Operational Semantics

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Reference: Semantics with Applications

Chapter 2

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http://www.daimi.au.dk/~bra8130/Wiley_book/wiley.html

Syntax vs. Semantics

- The pattern of formation of sentences or phrases in a language
- Examples
  - Regular expressions
  - Context free grammars

- The study or science of meaning in language
- Examples
  - Interpreter
  - Compiler
  - Better mechanisms will be given today

Benefits of Formal Semantics

- Programming language design
  - hard- to-define= hard-to-implement=hard-to-use
- Programming language implementation
- Programming language understanding
- Program correctness
- Program equivalence
- Compiler Correctness
  - Correctness of Static Analysis
  - Design of Static Analysis
- Automatic generation of interpreter
- But probably not
  - Automatic compiler generation

Alternative Formal Semantics

- Operational Semantics
  - The meaning of the program is described "operationally"
  - Natural Operational Semantics
  - Structural Operational Semantics
- Denotational Semantics
  - The meaning of the program is an input/output relation
  - Mathematically challenging but complicated
- Axiomatic Semantics
  - The meaning of the program are observed properties

$[x \mapsto 3]$

$[x \mapsto 3, z \mapsto \perp, y \mapsto \perp]$

```
int fact(int x) {
  int z, y;
```

$[x \mapsto 3, z \mapsto 1, y \mapsto \perp]$

```
  z = 1;
```

$[x \mapsto 3, z \mapsto 1, y \mapsto 3]$

```
  y= x
   while (y>0)   {
```

$[x \mapsto 3, z \mapsto 1, y \mapsto 3]$

```
        z = z * y ;
```

$[x \mapsto 3, z \mapsto 3, y \mapsto 3]$

```
        y = y – 1;
    }
```

$[x \mapsto 3, z \mapsto 3, y \mapsto 2]$

```
return z
}
```

```
int fact(int x) {
  int z, y;
  z = 1;
  y= x
   while (y>0)   {
        z = z * y ;
        y = y – 1;
    }
return z
}
```

[x↦3, z↦3, y↦2]

[x↦3, z↦3, y↦2]

[x↦3, z↦6, y↦2]

[x↦3, z↦6, y↦1]

```
int fact(int x) {
  int z, y;
  z = 1;
  y= x
   while (y>0)   {
        z = z * y ;
        y = y – 1;
   }
return z
}
```

$[x \mapsto 3, z \mapsto 6, y \mapsto 1]$

$[x \mapsto 3, z \mapsto 6, y \mapsto 1]$

$[x \mapsto 3, z \mapsto 6, y \mapsto 1]$

$[x \mapsto 3, z \mapsto 6, y \mapsto 0]$

```
int fact(int x) {
  int z, y;
  z = 1;
  y= x
   while (y>0)   {
        z = z * y ;
        y = y – 1;
    }
return z
}
```

$[x \mapsto 3,\ z \mapsto 6,\ y \mapsto 0]$

$[x \mapsto 3,\ z \mapsto 6,\ y \mapsto 0]$

```
int fact(int x) {
  int z, y;
  z = 1;
  y= x;
   while (y>0)   {
          z = z * y ;
          y = y – 1;
    }
return 6
}
```

$[x \longmapsto 3,\ z \longmapsto 6,\ y \longmapsto 0]$

$[x \longmapsto 3,\ z \longmapsto 6,\ y \longmapsto 0]$

Denotational Semantics

```
int fact(int x) {
  int z, y;
  z = 1;
  y= x ;
   while (y>0)   {
        z = z * y ;
        y = y – 1;
    }
return z;
}
```

$f=\lambda x.\ \text{if } x = 0 \text{ then } 1 \text{ else } x * f(x - 1)$

Axiomatic Semantics

$\{x=n\}$

int fact(int x) { int z, y;

z = 1;

$\{x=n \wedge z=1\}$

y= x

$\{x=n \wedge z=1 \wedge y=n\}$

while

$\{x=n \wedge y \geq 0 \wedge z=n! / y!\}$

(y>0) {

$\{x=n \wedge y > 0 \wedge z=n! / y!\}$

z = z * y ;

$\{x=n \wedge y>0 \wedge z=n!/(y-1)!\}$

y = y – 1;

