Iterative Program Analysis

Mooly Sagiv
http://www.cs.tau.ac.il/~msagiv/courses/pa.html
Tel Aviv University
640-6706

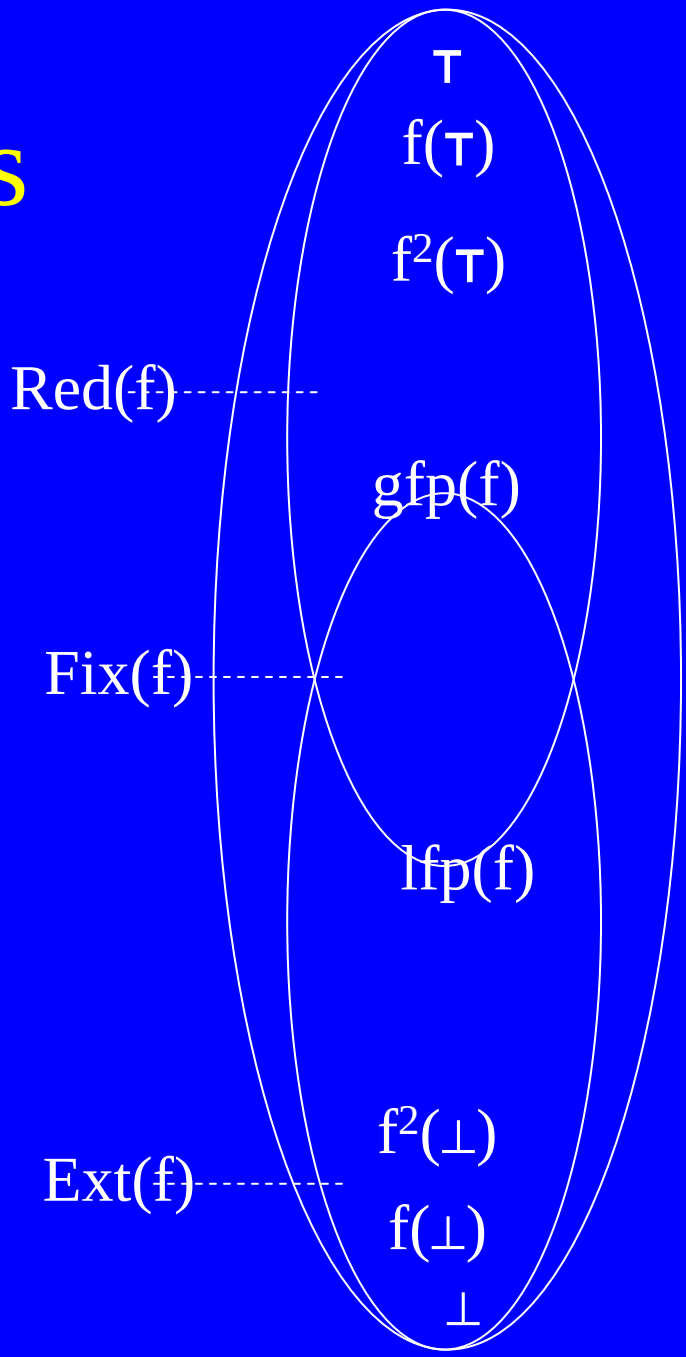
Textbook: Principles of Program Analysis
Chapter 2.1 +6 (modified)
Appendix A

