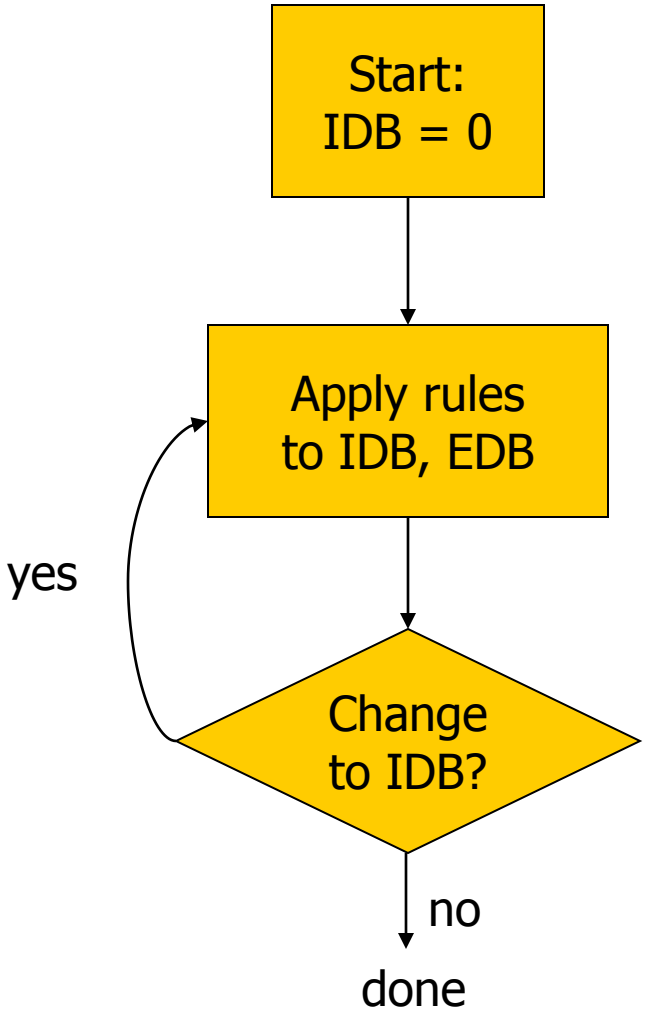
- A collection of rules is a (Datalog) *program*
  - *Order of rules and conjuncts is immaterial*
- Each program has a distinguished IDB predicate that represents the result of the program
- Sometimes given query
  - flight(sfo, X), flight(X, chi).

Datalog Evaluation

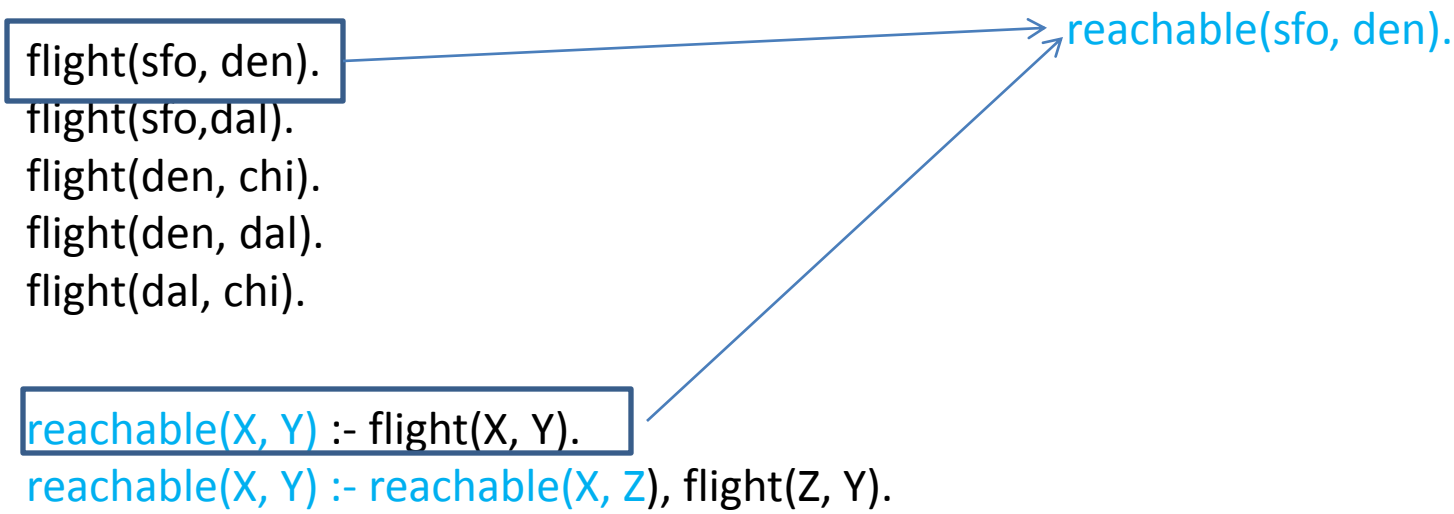
- Bottom-up Evaluation
  - Start with the EDB relations
  - Apply rules till convergence
  - Construct the minimal Herbrand model
- Top-down Evaluation (Prolog)
  - Start with a query
  - Construct a proof tree by left to right-evaluation
  - Limited resolution

The “Naïve” Evaluation Algorithm

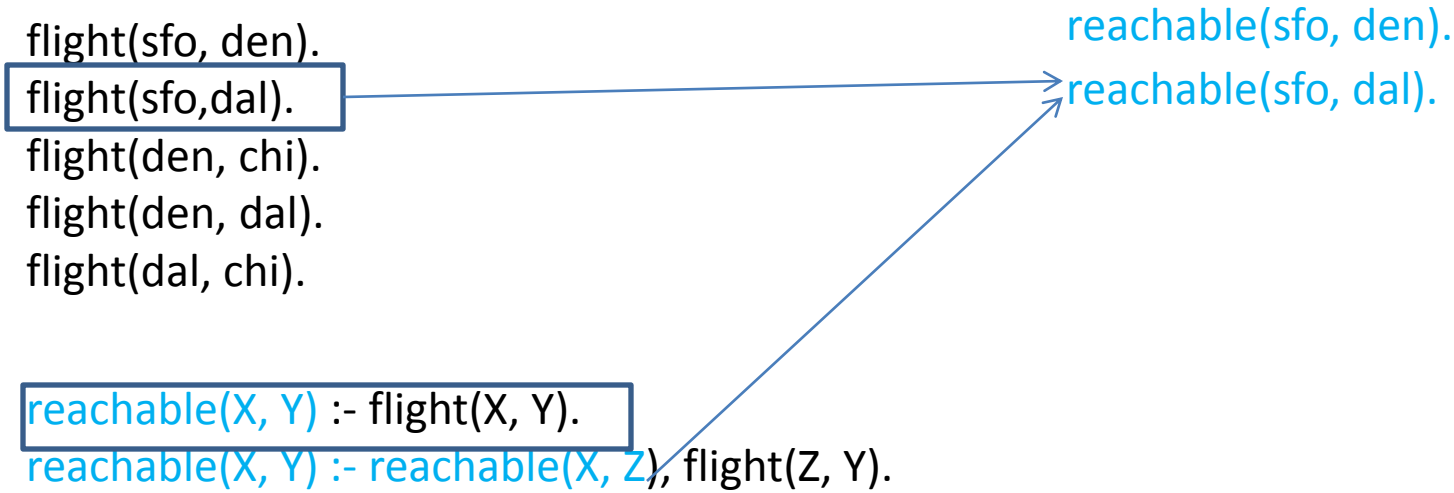
1. Start by assuming all IDB relations are empty
2. Repeatedly evaluate the rules using the EDB and the previous IDB, to get a new IDB
3. Terminate when no change to IDB



