Counter Example Guided Refinement CEGAR

Mooly Sagiv

Recap

- Many abstract domains
  - Signs
  - Odd/Even
  - Constant propagation
  - Intervals
  - [Polyhedra]
  - Canonic abstraction
  - Domain constructors
  - …
- Static Algorithms
  - Iterative Chaotic Iterations
  - Widening/Narrowing
  - Interprocedural Analysis
  - Concurrency
  - Modularity
  - Non-Iterative methods

A Lattice of Abstractions

- Every element is an abstract domain
- A $\sqsubseteq$ A' if there exists a Galois Connection from A to A'

But how to find the appropriate abstract domain

- Precision vs. Scalability
- Sometimes precision improves scalability
- Specialize the abstraction for the desired property

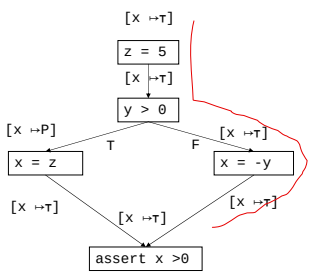
Counter Example Guided Refinement (CEGAR)

- Run the analysis with a simple abstract domain
- When the analysis verifies the property declare done
- If the analysis reports an error employs a theorem prover to identify if the error is feasible
  - If the error is feasible generate a concrete trace
  - If the error is spurious refine the abstract domain and repeat

A Simple Example

```
z =5
if (y >0)
   x = z;
 else
  x = -y;
assert x >0
```

sign(x)



A Simple Example

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if (y >0)
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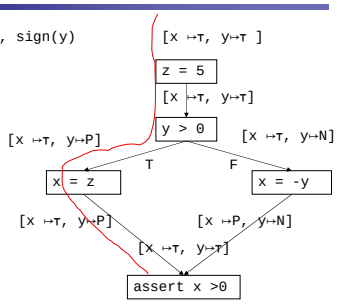
sign(x), sign(y)



A Simple Example

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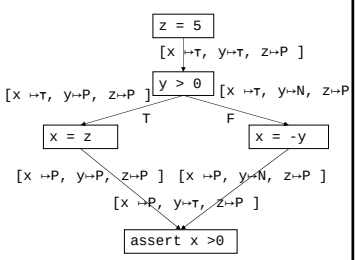
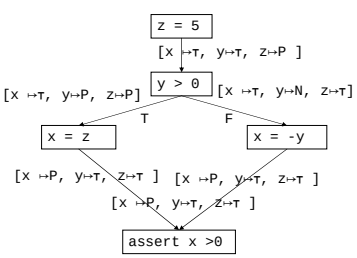
sign(x), sign(y), sign(z)



Simple Example (local abstractions)

```
z =5
if (y >0)
   x = z;
 else
  x = -y;
assert x >0
```

sign(x), sign(y), sign(z)



Plan

- CEGAR in BLAST (inspired by SLAM) POPL'04
- Limitations

Abstractions from Proofs

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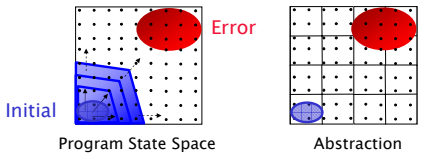
Scalable Program Verification

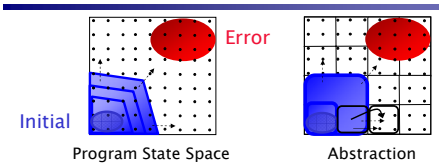
- *Little theorems* about *big programs*
  - Partial Specifications
    - Device drivers use kernel API correctly
    - Applications use root privileges correctly
  - Behavioral, path-sensitive properties

Predicate Abstraction: A crash course



- Abstraction: ***Predicates*** on program state
  - Signs: $x > 0$
  - Aliasing: $\&x \neq \&y$
- States satisfying the same predicates are equivalent
  - Merged into single abstract state

(Predicate) Abstraction: A crash course



Q1*: **Which predicates*** are required to verify a property ?