Subjects

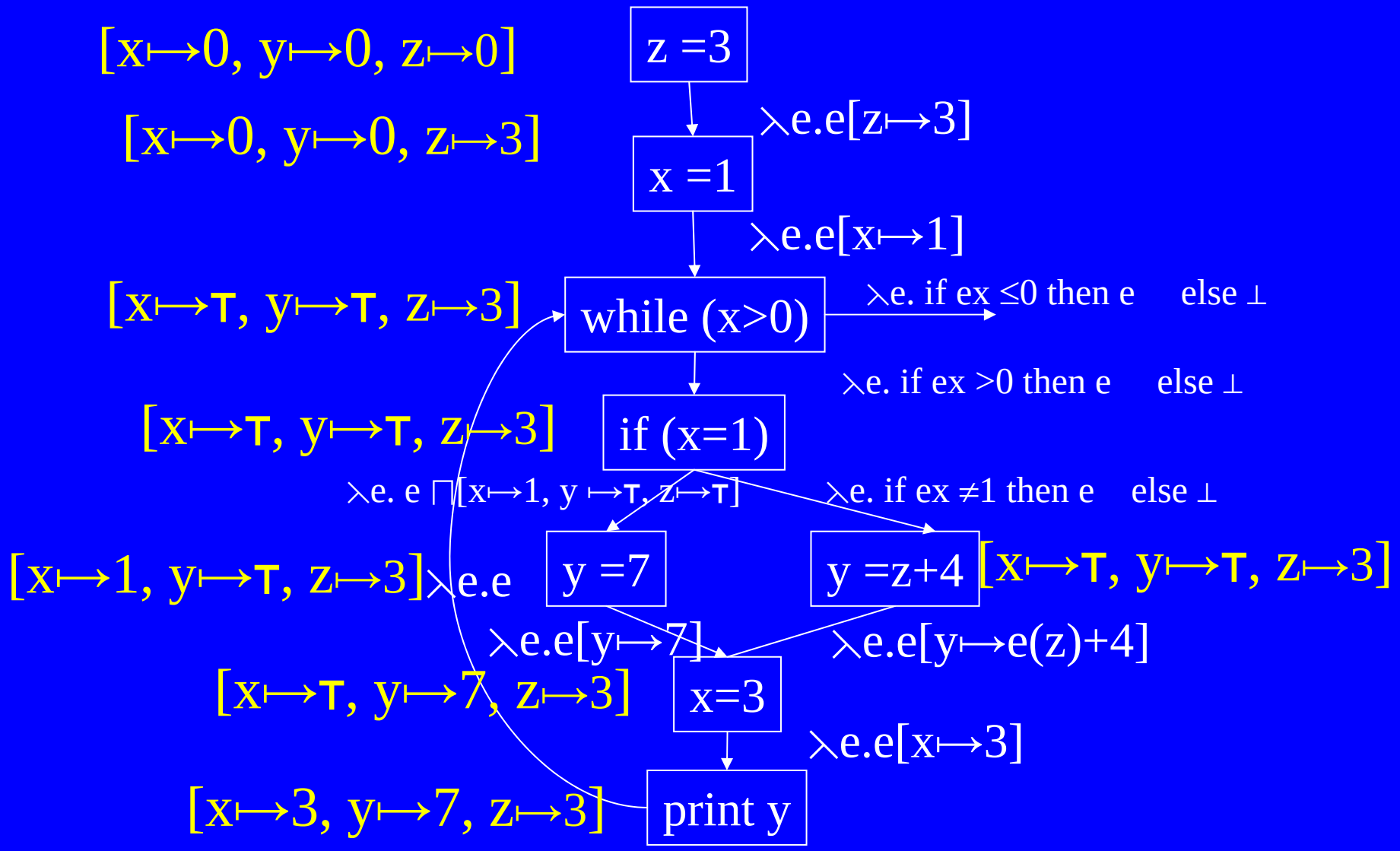
- **Why least fixed point**
- **The Monotone Framework**

Fixed Points

- A monotone function f: L $\rightarrow$ L where (L, $\sqsubseteq$, $\sqcup$, $\sqcap$, $\bot$, $\top$) is a complete lattice
- Fix(f) = { l: l $\in$ L, f(l) = l}
- Red(f) = {l: l $\in$ L, f(l) $\sqsubseteq$ l}
- Ext(f) = {l: l $\in$ L, l $\sqsubseteq$ f(l)}
  - $l_1 \sqsubseteq l_2 \Rightarrow f(l_1) \sqsubseteq f(l_2)$
- Tarski's Theorem 1955: if f is monotone then:
  - lfp(f) = $\sqcap$ Fix(f) = $\sqcap$ Red(f) $\in$ Fix(f)
  - gfp(f) = $\sqcup$ Fix(f) = $\sqcup$ Ext(f) $\in$ Fix(f)