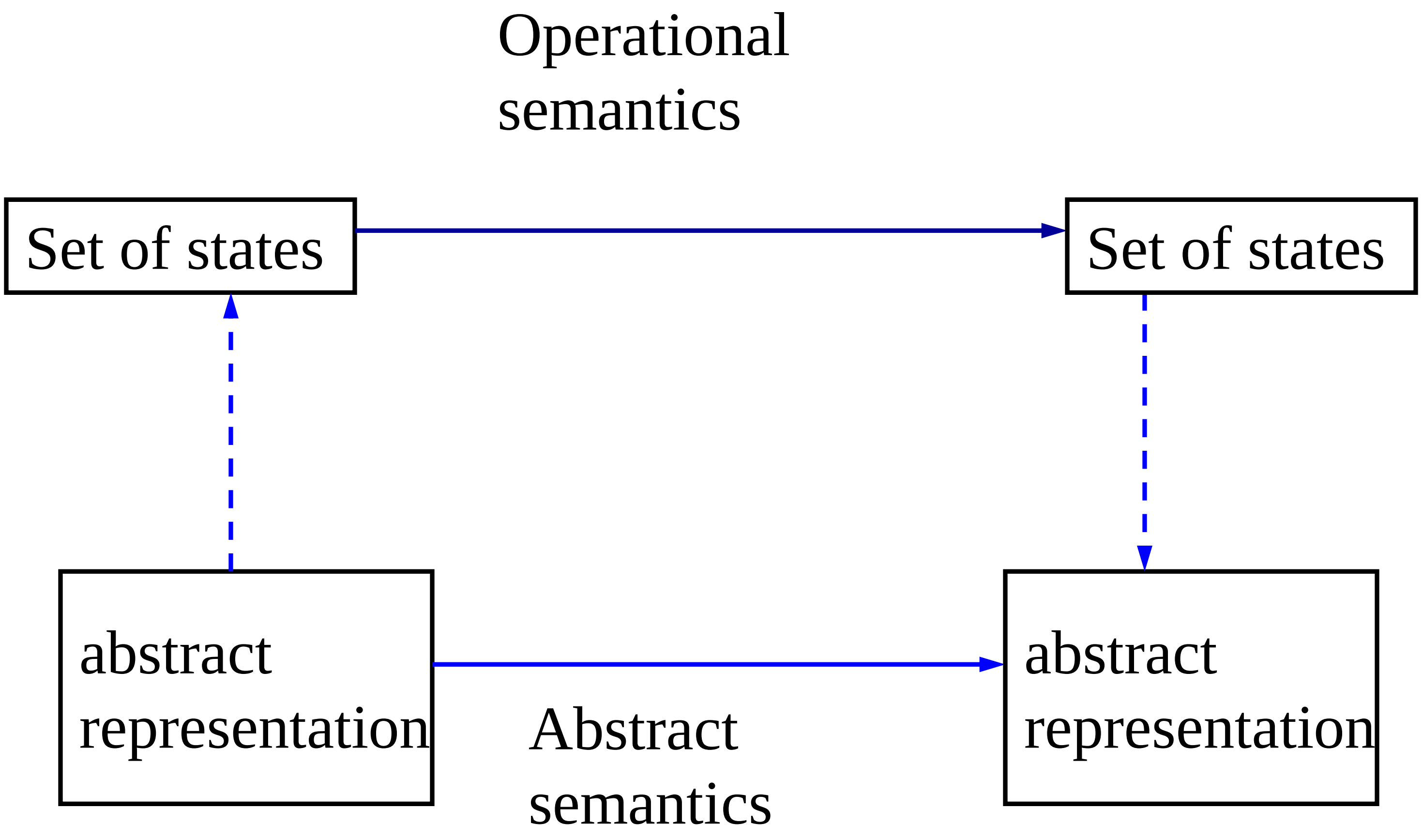
$\{x=n \wedge y \geq 0 \wedge z=n!/y!\}$

} return z} $\{x=n \wedge z=n!\}$

Static Analysis

- Automatic derivation of static properties which hold on every execution leading to a program location