Naïve Evaluation



Naïve Evaluation

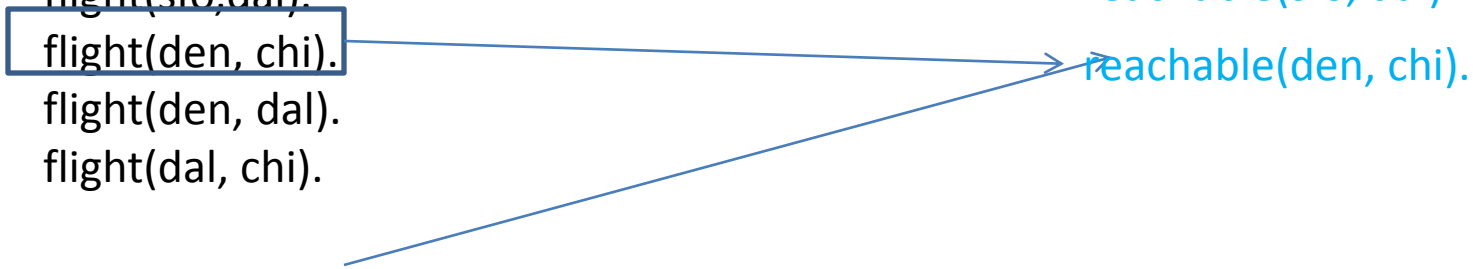


Naïve Evaluation

flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

reachable(sfo, den).
reachable(sfo, dal).
reachable(den, chi).

reachable(X, Y) :- flight(X, Y).
reachable(X, Y) :- reachable(X, Z), flight(Z, Y).



Naïve Evaluation

flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

reachable(X, Y) :- flight(X, Y).
reachable(X, Y) :- reachable(X, Z), flight(Z, Y).

reachable(sfo, den).

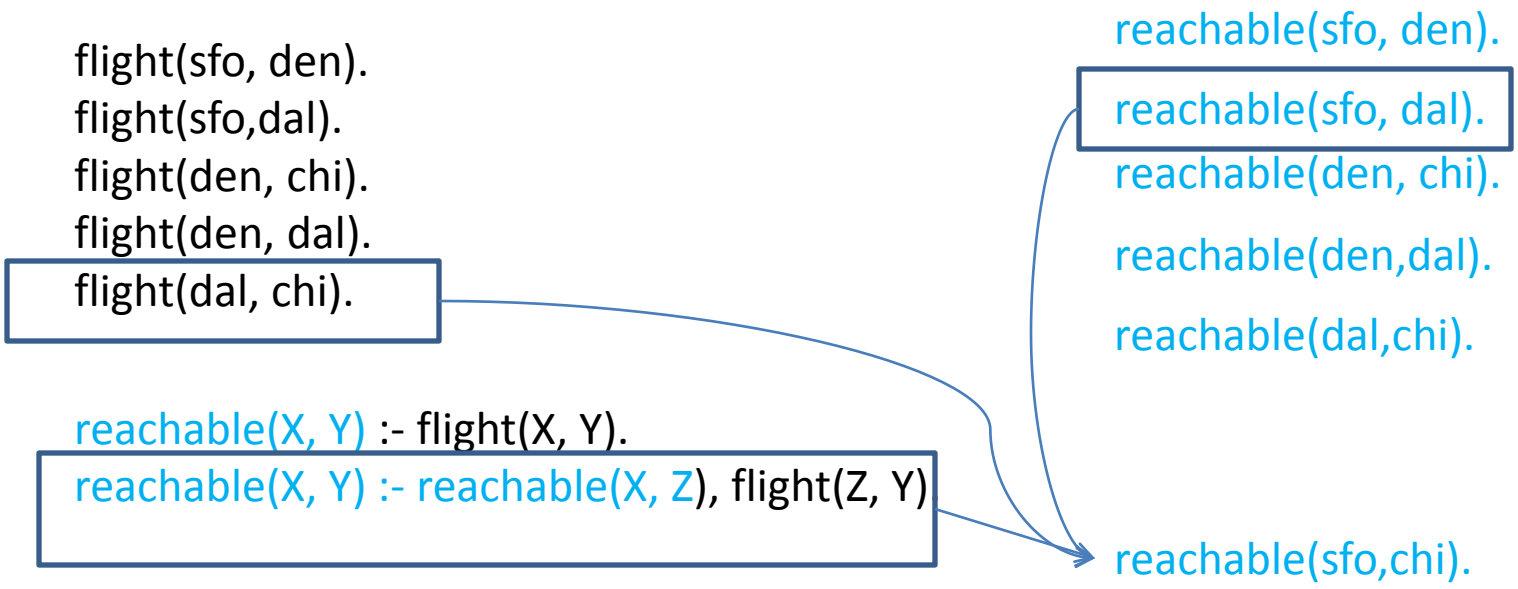
reachable(sfo, dal).

reachable(den, chi).

reachable(den,dal).

reachable(dal,chi).

Naïve Evaluation



Seminaïve Evaluation

- Key idea: to get a new tuple for relation P on one round, the evaluation must use some tuple for some relation of the body that was obtained on the previous round
- Maintain ΔP = new tuples added to P on previous round
- "Differentiate" rule bodies to be union of bodies with one IDB subgoal made "Δ."

Seminaïve Evaluation

flight(sfo, den).
flight(sfo,dal).
flight(den, chi).
flight(den, dal).
flight(dal, chi).

reachable(X, Y) :- flight(X, Y).
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reachable(sfo, den).

reachable(sfo, dal).

reachable(den, chi).

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reachable(dal,chi).

Seminaïve Evaluation

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reachable(sfo, den).

reachable(sfo, dal).

reachable(den, chi).

reachable(den,dal).

reachable(dal,chi).

reachable(sfo,chi).