Least Fixed Point



Monotone Forward Frameworks

- A complete lattice $(L, \sqsubseteq, \sqcup, \sqcap, \bot, \top)$ describes the "potential pieces of information"
- The initial value at entry is specified by $\iota \in L$
- The effect of every control flow edge e is described by a monotone function $f_e : L \rightarrow L$ (transfer function)
- Compute lfp of the following system of equations (forward)

$$DF(entry) = \iota$$

$$DF(v) = \sqcup\{f_{(u,v)}(DF(u) \mid (u, v) \in E\}$$

DF(1)=ι

DF(2)=λe. e[x↦3] DF(1)

DF(3)=λe. e[x↦1] DF(2)
⊔
λe. e DF(8)

DF(4)= λe. if ex >0 then e else ⊥ DF(3)

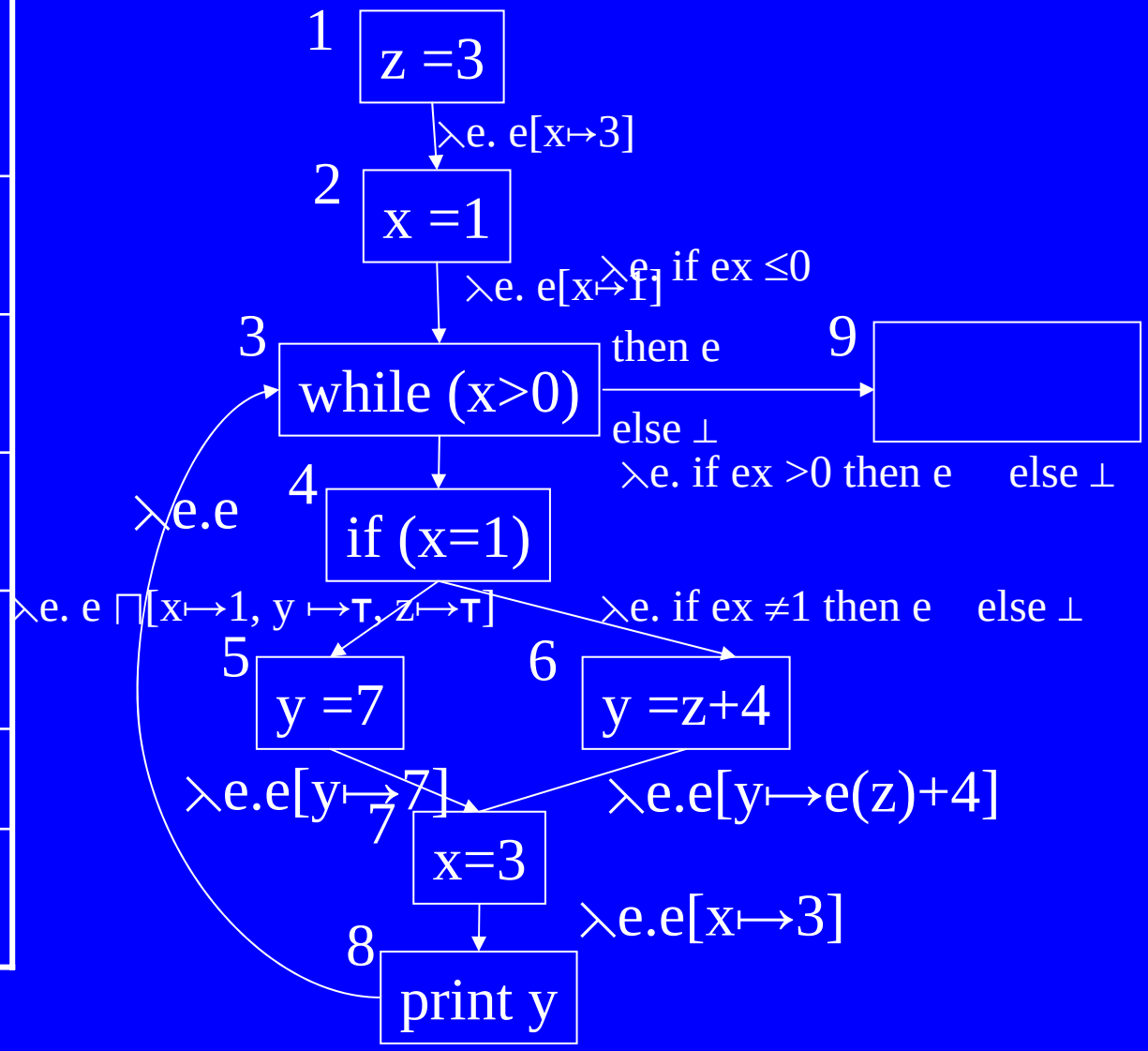
DF(5)=λe. e ⊓[x⟼1, y ⟼⊤, z⟼⊤] DF(4)

