Iterative Program Analysis
Part I

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Textbook: Principles of Program Analysis
Chapter 2.1 (modified)

A Simple Example Program

$[x \mapsto 0, y \mapsto 0, z \mapsto 0]$

z = 3

$[x \mapsto 0, y \mapsto 0, z \mapsto 3]$

x = 1

$[x \mapsto 1, y \mapsto 0, z \mapsto 3]$

while (x > 0) (

$[x \mapsto 1, y \mapsto 0, z \mapsto 3]$

if (x = 1) then y = 7

$[x \mapsto 1, y \mapsto 7, z \mapsto 3]$

else y = z + 4

$[x \mapsto 1, y \mapsto 7, z \mapsto 3]$

x = 3

$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

print y

$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

)

A Simple Example Program

$[x \mapsto 0, y \mapsto 0, z \mapsto 0]$

z = 3

$[x \mapsto 0, y \mapsto 0, z \mapsto 3]$

x = 1

$[x \mapsto 1, y \mapsto 0, z \mapsto 3]$

while (x > 0) (

$[x \mapsto \top, y \mapsto 0, z \mapsto 3]$

if (x = 1) then y = 7

$[x \mapsto 1, y \mapsto 7, z \mapsto 3]$

else y = z + 4

$[x \mapsto 1, y \mapsto 7, z \mapsto 3]$

x = 3

$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

print y

$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

)

A Simple Example Program

$[x \mapsto 0, y \mapsto 0, z \mapsto 0]$

z = 3

$[x \mapsto 0, y \mapsto 0, z \mapsto 3]$

x = 1

$[x \mapsto 1, y \mapsto 0, z \mapsto 3]$

while (x > 0) (

$[x \mapsto \top, y \mapsto 0, z \mapsto 3]$

if (x = 1) then y = 7

$[x \mapsto \top, y \mapsto 7, z \mapsto 3]$

else y = z + 4

$[x \mapsto 1, y \mapsto 7, z \mapsto 3]$

x = 3

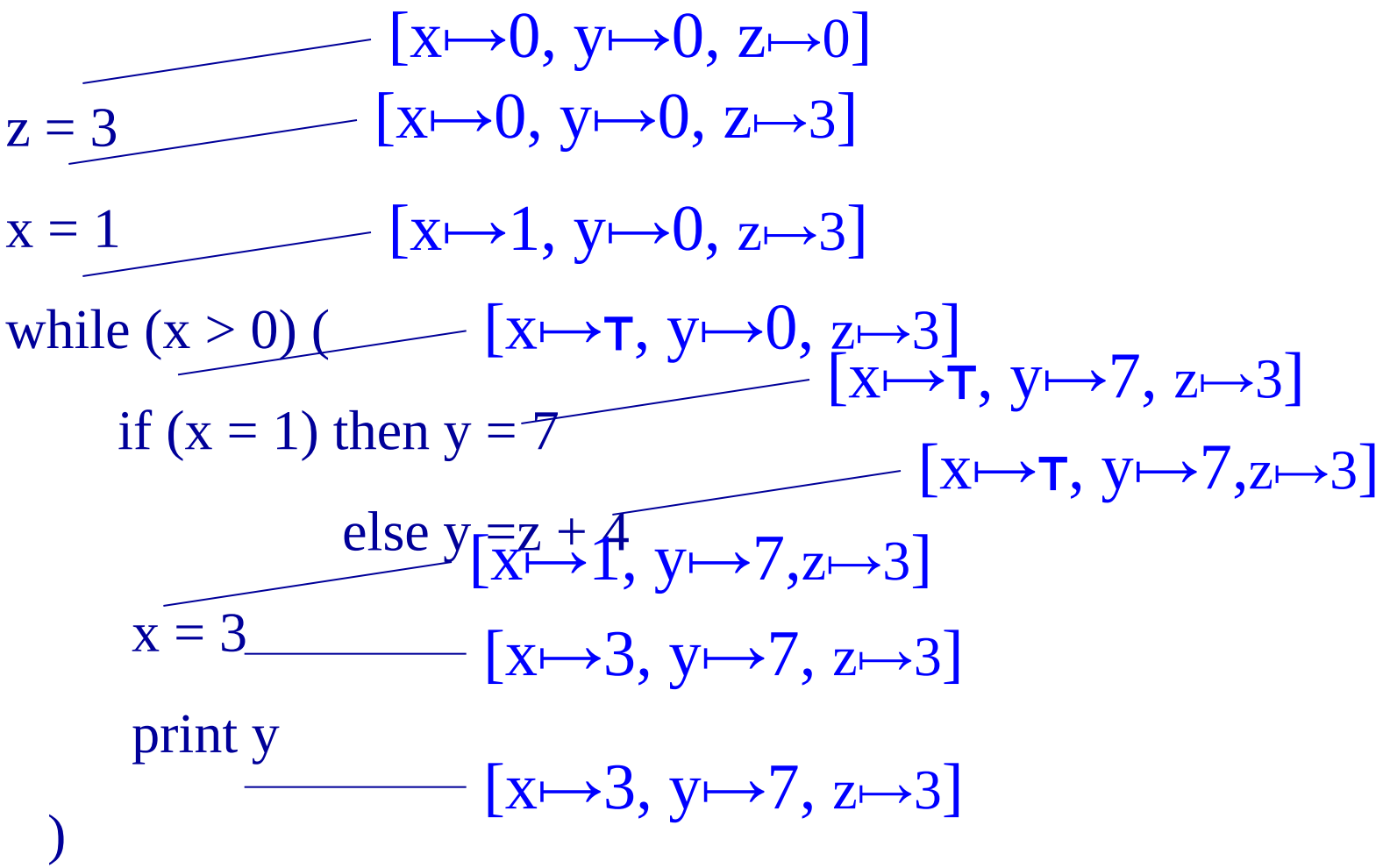
$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

print y

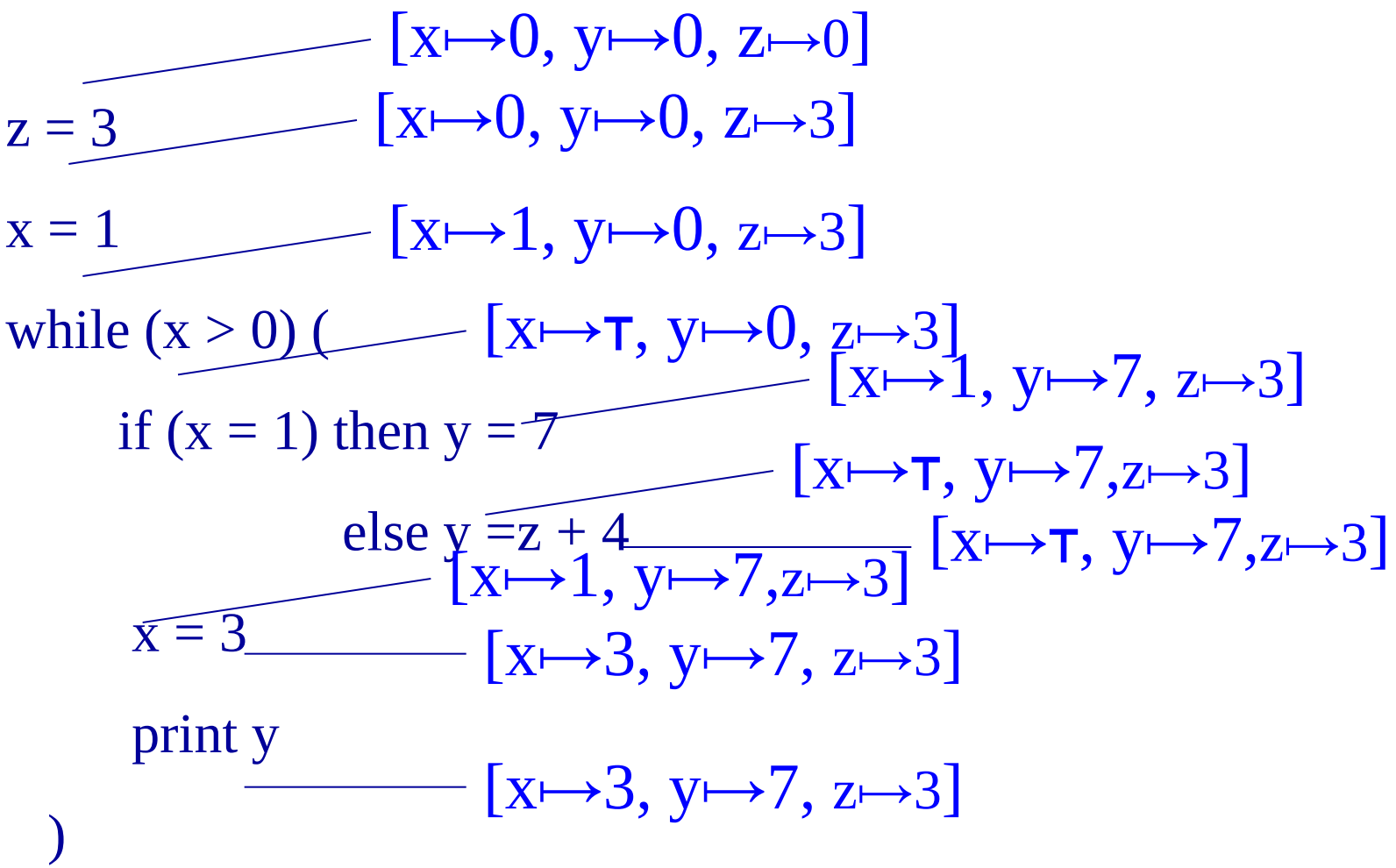
$[x \mapsto 3, y \mapsto 7, z \mapsto 3]$