Summary Bottom-Up Evaluation

- Efficient
- Useful
  - Databases
  - Program analysis
  - Semantic web
  - Network programming
- Tools
  - Java Iris
  - pyDatalog
  - Inter4Q (C++)
- But limited

Prolog

- Short for Programmation en logique
- Created by Alain Colmerauer in 1972 during research on Natural Language Processing
- A Turing complete language
  - Beyond Datalog
  - Not guaranteed to terminate
- Employs a goal-directed, backtracking depth-first search to answer queries in top-down evaluation to answer queries
  - Semantics sensitive to the order of rules and conjuncts
- Public domain version: http://www.swi-prolog.org/

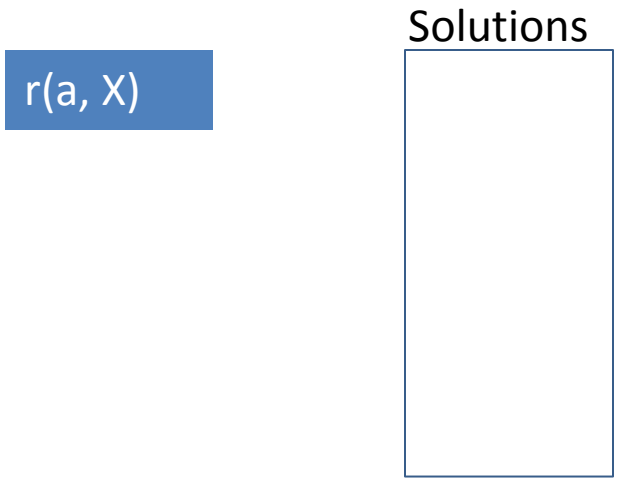
Prolog Search Strategy

- Tries to prove a goal g(…) by investigating each rules of g in order

```
g(a).
g(X) :- f(X), h(X, Y).
g(X):- …
```

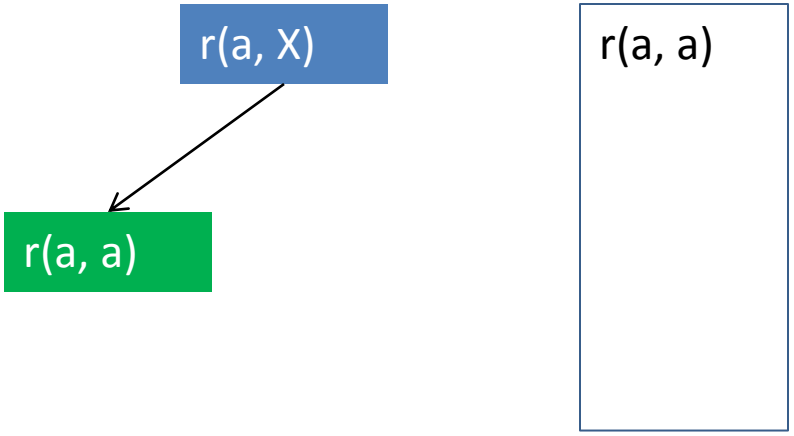
- Conjuncts are proved left-to-right

Example Flight Planning : “? r(a, X)”



f(a, b).
f(a, c).
r(S, S).
r(S, Y) :- f(S, Z), r(Z, Y).

Example Flight Planning: “? r(a, X)”

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f(a, b).
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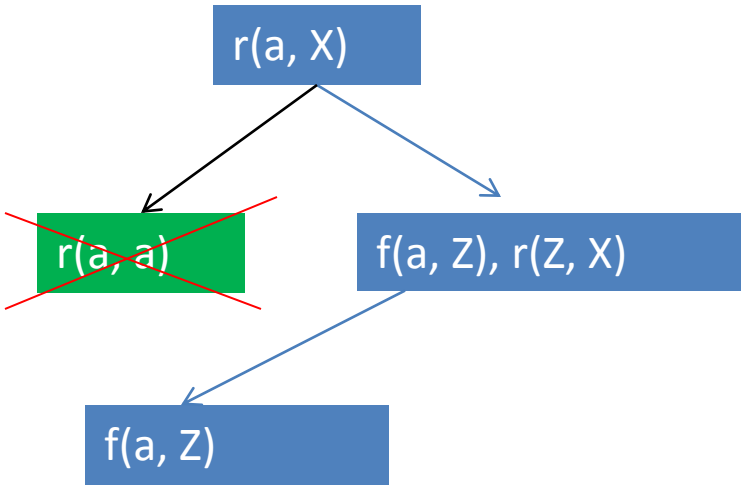
Example Flight Planning: “? r(a, X)”

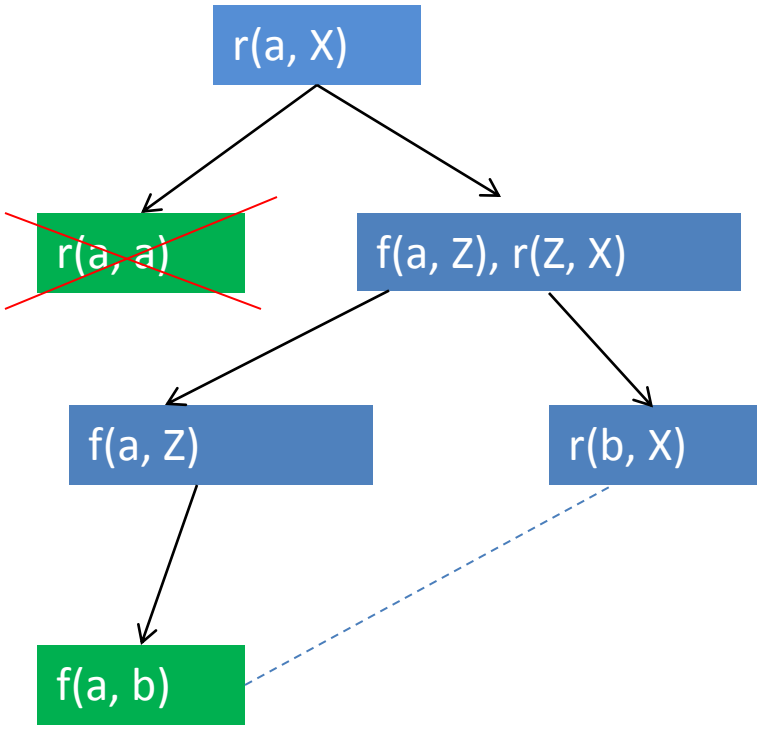


Solutions

r(a, a)

```
f(a, b).
f(a, c).
r(S, S).
r(S, Y) :- f(S, Z), r(Z, Y).
```

Example Flight Planning: “? r(a, X)”