DF(6)= λe. if ex ≠1 then e else ⊥ DF(4)

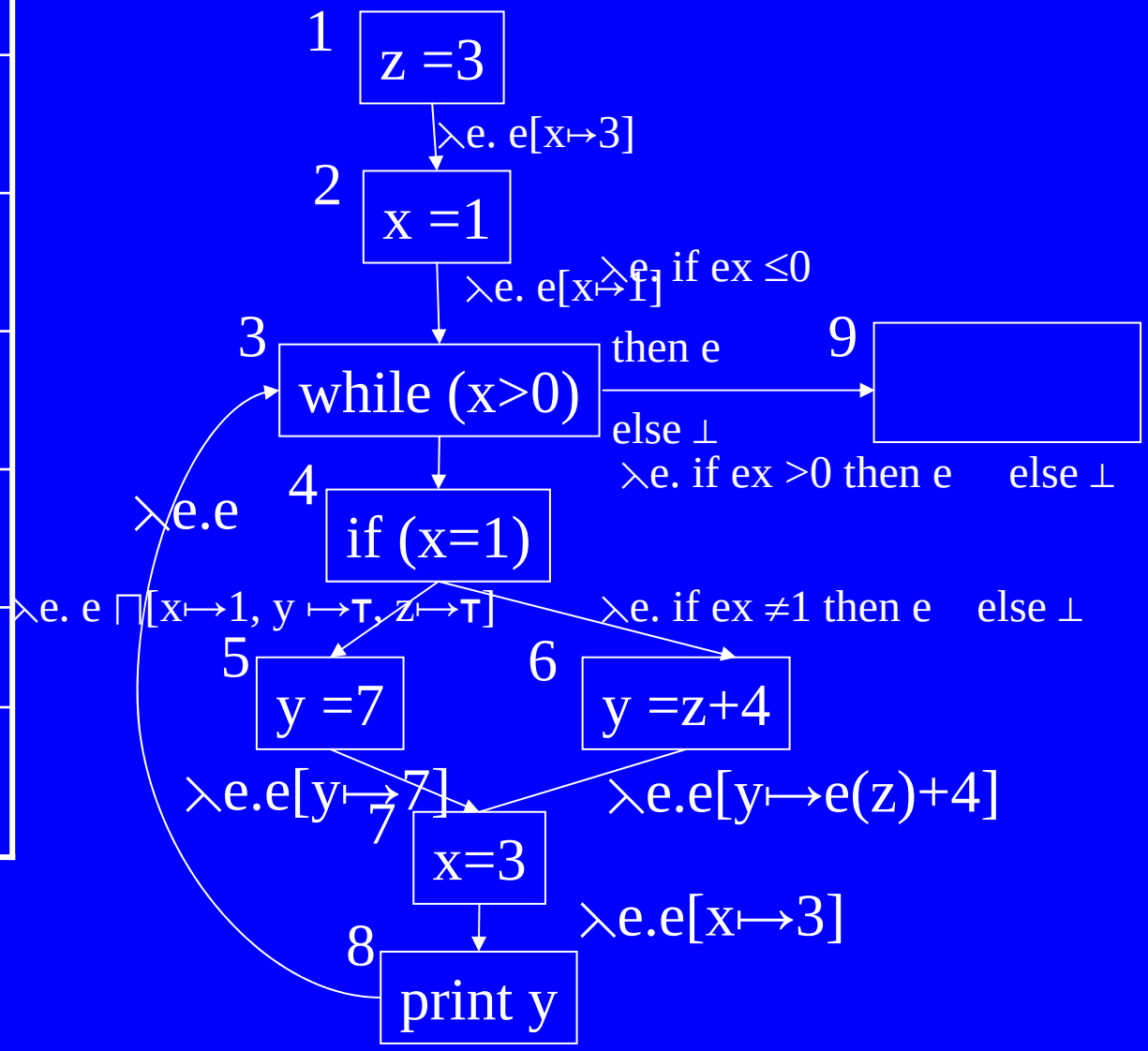
DF(7)=λe.e[y⟼7] DF(5)
⊔λe.e[y⟼e(z)+4] DF(6)