)

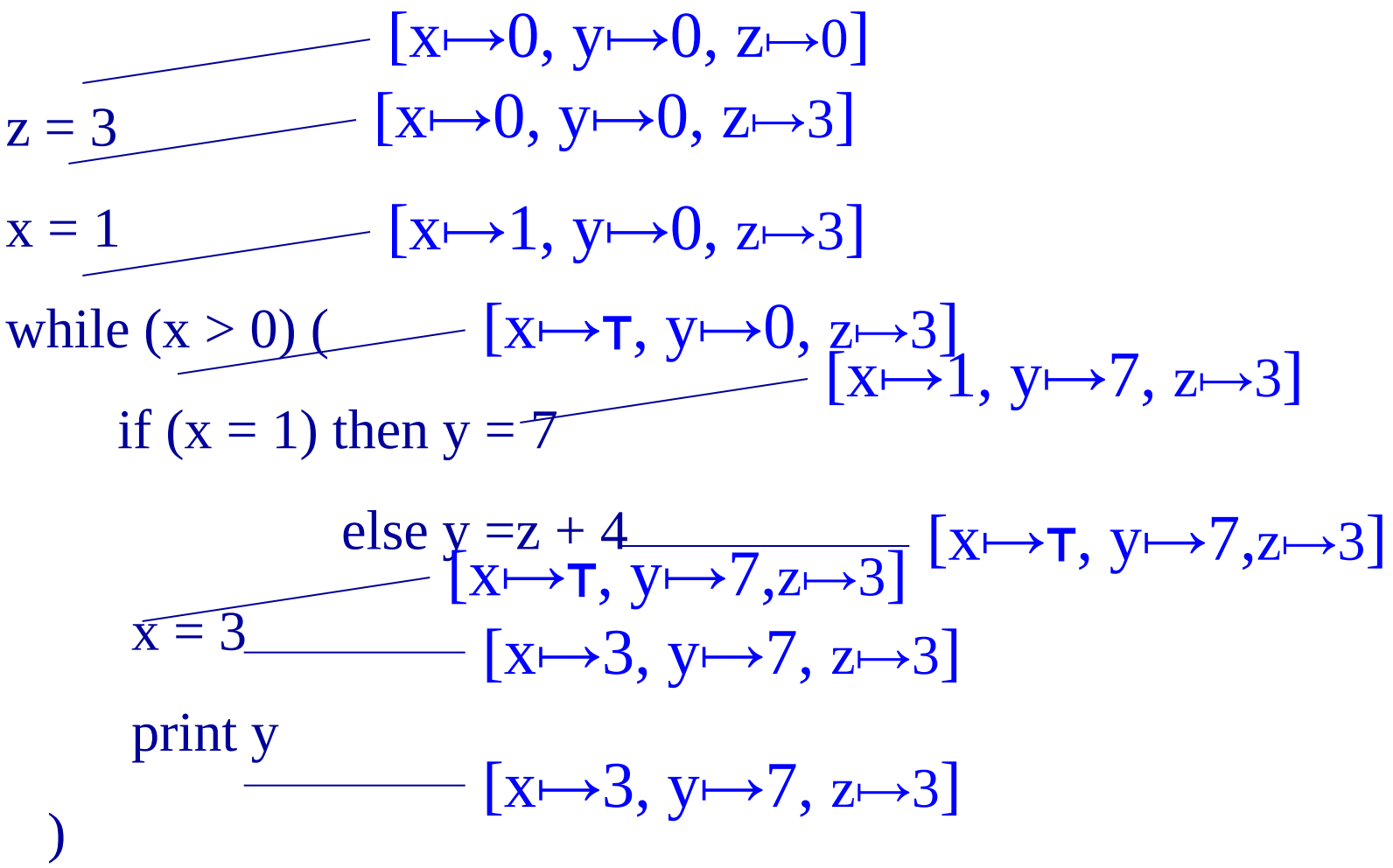
A Simple Example Program



A Simple Example Program



A Simple Example Program

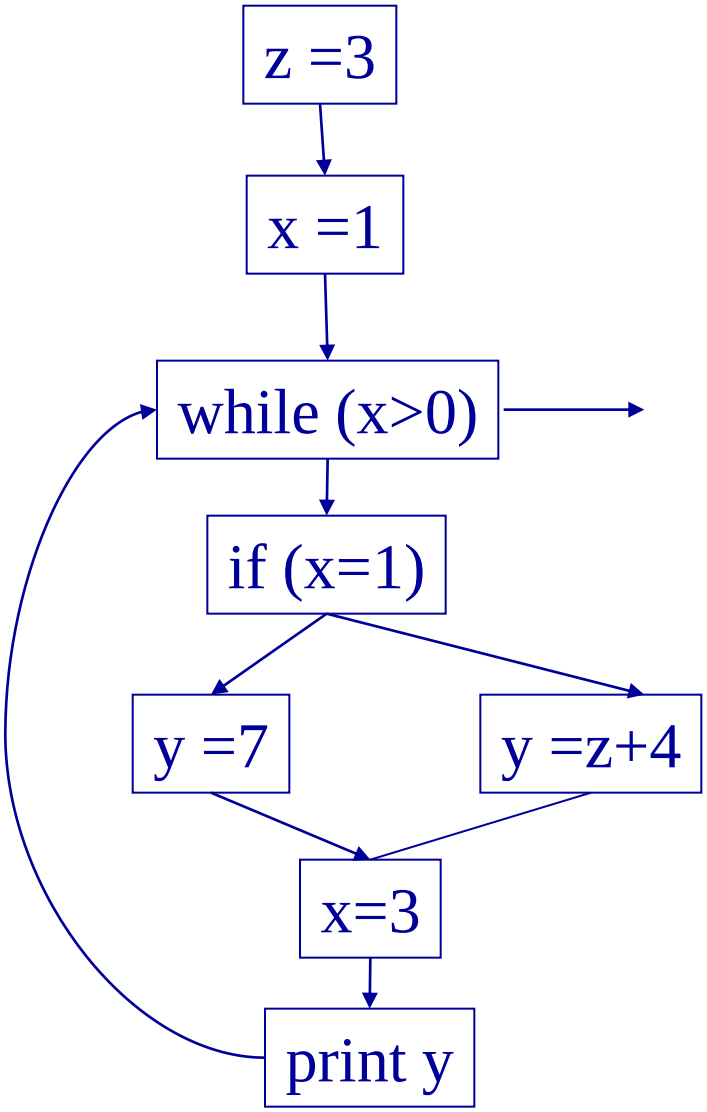


Computing Constants

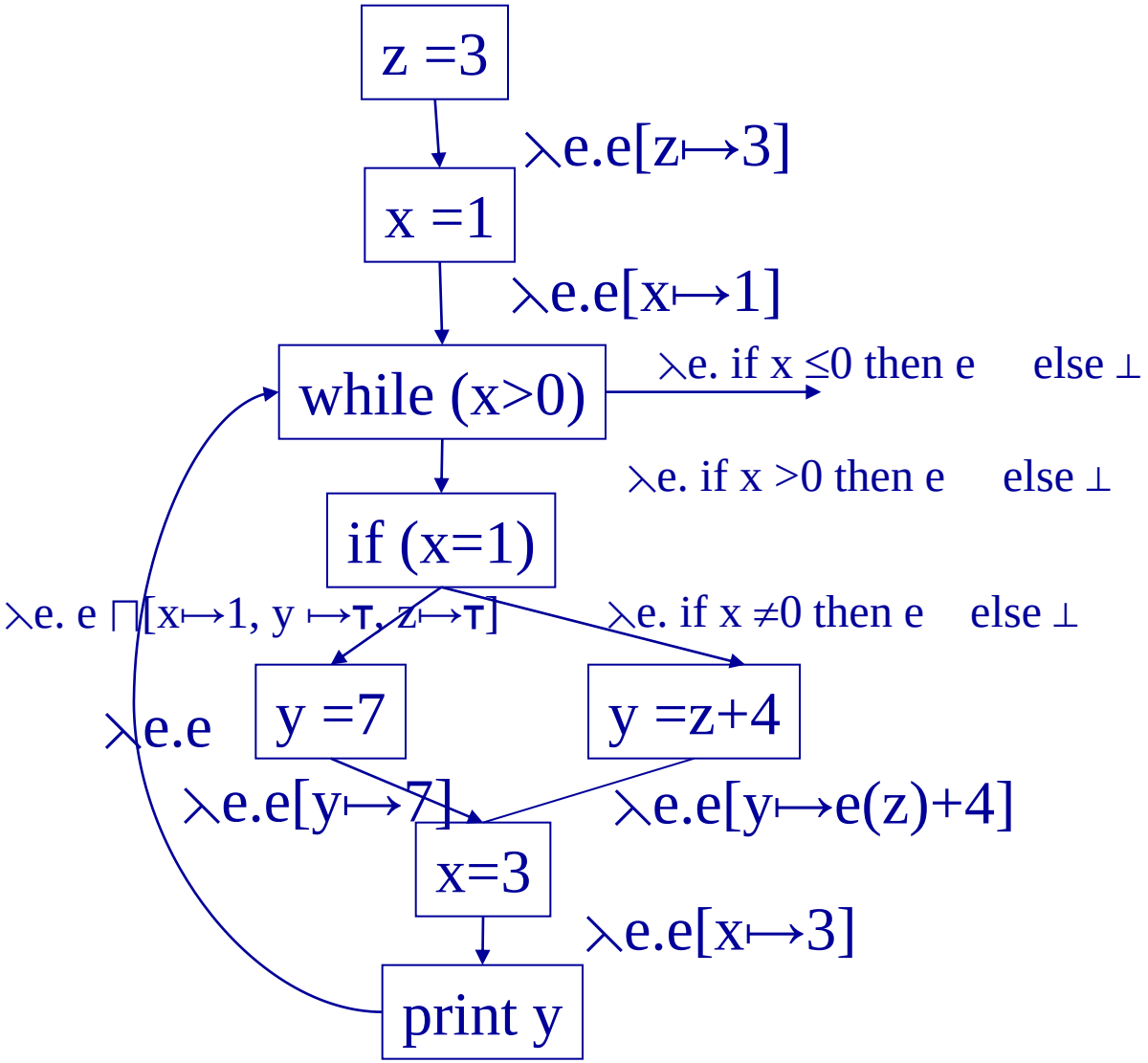
- Construct a control flow graph (CFG)
- Associate transfer functions with control flow graph edges