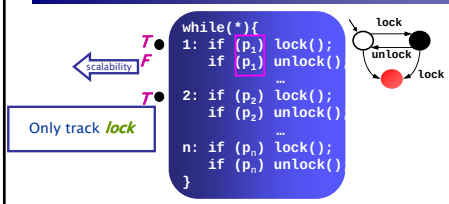
The Predicate Abstraction Domain

- Fixed set of predicates Pred
- The relational domain is
  $<P(P(Pred)), \varnothing, P(Pred), \cup, \cap>$
  - Join is set union
  - State space explosion
- Special case of canonic abstraction

Scalability vs. Verification

- Few predicates tracked
  - *e.g.* type of variables
- Imprecision hinders Verification
  - Spurious counterexamples
- Many predicates tracked
  - *e.g.* values of variables
- State explosion
  - Analysis drowned in detail

Example

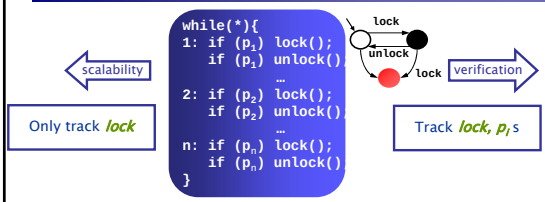


Bogus Counterexample
- Must *correlate* branches

Predicate p_1 makes trace *abstractly infeasible*

p_1 required for verification

Example

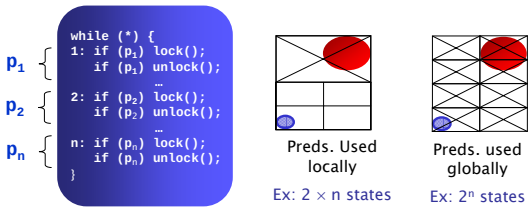


Bogus Counterexample
- Must *correlate* branches

State Explosion
- $> 2^n$ distinct states
- intractable

How can we get scalable verification ?

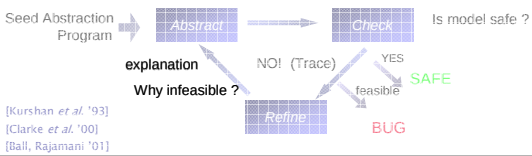
By Localizing Precision



Q2: ***Where*** are the predicates required ?

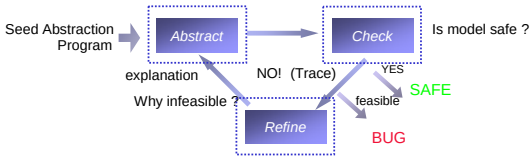
Counterexample Guided Refinement

1. ***What predicates*** remove trace ?
   - Make it abstractly infeasible
2. ***Where*** are predicates needed ?

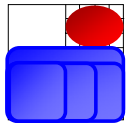


[Kurshan *et al.* '93]
[Clarke *et al.* '00]
[Ball, Rajamani '01]

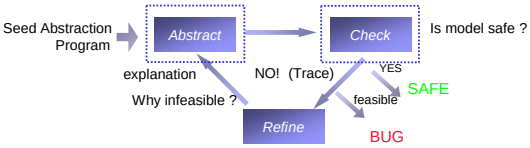
Counterexample Guided Refinement



Counterexample Guided Refinement

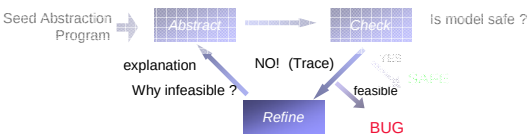


safe



This Talk: Counterexample Analysis

1. *What predicates* remove trace ?
   - Make it abstractly infeasible
2. *Where* are predicates needed ?



Plan

1. Motivation
2. Refinement using Traces
   - Simple
   - Procedure calls
3. Results

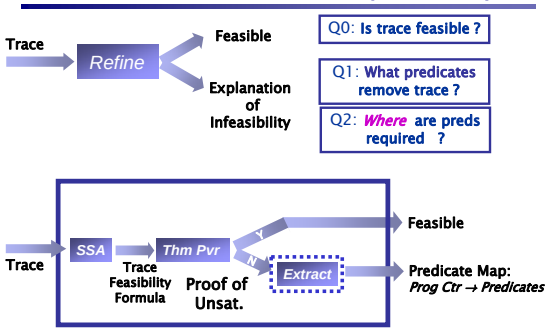
Trace Formulas

- A single abstract trace represents infinite number of traces
  - Different loop iterations
  - Different concrete values
- Solution
  - Only considers concrete traces with the same number of executions
  - Use formulas to represent sets of states