DF(8)= λe.e[x⟼3] DF(7)

DF(9)=e. if ex ≤0
then e else ⊥ DF(3)



DF(1)=ι
DF(2)=DF(1)[x↦3]
DF(3)=DF(2)[x↦1] ⊔DF(8)
DF(4)= if DF(3)x >0 then DF(3) else ⊥
DF(5)= DF(4) ⊓ [x↦1, y ↦⊤, z↦⊤]
DF(6)= if DF(4)x ≠1 then DF(4) else ⊥
DF(7)= DF(5) [y↦7] ⊔ DF(6)[y↦e(z)+4]
DF(8)= DF(7)[x↦3]
DF(9)=if DF(3)x ≤0 then DF(3) else ⊥



Chaotic Iterations for forward problems

for $v \in N$ **do**
    $DF(v) := \perp$
$DF(entry) := \iota$
$WL = N$
while $WL \neq \emptyset$ do
    Select and remove an arbitrary $u \in WL$
    for every edge $(u, v) \in E$
        $temp = DF(v) \sqcup f_e(DF(u))$
        if $(temp \neq DF(v))$
            $DF(v) := temp$
            $WL := WL \cup \{v\}$

Monotone Backward Frameworks

- A complete lattice $(L, \sqsubseteq, \sqcup, \sqcap, \bot, \top)$ describes the "potential pieces of information"
- The initial value at exit is specified by $\iota \in L$
- The (backward) effect of every control flow edge e is described by a monotone function $f_e : L \rightarrow L$ (transfer function)
- Compute lfp of the following system of equations (backward)

$DF(exit) = \iota$

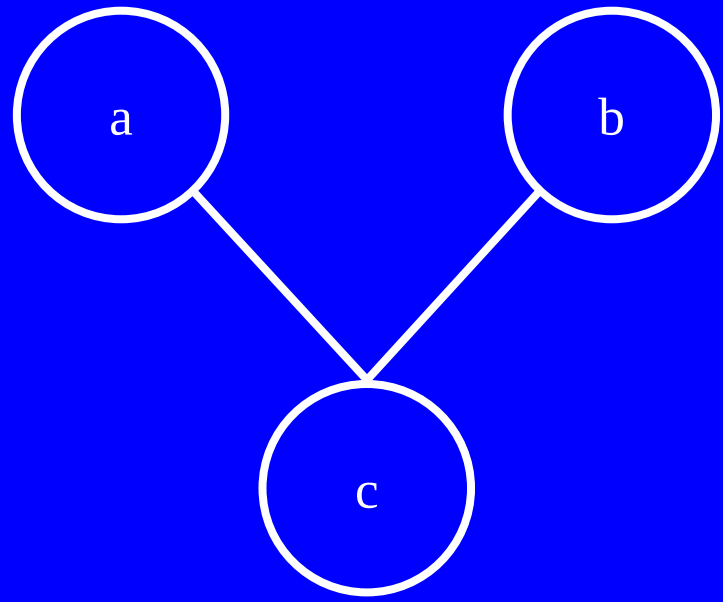
$DF(v) = \sqcup\{f(u,v)(DF(u)) \mid (v, u) \in E\}$

Chaotic Iterations for backward problems

for v ∈ N **do**
    DF(v) := ⊥
DF(exit) := ι
WL= N
while WL != ∅ do
    Select and remove an arbitrary u ∈ WL
    for every edge (v, u) ∈ E
        temp = DF(v) ⊔ f_e(DF(u))
        if (temp != DF(v))
            DF(v) := temp
            WL := WL ∪ {v}