Abstract (Conservative) interpretation

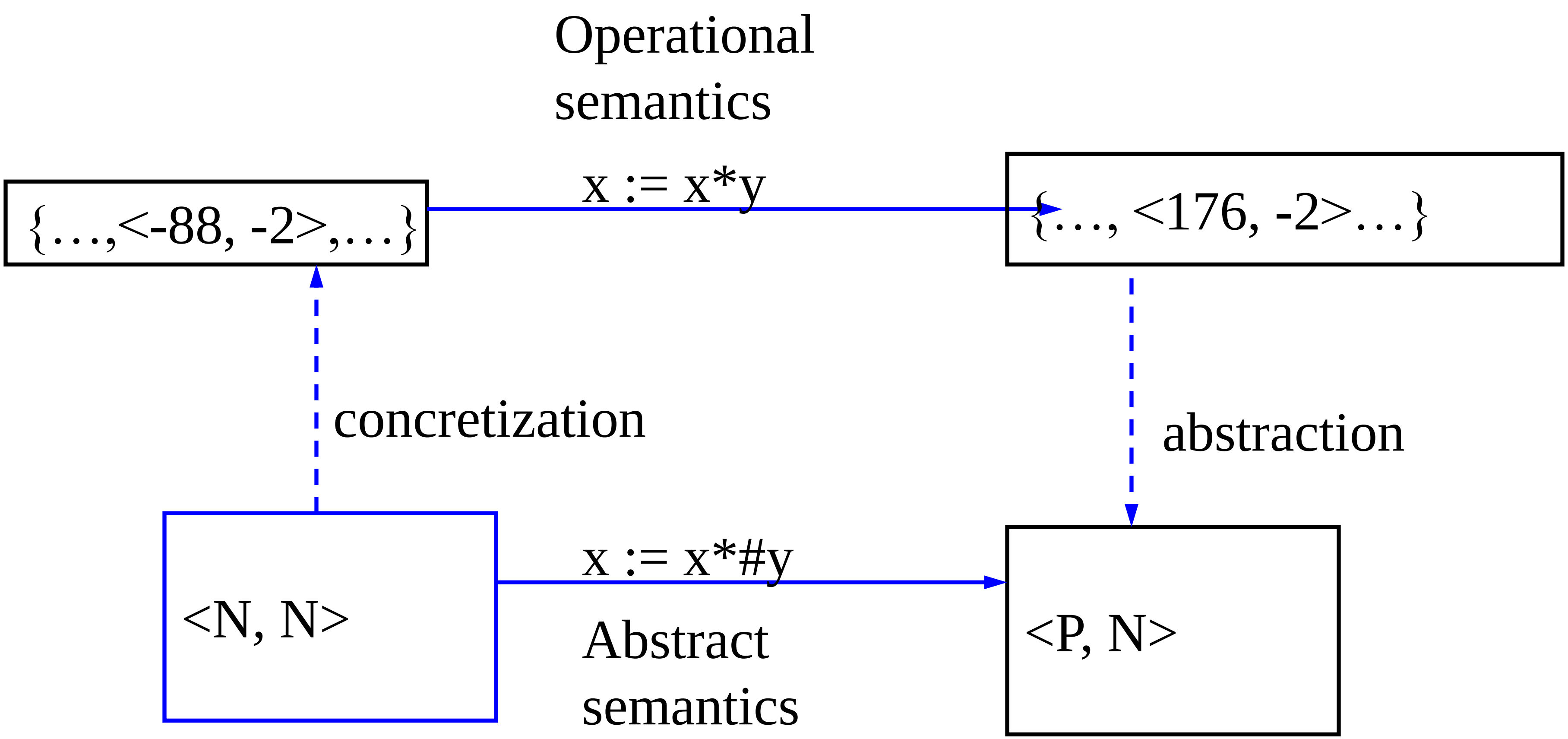


Example rule of signs

- Safely identify the sign of variables at every program location
- Abstract representation {P, N, ?}
- Abstract (conservative) semantics of *

*#	P	N	?
P	P	N	?
N	N	P	?
?	?	?	?