- Iterate until a solution is found
- The solution is unique
  - But order of evaluation may affect the number of iterations

Constructing CFG

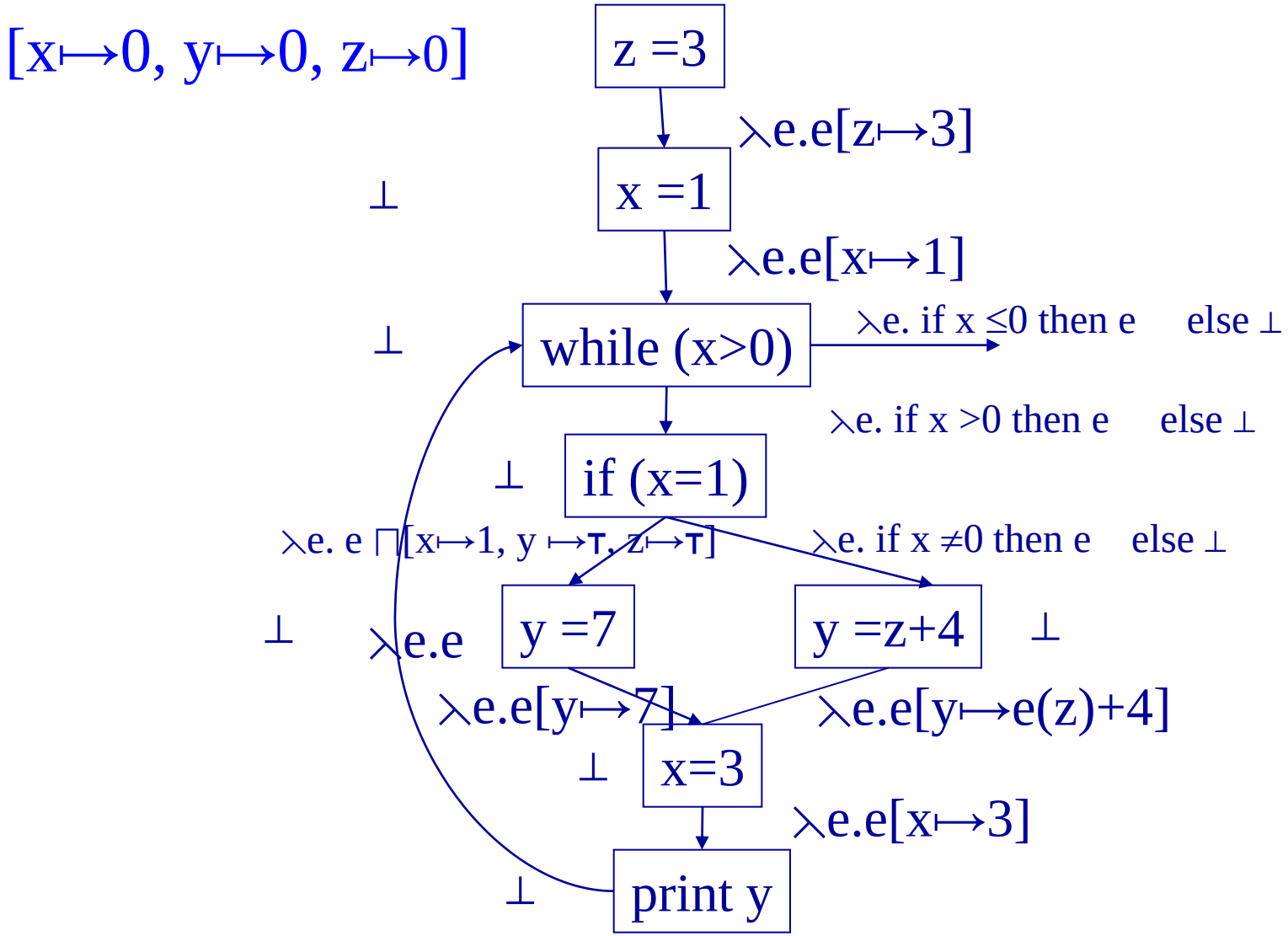
```
z = 3
x = 1
while (x > 0) (
        if (x = 1) then y = 7
                   else y =z + 4
        x = 3
        print y
   )
```



