Abstract Interpretation Part II

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Textbook: Chapter 4

CC79, CC92

Tentative Schedule

24/5	Operational Semantics
31/5 7/6	Abstract Interpretation
14/6	No class
21/6 22/6 9-12 309	Shape Analysis
27/6	Predicate Abstraction
3/8 9-12 309	Advanced Topics

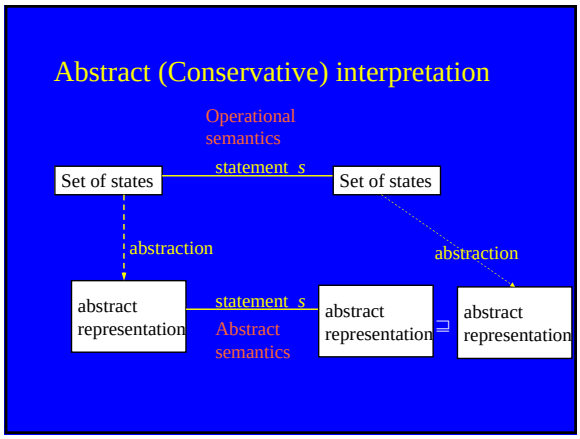
Targil 2

Course Project

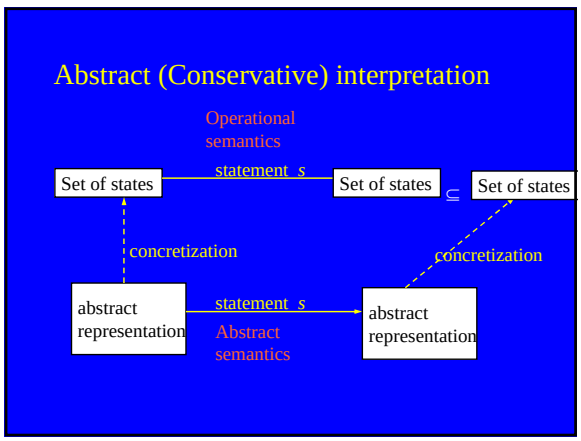
Outline

- The Soundness Theorem
- Intuition about abstract interpretation
- Methodologies for creating abstractions

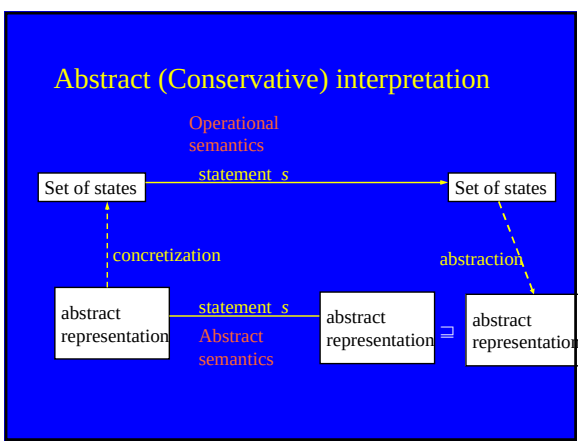
Abstract (Conservative) interpretation



Abstract (Conservative) interpretation



Abstract (Conservative) interpretation



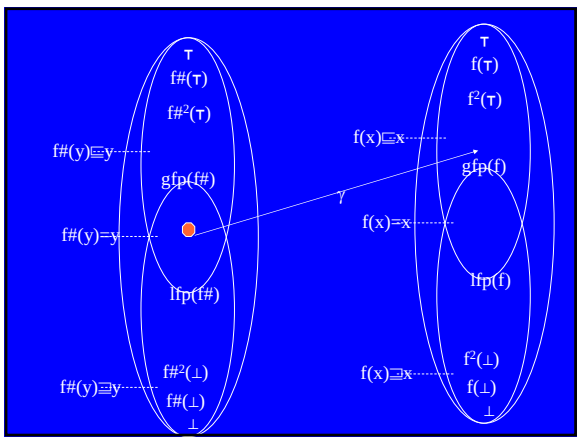
Soundness Theorem

1. Let (α, γ) form Galois connection from C to A
2. $f: C \to C$ be a monotone function
3. $f^{\#}: A \to A$ be a monotone function
4. $\forall a \in A: f(\gamma(a)) \sqsubseteq \gamma(f^{\#}(a))$
5. $\forall c \in C: \alpha(f(c)) \sqsubseteq f^{\#}(\alpha(a))$
6. $\forall a \in A: \alpha(f(\gamma(a)) \sqsubseteq f^{\#}(a)$

