SLAM

- A Microsoft tool for checking safety of device drivers
- Inspired BLAST

BLAST

Berkeley Lazy Abstraction Software * Tool

www.eecs.berkeley.edu/~blast/

Counter Example Guided Refinement CEGAR

Mooly Sagiv

Recap

- Many abstract domains
  - Signs
  - Odd/Even
  - Constant propagation
  - Intervals
  - [Polyhedra]
  - Canonic abstraction
  - Domain constructors
  - …
- Static Algorithms
  - Iterative Chaotic Iterations
  - Widening/Narrowing
  - Interprocedural Analysis
  - Concurrency
  - Modularity
  - Non-Iterative methods

A Lattice of Abstractions

- Every element is an abstract domain
- $A \sqsubseteq A'$ if there exists a Galois Connection from A to A'

But how to find the appropriate abstract domain

- Precision vs. Scalability
- Sometimes precision improves scalability
- Specialize the abstraction for the desired property

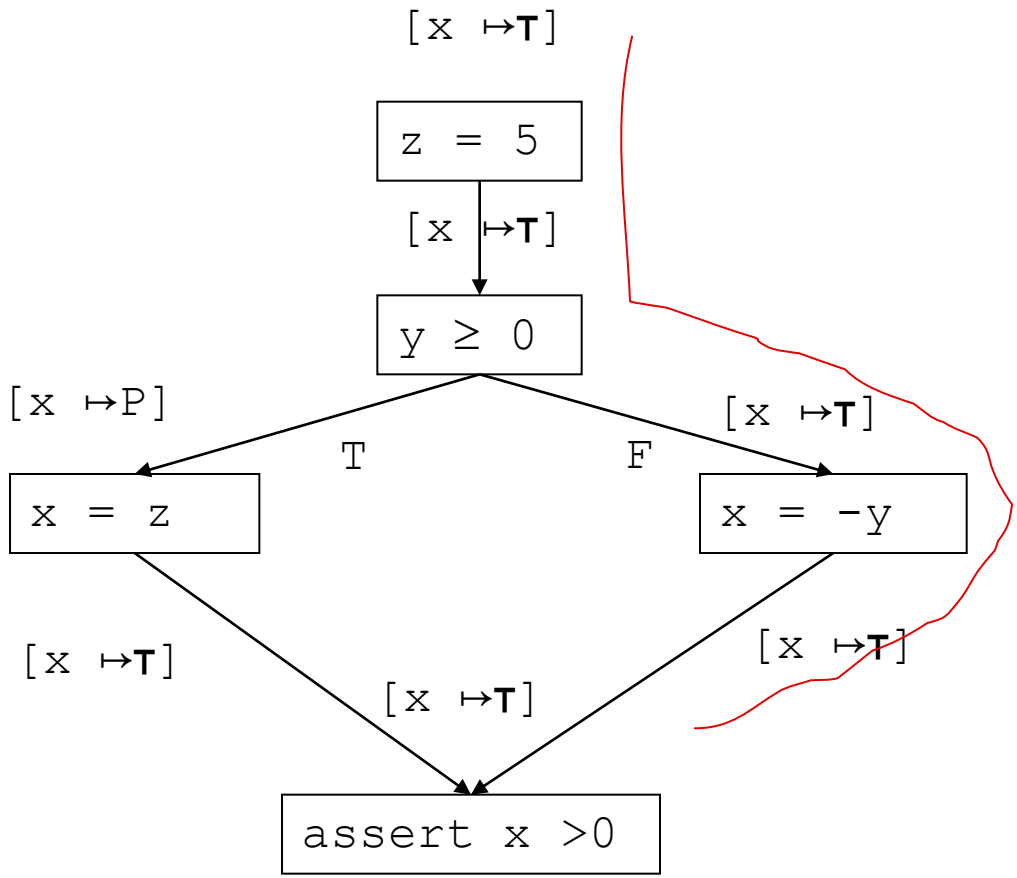
Counter Example Guided Refinement (CEGAR)

- Run the analysis with a simple abstract domain
- When the analysis verifies the property declare done
- If the analysis reports an error employs a theorem prover to identify if the error is feasible
  - If the error is feasible generate a concrete trace
  - If the error is spurious refine the abstract domain and repeat

A Simple Example

```
z =5
if (y >0)
   x = z;
 else
  x = -y;
assert x >0
```

sign(x)



A Simple Example

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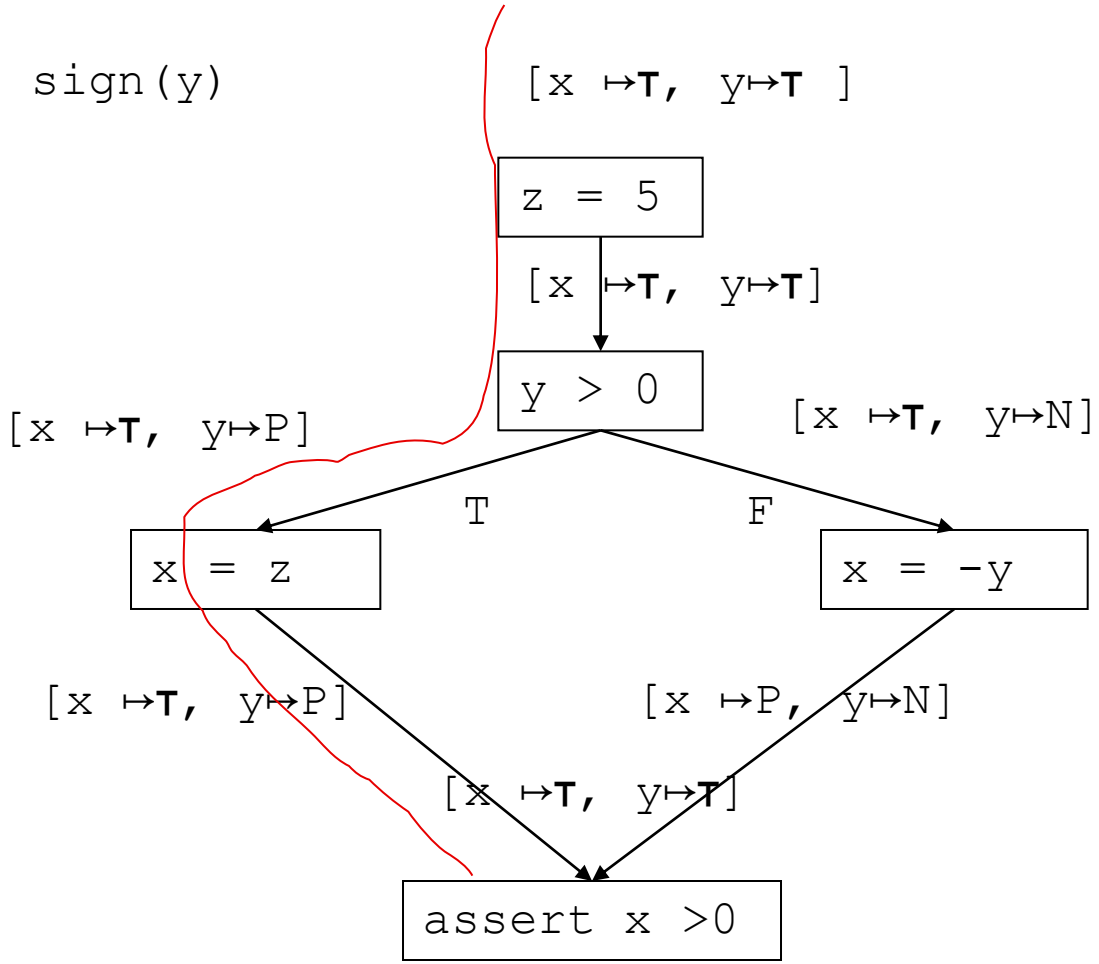
```

sign(x), sign(y)



A Simple Example

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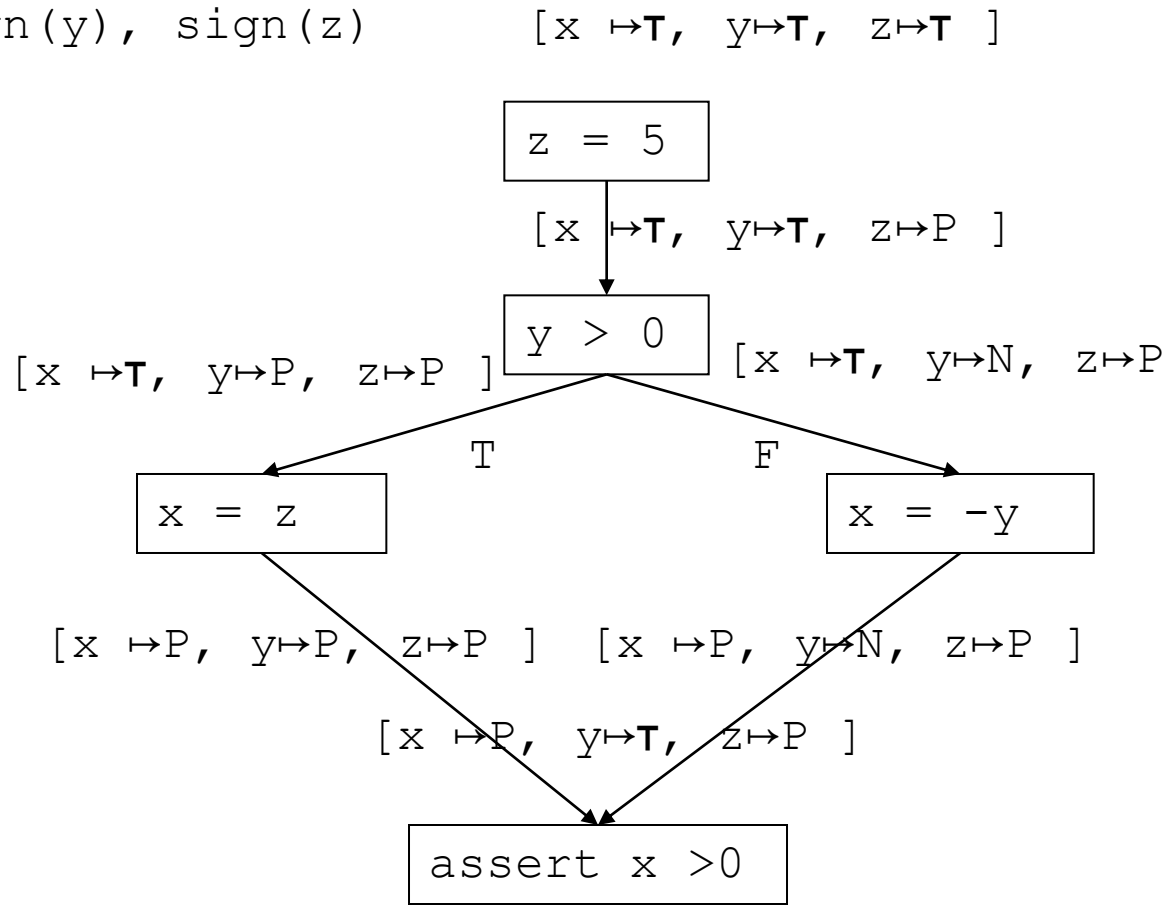
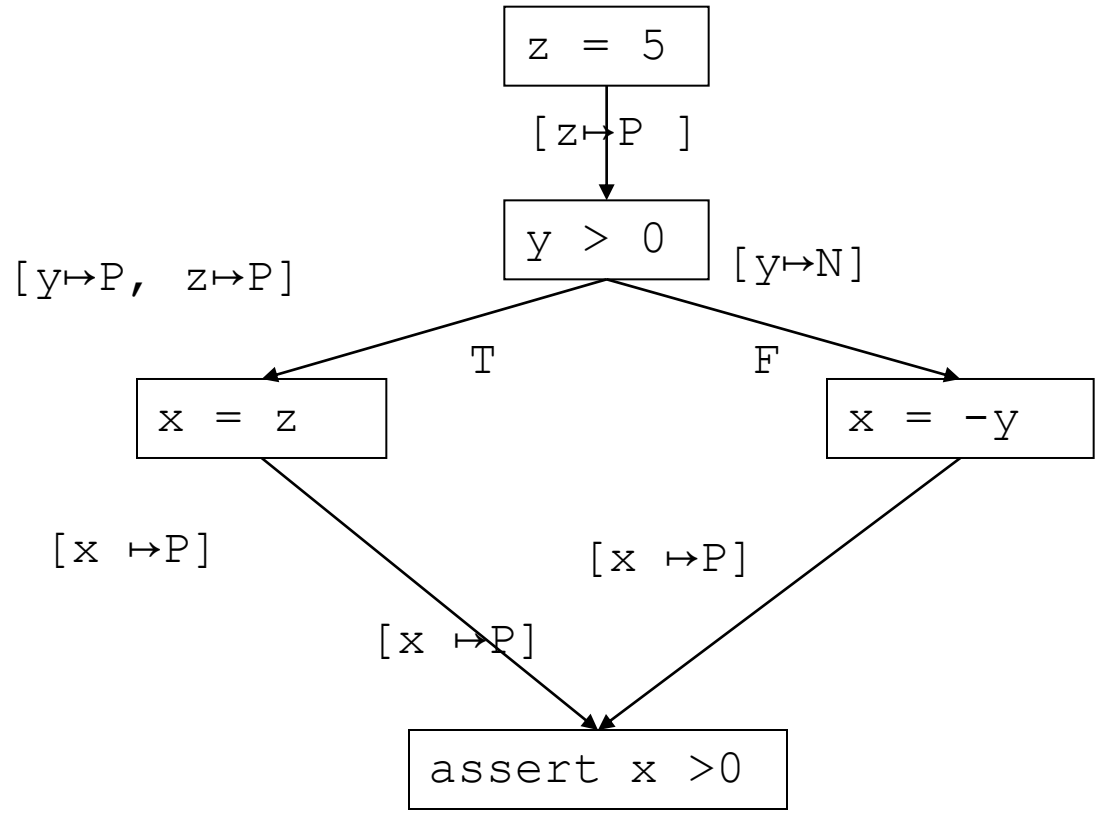


Simple Example (local abstractions)