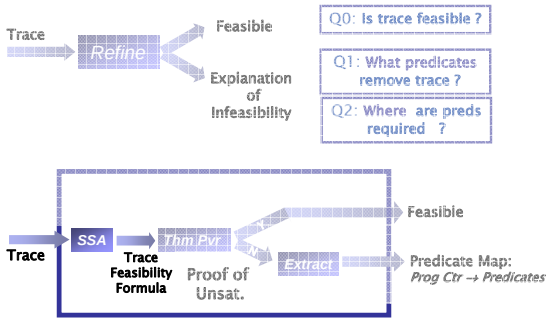
Representing States as *Formulas*

$[F]$ states satisfying F $\{s \mid s \models F\}$	F FO fmla over prog. vars
$[F_1] \cap [F_2]$	$F_1 \wedge F_2$
$[F_1] \cup [F_2]$	$F_1 \vee F_2$
$\overline{[F]}$	$\neg F$
$[F_1] \subseteq [F_2]$	F_1 implies F_2 i.e. $F_1 \wedge \neg F_2$ unsatisfiable

Counterexample Analysis



Counterexample Analysis



Traces

```
pc1: x = ctr;
pc2: ctr = ctr + 1;
pc3: y = ctr;
pc4: if (x = i-1){
pc5:   if (y != i){
         ERROR: }
     }
```

pc_1: x = ctr
pc_2: ctr = ctr + 1
pc_3: y = ctr
pc_4: assume(x = i-1)
pc_5: assume(y ≠ i)

$y = x + 1$

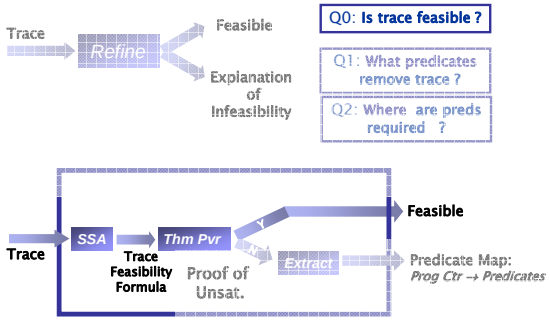
Trace Feasibility Formulas

Trace	SSA Trace	Trace Feasibility Formula
pc_1: x = ctr	pc_1: $x_1 = ctr_0$	$x_1 = ctr_0$
pc_2: ctr = ctr+1	pc_2: $ctr_1 = ctr_0+1$	$\wedge\ ctr_1 = ctr_0 + 1$
pc_3: y = ctr	pc_3: $y_1 = ctr_1$	$\wedge\ y_1 = ctr_1$
pc_4: assume(x=i-1)	pc_4: assume(x_1=i_0-1)	$\wedge\ x_1 = i_0 - 1$
pc_5: assume(y≠i)	pc_5: assume(y_1≠i_0)	$\wedge\ y_1 \neq i_0$

Theorem: Trace is *Feasible* ⇔ TFF is *Satisfiable*

Compact Verification Conditions [Flanagan,Saxe '00]

Counterexample Analysis



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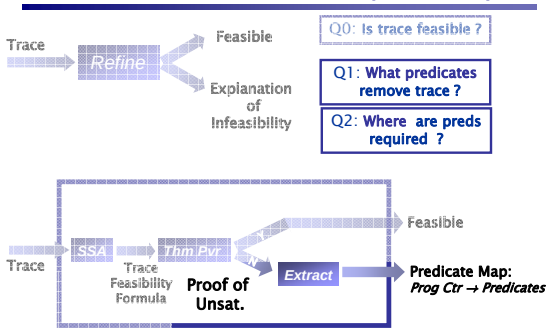

Proof of Unsatisfiability

$x_1 = ctr_0$
$\wedge\ ctr_1 = ctr_0 + 1$
$\wedge\ y_1 = ctr_1$
$\wedge\ x_1 = i_0 - 1$
$\wedge\ y_1 \neq i_0$

Trace Formula

$\dfrac{\dfrac{\dfrac{\dfrac{x_1 = ctr_0 \quad x_1 = i_0 - 1}{ctr_0 = i_0 - 1 \quad ctr_1 = ctr_0 + 1}}{ctr_1 = i_0 \quad y_1 = ctr_1}}{y_1 = i_0 \quad y_1 \neq i_0}}{\emptyset}$

Proof of Unsatisfiability

PROBLEM

Proof uses entire *history* of execution

- Information flows up and down

No *localized* or *state* information !