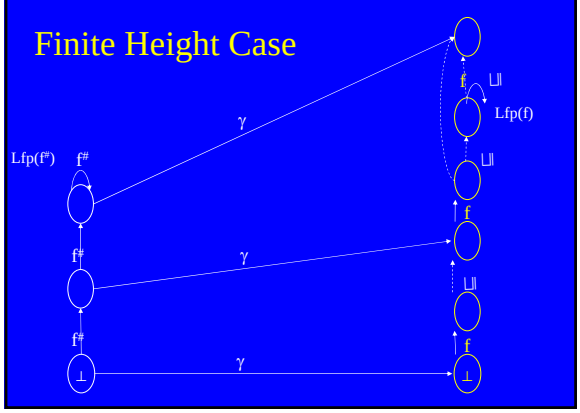
$\Vee$

$lfp(f) \sqsubseteq \gamma(lfp(f^{\#}))$

$\alpha(lfp(f)) \sqsubseteq lfp(f^{\#})$



Finite Height Case



Local Concrete Semantics

- For every atomic statement S
  - $[\![S]\!] : [Var_* \to Z] \to [Var_* \to Z]$
  - $[\![x := a]\!]s = s[x \mapsto \mathbf{A}[\![a]\!]s]$
  - $[\![skip]\!]s = s$
- For Boolean conditions …

Local Abstract Semantics(CP)

- For every atomic statement S
  - $[\![S]\!]^{\#} : Var_* \to L \to Var_* \to L$
  - $[\![x := a]\!]^{\#}(e) = e\,[x \mapsto [\![a]\!]^{\#}(e)$
  - $[\![skip]\!]^{\#}(e) = e$
- For Booleans …

Lemma 1

Consider a lattice L.

f: $L \to L$ is monotone iff

for all $X \subseteq L$: $\sqcup\{f(z) \mid z \in X\} \sqsubseteq f(\sqcup\{z \mid z \in X\})$

Assignments in constant propagation

- Monotone
  - $df_1 \sqsubseteq df_2 \rightarrow [\![x := e]\!]^\#(df_1) \sqsubseteq [\![x := e]\!]^\#(df_2)$
- Local Soundness
  - $\alpha(\{[\![x := e]\!]\sigma \mid \sigma \in CS\}) \sqsubseteq [\![x := e]\!]^\#(\alpha(CS))$
- Best Transformer
- Homomorphic

Proof of Soundness (Summary)

- Define an "appropriate" operational semantics
- Define "collecting" operational semantics
- Establish a Galois connection between collecting states and abstract states
- (Local correctness) Show that the abstract interpretation of every atomic statement is sound w.r.t. the collecting semantics
- (Global correctness) Conclude that the result of the iterative analysis is sound w.r.t. the collecting semantics
- Can be applied between different abstractions

Induced Analysis (Relatively Optimal)

- It is sometimes possible to show that a given analysis is not only sound but optimal w.r.t. the chosen abstraction
  - but not necessarily optimal!
- Define
  $[\![S]\!]^\#(df) = \alpha(\{[\![S]\!]\sigma \mid \sigma \in \gamma(df)\})$
- But this $[\![S]\!]^\#$ may not be computable
- Derive (at compiler-generation time) an alternative form for $[\![S]\!]^\#$
- A useful measure to decide if the abstraction must lead to overly imprecise results

Properties of Abstractions

- Eagerly forget parts of the state
- Reduce state space
- Abstract traces do not necessarily correspond to concrete trace
  - even when best transformer is used
- Executes the program on traces with “fabricated” states
- When the abstraction succeeds prove stronger properties

Notions of precision

- CS = γ (df)
- α(CS) = df
- Meet(Join) over all paths
- Using best transformers
- Good enough

Summary

- Abstract interpretation relates runtime semantics and static information
- The concrete semantics serves as a tool in designing abstractions
- Understanding concretization is a must
- Understand what is preserved/lost

Combining Data Flow Analyzes

- Develop new algorithms from old
- If I know how to conservatively represent
  - Pointers
  - Integers
- Do I know how to handle C programs with integers and pointers?