```
z =5
if (y >0)
   x = z;
 else
  x = -y;
assert x >0
```

sign(x), sign(y), sign(z) []



Plan

- CEGAR in BLAST (inspired by SLAM) POPL'04
- Limitations

Abstractions from Proofs

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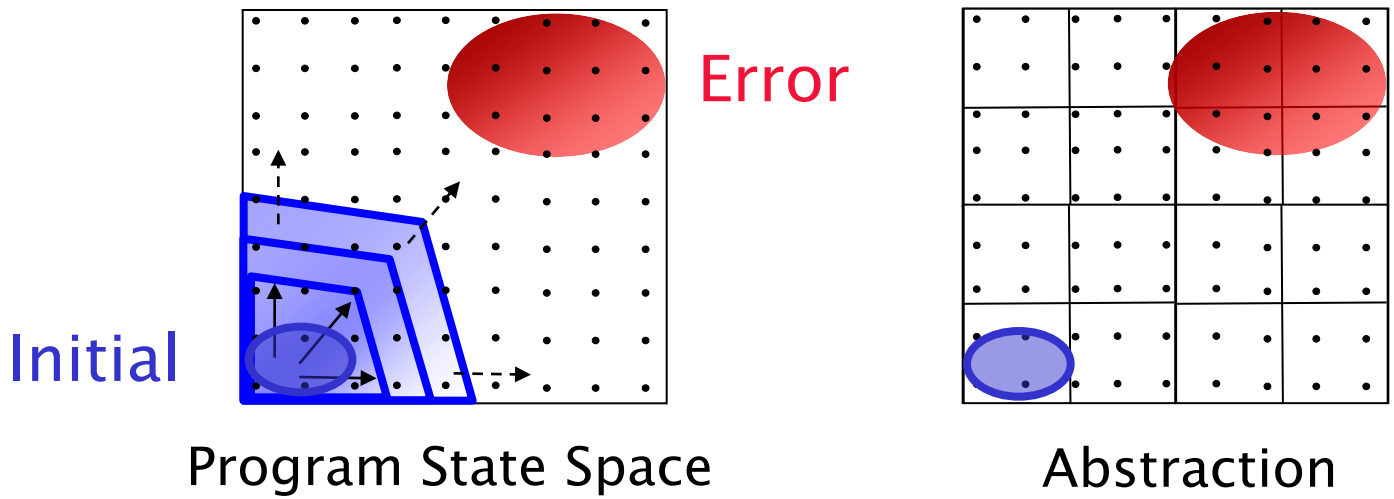
[UC Los Angeles]

Kenneth L. McMillan
[Cadence Berkeley Labs]

Scalable Program Verification

- ***Little theorems*** **about** ***big programs***
  - Partial Specifications
    - Device drivers use kernel API correctly
    - Applications use root privileges correctly
  - Behavioral, path-sensitive properties

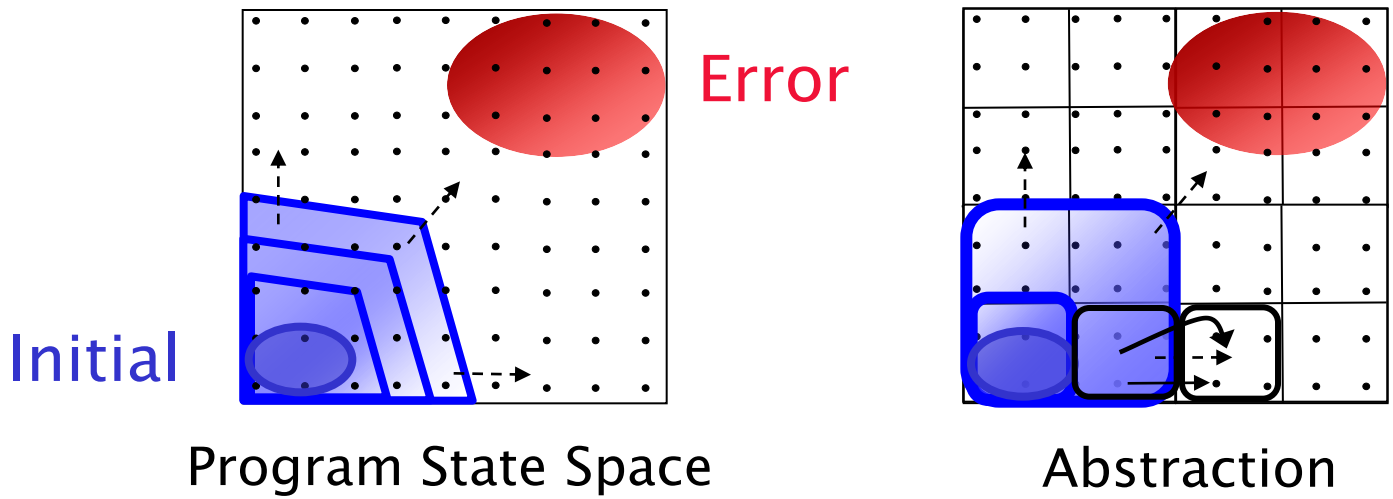
Predicate Abstraction: A crash course



- Abstraction: ***Predicates*** on program state
  - Signs: $x > 0$
  - Aliasing: $\&x \neq \&y$

- States satisfying the same predicates are equivalent
  - Merged into single abstract state

(Predicate) Abstraction: A crash course



Q1 *: Which predicates* are required to verify a property ?

The Predicate Abstraction Domain

- Fixed set of predicates Pred
- The relational domain is
  $<P(P(Pred)), \emptyset, P(Pred), \cup, \cap>$
  - Join is set union
  - State space explosion
- Special case of canonic abstraction

Scalability vs. Verification

- Few predicates tracked
  - *e.g.* type of variables
- Imprecision hinders Verification
  - Spurious counterexamples
- Many predicates tracked
  - *e.g.* values of variables
- State explosion
  - Analysis drowned in detail

Example

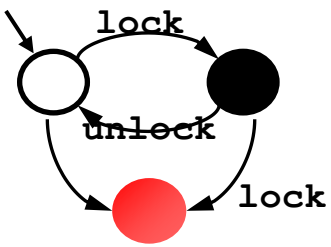
scalability

Only track *lock*

```
while(*){
1: if (p1) lock();
   if (p1) unlock()
                 …
2: if (p2) lock();
   if (p2) unlock()
                 …
n: if (pn) lock();
   if (pn) unlock()
}
```

T ● *F*

T ●



Bogus Counterexample

- Must *correlate* branches

Predicate p_1 makes trace *abstractly infeasible*

p_i required for verification

Example

scalability

Only track *lock*

```
while(*){
1: if (p1) lock();
   if (p1) unlock()
                 …
2: if (p2) lock();
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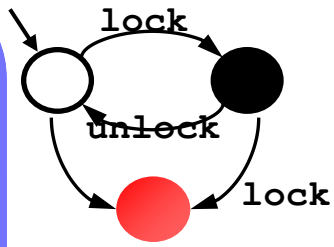


verification

Track *lock*, p_i s

Bogus Counterexample

- Must *correlate* branches

State Explosion

- $> 2^n$ distinct states
- intractable

How can we get scalable verification ?