Abstract (conservative) interpretation



Example rule of signs (cont)

- Safely identify the sign of variables at every program location
- Abstract representation {P, N, ?}
- α(C) = if all elements in C are positive
  then return P
  else if all elements in C are negative
  then return N
  else return ?
- γ(a) = if (a==P) then
  return{0, 1, 2, … }
  else if (a==N)
  return {-1, -2, -3, …, }
  else return Z

Benefits of Operational Semantics for Static Analysis

- Correctness (soundness) of the analysis
  - The compiler will never change the meaning of the program
  - All impacts are identified
- Establish the right mindset
- Design the analysis
- Becomes familiar with mathematical notations used in programming languages

The **While** Programming Language

- Abstract syntax
  S::= x := a | **skip** | S_1 ; S_2 | **if** b **then** S_1 **else** S_2 |
  while b do S
- Use parenthesizes for precedence
- Informal Semantics
  - **skip** behaves like no-operation
  - Import meaning of arithmetic and Boolean operations

Example While Program

```
y := 1;
while ¬(x=1) do (
        y := y * x;
        x := x - 1
)
```

General Notations

- Syntactic categories
  - Var the set of program variables
  - Aexp the set of arithmetic expressions
  - Bexp the set of Boolean expressions
  - Stm set of program statements
- Semantic categories
  - Natural values N={0, 1, 2, …}
  - Truth values T={ff, tt}
  - States State = Var $\rightarrow$ N
  - Lookup in a state s: s x
  - Update of a state s: s [x $\mapsto$ 5]

Example State Manipulations

- $[x \mapsto 1, y \mapsto 7, z \mapsto 16]\ y =$
- $[x \mapsto 1, y \mapsto 7, z \mapsto 16]\ t =$
- $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5] =$
- $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5]\ x =$
- $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5]\ y =$

Semantics of arithmetic expressions

- Assume that arithmetic expressions are side-effect free
- $A[\![\text{Aexp}]\!] : \text{State} \rightarrow N$
- Defined by induction on the syntax tree
  - $A[\![n]\!] s = n$
  - $A[\![x]\!] s = s\ x$
  - $A[\![e_1 + e_2]\!] s = A[\![e_1]\!] s + A[\![e_2]\!] s$
  - $A[\![e_1 * e_2]\!] s = A[\![e_1]\!] s * A[\![e_2]\!] s$
  - $A[\![(e_1)]\!] s = A[\![e_1]\!] s$ --- not needed
  - $A[\![-e_1]\!] s = -A[\![e_1]\!] s$
- Compositional
- Properties can be proved by structural induction

Semantics of Boolean expressions

- Assume that Boolean expressions are side-effect free
- $B[\![\text{Bexp}]\!] : \text{State} \to T$
- Defined by induction on the syntax tree
  - $B[\![\text{true}]\!] s = tt$
  - $B[\![\text{false}]\!] s = ff$
  - $B[\![e_1 = e_2]\!] s =$
  - $B[\![e_1 \wedge e_2]\!] s = \begin{cases} tt \text{ if } A[\![e_1]\!] s = A[\![e_2]\!] s \\ ff \text{ if } A[\![e_1]\!] s \neq A[\![e_2]\!] s \end{cases}$

$$\begin{cases} tt \text{ if } B[\![e_1]\!] s = tt \text{ and } B[\![e_2]\!]=tt \\ ff \text{ if } B[\![e_1]\!] s=ff \text{ or } B[\![e_2]\!] s=ff \end{cases}$$

  - $B[\![e_1 \geq e_2]\!] s =$

Natural Operational Semantics

- Describe the “overall” effect of program constructs
- Ignores non terminating computations

Natural Semantics

- Notations
  - <S, s> - the program statement S is executed on input state s
  - s representing a terminal (final) state
- For every statement S, write meaning rules
  $<S, i> \rightarrow o$
  "If the statement S is executed on an input state *i*, it terminates and yields an output state *o*"
- The meaning of a program P on an input state s is the set of outputs states *o* such that $<P, i> \rightarrow o$
- The meaning of compound statements is defined using the meaning immediate constituent statements

Natural Semantics for While

axioms

$[\text{ass}_{ns}]$ $\langle x := a, s\rangle \to s[x \mapsto \mathbf{A}[\![a]\!]s]$