Solutions

r(a, a)

f(a, b).
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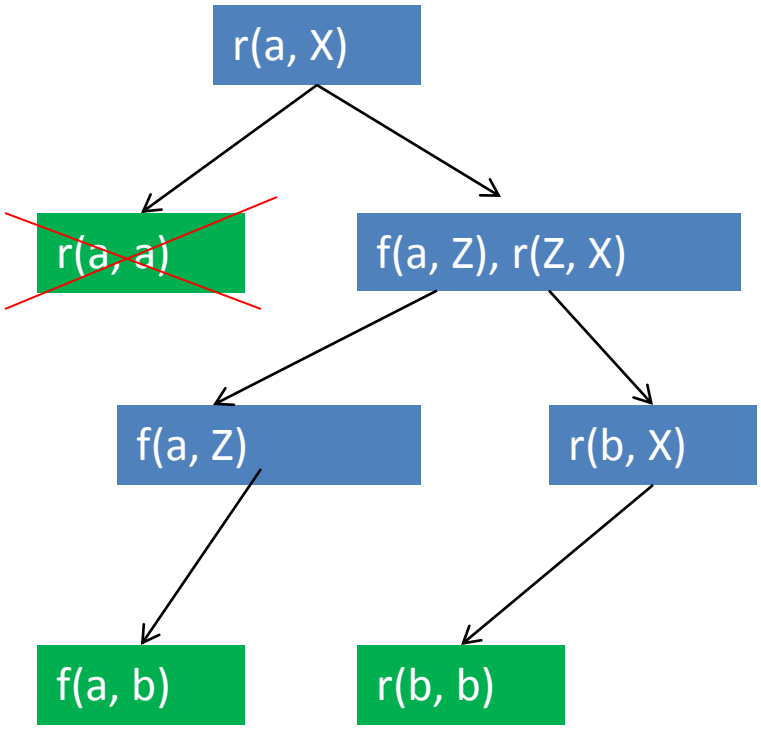
Example Flight Planning: “? r(a, X)”



Solutions

r(a, a)
r(a, b)

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f(a, b).
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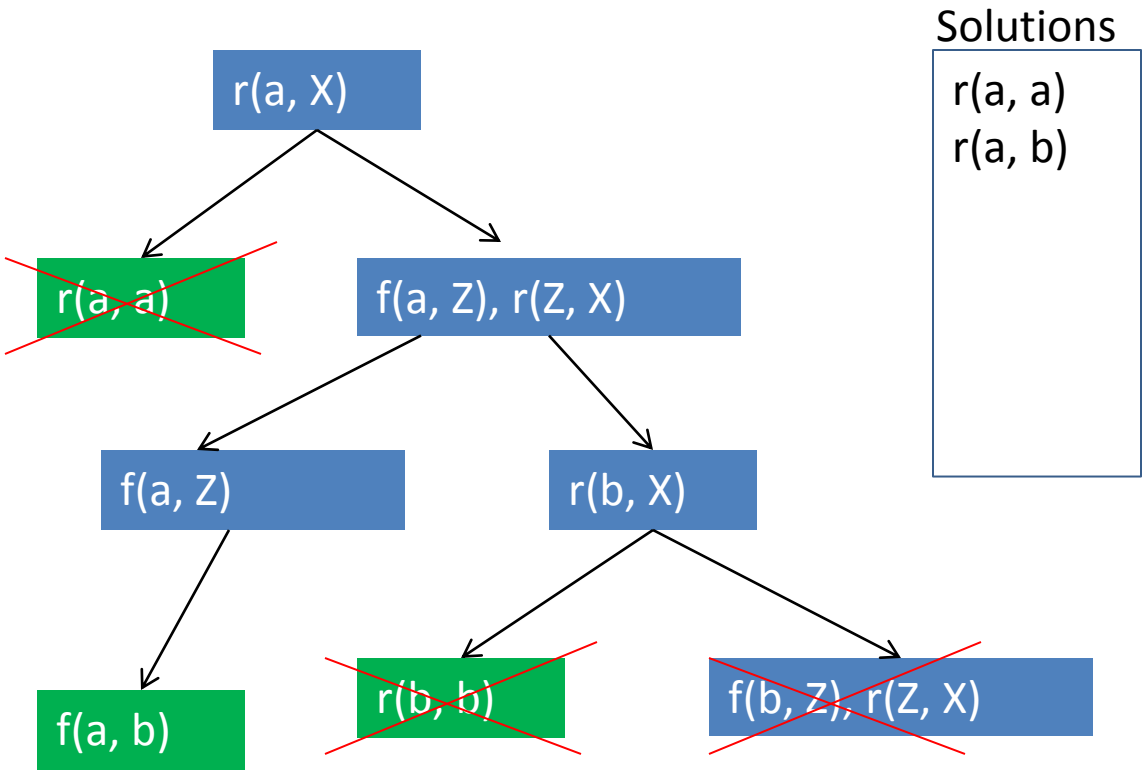

Solutions

r(a, a)
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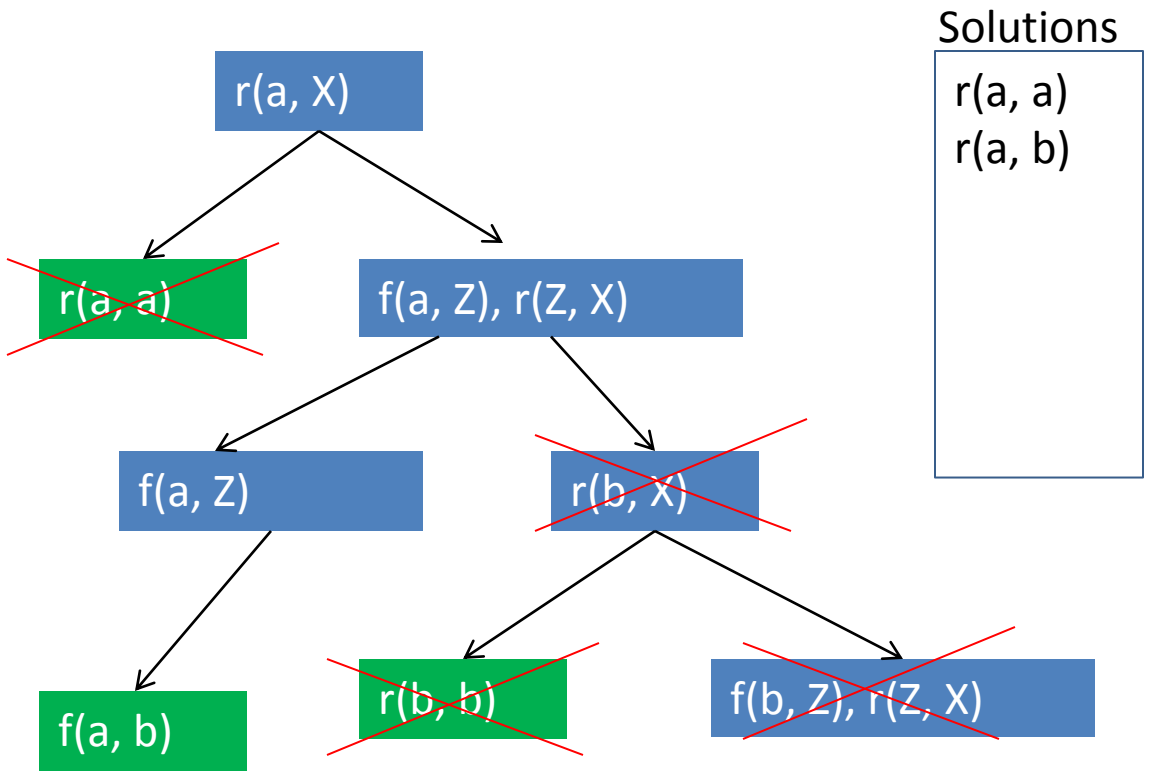