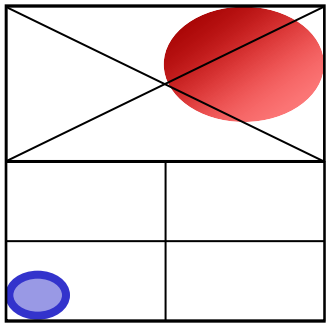
By Localizing Precision

p_1

p_2

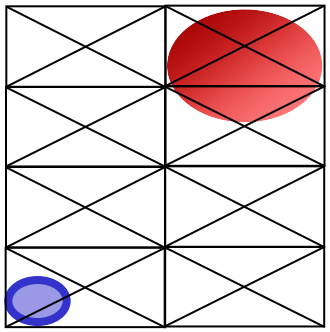
p_n

```
while (*) {
1: if (p1) lock();
   if (p1) unlock();
           …
2: if (p2) lock();
   if (p2) unlock();
           …
n: if (pn) lock();
   if (pn) unlock();
}
```



Preds. Used locally

Ex: 2 £ n states

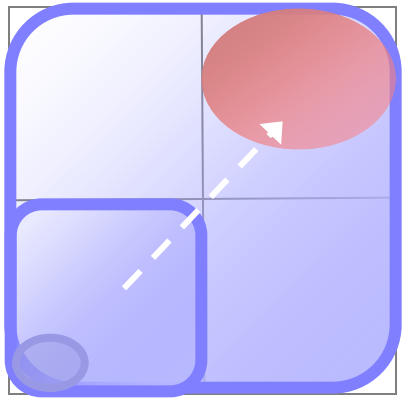


Preds. used globally

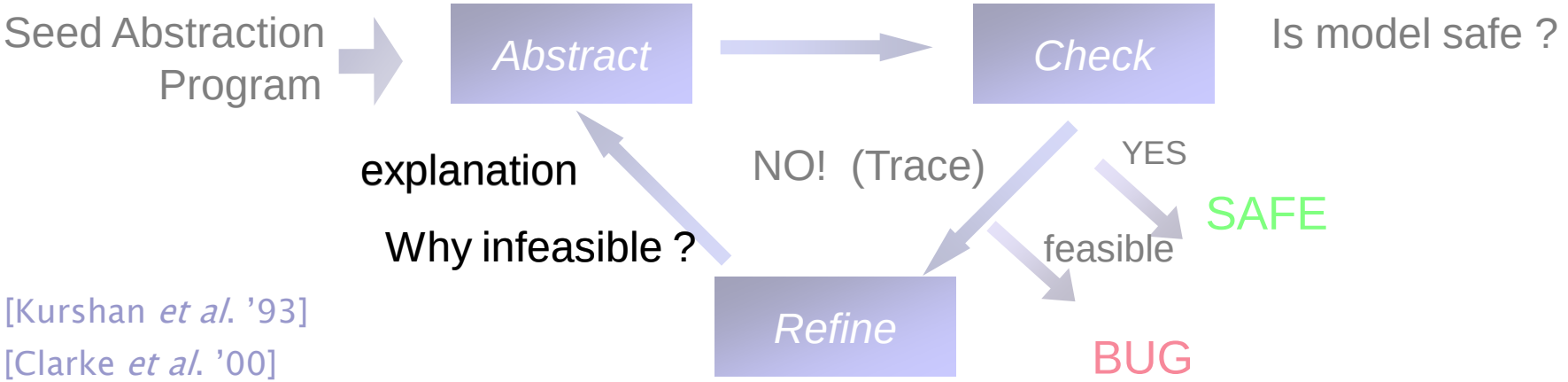
Ex: 2^n states

Q2: *Where* are the predicates required ?

Counterexample Guided Refinement

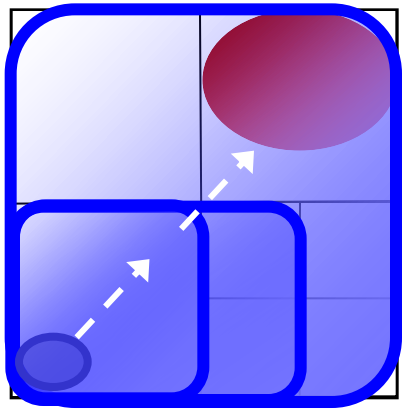


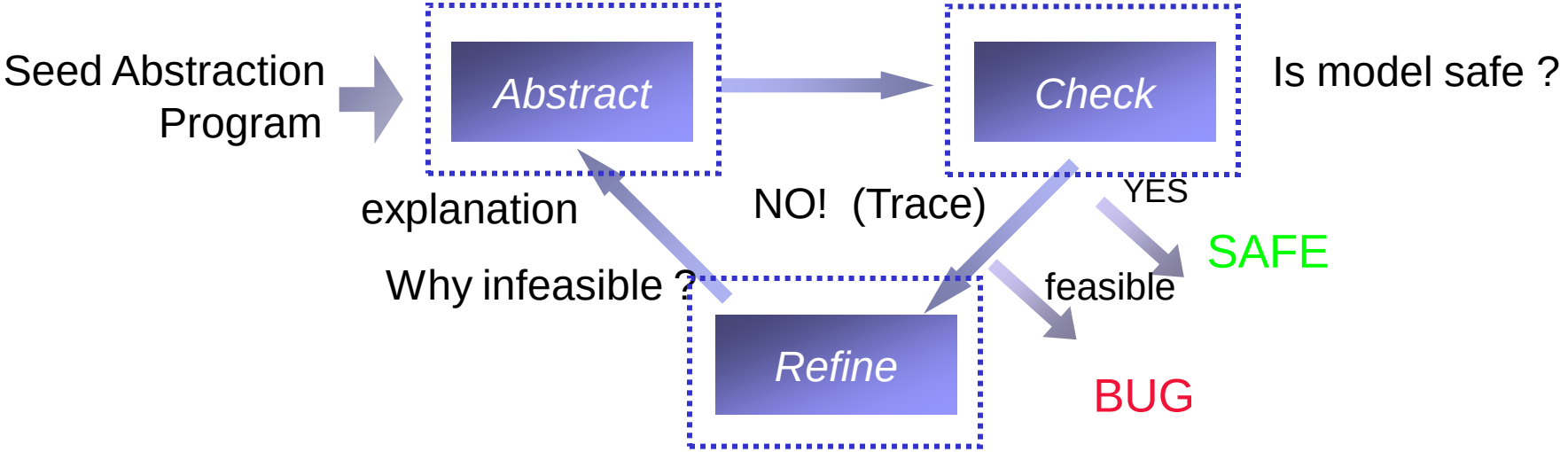
1. *What predicates* remove trace ?
   - Make it abstractly infeasible
2. *Where* are predicates needed ?



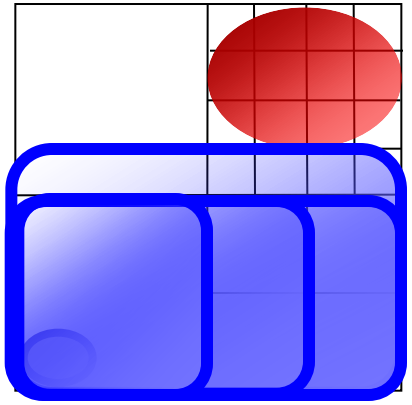
[Kurshan *et al.* '93]
[Clarke *et al.* '00]
[Ball, Rajamani '01]