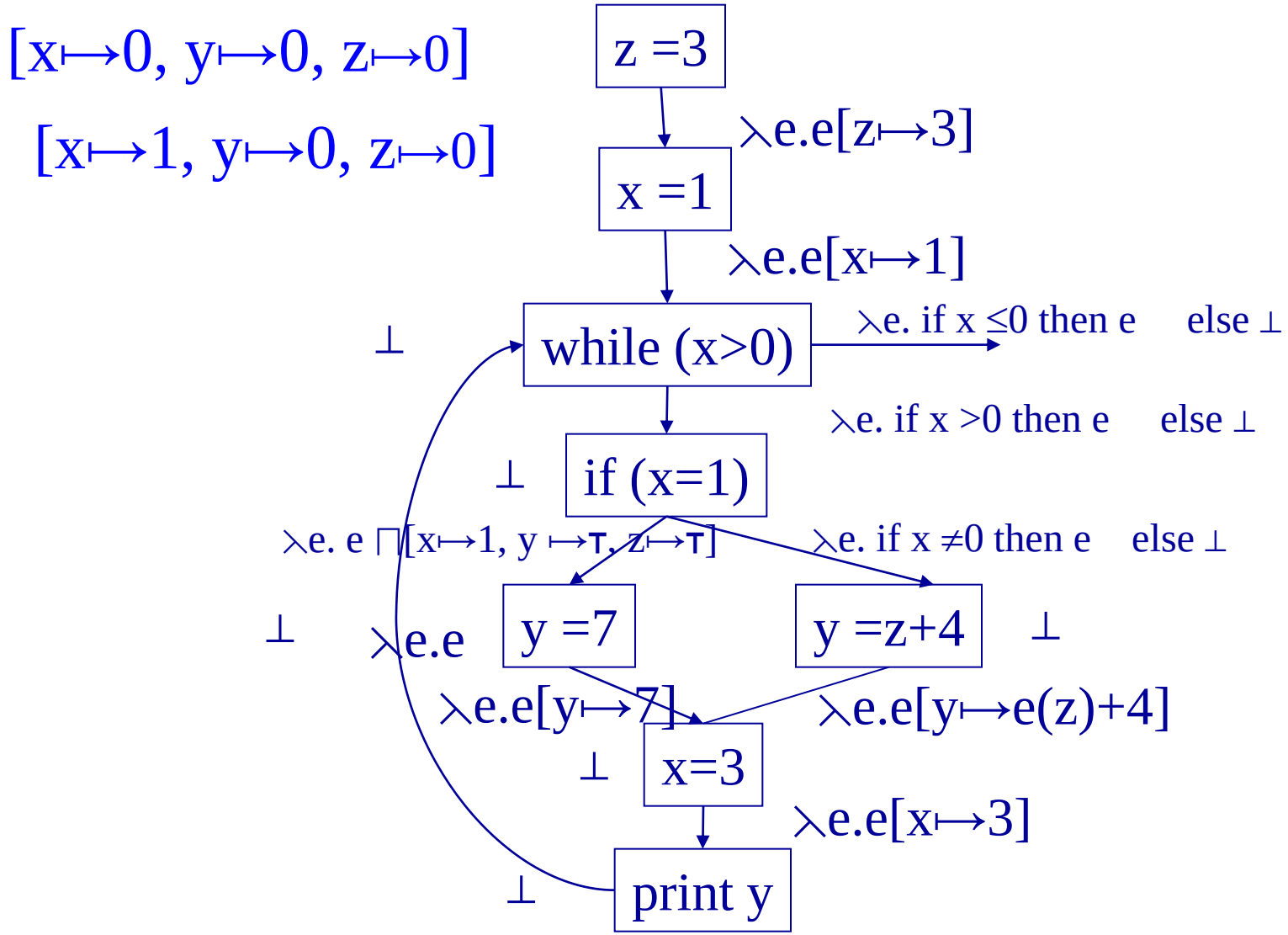
Associating Transfer Functions



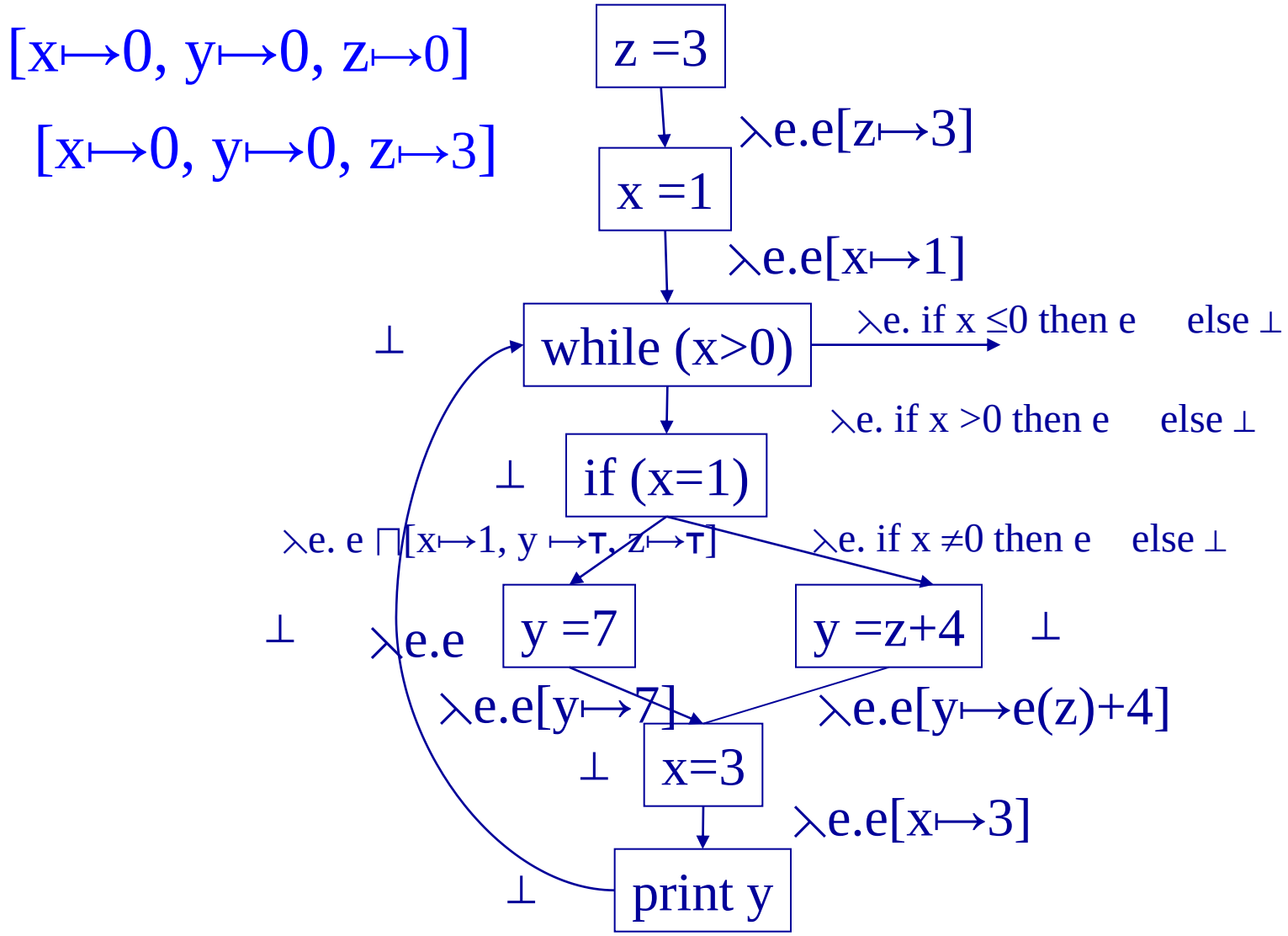
Iterative Computation



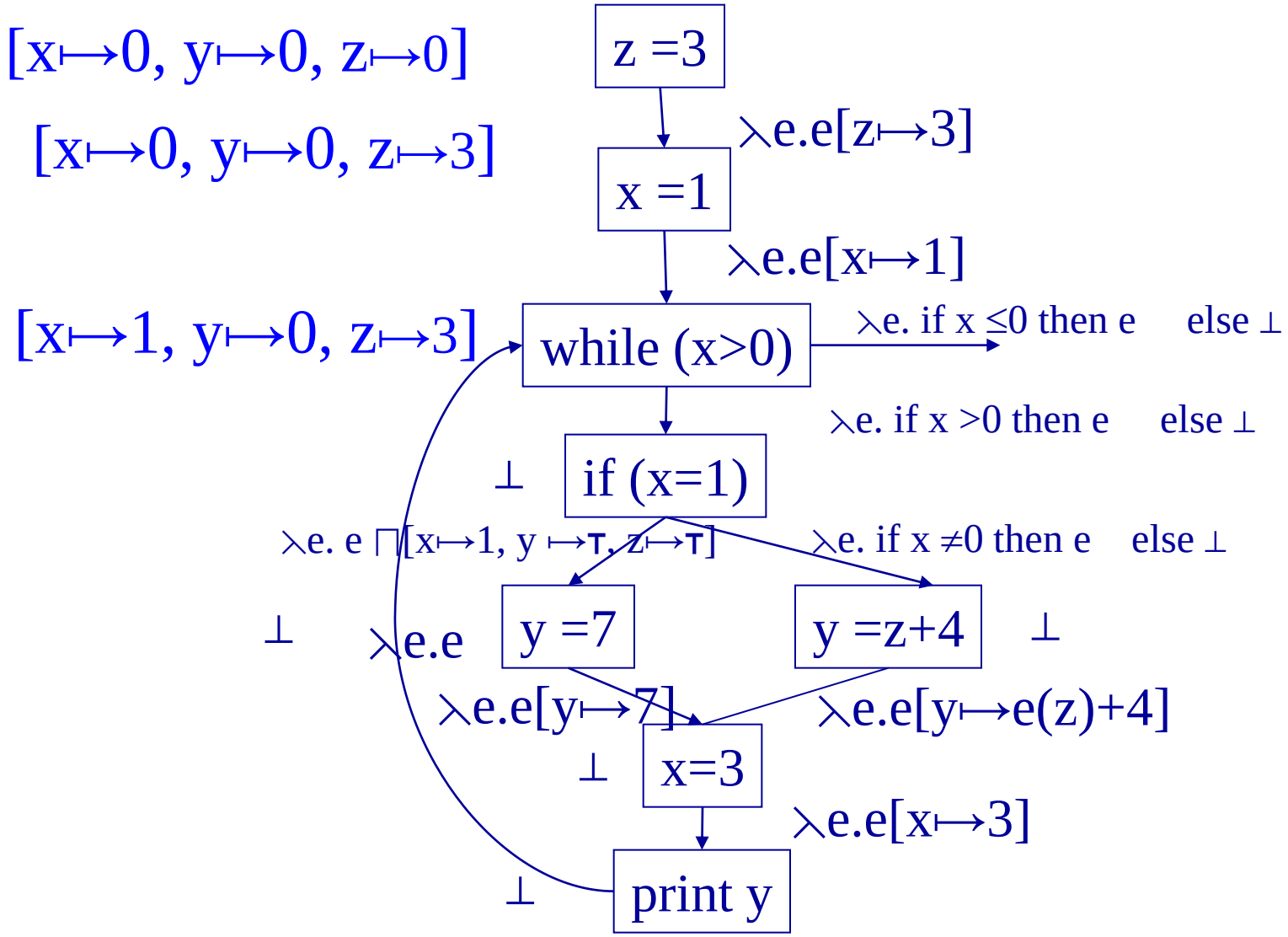
Iterative Computation



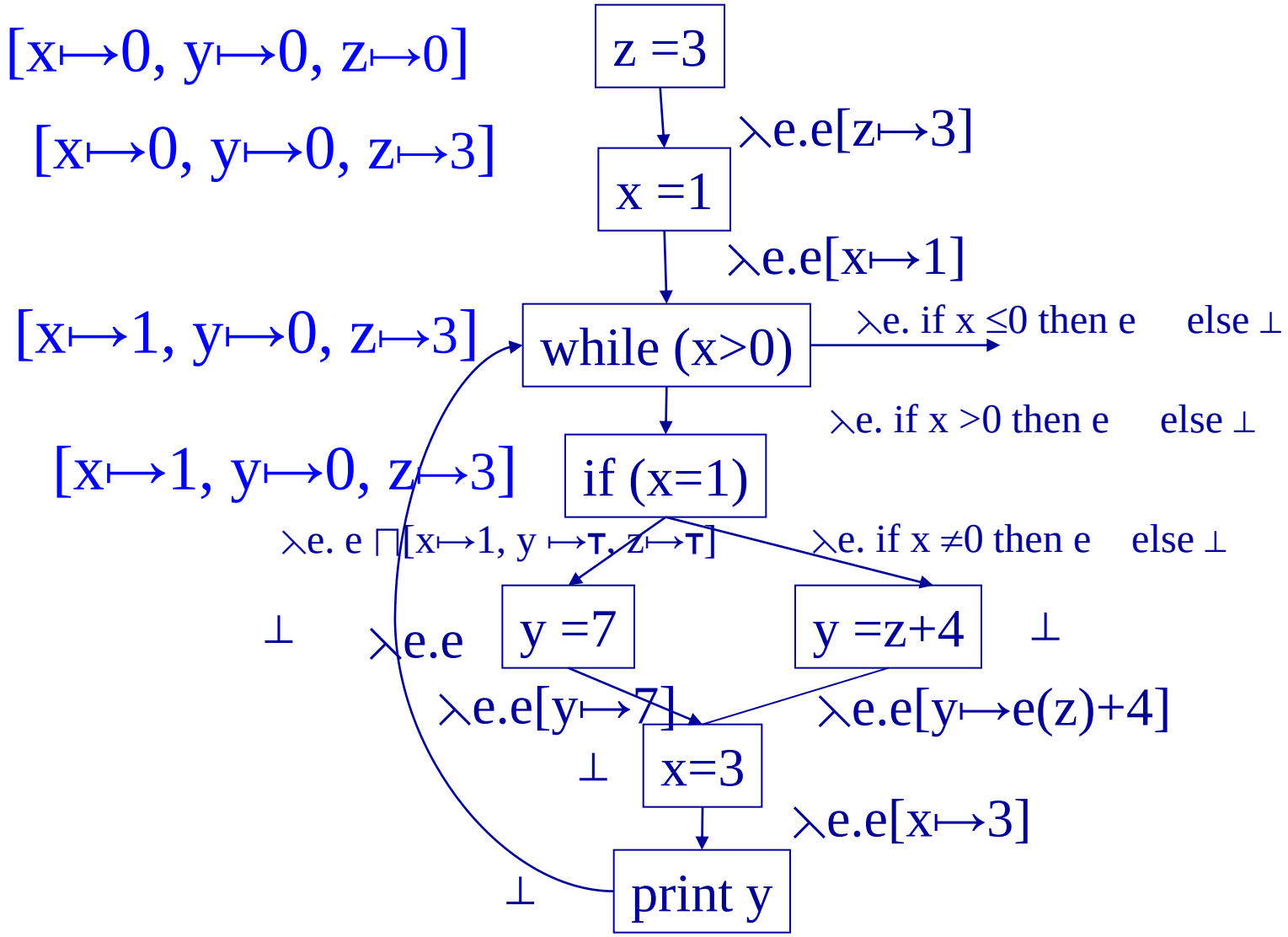
Iterative Computation



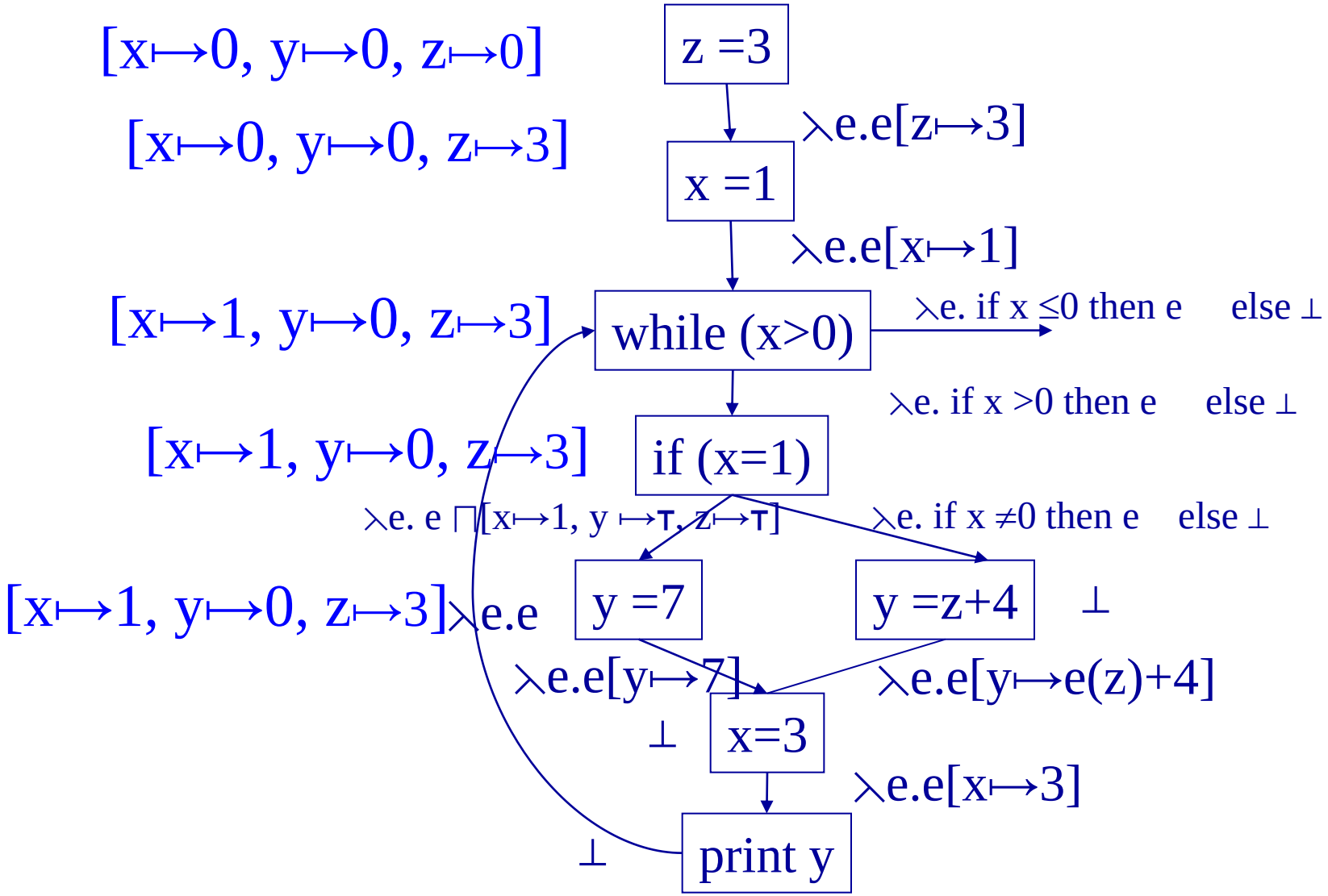
Iterative Computation



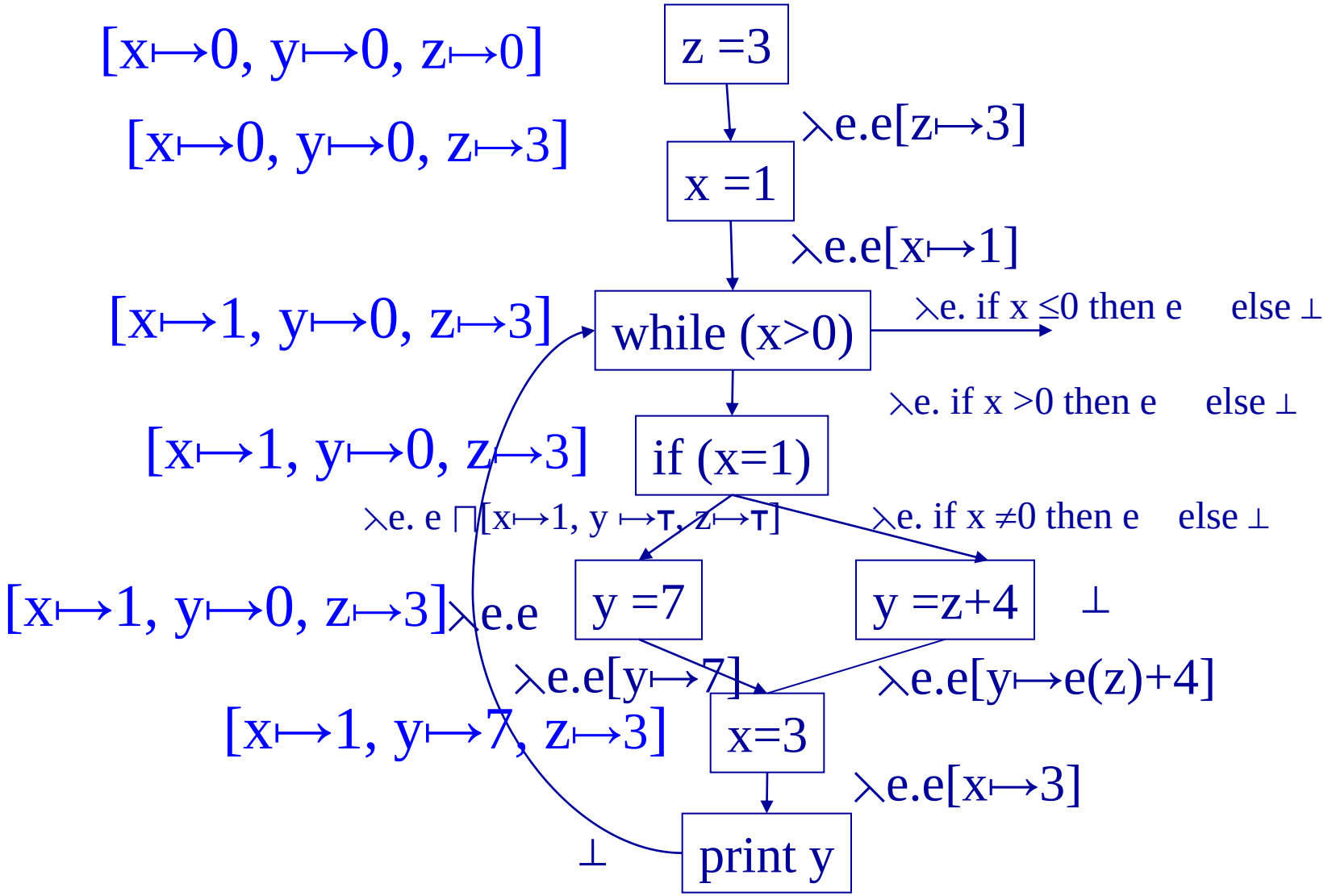