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```

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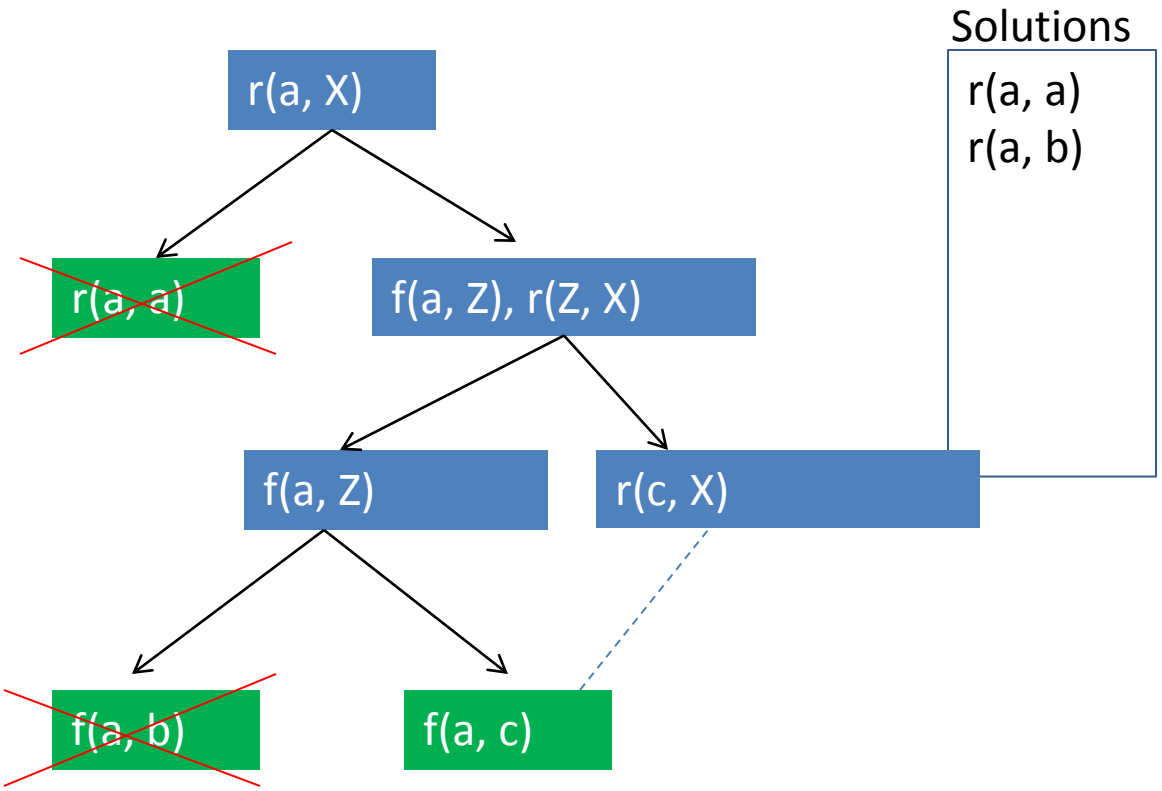

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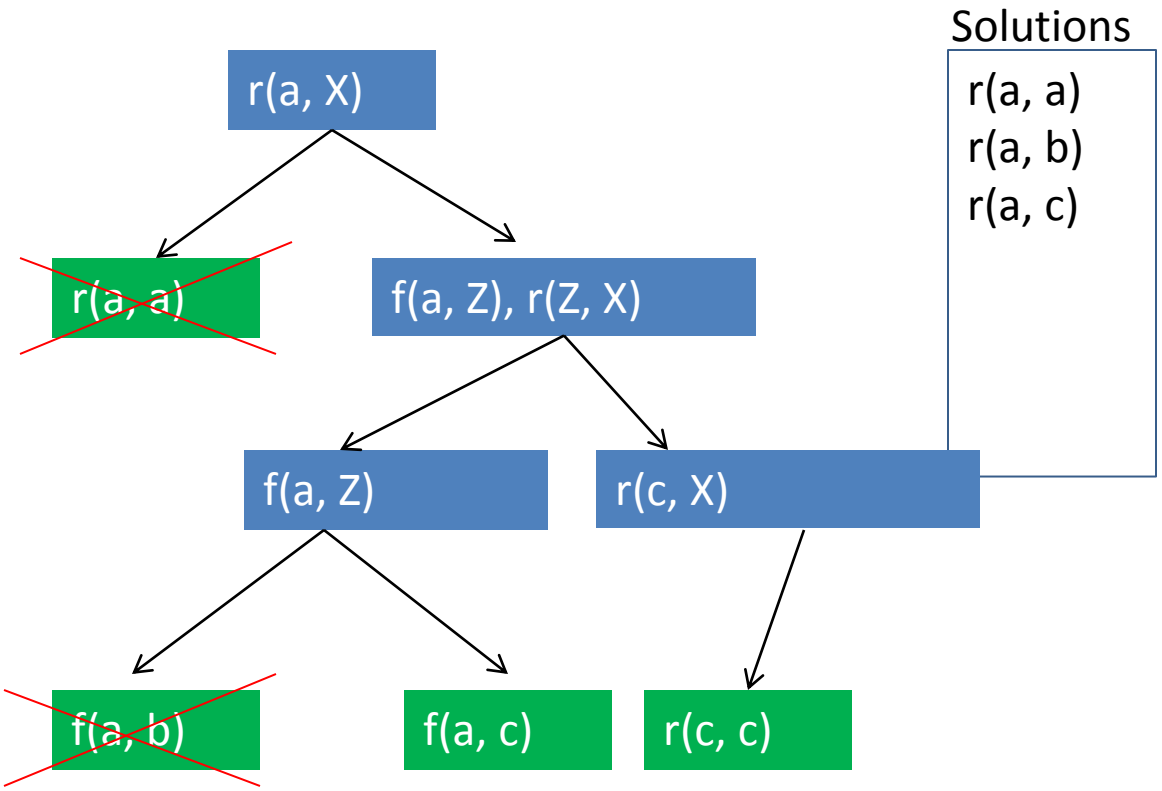