Counterexample Guided Refinement

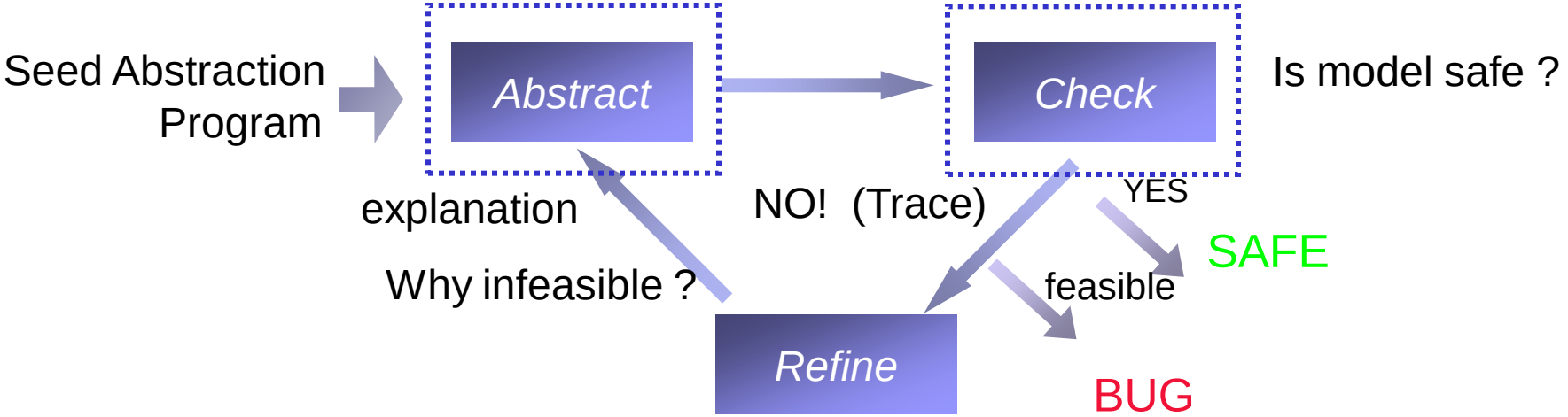




Counterexample Guided Refinement

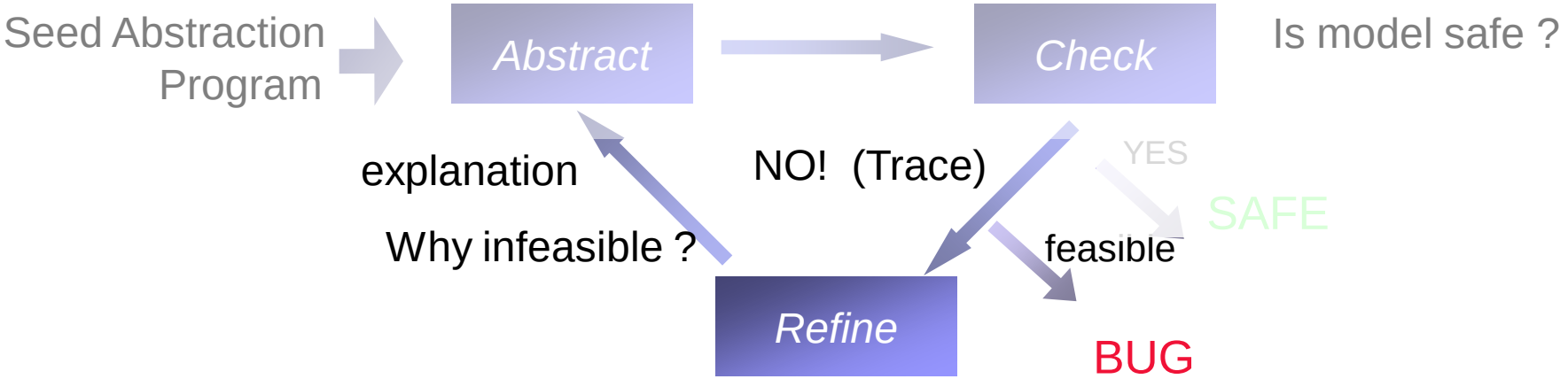


safe



This Talk: Counterexample Analysis

1. *What predicates* remove trace ?
   - Make it abstractly infeasible
2. *Where* are predicates needed ?



Plan

1. Motivation

2. Refinement using Traces
   - Simple
   - Procedure calls

3. Results

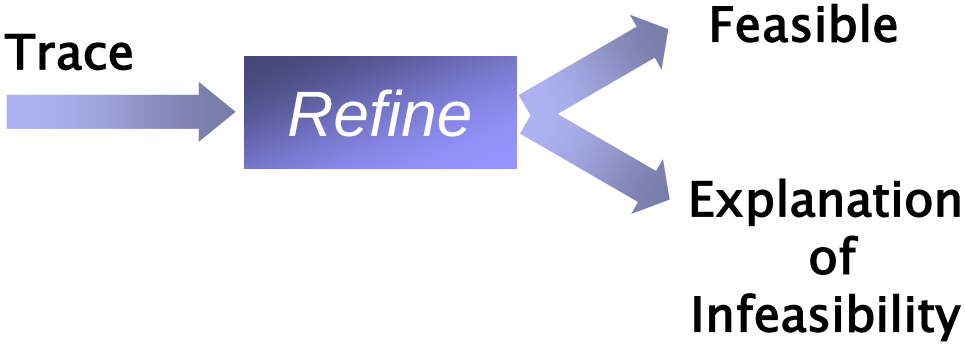
Trace Formulas

- A single abstract trace represents infinite number of traces
  - Different loop iterations
  - Different concrete values
- Solution
  - Only considers concrete traces with the same number of executions
  - Use formulas to represent sets of states

Representing States as *Formulas*

$[F]$ states satisfying F $\{s \mid s \models F\}$	F FO formula over prog. vars
$[F_1] \cap [F_2]$	$F_1 \wedge F_2$
$[F_1] \cup [F_2]$	$F_1 \vee F_2$
$\overline{[F]}$	$\neg F$
$[F_1] \subseteq [F_2]$	F_1 implies F_2 i.e. $F_1 \wedge \neg F_2$ unsatisfiable

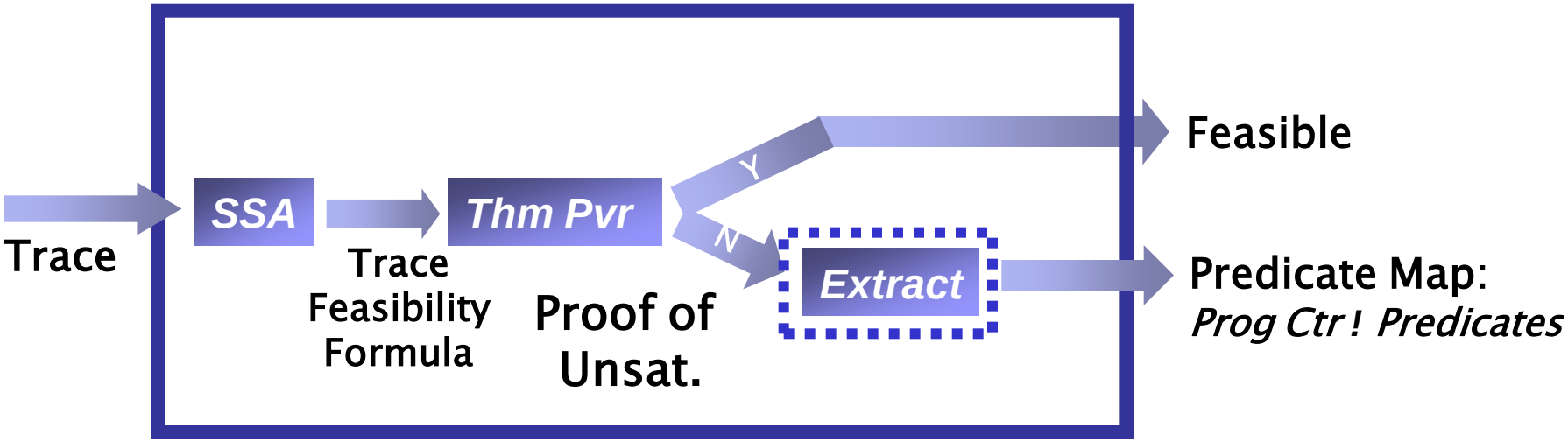
Counterexample Analysis



Q0: Is trace feasible ?

Q1: What predicates remove trace ?

Q2: *Where* are preds required ?



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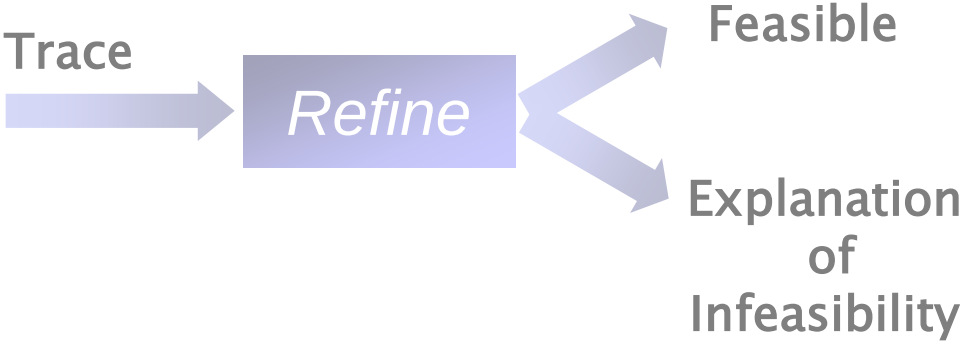

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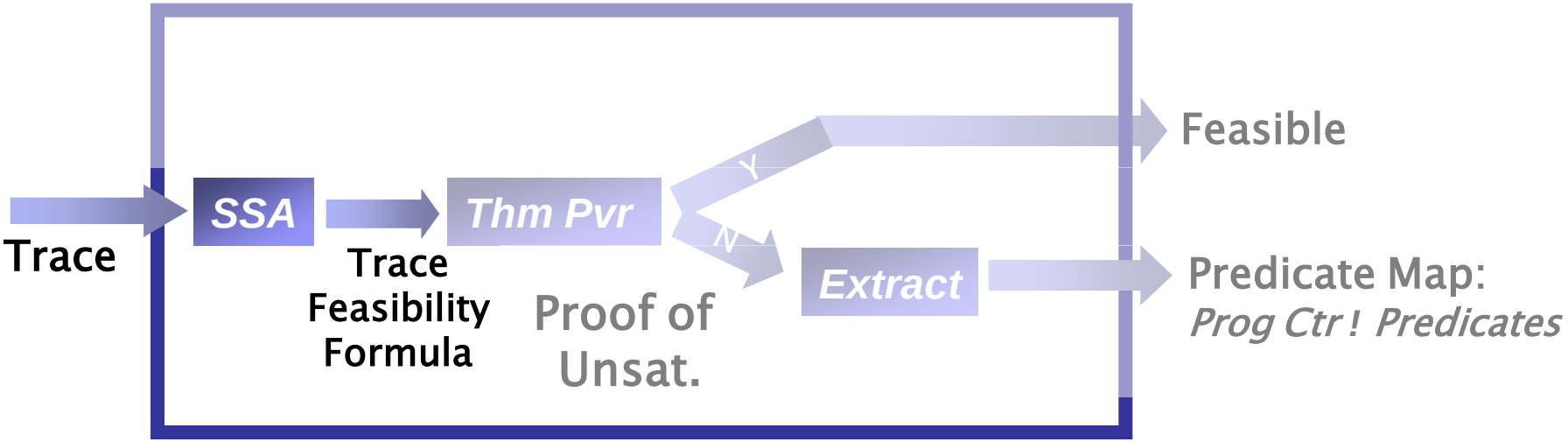


Traces

```
pc1: x = ctr;
pc2: ctr = ctr + 1;
pc3: y = ctr;
pc4: if (x = i-1){
pc5:   if (y != i){
        ERROR: }
     }
```

```
pc1: x = ctr
pc2: ctr = ctr + 1
pc3: y = ctr
pc4: assume(x = i-1)
pc5: assume(y ≠ i)
```