$[\text{skip}_{ns}]$ $\langle \mathbf{skip}, s\rangle \to s$

rules

$$[\text{comp}_{ns}]\ \frac{\langle S_1, s\rangle \to s',\ \langle S_2, s'\rangle \to s''}{\langle S_1; S_2, s\rangle \to s''}$$

$$[\text{if}^{tt}_{ns}]\ \frac{\langle S_1, s\rangle \to s'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s\rangle \to s'} \quad \text{if } \mathbf{B}[\![b]\!]s = tt$$

$$[\text{if}^{ff}_{ns}]\ \frac{\langle S_2, s\rangle \to s'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s\rangle \to s'} \quad \text{if } \mathbf{B}[\![b]\!]s = ff$$

Natural Semantics for While (More rules)

$$[\text{while}^{ff}_{ns}]\quad \frac{}{\langle \text{while } b \text{ do } S, s\rangle \to s} \quad \text{if } \mathbf{B}[\![b]\!]s=ff$$

$$[\text{while}^{tt}_{ns}]\quad \frac{\langle S\,, s\rangle \to s',\ \langle \text{while } b \text{ do } S, s'\rangle \to s''}{\langle \text{while } b \text{ do } S, s\rangle \to s''} \quad \text{if } \mathbf{B}[\![b]\!]s=tt$$

A Derivation Tree

- A "proof" that $\langle S, s\rangle \to s'$
- The root of tree is $\langle S, s\rangle \to s'$
- Leaves are instances of axioms
- Internal nodes rules
  - Immediate children match rule premises
- Simple Example

$$\dfrac{\langle skip, s_0\rangle \to s_0 \qquad \langle x := x+1, s_0\rangle \to s_0[x \mapsto 1]\rangle}{\langle skip;\ x := x+1, s_0\rangle \to s_0[x \mapsto 1]\rangle}\ \text{comp}_{\text{ns}}$$

An Example Derivation Tree

<(x :=x+1; y :=x+1) ; z := y), s0> →s0[x ↦1][y ↦2][z ↦2]

comp$_{ns}$

<x :=x+1; y :=x+1, s0> →s0[x ↦1][y ↦2]

<z :=y,s0[x ↦1][y ↦2]>→s0[x↦1][y↦2][z ↦2]

comp$_{ns}$

<x :=x+1; s0> →s0[x ↦1]

<y :=x+1, s0[x ↦1]> →s0[x ↦1][y ↦2]

ass$_{ns}$

ass$_{ns}$

Top Down Evaluation of Derivation Trees

- Given a program S and an input state s
- Find an output state s' such that
  $\langle S, s\rangle \rightarrow s'$
- Start with the root and repeatedly apply rules until the axioms are reached
- Inspect different alternatives in order
- In While s' and the derivation tree is unique

Example of Top Down Tree Construction

- Input state s such that s x = 2
- Factorial program

$\langle y := 1; \text{while } \neg(x=1) \text{ do } (y := y * x; x := x - 1), s\rangle \to s[y \mapsto 2][x \mapsto 1]$

comp_{ns}

$\langle y := 1, s\rangle \to s[y \mapsto 1]$

ass_{ns}

$\langle W, s[y \mapsto 1]\rangle \to s[y \mapsto 2][x \mapsto 1]$

while^{tt}_{ns}

$\langle (y := y * x; x := x - 1, s[y \mapsto 1]\rangle \to s[y \mapsto 2][x \mapsto 1]$

comp_{ns}

$\langle y := y * x; s[y \mapsto 1]\rangle \to s[y \mapsto 2]$

ass_{ns}

$\langle x := x - 1, s[y \mapsto 2]\rangle \to s[y \mapsto 2][x \mapsto 1]$

ass_{ns}

$\langle W, s[y \mapsto 2][x \mapsto 1]\rangle \to s[y \mapsto 2][x \mapsto 1]$

while^{ff}_{ns}

Semantic Equivalence

- S_1 and S_2 are semantically equivalent if for all s and s'
  $\langle S_1, s\rangle \to s'$ if and only if $\langle S_2, s\rangle \to s'$
- Simple example
  "while b do S"
  is semantically equivalent to:
  "if b then (S ; while b do S) else skip"