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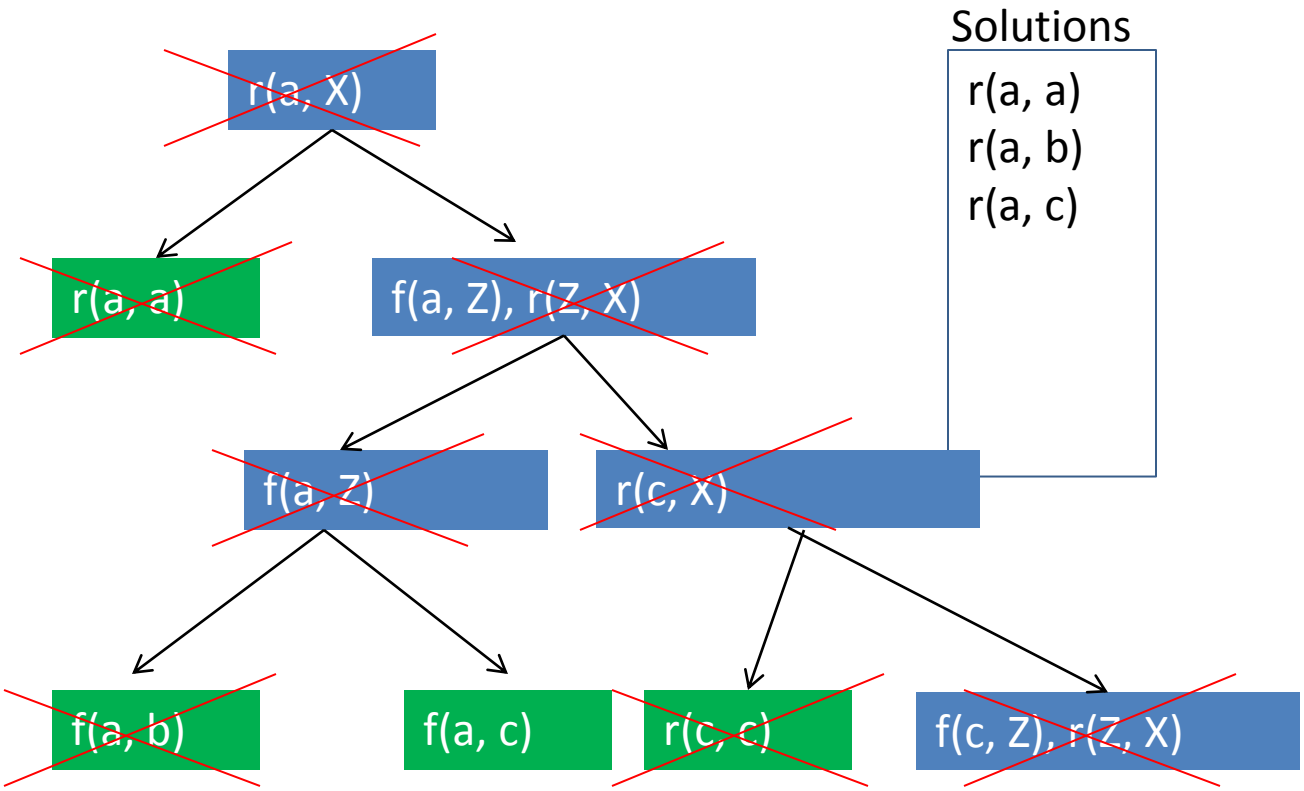


f(a, b).
f(a, c).
r(S, S).
r(S, Y) :- f(S,Z), r(Z, Y).

Computation and Proof Search

- We can think of proof search as performing computation
- The inputs and outputs of the computation happen through assignments made to free variables

Will Prolog's proof search always work?

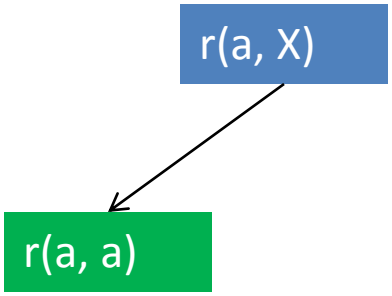
- Old example

```
f(a, b).
f(a, c).
r(S, S).
r(S, Y) :- f(S, Z), r(Z, Y).
```

- New example

```
f(a, b).
f(a, c).
r(S, S).
r(S, Y) :-  r(S, Z), f(Z, Y).
```

Example Flight Planning : “? r(a, X)”



Solutions

r(a, a).

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f(a, b).
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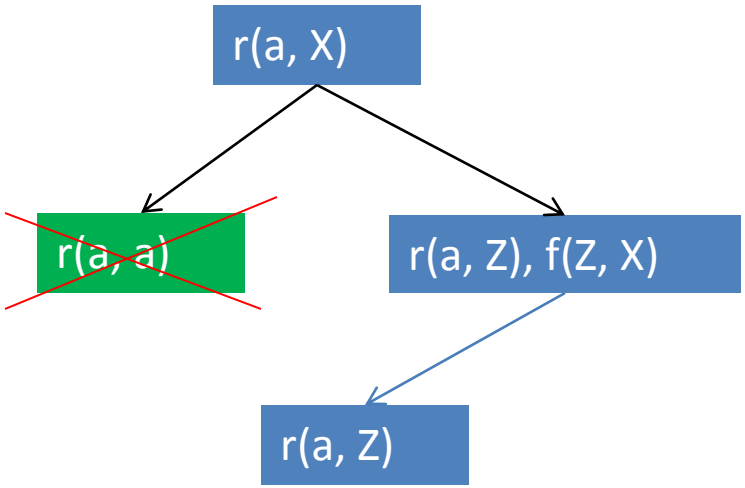
Example Flight Planning : “? r(a, X)”



Solutions

r(a, a).

Infinite loop

```
f(a, b).
f(a, c).
r(S, S).
r(S, Y) :- r(S, Z), f(Z, Y).
```

Soundness and Completeness

- Prolog's proof search procedure is sound but not complete
  - Sound
    - If we can prove something, then it is true in the logical reading of the program
  - Complete
    - We can prove everything that is logically true

Soundness and Completeness: Garbage Collection

- A garbage collection algorithm can be thought of as proving that blocks of memory are garbage
- Example: treat elements which are not reachable from the roots (reference variables and globals)
  - Sound?
  - Complete?