$y = x + 1$

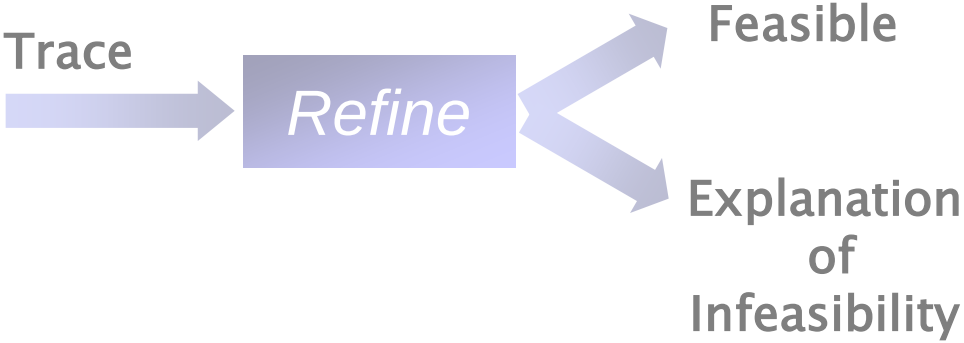
Trace Feasibility Formulas

Trace	SSA Trace	Trace Feasibility Formula
pc_1: `x = ctr`	pc_1: x_1 = ctr_0	$x_1 = ctr_0$
pc_2: `ctr = ctr+1`	pc_2: ctr_1 = ctr_0+1	$\wedge\ ctr_1 = ctr_0 + 1$
pc_3: `y = ctr`	pc_3: y_1 = ctr_1	$\wedge\ y_1 = ctr_1$
pc_4: `assume(x=i-1)`	pc_4: assume(x_1=i_0-1)	$\wedge\ x_1 = i_0 - 1$
pc_5: `assume(y≠i)`	pc_5: assume($y_1 \neq i_0$)	$\wedge\ y_1 \neq i_0$

Theorem: Trace is ***Feasible***, TFF is ***Satisfiable***

Compact Verification Conditions [Flanagan,Saxe '00]

Counterexample Analysis



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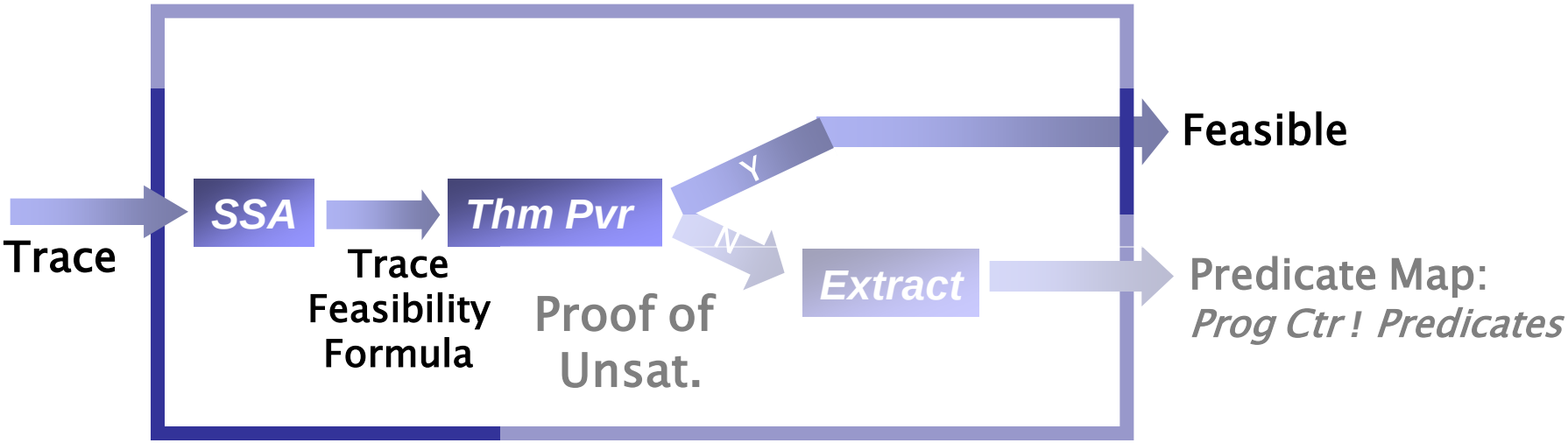
Q1: What predicates remove trace ?

Q2: Where are preds required ?



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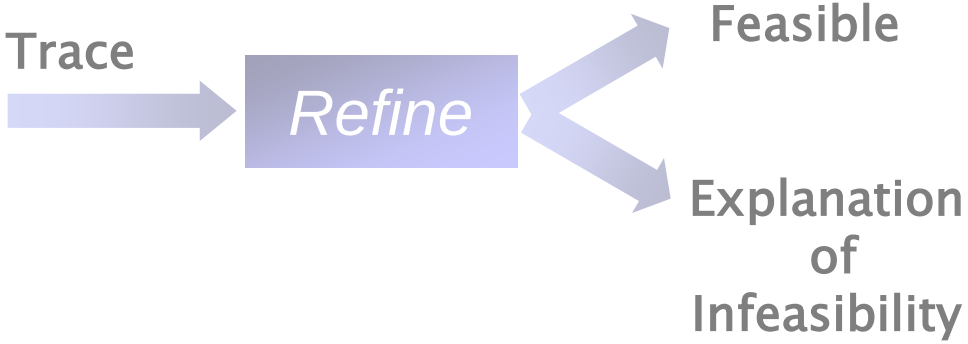

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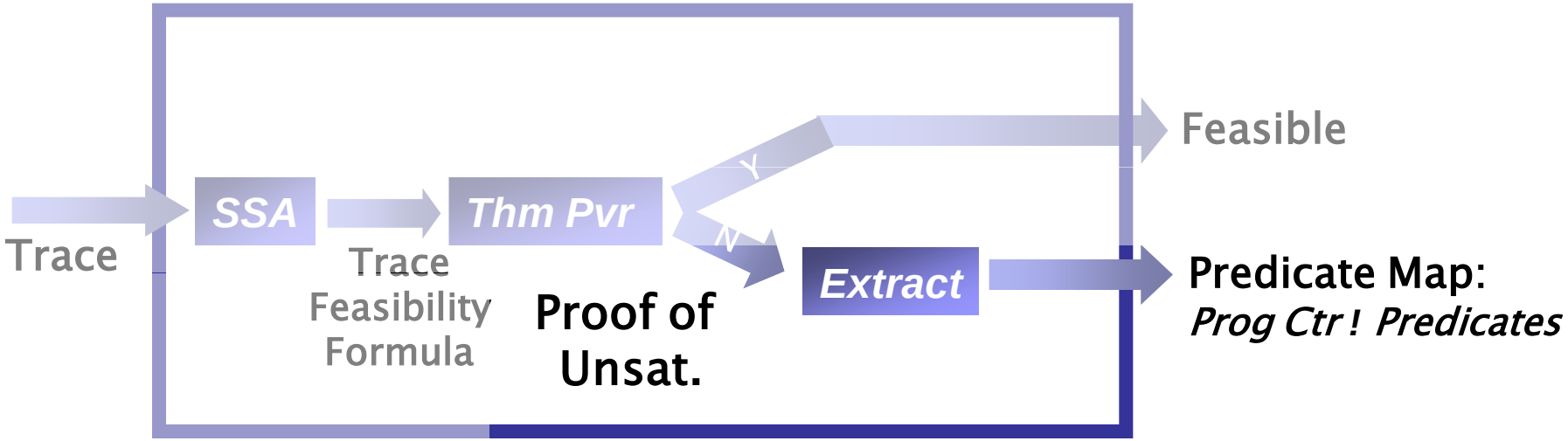
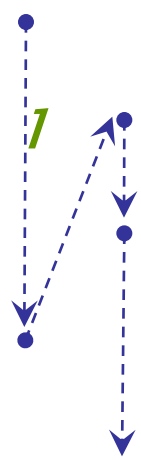


Proof of Unsatisfiability

$$x_1 = ctr_0$$
$$\wedge ctr_1 = ctr_0 + 1$$
$$\wedge y_1 = ctr_1$$
$$\wedge x_1 = i_0 - 1$$
$$\wedge y_1 \neq i_0$$



Trace Formula

$$\dfrac{\dfrac{\dfrac{\dfrac{x_1 = ctr_0 \quad x_1 = i_0 - 1}{ctr_0 = i_0 - 1} \quad ctr_1 = ctr_0 + 1}{ctr_1 = i_0} \quad y_1 = ctr_1}{y_1 = i_0} \quad y_1 \neq i_0}{;}$$

Proof of Unsatisfiability

PROBLEM

Proof uses entire *history* of execution

- Information flows up and down

No *localized* or *state* information !

The Present State…

Trace

```
pc1: x = ctr
pc2: ctr = ctr + 1
pc3: y = ctr
pc4: assume(x = i-1)
pc5: assume(y ≠ i)
```

… is all the information the executing program has ***here***

State…

1. … after executing trace ***prefix***
2. … knows ***present values*** of variables
3. … makes trace ***suffix*** infeasible

At pc_4, which predicate on ***present state*** shows infeasibility of ***suffix***?

What Predicate is needed ?

Trace

pc_1: `x = ctr`

pc_2: `ctr = ctr + 1`

pc_3: `y = ctr`

pc_4: `assume(x = i-1)`

pc_5: `assume(y ≠ i)`

Trace Formula (TF)

$$
\begin{aligned}
& x_1 = ctr_0 \\
\wedge\ & ctr_1 = ctr_0 + 1 \\
\wedge\ & y_1 = ctr_1 \\
\wedge\ & x_1 = i_0 - 1 \\
\wedge\ & y_1 \neq i_0
\end{aligned}
$$

State…

1. … after executing trace *prefix*
2. … has *present values* of variables
3. … makes trace *suffix* infeasible

Predicate …

… implied by TF *prefix*

What Predicate is needed ?