Deterministic Semantics for While

- If $<S, s> \rightarrow s_1$ and $<S, s> \rightarrow s_2$ then $s_1=s_2$
- The proof uses induction on the shape of derivation trees
  - Prove that the property holds for all simple derivation trees by showing it holds for axioms
  - Prove that the property holds for all composite trees:
    - » For each rule assume that the property holds for its premises (induction hypothesis) and prove it holds for the conclusion of the rule

The Semantic Function S_{ns}

- The meaning of a statement S is defined as a partial function from **State** to **State**
- S_{ns}: $\mathbf{Stm} \rightarrow (\mathbf{State} \hookrightarrow \mathbf{State})$
- $S_{ns}[\![S]\!]s = s'$ if $\langle S, s\rangle \rightarrow s'$ and otherwise $S_{ns}[\![S]\!]s$ is undefined
- Examples
  - $S_{ns}[\![\text{skip}]\!]s = s$
  - $S_{ns}[\![x := 1]\!]s = s\,[x \mapsto 1]$
  - $S_{ns}[\![\text{while true do skip}]\!]s = \text{undefined}$

Structural Operational Semantics

- Emphasizes the individual steps
- Usually more suitable for analysis
- For every statement S, write meaning rules $\langle S, i\rangle \Rightarrow \gamma$
  "If the **first** step of executing the statement S on an input state *i* leads to γ"
- Two possibilities for γ
  - $\gamma = \langle S', s'\rangle$ The execution of S is not completed, S' is the remaining computation which need to be performed on s'
  - $\gamma = o$ The execution of S has terminated with a final state o
  - γ is a stuck configuration when there are no transitions
- The meaning of a program P on an input state s is the set of final states that can be executed in arbitrary finite steps

Structural Semantics for While

axioms

$[\text{ass}_{sos}]\ \langle x := a, s\rangle \Rightarrow s[x \mapsto \mathbf{A}[\![a]\!]s]$

$[\text{skip}_{sos}]\ \langle \mathbf{skip}, s\rangle \Rightarrow s$

rules

$$[\text{comp}^1{}_{sos}]\ \frac{\langle S_1 , s\rangle \Rightarrow \langle S'_1, s'\rangle}{\langle S_1; S_2, s\rangle \Rightarrow \langle S'_1; S_2, s'\rangle}$$

$$[\text{comp}^2{}_{sos}]\ \frac{\langle S_1 , s\rangle \Rightarrow s'}{\langle S_1; S_2, s\rangle \Rightarrow \langle S_2, s'\rangle}$$

Structural Semantics for While
if construct

$[\text{if}^{tt}_{sos}]$ $\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s\rangle \Rightarrow \langle S_1, s\rangle$ if $\mathbf{B}[\![b]\!]s = tt$

$[\text{if}^{ff}_{os}]$ $\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s\rangle \Rightarrow \langle S_2, s\rangle$ if $\mathbf{B}[\![b]\!]s = ff$

Structural Semantics for While
while construct

[while$_{sos}$] <while b do S, s> $\Rightarrow$
<if b then (S; while b do S) else skip, s>

Derivation Sequences

- A finite derivation sequence starting at <S, s>
  $\gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_k$ such that
  - $\gamma_0 = \langle S, s \rangle$
  - $\gamma_i \Rightarrow \gamma_{i+1}$
  - γ_k is either stuck configuration or a final state
- An infinite derivation sequence starting at <S, s>
  $\gamma_0, \gamma_1, \gamma_2 \ldots$ such that
  - $\gamma_0 = \langle S, s \rangle$
  - $\gamma_i \Rightarrow \gamma_{i+1}$
- $\gamma_0 \Rightarrow^i \gamma_i$ in i steps
- $\gamma_0 \Rightarrow^* \gamma_i$ in finite number of steps
- For each step there is a derivation tree

Example

- Let s_0 such that
  $s_0\, x = 5$
  and
  $s_0\, y = 7$
- $S = (z:=x;\ x := y);\ \ y := z$

Factorial Program

- Input state s such that s x = 3

$$y := 1; \text{while } \neg(x=1) \text{ do } (y := y * x; x := x - 1)$$

$\langle y := 1 ; W, s\rangle$

$\Rightarrow \langle W, s[y \mapsto 1]\rangle$

$\Rightarrow \langle \text{if } \neg\neg (x = 1) \text{ then skip else } ((y := y * x ; x := x - 1); W), s[y \mapsto 1]\rangle$