A Simple Example

```
/* c  */
L0:        a := 0
/* ac */
L1:        b := a + 1
/* bc */
           c := c + b
/* bc */
           a := b * 2
/* ac */
           if c < N goto L1
/* c  */
           return c
```



Instances of Monotone Frameworks

- Kill/Gen Problems
  - Reaching Definitions
  - Available Expressions
  - Live Variables
  - $\sqcup = \cup$ or $\sqcup = \cap$
  - $f_e(\text{entry})) = (\text{entry} - \text{kill}(e)) \cup \text{gen}(e)$
- Forward
  - May be uninitialized (garbage) variables
  - Constant propagation
  - Sign-analysis
  - Parity analysis
  - Points-to analysis
  - Shape Analysis
  - Driver verification
- Backward
  - Truly-live variables

May-be-garbage variables

- A variable may-be-garbage at a label *l* if there may be a path to *l* in which it is either uninitialized or set using an uninitialized variable

```
x := 5 ;
if z > 2
      then y := 17 ;
      else skip ;
t := y + x ;
```

May-be-garbage variables(cont)

- L = (P(Var), $\subseteq$, $\cup$, $\cap$, $\varnothing$, Var)
- Initial value $\iota = Var_*$
- Transfer functions $f_e(DF)$

x := a	if $FV(a) \cap DF \neq \varnothing$ then $DF \cup \{x\}$ else $DF - \{x\}$
skip	DF
b	DF

The Factorial Program

```
y := x;
z := 1;

while y>1 do (

        z:= z * y;

        y := y - 1;

)

y := 0;
```

Over Conservative Solution

```
if y >1
    then z := 1;
…
if y >1
    then y := z;
```

Constant Propagation

- Determine variables with constant values
- Information Lattice
  - Extended integer lattice $(L_1, \sqsubseteq_1, \sqcup_1, \sqcap_1, \bot_1, \top_1)$
    - » $L_1 = Z \cup \{\bot_1, \top_1\}$
    - » $\bot_1 \sqsubseteq_1 z \sqsubseteq_1 \top_1$
  - Define $L = (S \rightarrow L_1, \sqsubseteq)$ where $S = Var_*$
  - Initial value ι ?
- Transfer functions A_{cp}: $AExp \rightarrow (L \rightarrow L_1)$

	$f_e(DF)$
x := a	$DF[x \mapsto A_{cp}[\![a]\!](DF)]$
skip	DF
x = c	$DF \sqcap [x \mapsto c, y \mapsto \top]$

```
y := 5;
x := 8;
while x>1 do (
        x:= y+3;
        x := (y * y) -17;
)
```

Points-To Analysis

- Determine if a pointer variable p may point to q
- L = (P(Var× Var), $\subseteq$, $\cup$, $\cap$, $\varnothing$, Var ×Var)
- (x, y) $\in$l --- x may hold the address of y
- The initial value ι= $\varnothing$
- Transfer functions

$f_e(DF)$

x := &y

x := y

x := *y

*x := y

Usage of Points-To-Information

- "Adapt" other optimizations
  - Constant propagation

    x = 5;

    *p = 7 ;

    … x …
- Pointer aliases
  - Variables p and q are may-aliases at l if the points-to set at l contains entries (p, x) and (q, x)
- Side effect analysis

  *p = *q + * * t

```
t := &a;
y := &b;

z := &c;

 if x> 0;
        then p:= &y;
        else p:= &z;

 p := t;
```

```
/*  ∅ */ t := &a; /* {(t, a)}*/
 /* {(t, a)}*/  y := &b; /* {(t, a), (y, b) }*/
/* {(t, a), (y, b)}*/ z := &c; /* {(t, a), (y, b), (z, c) }*/
 if x> 0;
        then p:= &y; /* {(t, a),  (y, b), (z, c), (p, y)}*/

        else p:= &z; /* {(t, a),  (y, b), (z, c), (p, z)}*/
/* {(t, a),  (y, b), (z, c), (p, y), (p, z)}*/
 *p := t;
/* {(t, a), (y, b), (y, c), (p, y), (p, z), (y, a), (z, a)}*/
```