Trace

pc_1: `x = ctr`

pc_2: `ctr = ctr + 1`

pc_3: `y = ctr`

pc_4: `assume(x = i-1)`

pc_5: `assume(y ≠ i)`

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State…

1. … after executing trace *prefix*
2. … has *present values* of variables
3. … makes trace *suffix* infeasible

Predicate …

… implied by TF *prefix*

… on *common* variables

What Predicate is needed ?

Trace

```
pc1: x = ctr
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Trace Formula (TF)

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State…

1. … after executing trace *prefix*
2. … has *present values* of variables
3. … makes trace *suffix* infeasible

Predicate …

… implied by TF *prefix*

… on *common* variables

… & TF *suffix* is *unsatisfiable*

What Predicate is needed ?

Trace

pc_1: `x = ctr`

pc_2: `ctr = ctr + 1`

pc_3: `y = ctr`

pc_4: `assume(x = i-1)`

pc_5: `assume(y ≠ i)`

Trace Formula (TF)

$$x_1 = ctr_0$$
$$\wedge \quad ctr_1 = ctr_0 + 1$$
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$$\wedge \quad x_1 = i_0 - 1$$
$$\wedge \quad y_1 \neq i_0$$

State…

1. … after executing trace *prefix*
2. … knows *present values* of variables
3. … makes trace *suffix* infeasible

Predicate …

… implied by TF *prefix*

… on *common* variables

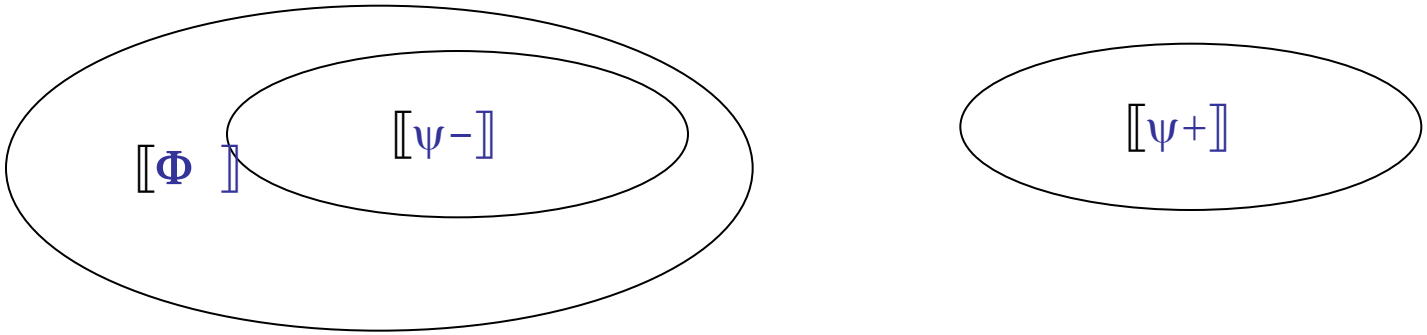
… & TF *suffix* is *unsatisfiable*

Craig's Interpolation Theorem [Craig '57]

Given formulas ψ^- , ψ^+ s.t. $\psi^- \wedge \psi^+$ is ***unsatisfiable***

There exists an ***Interpolant*** Φ for ψ^- , ψ^+ , s.t.

1. ψ^- ***implies*** Φ
2. Φ has symbols ***common*** to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is ***unsatisfiable***



Examples of Craig's Interpolation

- $\psi^- = b \wedge (\neg b \vee c)$
  $\psi^+ = \neg c$
- $\psi^- = x_1 = ctr_0 \wedge ctr_1 = ctr_0 + 1 \wedge y_1 = ctr_1$
  $\psi^+ = x_1 = i_0 - 1 \wedge y_1 \neq i_0$
  - $y_1 = x_1 + 1$

Craig's Interpolation Theorem [Craig '57]

Given formulas ψ^- , ψ^+ s.t. ψ^-Æ ψ^+ is ***unsatisfiable***

There exists an ***Interpolant*** Φ for ψ^- , ψ^+ , s.t.

1. ψ^- ***implies*** Φ
2. Φ has only symbols ***common*** to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is ***unsatisfiable***

Φ computable from ***Proof of Unsat.*** of $\psi^- \wedge \psi^+$

[Krajicek '97] [Pudlak '97]

(boolean) SAT-based Model Checking [McMillan '03]

Interpolant = Predicate !

Trace	Trace Formula	
pc_1: `x = ctr`	$x_1 = ctr_0$	
pc_2: `ctr = ctr + 1`	$\wedge \quad ctr_1 = ctr_0 + 1$	ψ^-
pc_3: `y = ctr`	$\wedge \quad y_1 = ctr_1$	
pc_4: `assume(x = i-1)`	$\wedge \quad x_1 = i_0 - 1$	ψ^+
pc_5: `assume(y ≠ i)`	$\wedge \quad y_1 \neq i_0$	

Interpolate → Φ

Require:

1. Predicate *implied* by trace *prefix*
2. Predicate on *common* variables
   common = *current* value
3. Predicate & *suffix* yields a *contradiction*

Interpolant:

1. ψ^- *implies* Φ
2. Φ has symbols *common* to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is *unsatisfiable*

Interpolant = Predicate !

Trace	Trace Formula	
pc_1: `x = ctr`	$x_1 = ctr_0$	
pc_2: `ctr = ctr + 1`	$\wedge \quad ctr_1 = ctr_0 + 1$	ψ^-
pc_3: `y = ctr`	$\wedge \quad y_1 = ctr_1$	
pc_4: `assume(x = i-1)`	$\wedge \quad x_1 = i_0 - 1$	ψ^+
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Interpolate → Φ

$y_1 = x_1 + 1$

Require:

1. Predicate *implied* by trace *prefix*
2. Predicate on *common* variables
3. Predicate & *suffix* yields a *contradiction*

Interpolant:

1. ψ^- *implies* Φ
2. Φ has symbols *common* to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is *unsatisfiable*

Interpolant = Predicate !

Trace	Trace Formula	
pc_1: `x = ctr`	$x_1 = ctr_0$	
pc_2: `ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$	ψ^-
pc_3: `y = ctr`	$\wedge\ y_1 = ctr_1$	
pc_4: `assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$	ψ^+
pc_5: `assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$	

Interpolate → Φ

$y_1 = x_1 + 1$

Predicate at pc_4:
$y = x + 1$

Require:

1. Predicate *implied* by trace *prefix*
2. Predicate on *common* variables
3. Predicate & *suffix* yields a *contradiction*

Interpolant:

1. ψ^- *implies* Φ
2. Φ has symbols *common* to ψ^-, ψ^+
3. Φ Æ ψ^+ is *unsatisfiable*

Building Predicate Maps

Predicate Map
pc_2: $x = ctr$

Trace	Trace Formula
pc_1: `x = ctr`	$x_1 = ctr_0$
pc_2: `ctr = ctr + 1`	$\wedge \quad ctr_1 = ctr_0 + 1$
pc_3: `y = ctr`	$\wedge \quad y_1 = ctr_1$
pc_4: `assume(x = i-1)`	$\wedge \quad x_1 = i_0 - 1$
pc_5: `assume(y ≠ i)`	$\wedge \quad y_1 \neq i_0$

ψ^-

Interpolate → $x_1 = ctr_0$

ψ^+