Iterative Computation



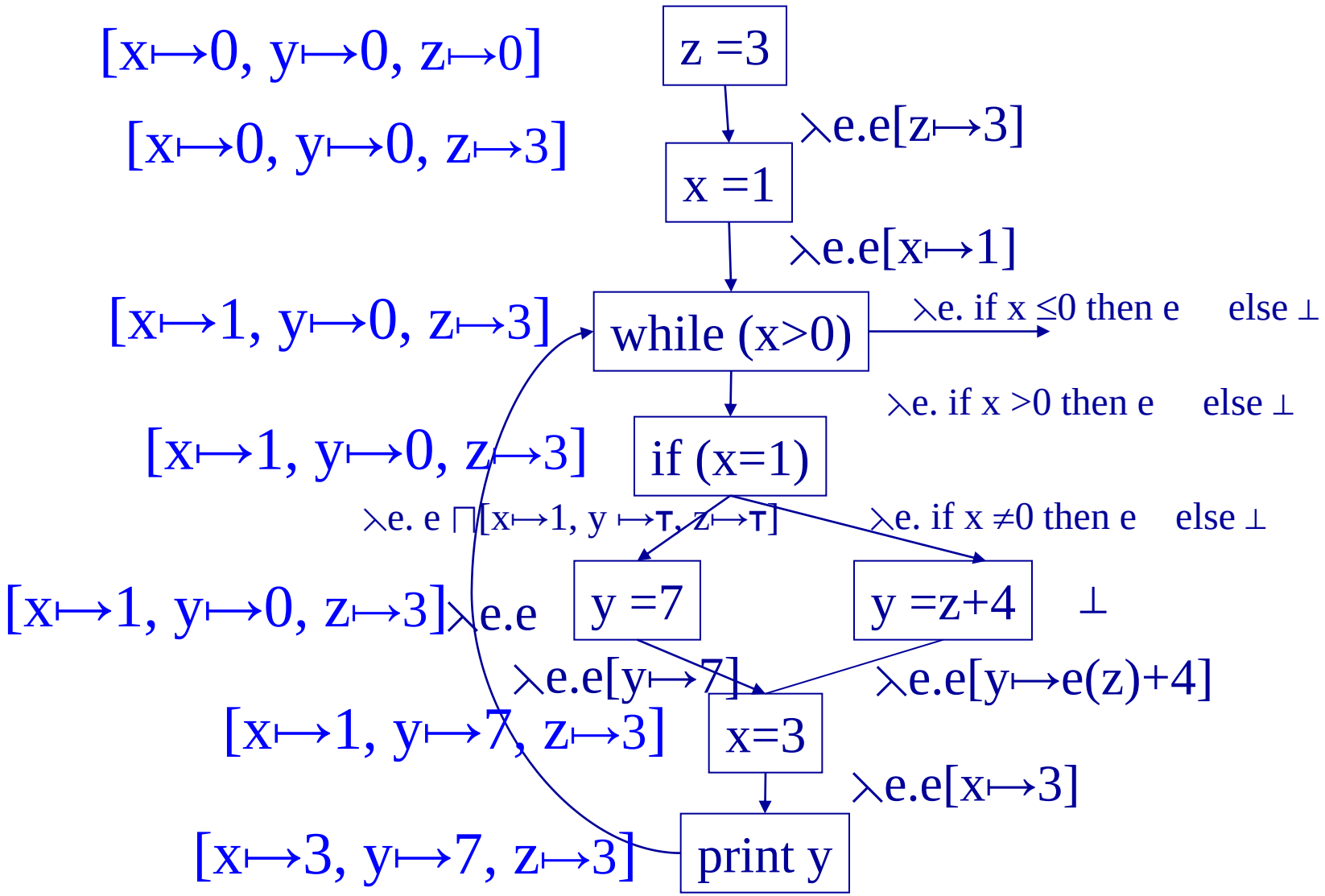
Iterative Computation



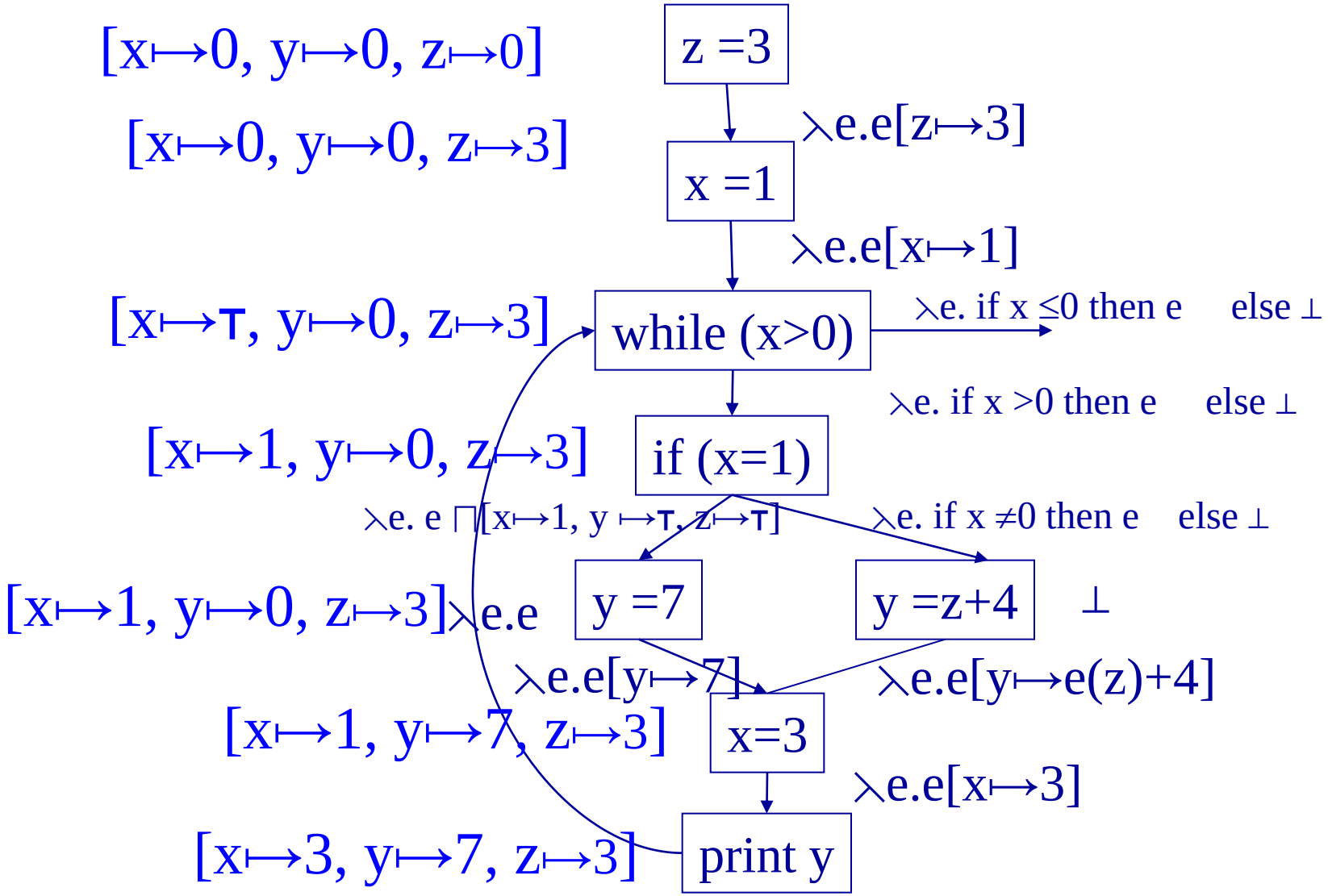
Iterative Computation



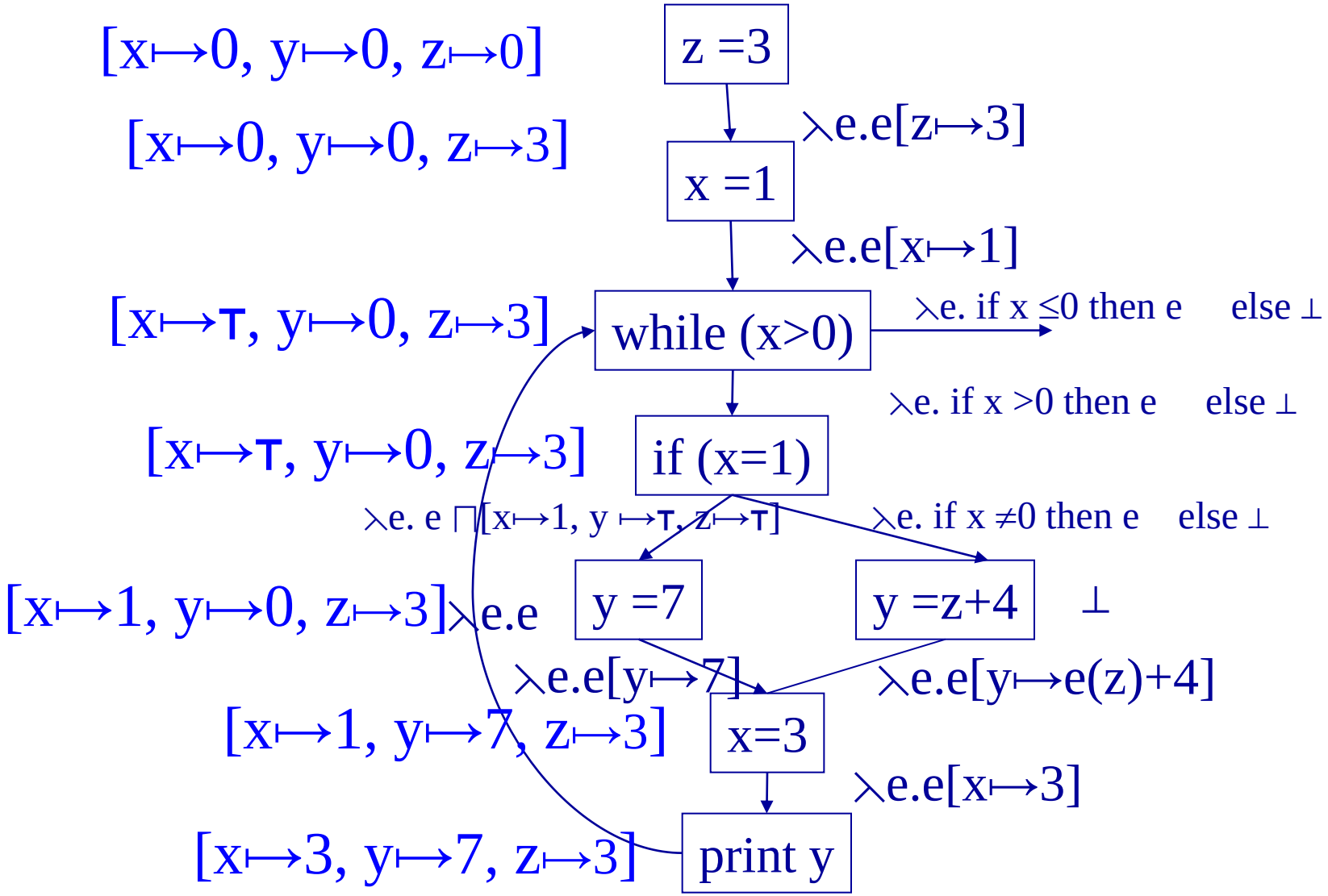
Iterative Computation



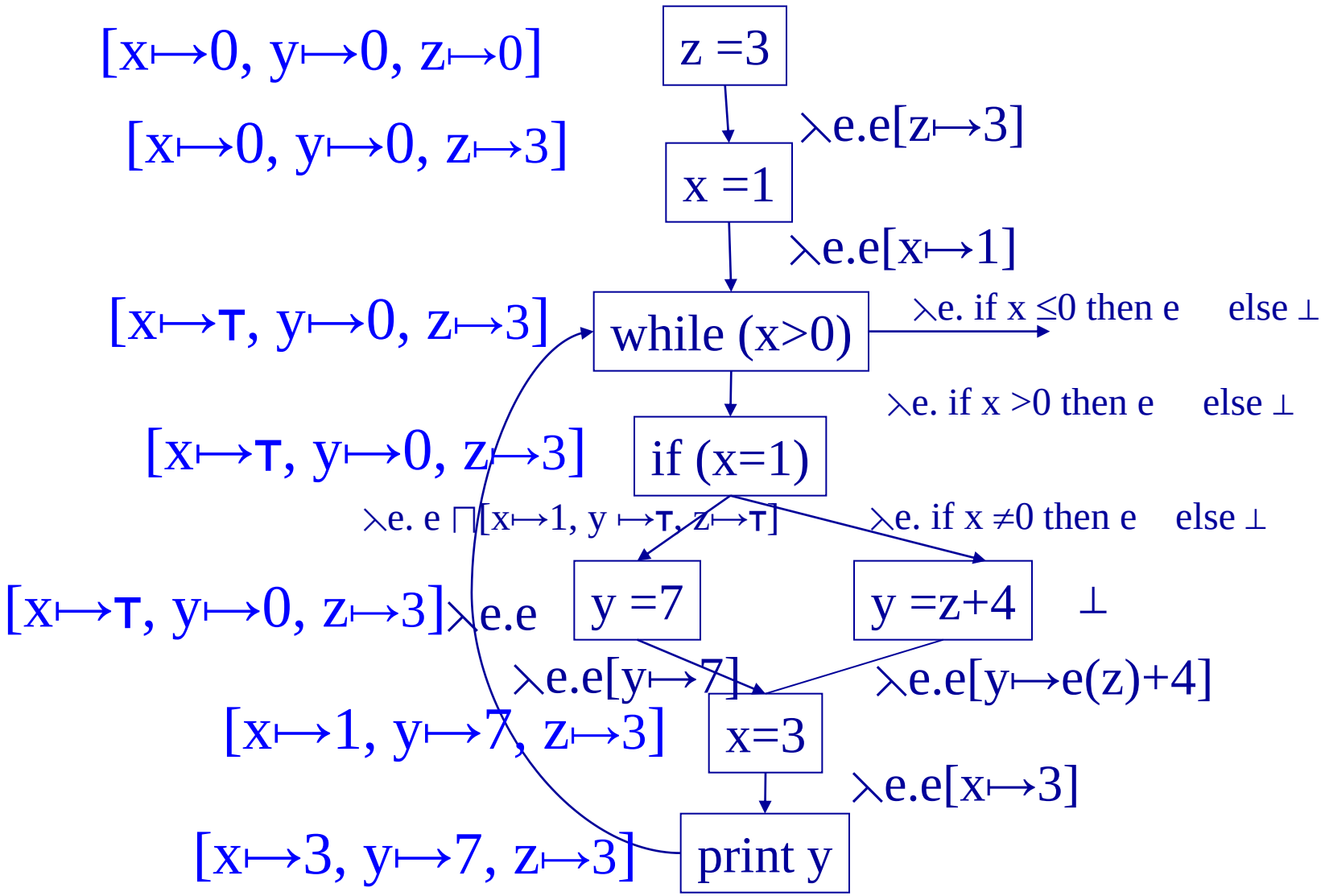
Iterative Computation



Iterative Computation



Iterative Computation