Flow insensitive points-to-analysis Steengard 1996

- Ignore control flow
- One set of points-to per program
- Can be represented as a directed graph
- Conservative approximation
  - Accumulate pointers
- Can be computed in almost linear time

```
t := &a;
y := &b;

z := &c;

 if x> 0;
        then p:= &y;
        else p:= &z;

 *p := t;
```

Complexity of Chaotic Iterations

- Parameters:
  - N number of nodes
  - k is the maximum outdegree
  - A lattice of height h
  - c is the maximum cost of
    - » applying f_l
    - » $\sqcup$
    - » L comparisons
- Complexity
  O(N * h * c * k)

Soundness of Chaotic Iterations

- Later with abstract interpretation

Precision of Chaotic Iterations

- Optimal
- Join-over-all-paths - No loss of information w.r.t. straight line code
- Relatively optimal (induced) w.r.t. the abstraction
- Compare at run-time
- Good enough for the used optimization

The Join-Over-All-Paths (JOP)

- Let paths(entry, v) denote the potentially infinite set of paths from entry to v (written as sequences of edges)
- For a sequence of edges [e_1, e_2, …, e_n] define f [e_1, e_2, …, e_n]: L $\rightarrow$ L by composing the effects of edges
  - $f[\varepsilon](s)=s$
  - $f[p.e](s) = f[p](f_e(s))$
- $JOP_1 = \sqcup\{f[e_1, e_2, \ldots, e](\iota)$
  $[e_1, e_2, \ldots, e] \in$ paths(entry, l)$\}$

JOP vs. Least Solution

- The DF solution obtained by Chaotic iteration satisfies for every v:
  - $\mathrm{JOP}_v \sqsubseteq \mathrm{DF}(v)$
- If every f_e is additive (distributive) for all the labels l
  - $\mathrm{JOP}_v = \mathrm{DF}(v)$

Additive (Distributive) Monotone Problems

- Kill/Gen Problems
- May-be uninitialized
- Truly-live variables
- Linear constant propagation with one variable
- Points-to analysis with one level pointers

Non Distributive Monotone Problems

- Points-to-analysis
- Constant Propagation on arbitrary expressions

Converting into Distributive Frameworks

- Consider
  - a finite lattice $(L, \sqsubseteq, \sqcup, \sqcap, \bot, \top)$
  - An initial value at entry $\iota \in L$
  - The effect of every edge at e is described by a monotone function $f_e : L \rightarrow L$ (transfer function)
  - Define a distributive function on $(P(L), \subseteq, \cup, \cap, \varnothing, L)$ by $F(X) = \{ f_l(x) : x \in X \}$
  - Solve the following system of equations over P(L)

$$DF(entry) = \{\iota\}$$

$$DF(v) = \cup\{ f_{(u,v)}(x) \mid (u, v) \in E, x \in DF(u) \}$$

The Constant Propagation

- It is undecidable to find the JOP in the constant propagation problem
- A Sketch of a proof

```
while (...) (
        if (...) x_1 = x_1 + 1;
        if (...) x_2 = x_2 + 1;
        ...
       if  (...) x_n = x_n + 1;
        }
y =  truncate (1/ (1 + p^2(x_1, x_2, ..., x_n))
/* Is y=0 here? */
```

Static Analysis problems beyond Monotone Frameworks

- Infinite heights
  - integer intervals
  - Linear relationships between variables
- Bi-directional problems
  - Partial-redundant expressions
  - Automatic inference of variable types in imperative (weakly typed) programs

    x := b[z]

    a [b[y]] := x
- Procedures

Historical Perspective

- 1973 Kildall defined the basic framework but required distributive frameworks
- 1976 Kam and Ulmann defined Monotone Framework
- 1980 Tarjan suggested an almost linear time algorithm for reducible flow graph
- 1980 Rosen suggested a linear time algorithm for high level language

Static Analysis problems beyond Monotone Frameworks

- Infinite heights
  - integer intervals
  - Linear relationships between variables
- Bi-directional problems
  - Partial-redundant expressions
  - Automatic inference of variable types in imperative (weakly typed) programs

    x := b[z]

    a [b[y]] := x
- Procedures

Conclusions

- Chaotic iterations is a powerful technique
- Easy to implement
- Rather precise
- But expensive
  - More efficient methods exist for structured programs