- Cut + Interpolate at *each* point
- Pred. Map: $pc_i \mapsto$ Interpolant from cut i

Building Predicate Maps

Predicate Map

pc_2: $x = ctr$

pc_3: $x = ctr - 1$

Trace	Trace Formula
pc_1: `x = ctr`	$x_1 = ctr_0$
pc_2: `ctr = ctr + 1`	$\wedge \quad ctr_1 = ctr_0 + 1$
pc_3: `y = ctr`	$\wedge \quad y_1 = ctr_1$
pc_4: `assume(x = i-1)`	$\wedge \quad x_1 = i_0 - 1$
pc_5: `assume(y ≠ i)`	$\wedge \quad y_1 \neq i_0$

ψ^-

ψ^+

Interpolate → $x_1 = ctr_1 - 1$

- Cut + Interpolate at *each* point
- Pred. Map: $pc_i \mapsto$ Interpolant from cut i

Building Predicate Maps

Trace		Trace Formula
pc_1: `x = ctr`		$x_1 = ctr_0$
pc_2: `ctr = ctr + 1`	$\wedge$	$ctr_1 = ctr_0 + 1$
pc_3: `y = ctr`	$\wedge$	$y_1 = ctr_1$
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pc_5: `assume(y ≠ i)`	$\wedge$	$y_1 \neq i_0$

ψ^-

ψ^+

Interpolate $\rightarrow$ $y_1 = i_0$

Predicate Map

pc_2: $x = ctr$

pc_3: $x = ctr-1$

pc_4: $y = x+1$

pc_5: $y = i$

- Cut + Interpolate at *each* point
- Pred. Map: $pc_i \mapsto$ Interpolant from cut i

Building Predicate Maps

Trace	Trace Formula
pc_1: `x = ctr`	$x_1 = ctr_0$
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pc_5: `assume(y ≠ i)`	$\wedge \quad y_1 \neq i_0$

Predicate Map

pc_2: $x = ctr$
pc_3: $x = ctr-1$
pc_4: $y = x+1$
pc_5: $y = i$

Theorem: *Predicate map* makes trace *abstractly infeasible*

Plan

1. Motivation

2. Refinement using Traces
   - Simple
   - Procedure calls

3. Results

Traces with Procedure Calls

Trace **Trace Formula**

pc_1: $x_1 = 3$
pc_2: assume $(x_1>0)$
pc_3: $x_3 = f_1(x_1)$
pc_4: $y_1 = y_1$
pc_5: $y_3 = f_2(y_2)$
pc_6: $z_2 = z_1+1$
pc_7: $z_3 = 2*z_2$
pc_8: return z_3
pc_9: return y_3
pc_{10}: $x_4 = x_3+1$
pc_{11}: $x_5 = f_3(x_4)$
pc_{12}: assume $(w_1<5)$
pc_{13}: return w_1
pc_{14}: assume $x_4>5$
pc_{15}: assume $(x_1=x_3+2)$

pc_3: $x_3 = f_1(x_1)$
pc_5: $y_3 = f_2(y_2)$
pc_6: $z_2 = z_1+1$
pc_7: $z_3 = 2*z_2$
pc_8: return z_3
pc_9: return y_3
pc_{11}: $x_5 = f_3(x_4)$
pc_{12}: assume $(w_1<5)$
pc_{13}: return w_1

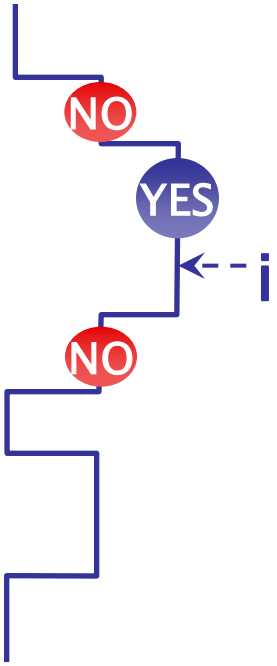
i

i

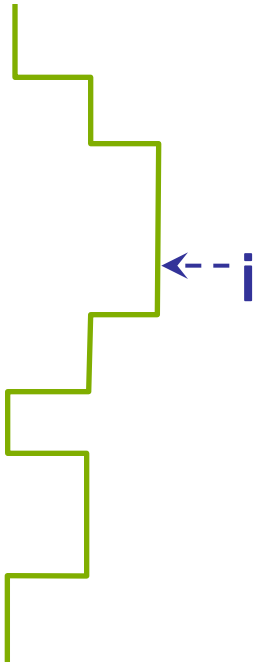
Find predicate needed at point i

Interprocedural Analysis

Trace



Trace Formula



Find predicate needed at point i

Require at each point i :

Well-scoped predicates

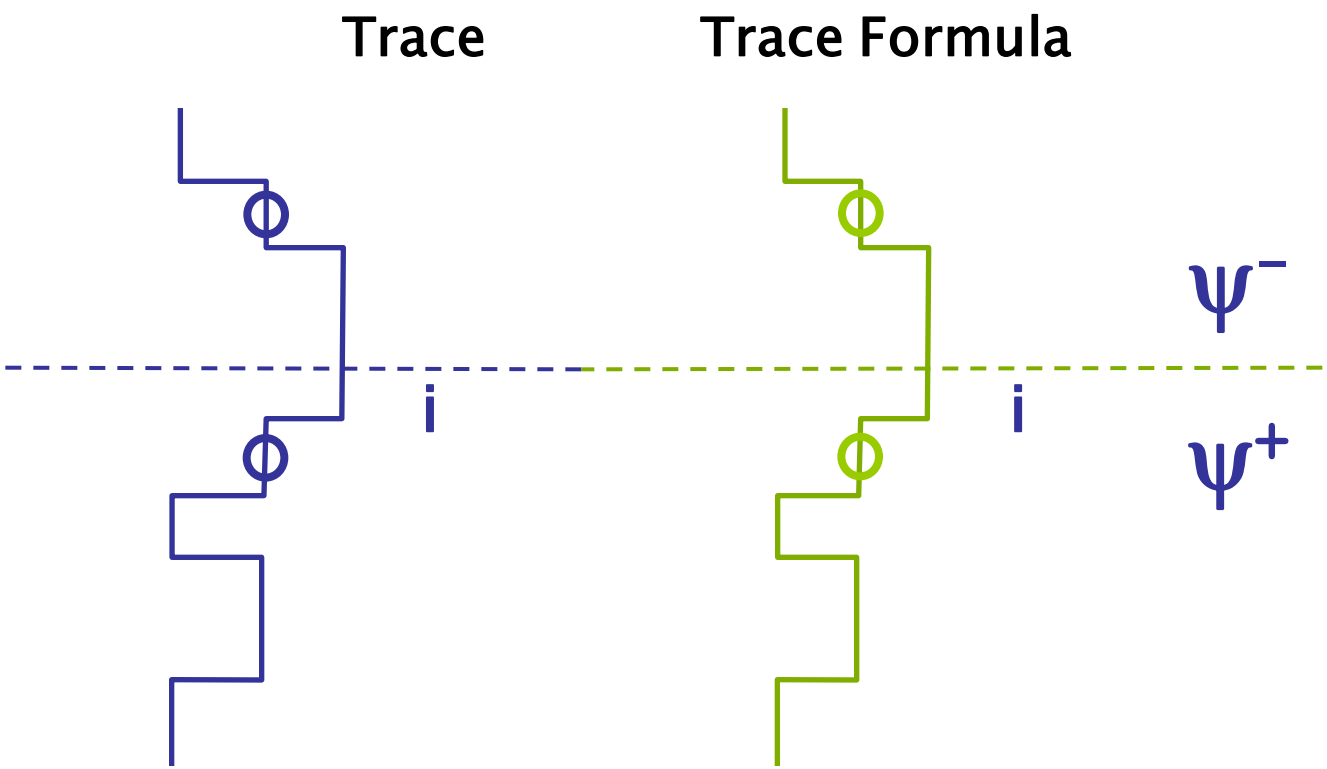
YES: Variables ***visible*** at i

NO: Caller's local variables

Procedure Summaries [Reps,Horwitz,Sagiv '95]

Polymorphic Predicate Abstraction [Ball,Millstein,Rajamani '02]

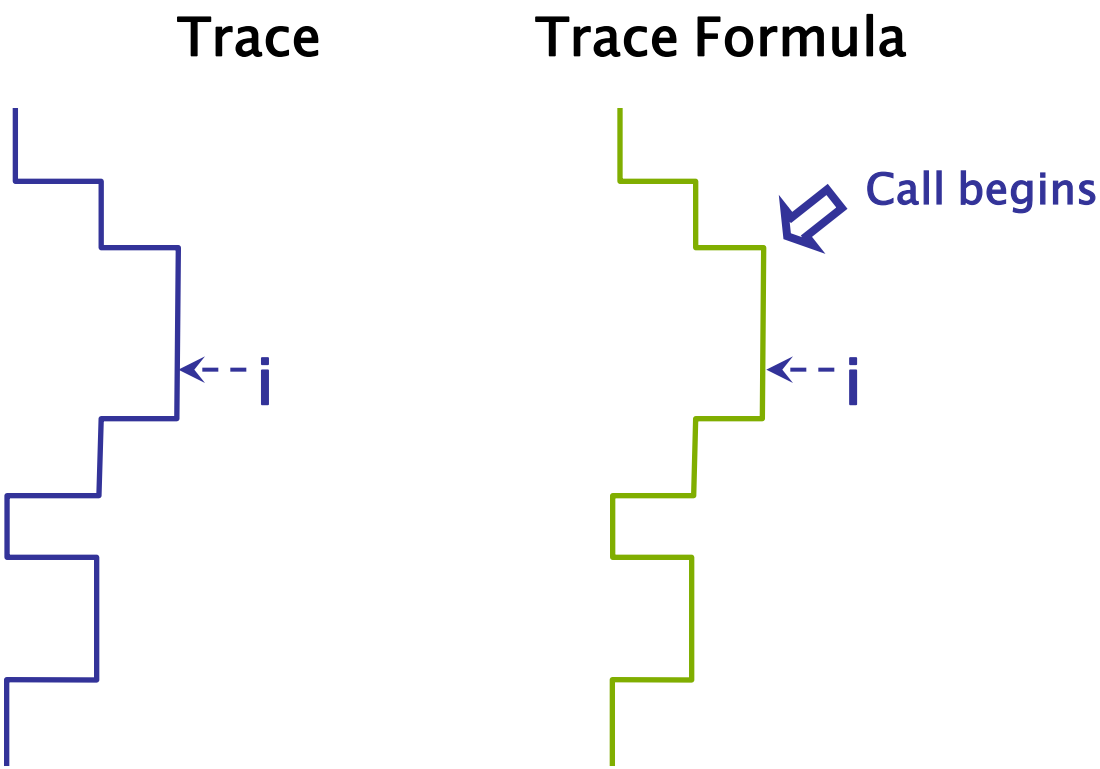
Problems with Cutting



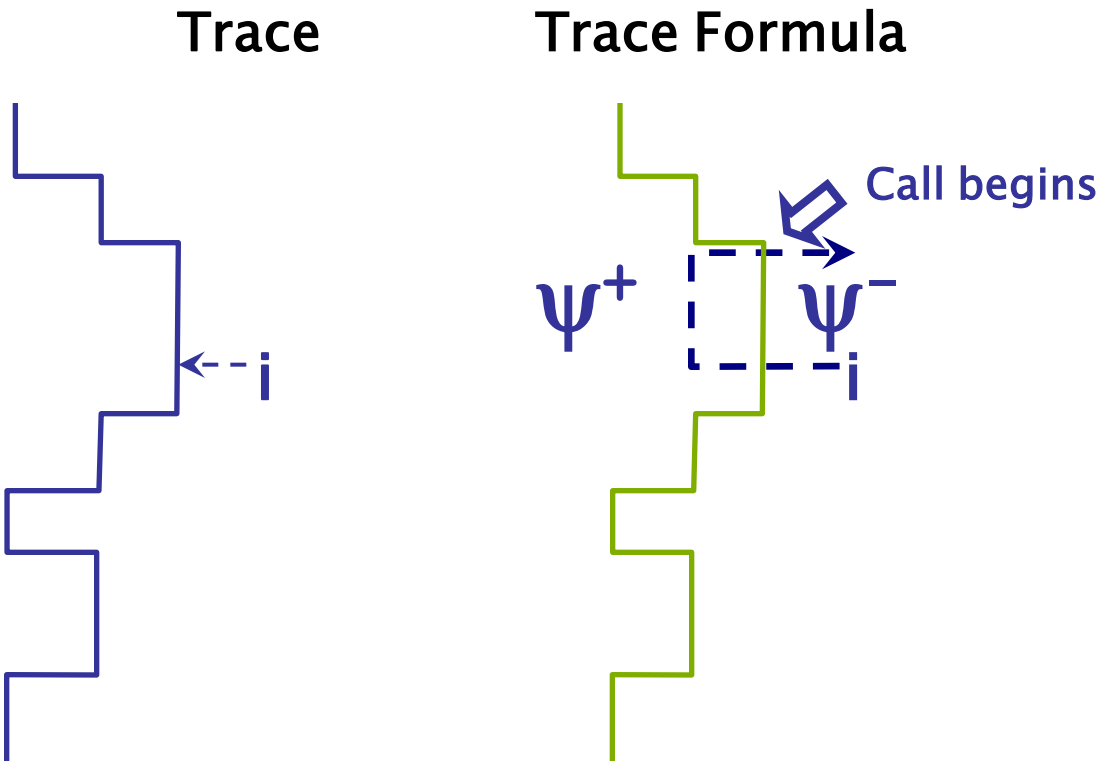
Caller variables common to ψ^- and ψ^+

- Unsuitable interpolant: not well-scoped

Interprocedural Cuts

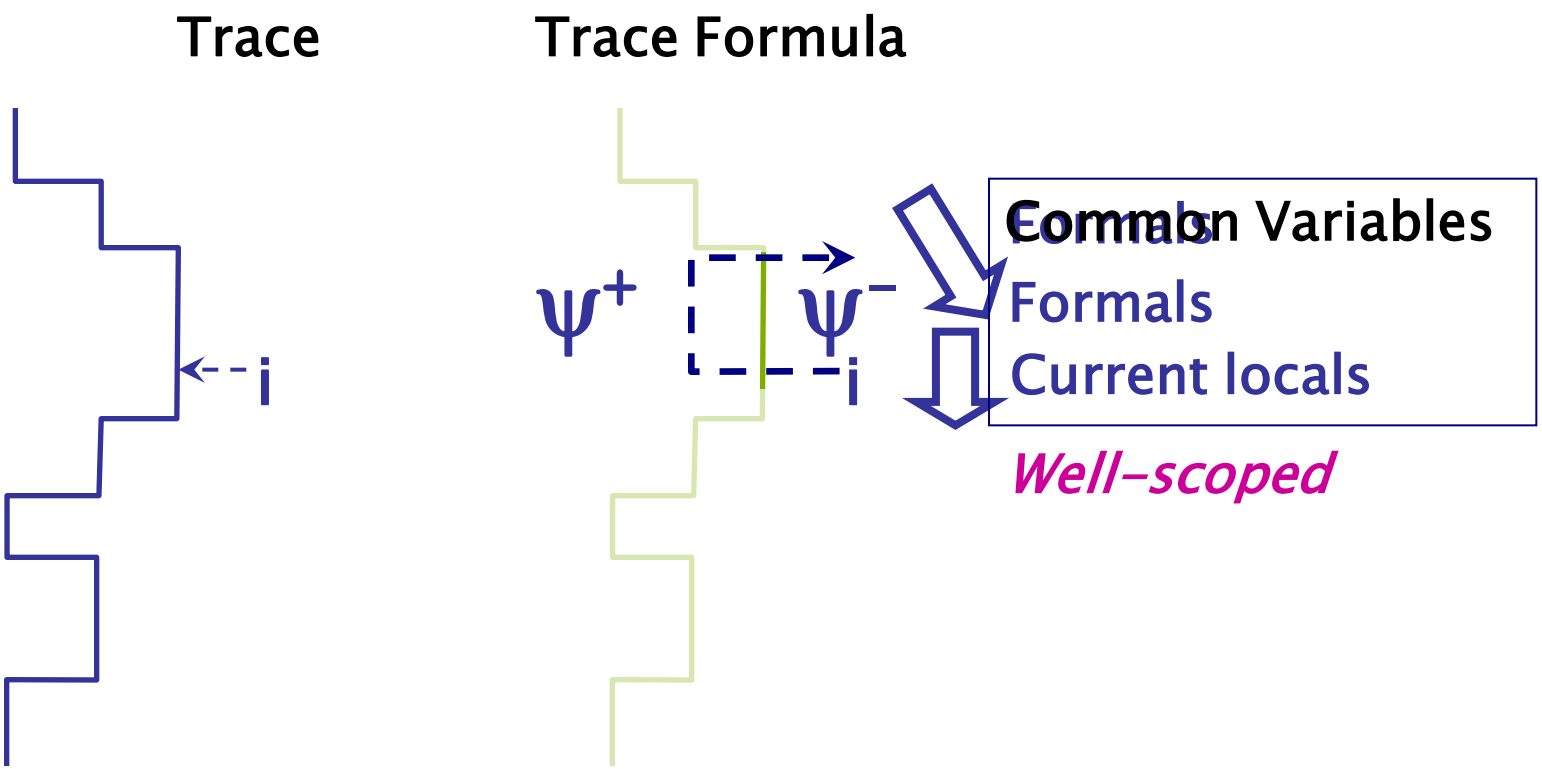


Interprocedural Cuts



Predicate at pc_i = Interpolant from cut i

Common Variables



Predicate at pc_i = Interpolant from i-cut

Plan

1. Motivation

2. Refinement using Traces
   - Simple
   - Procedure calls

3. Results

Implementation

- Algorithms implemented in BLAST
  - Verifier for C programs, Lazy Abstraction [POPL '02]
- FOCI : Interpolating decision procedure
- Examples:
  - Windows Device Drivers (DDK)
  - IRP Specification: 22 state FSM
  - Current: Security properties of Linux programs

Windows DDK

IRP

22 state

Results

Program	*LOC**	*Previous Time*	*New Time*	*Predicates*	
				Total	*Average*
kbfiltr	12k	1m12s	3m48s	72	6.5
floppy	17k	7m10s	25m20s	240	7.7
diskperf	14k	5m36s	13m32s	140	10
cdaudio	18k	20m18s	23m51s	256	7.8
parport	61k	*DNF*	*74m58s*	753	8.1
parclass	138k	*DNF*	*77m40s*	382	7.2

** Pre-processed*

Windows DDK

IRP

22 state

Localizing works…

Program	*LOC**	*Previous Time*	*New Time*	*Predicates*	
				Total	*Average*
kbfiltr	12k	1m12s	3m48s	72	*6.5*
floppy	17k	7m10s	25m20s	240	*7.7*
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cdaudio	18k	20m18s	23m51s	256	*7.8*
parport	61k	DNF	74m58s	753	*8.1*
parclass	138k	DNF	77m40s	382	*7.2*

** Pre-processed*

Conclusion

- Scalability *and* Precision by *localizing*
- Craig Interpolation
  - Interprocedural cuts give well-scoped predicates

- Some Current and Future Work:
  - Multithreaded Programs
    - Project local info of thread to predicates over globals
  - Hierarchical trace analysis

Limitations of CEGAR

- Limited to powerset/relational abstract domains
- Interpolant computations
- Interactions with widening
- Starting on the right foot
- Unnecessary refinement steps
- Long and infinite number of refinement steps
- Long traces

Unnecessary Refinements