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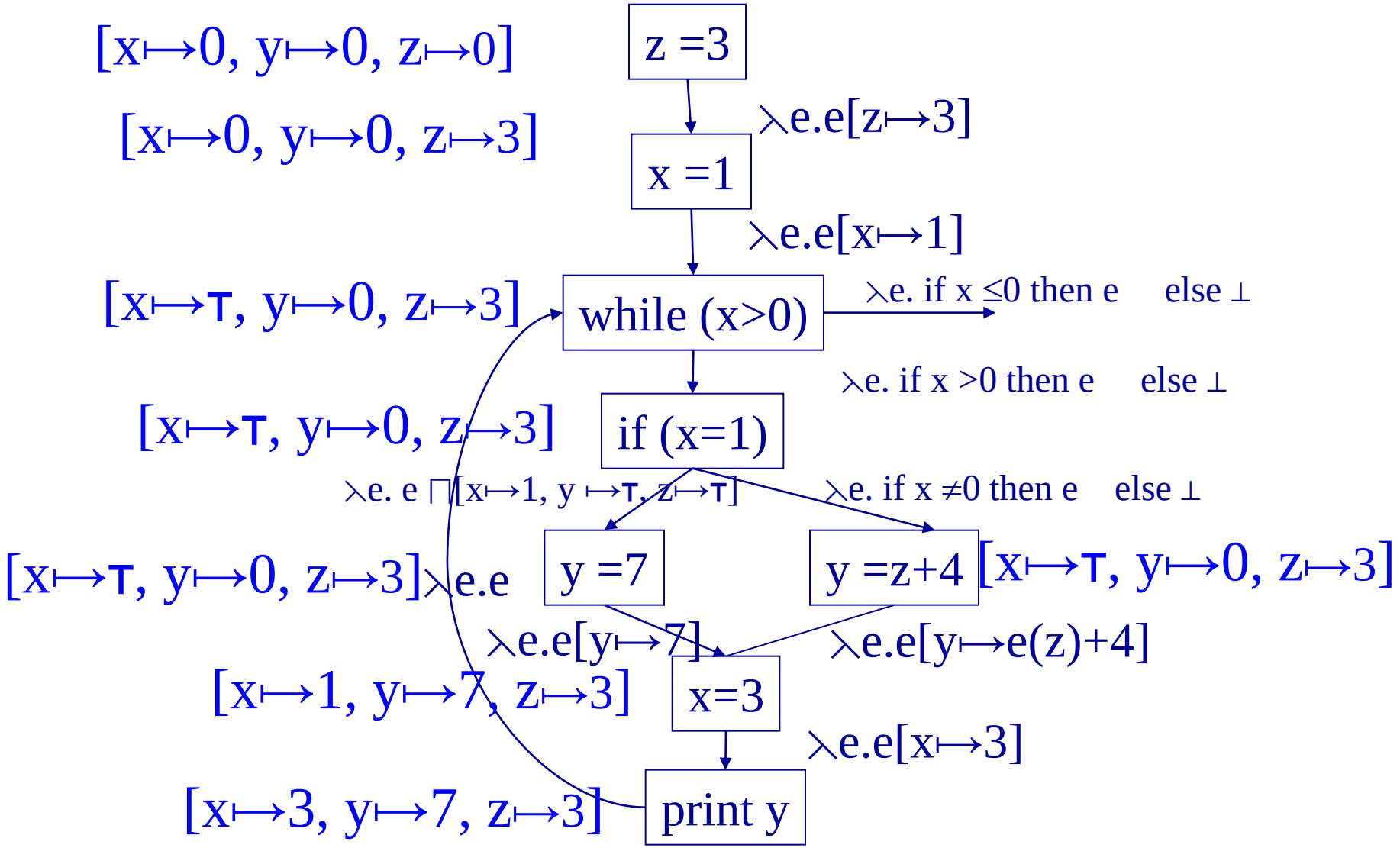

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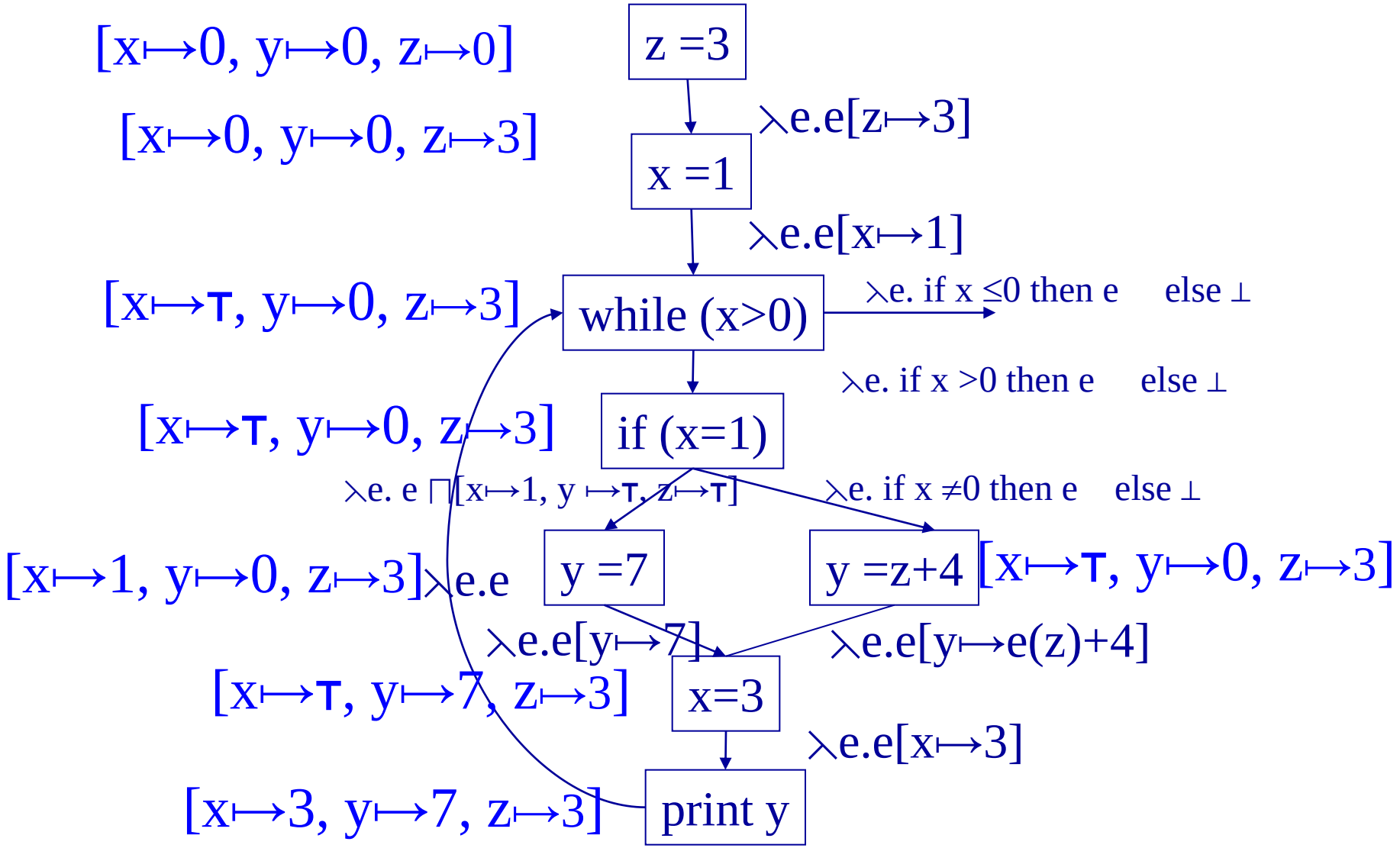

Low Level View

- Explicitly represent the program counter
- Create an abstract transition system which represents the analysis
- Execute transitions in arbitrary order

Low Level View (Example)

State : PC → (Var → Val)

Transformer: State → State

1: z = 3

2: x = 1

while 3: (x > 0) (

4: if (x = 1) then 5: y = 7

else 6: y =z + 4

7: x = 3

8: print y

)

$$\lambda S.\ \lambda pc.\ \lambda v:$$

$$\begin{cases} 0 & pc = 1 \\ S\ 1\ [z \mapsto 3]\ v & pc = 2 \\ (S\ 2\ [x \mapsto 1] \sqcup S\ 8)\ v & pc = 3 \\ S\ 3\ \ v & pc = 4 \\ (S\ 4 \sqcap [x \mapsto 1,\ y \mapsto \top,\ z \mapsto \top])\ v & pc = 5 \\ S\ 4\ v & pc = 6 \\ (S\ 5\ [y \mapsto 7] \sqcup (S\ 6\ [y \mapsto (S\ 6\ z) + 4] & pc = 7 \\ S\ 7\ [x \mapsto 3]\ v & pc = 8 \end{cases}$$

Summary

- Chaotic iterations is a powerful technique
- Easy to implement
- Rather precise
- But expensive
  - More efficient methods exist for structured programs