The Present State…

Trace

pc_1: x = ctr
pc_2: ctr = ctr + 1
pc_3: y = ctr
pc_4: assume(x = i-1)
pc_5: assume(y ≠ i)

… is all the information the executing program has ***here***

State…

1. … after executing trace ***prefix***
2. … knows ***present values*** of variables
3. … makes trace ***suffix*** infeasible

At pc_4, which predicate on ***present state*** shows infeasibility of ***suffix***?

What Predicate is needed ?

Trace

pc_1: x = ctr
pc_2: ctr = ctr + 1
pc_3: y = ctr
pc_4: assume(x = i-1)
pc_5: assume(y ≠ i)

Trace Formula (TF)

$x_1 = ctr_0$
$\wedge\ ctr_1 = ctr_0 + 1$
$\wedge\ y_1 = ctr_1$
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State…

1. … after executing trace ***prefix***
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Predicate …

… implied by TF ***prefix***

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State…

1. … after executing trace ***prefix***
2. … has ***present values*** of variables
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Predicate …

… implied by TF ***prefix***

… on ***common*** variables

What Predicate is needed ?

Trace	Trace Formula (TF)
pc_1: x = ctr	$x_1 = ctr_0$
pc_2: ctr = ctr + 1	$\wedge$ $ctr_1 = ctr_0 + 1$
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pc_5: assume(y ≠ i)	$\wedge$ $y_1 \neq i_0$

State…	Predicate …
1. … after executing trace *prefix*	… implied by TF *prefix*
2. … has *present values* of variables	… on *common* variables
3. … makes trace *suffix* infeasible	… & TF *suffix* is *unsatisfiable*

What Predicate is needed ?

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State…	Predicate …
1. … after executing trace *prefix*	… implied by TF *prefix*
2. … knows *present values* of variables	… on *common* variables
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Craig's Interpolation Theorem [Craig '57]

Given formulas ψ^- , ψ^+ s.t. $\psi^- \wedge \psi^+$ is *unsatisfiable*

There exists an *Interpolant* Φ for ψ^- , ψ^+ , s.t.

1. ψ^- *implies* Φ
2. Φ has symbols *common* to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is *unsatisfiable*

Examples of Craig's Interpolation

- $\psi^- = b \wedge (\neg b \vee c)$
  $\psi^+ = \neg c$
- $\psi^- = x_1 = ctr_0 \wedge ctr_1 = ctr_0 + 1 \wedge y_1 = ctr_1$
  $\psi^+ = x_1 = i_0 - 1 \wedge y_1 \neq i_0$
  - $y_1 = x_1 + 1$

Craig's Interpolation Theorem [Craig '57]

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2. Φ has symbols ***common*** to ψ^-, ψ^+
3. $\Phi \wedge \psi^+$ is ***unsatisfiable***

Φ computable from ***Proof of Unsat.*** of $\psi^- \wedge \psi^+$

[Krajicek '97] [Pudlak '97]
(boolean) SAT-based Model Checking [McMillan '03]

Interpolant = Predicate !

Trace		Trace Formula	
pc_1: x = ctr		$x_1 = ctr_0$	
pc_2: ctr = ctr + 1	$\wedge$	$ctr_1 = ctr_0 + 1$	ψ^-
pc_3: y = ctr	$\wedge$	$y_1 = ctr_1$	
pc_4: assume(x = i-1)	$\wedge$	$x_1 = i_0 - 1$	ψ^+
pc_5: assume(y ≠ i)	$\wedge$	$y_1 \neq i_0$	

Interpolate → Φ

Require:

1. Predicate ***implied*** by trace ***prefix***
2. Predicate on ***common*** variables
   common = ***current*** value
3. Predicate & ***suffix*** yields a ***contradiction***

Interpolant:

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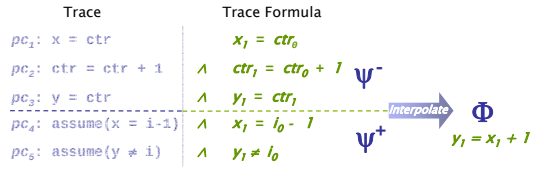
Interpolant = Predicate !