$\Rightarrow \langle ((y := y * x ; x := x - 1); W), s[y \mapsto 1]\rangle$

$\Rightarrow \langle (x := x - 1 ; W), s[y \mapsto 3]\rangle$

$\Rightarrow \langle W , s[y \mapsto 3][x \mapsto 2]\rangle$

$\Rightarrow \langle \text{if } \neg\neg (x = 1) \text{ then skip else } ((y := y * x ; x := x - 1); W), s[y \mapsto 3][x \mapsto 2]\rangle$

$\Rightarrow \langle ((y := y * x ; x := x - 1); W), s[y \mapsto 3] [x \mapsto 2] \rangle$

$\Rightarrow \langle (x := x - 1 ; W) , s[y \mapsto 6] [x \mapsto 2] \rangle$

$\Rightarrow \langle W, s[y \mapsto 6][x \mapsto 1]\rangle$

$\Rightarrow \langle \text{if } \neg\neg (x = 1) \text{ then skip else } ((y := y * x ; x := x - 1); W), s[y \mapsto 6][x \mapsto 1]\rangle$

$\Rightarrow \langle \text{skip}, s[y \mapsto 6][x \mapsto 1]\rangle \Rightarrow s[y \mapsto 6][x \mapsto 1]$

Program Termination

- Given a statement S and input s
  - S terminates on s if there exists a finite derivation sequence starting at <S, s>
  - S terminates successfully on s if there exists a finite derivation sequence starting at <S, s> leading to a final state
  - S loops on s if there exists an infinite derivation sequence starting at <S, s>

Properties of the Semantics

- S_1 and S_2 are semantically equivalent if:
  - for all s and γ which is either final or stuck $<S_1, s> \Rightarrow^* \gamma$ if and only if $<S_2, s> \Rightarrow^* \gamma$
  - there is an infinite derivation sequence starting at $<S_1, s>$ if and only if there is an infinite derivation sequence starting at $<S_2, s>$
- Deterministic
  - If $<S, s> \Rightarrow^* s_1$ and $<S, s> \Rightarrow^* s_2$ then $s_1=s_2$
- The execution of $S_1; S_2$ on an input can be split into two parts:
  - execute S_1 on s yielding a state s'
  - execute S_2 on s'

Sequential Composition

- If $\langle S_1; S_2, s\rangle \Rightarrow^k s''$ then there exists a state s' and numbers k_1 and k_2 such that
  - $\langle S_1, s\rangle \Rightarrow^{k1} s'$
  - $\langle S_2, s'\rangle \Rightarrow^{k2} s''$
  - and $k = k_1 + k_2$
- The proof uses induction on the length of derivation sequences
  - Prove that the property holds for all derivation sequences of length 0
  - Prove that the property holds for all other derivation sequences:
    - » Show that the property holds for sequences of length k+1 using the fact it holds on all sequences of length k (induction hypothesis)

The Semantic Function S_{sos}

- The meaning of a statement S is defined as a partial function from **State** to **State**
- S_{sos}: $\mathbf{Stm} \rightarrow (\mathbf{State} \hookrightarrow \mathbf{State})$
- $S_{sos}[\![S]\!]s = s'$ if $\langle S, s\rangle \Rightarrow^{*} s'$ and otherwise $S_{sos}[\![S]\!]s$ is undefined

An Equivalence Result

- For every statement S of the While language
  - $S_{nat}[\![S]\!] = S_{sos}[\![S]\!]$

Extensions to While

- Abort statement (like C exit w/o return value)
- Non determinism
- Parallelism
- Local Variables
- Procedures
  - Static Scope
  - Dynamic scope

The **While** Programming Language with Abort

- Abstract syntax
  S::= x := a | **skip** | S_1 ; S_2 | **if** b **then** S_1 **else** S_2 |
  while b do S| **abort**
- Abort terminates the execution
- No new rules are needed in natural and structural operational semantics
- Statements
  - if x = 0 then abort else y := y / x
  - skip
  - abort
  - while true do skip