Terms

- The values of Prolog are called terms
- The values a, b, sfo, den, . . . are a special type of term called an atom
- Variables like X, Y are also a type of term

Compound Terms

- Prolog supports terms beyond Datalog
- Can build compound terms
- Syntax: function_symbol(t_1, t_2, …, t_n)
- Examples:
  - f(sfo)
  - f(f(sfo))
  - g(sfo, f(den))
- Lists
  - Special kind of terms
  - [A,B, C]
  - [A, B, C | Tail]
- Terms are not the same as predicates
  - Term: a value: a name of something
  - Predicate: Either true or false

Matching Terms

- Previously we glossed over what it meant for two terms to match because it was obvious:
  - sfo =sfo ☑
  - dal=dal ☑
  - den=dal ☒
  - sfo=chi ☒

Matching Compound Terms

- What happens in the case of compound terms?
  - f(X) =f(3)
  - f(X)=f(f(Y))
  - g(X, Y)=f(3)
  - g(X, Y)=g(Z, Z)
  - g(f(3), Y)= g(h(X), Z))

Matching Compound Terms

- In Prolog we say two terms are unifiable if we can apply a variable substitution such that the two terms become the same.
  - f(X) =f(3) ☑ {X = 3}
  - f(X)=f(f(Y)) ☑ {X = f(Y)}
  - g(X, Y)=f(3) ☒
  - g(X, Y)=g(Z, Z) ☑ {X =Z, Y =Z)}
  - g(f(3), Y)= g(h(X), Z)) ☒

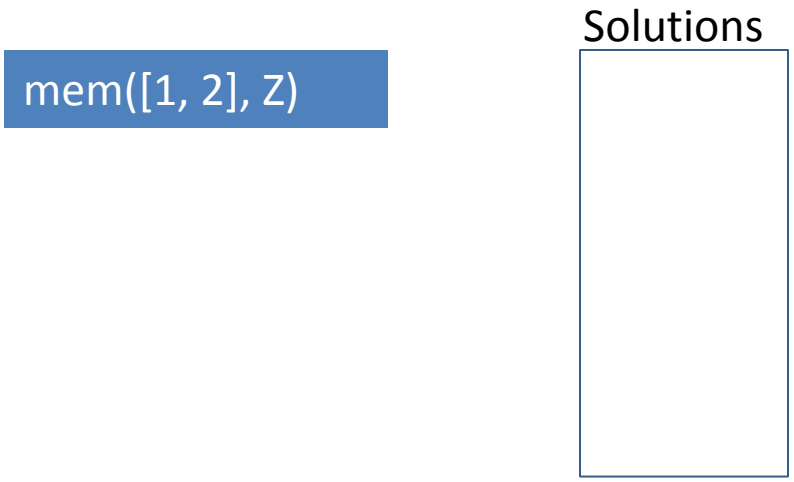
Proofs and Unification

- Lets illustrate how proofs and unication interact using a list membership predicate mem
- mem(Ys, X) holds if X is a member of list Ys

```
% base case
mem([X|_], X).

%recursive case
mem([_|Ys], X) :- mem(Ys, X).
```

List Membership

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```
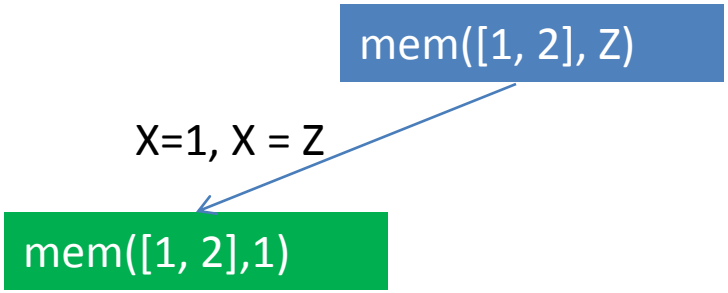

List Membership

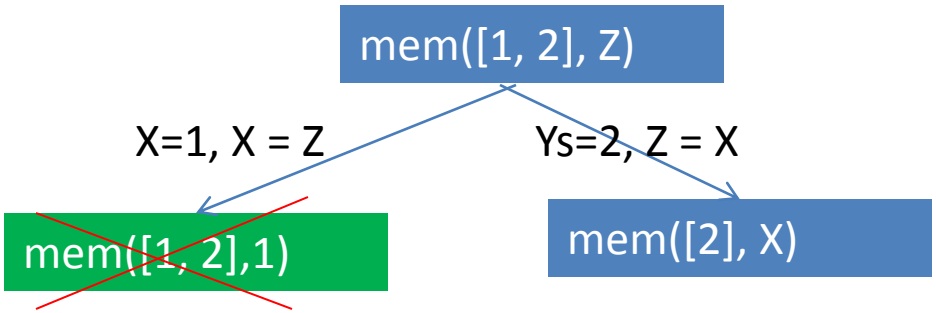


Solutions

mem([1, 2], 1)

```
mem([X|_], X).
mem([_|Ys], X) :- mem(Ys, X).
```

List Membership



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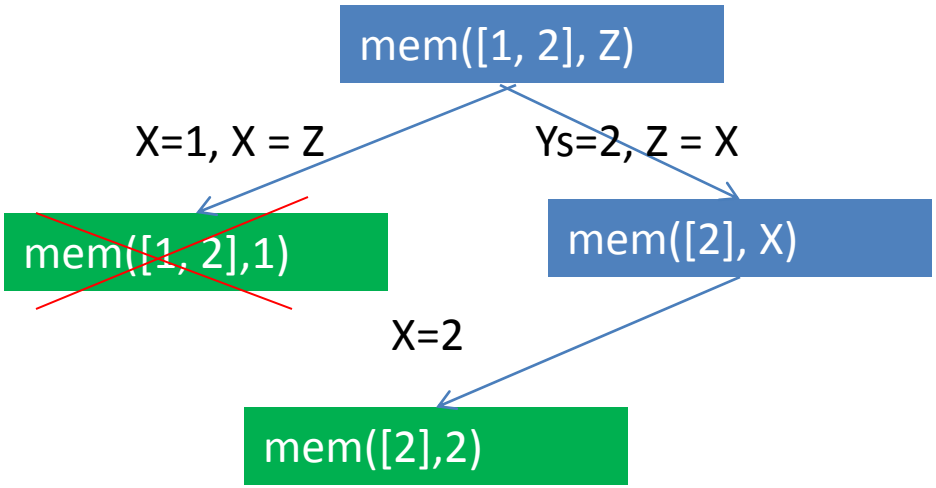
List Membership



Solutions

mem([1, 2], 1)
mem([1, 2], 2)