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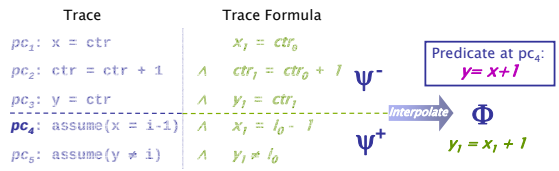

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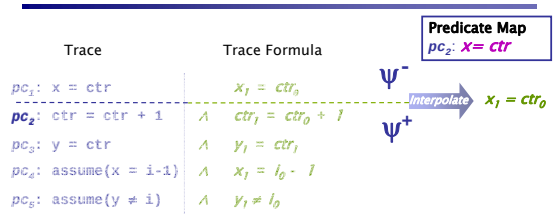
Building Predicate Maps



- Cut + Interpolate at ***each*** point
- Pred. Map: $pc_i \mapsto$ Interpolant from cut i

Building Predicate Maps

Trace		Trace Formula
pc_1: x = ctr		$x_1 = ctr_0$
pc_2: ctr = ctr + 1	$\wedge$	$ctr_1 = ctr_0 + 1$
pc_3: y = ctr	$\wedge$	$y_1 = ctr_1$
pc_4: assume(x = i-1)	$\wedge$	$x_1 = i_0 - 1$
pc_5: assume(y ≠ i)	$\wedge$	$y_1 \neq i_0$

ψ^- | ψ^+ — Interpolate: $x_1 = ctr_1 - 1$

Predicate Map
pc_2: $x = ctr$
pc_3: $x = ctr-1$

- Cut + Interpolate at ***each*** point
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Building Predicate Maps

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ψ^- | ψ^+ — Interpolate: $y_1 = i_0$

Predicate Map
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Predicate Map
pc_2: $x = ctr$
pc_3: $x = ctr-1$
pc_4: $y = x+1$
pc_5: $y = i$

Theorem: ***Predicate map*** makes trace ***abstractly infeasible***

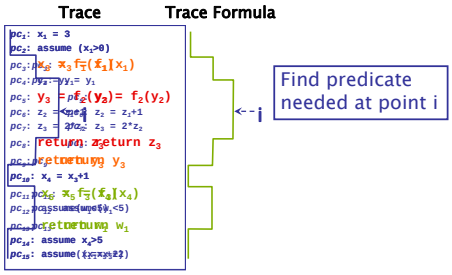
Plan

1. Motivation

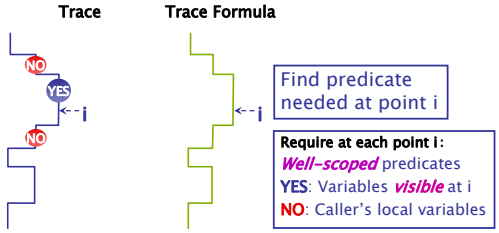
2. Refinement using Traces
   - Simple
   - Procedure calls

3. Results

Traces with Procedure Calls

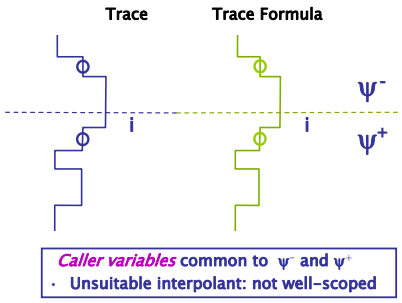


Interprocedural Analysis



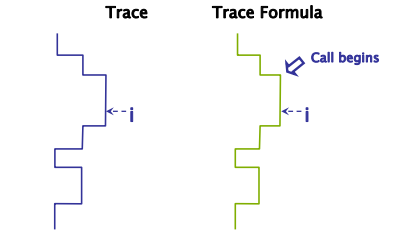
Procedure Summaries [Reps,Horwitz,Sagiv '95]
Polymorphic Predicate Abstraction [Ball,Millstein,Rajamani '02]