Conclusion

- The natural semantics cannot distinguish between looping and abnormal termination (unless the states are modified)
- In the structural operational semantics looping is reflected by infinite derivations and abnormal termination is reflected by stuck configuration

The **While** Programming Language with Non-Determinism

- Abstract syntax
  S::= x := a | **skip** | S_1 ; S_2 | **if** b **then** S_1 **else** S_2 |
  while b do S| S_1 or S_2
- Either S_1 or S_2 is executed
- Example
  - x := 1 or (x :=2 ; x := x+2)

The While Programming Language with Non-Determinism Natural Semantics

$$[or^1_{ns}]\ \frac{\langle S_1 , s\rangle \to s'}{\langle S_1 \text{ or } S_2, s\rangle \to s'}$$

$$[or^2_{ns}]\ \frac{\langle S_2 , s\rangle \to s'}{\langle S_1 \text{ or } S_2, s\rangle \to s'}$$

The While Programming Language with Non-Determinism Structural Semantics

The While Programming Language with Non-Determinism Examples

- x := 1 or (x :=2 ; x := x+2)
- (while true do skip) or (x :=2 ; x := x+2)

Conclusion

- In the natural semantics non-determinism will suppress looping if possible (mnemonic)
- In the structural operational semantics non-determinism does suppress not termination configuration

The **While** Programming Language with Parallel Constructs

- Abstract syntax
  S::= x := a | **skip** | S_1 ; S_2 | **if** b **then** S_1 **else** S_2 |
  while b do S| S_1 par S_2
- All the interleaving of S_1 or S_2 are executed
- Example
  - x := 1 par (x :=2 ; x := x+2)

The **While** Programming Language with Parallel Constructs Structural Semantics

$$[\text{par}^1_{\text{sos}}]\ \frac{\langle S_1, s\rangle \Rightarrow \langle S'_1, s'\rangle}{\langle S_1 \text{ par } S_2, s\rangle \Rightarrow \langle S'_1 \text{ par } S_2, s'\rangle}$$

$$[\text{par}^2_{\text{sos}}]\ \frac{\langle S_1, s\rangle \Rightarrow s'}{\langle S_1 \text{ par } S_2, s\rangle \Rightarrow \langle S_2, s'\rangle}$$

$$[\text{par}^3_{\text{sos}}]\ \frac{\langle S_2, s\rangle \Rightarrow \langle S'_2, s'\rangle}{\langle S_1 \text{ par } S_2, s\rangle \Rightarrow \langle S_1 \text{ par } S'_2, s'\rangle}$$

$$[\text{par}^4_{\text{sos}}]\ \frac{\langle S_2, s\rangle \Rightarrow s'}{\langle S_1 \text{ par } S_2, s\rangle \Rightarrow \langle S_1, s'\rangle}$$

The **While** Programming Language with Parallel Constructs Natural Semantics

Conclusion

- In the natural semantics immediate constituent is an atomic entity so we cannot express interleaving of computations
- In the structural operational semantics we concentrate on small steps so interleaving of computations can be easily expressed

The **While** Programming Language with local variables and procedures

- Abstract syntax

  S::= x := a | **skip** | S_1 ; S_2 | **if** b **then** S_1 **else** S_2 |

  while b do S|

  begin D_v D_p S **end | call p**

  D_v ::= **var** x := a ; D_v | ε

  D_p ::= **proc** p **is** S ; D_p | ε

Conclusions Local Variables

- The natural semantics can “remember” local states
- Need to introduce stack or heap into state of the structural semantics

Transition Systems

- Low-level semantics
- Include program counter in the set of states Σ
- The meaning of a program is a relation $\tau \subseteq \Sigma \times \Sigma$
- Execution is a finite sequence of states

Example

```
1: y := 1;
 while 2: ¬(x=1) do (
        3: y := y * x;
        4: x := x - 1
)
5:
```

Summary

- SOS is powerful enough to describe imperative programs
  - Can define the set of traces
  - Can represent program counter implicitly
  - Handle gotos
- Natural operational semantics is an abstraction
- Different semantics may be used to justify different behaviors
- Thinking in concrete semantics is essential for a compiler writer