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mem([X|_], X).
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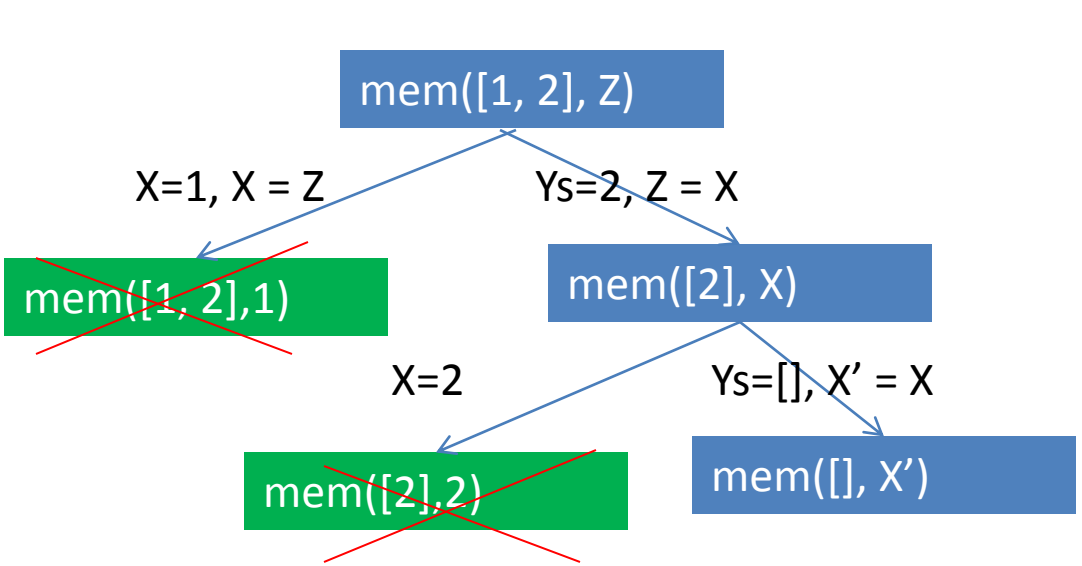
List Membership



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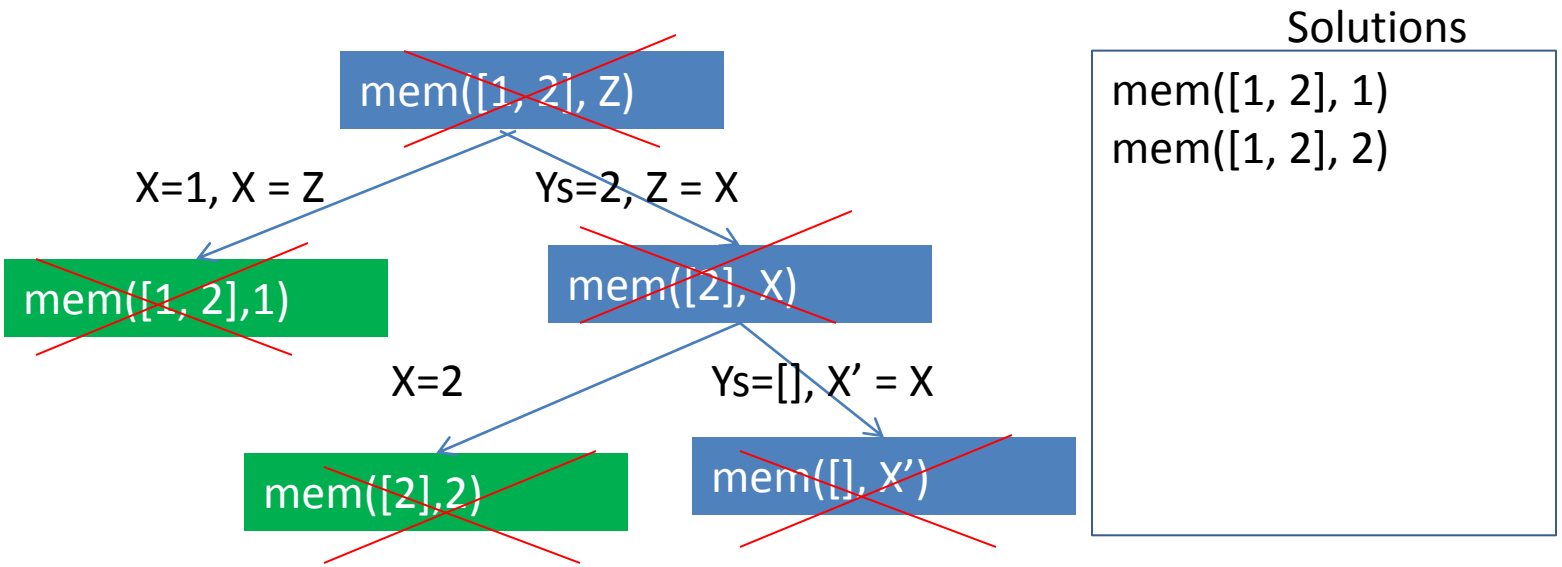
List Membership




```
mem([X|_], X).
mem([_|Ys], X) :- mem(Ys, X).
```

Directionality

- We've seen that mem([1, 2], X) returns 1 and 2
- What happens for mem(X, 42)?
  - X = [42 |_Z1]
  - X = [_Z1, 42| _Z2]
  - …
- Sometimes inversions is useful reachable(X, a).

Negation

- Negation can be useful
- Example: to compute all pairs of disconnected nodes in a graph

```
reachable(S,D) :- link(S,D).
reachable(S,D)  :- link(S,Z), reachable(Z,D).
unreachable(S,D) :- node(S), node(D),  \+reachable(S,D).
```

- Prolog makes a closed-world assumption. That is, things that we cannot prove true are false
- \+ mem([1, 2], 3]
- Datalog usually enforces further restrictions

Arithmetic

- Prolog supports arithmetic on ground terms

```
%base case
length([], 0).
% recursive case
length([H|T], L) :- length(T, L1),
                    L is L1 + 1.
```

N-Queens in Prolog

```
diagsafe(_, _, []).

diagsafe(Row, ColDist, [QR|QRs]) :-  RowHit1 is Row + ColDist, QR =\= RowHit1,
                                     RowHit2 is Row - ColDist, QR =\= RowHit2,
                                     ColDist1 is ColDist + 1,
                                     diagsafe(Row, ColDist1, QRs).

safe_position([_]).
safe_position([QR|QRs]) :- diagsafe(QR, 1, QRs), safe_position(QRs).

nqueens(N, Y) :- sequence(N, X), permute(X, Y), safe_position(Y).
```

Summary Prolog

- Can be powerful when used carefully
  - Inductive definitions
  - Parsers
  - Type checkers
  - Solving games
  - Search problems
- Implements a special case of resolution
- Includes primitive to restrict backtracking and side-effects (cut, fail)
  - Can be hard to understand

Summary Prolog vs. Datalog

- Datalog is more restricted
  - Restricted compound terms
  - Restricted negations
    - Stratified
  - Safe rules
    - Every variable in the rule must occur in a positive (non-negated) relational atom in the rule body
- But easier to understand
- More declarative
- Simpler semantics