Problems with Cutting

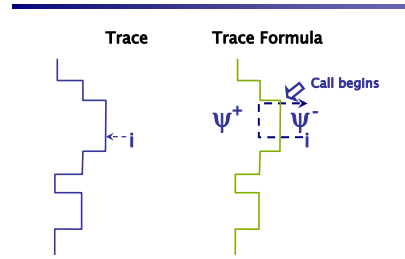


Caller variables common to ψ^- and ψ^+
- Unsuitable interpolant: not well-scoped

Interprocedural Cuts

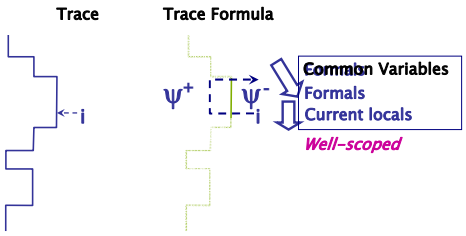


Interprocedural Cuts



Predicate at pc_i = Interpolant from cut i

Common Variables



Predicate at pc_i = Interpolant from i-cut

Plan

1. Motivation

2. Refinement using Traces
   - Simple
   - Procedure calls

3. Results

Implementation

- Algorithms implemented in BLAST
  - Verifier for C programs, Lazy Abstraction [POPL '02]
- FOCI : Interpolating decision procedure
- Examples:
  - Windows Device Drivers (DDK)
  - IRP Specification: 22 state FSM
  - Current: Security properties of Linux programs

Windows DDK
IRP
22 state

Results

Program	*LOC**	*Previous Time*	*New Time*	*Predicates Total*	*Predicates Average*
`kbfiltr`	12k	1m12s	3m48s	72	6.5
`floppy`	17k	7m10s	25m20s	240	7.7
`diskperf`	14k	5m36s	13m32s	140	10
`cdaudio`	18k	20m18s	23m51s	256	7.8
`parport`	61k	*DNF*	*74m58s*	753	8.1
`parclass`	138k	*DNF*	*77m40s*	382	7.2

** Pre-processed*

Windows DDK
IRP
22 state

Localizing works…

Program	*LOC**	*Previous Time*	*New Time*	*Predicates Total*	*Predicates Average*
`kbfiltr`	12k	1m12s	3m48s	72	*6.5*
`floppy`	17k	7m10s	25m20s	240	*7.7*
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`parport`	61k	DNF	74m58s	753	*8.1*
`parclass`	138k	DNF	77m40s	382	*7.2*

** Pre-processed*

Conclusion

- Scalability ***and*** Precision by ***localizing***
- Craig Interpolation
  - Interprocedural cuts give well-scoped predicates
- Some Current and Future Work:
  - Multithreaded Programs
    - Project local info of thread to predicates over globals
  - Hierarchical trace analysis

BLAST

Berkeley Lazy Abstraction Software * Tool

www.eecs.berkeley.edu/~blast/

SLAM

- A Microsoft tool for checking safety of device drivers
- Inspired BLAST

Limitations of CEGAR

- Limited to powerset/relational abstract domains
- Interpolant computations
- Interactions with widening
- Starting on the right foot
- Unnecessary refinement steps
- Long and infinite number of refinement steps
- Long traces

Unnecessary Refinements

```
x = 0
while (x < 10^6) do
    x = x + 1
assert x < 100
```

Unsuccessful Refinement Set

```
x = malloc();
y = x ;
while (…)
     t = malloc();
     t->next = x
     x = t;
…
while (x !=y) do
     assert x != null;
     x = x->next
```

Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

4:if (a>0) {
5:   if (x==0) {
ERR:  ;
     }
  }
}
```

- Assume f always terminates
- ERR is reachable
  - a and x are unconstrained
- Any feasible path to error must unroll the loop 1000 times AND find feasible paths through f
- Any other path must be dismissed as a false positive

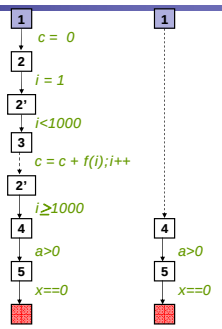
Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

4:if (a>0) {
5:   if (x==0) {
ERR:  ;
     }
  }
}
```

- Intuitively, the for loop is irrelevant
- ERR reachable as long as there exists some path from 2 to 4 that does not modify a or x
- Can we use static analysis to precisely report a statement is reachable *without* finding a feasible path?

Long Traces

```
Example ( ) {
1:c = 0;
2:for(i=1;i<1000;i++)
3:   c = c + f(i);

4:if (a>0) {
5:   if (x==0) {
ERR:  ;
     }
  }
}
```



Path Slice (PLDI'05)

The path slice of a program path π is a subsequence of the edges of π such that if the sequence of operations along the subsequence is:

1. infeasible, then π is infeasible, and
2. feasible, then the last location of π is reachable (but not necessarily along π)