```
x = 0
while (x < 10^6) do
    x = x + 1
assert x < 100
```

Unsuccessful Refinement Set

```
x = malloc();
y = x ;
while (…)
      t = malloc();
      t->next = x
      x = t;
…
while (x !=y) do
      assert x != null;
      x = x->next
```

Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

4:if (a>0) {
5:   if (x==0) {
ERR:  ;
     }
  }
}
```

- Assume f always terminates
- ERR is reachable
  - a and x are unconstrained
- Any feasible path to error must unroll the loop 1000 times AND find feasible paths through f
- Any other path must be dismissed as a false positive

Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

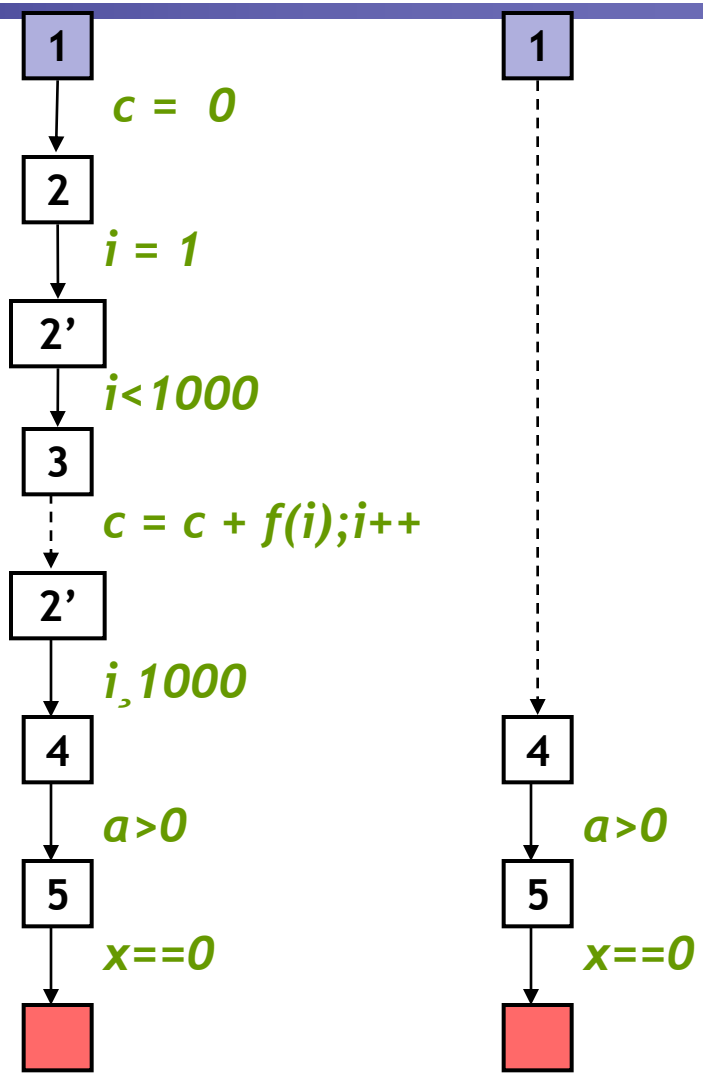
4:if (a>0) {
5:   if (x==0) {
ERR:  ;
      }
  }
}
```

- Intuitively, the for loop is irrelevant
- ERR reachable as long as there exists some path from 2 to 4 that does not modify a or x
- Can we use static analysis to precisely report a statement is reachable *without* finding a feasible path?

Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

4:if (a>0) {
5:   if (x==0) {
ERR:  ;
      }
  }
}
```



Path Slice (PLDI'05)

The path slice of a program path π is a subsequence of the edges of π such that if the sequence of operations along the subsequence is:

1. infeasible, then π is infeasible, and
2. feasible, then the last location of π is reachable (but not necessarily along π)