Combining Data Flow Analyzes

- Develop new algorithms from old
- If I know how to conservatively represent
  - Pointers
  - Integers
- Do I know how to handle C programs with integers and pointers?
- Improve the precision of an analysis
- Obtain a more efficient analysis

Combining Data Flow Analyzers

- Lattice constructors
  - $L_1 \times L_2$
  - $S \rightarrow L_1$
  - …
- Galois connection constructors
- Constructing the abstract effect of elementary statements
- Model the “relevant” parts of the program
- Abstract “irrelevant” parts of the program

Galois Connections

- For
  - A complete lattice $(L_1, \sqsubseteq_1) = (L_1, \sqsubseteq, \sqcup_1, \sqcap_1, \bot_1, \top_1)$
  - A complete lattice $(L_2, \sqsubseteq_2) = (, \sqsubseteq, \sqcup_2, \sqcap_2, \bot_2, \top_2)$
  - $\alpha: L_1 \to L_2$
  - $\gamma: L_2 \to L_1$
- We say that $(L_1, \alpha, \gamma, L_2)$ is a Galois connection
  - α and γ are monotone
  - For all $c \in L_1$: $\gamma(\alpha(c)) \sqsupseteq c$
  - For all $a \in L_2$: $\alpha(\gamma(a)) \sqsubseteq a$

Cartesian Products

- A complete lattice $(L_1, \sqsubseteq_1) = (L_1, \sqsubseteq, \sqcup_1, \sqcap_1, \bot_1, \top_1)$
- A complete lattice $(L_2, \sqsubseteq_2) = (, \sqsubseteq, \sqcup_2, \sqcap_2, \bot_2, \top_2)$
- Define a Poset $L = (L_1 \times L_2, \sqsubseteq)$ where
  - $(x_1, x_2) \sqsubseteq (y_1, y_2)$ if
    - » $x_1 \sqsubseteq y_1$ and
    - » $x_2 \sqsubseteq y_2$
- L is a complete lattice
- But what does an element in L represent?

Cartesian Products (cont)

- A complete lattice $(L_1, \sqsubseteq_1) = (L_1, \sqsubseteq, \sqcup_1, \sqcap_1, \bot_1, \top_1)$
- A complete lattice $(L_2, \sqsubseteq_2) = (, \sqsubseteq, \sqcup_2, \sqcap_2, \bot_2, \top_2)$
- Complete lattice $L = (L_1 \times L_2, \sqsubseteq)$
- A concrete lattice C (usually a powerset)
- A Galois connection $(C, \alpha_1, \gamma_1, L_1)$
- A Galois connection $(C, \alpha_2, \gamma_2, L_2)$
- Define $\alpha: C \to L_1 \times L_2$ and $\gamma: L_1 \times L_2 \to C$?
- Example: Parity × Sign

Cartesian Products (cont)

- A Galois connection $(C, \alpha_1, \gamma_1, L_1)$
- A Galois connection $(C, \alpha_2, \gamma_2, L_2)$
- A Galois connection $(C, \alpha, \gamma, L_1 \times L_2)$
  - $\alpha(c) = <\alpha_1(c), \alpha_2(c)>$
  - $\gamma(<a_1, a_2>) = \gamma_1(a_1) \sqcap \gamma_2(a_2)$
- Define
  - $L_1[\![st]\!]^{\#}: L_1 \to L_1$
  - $L_2[\![st]\!]^{\#}: L_2 \to L_2$
- How to define $L_1 \times L_2\ [\![st]\!]^{\#}: L_1 \times L_2 \to L_1 \times L_2$
  - Preserve soundness
  - Preserve relative optimality (induced)
  - Reasonable
- Example: Parity $\times$ Sign

Component-wise combinations

- Combine several analyses into a single analysis
- ✓ Cartesian products (Direct product)
- Independent attribute method
- Relational attribute method
- Total function space
- Monotone function space
- Direct tensor product

Independent Attribute Method

- A Galois connection $(C_1, \alpha_1, \gamma_1, L_1)$
- A Galois connection $(C_2, \alpha_2, \gamma_2, L_2)$
- A Galois connection $(C_1 \times C_2, \alpha, \gamma, L_1 \times L_2)$
  - $\alpha(<c_1, c_2>) = <\alpha_1(c_1), \alpha_2(c_2)>$
  - $\gamma(<a_1, a_2>) = <\gamma_1(a_1), \gamma_2(a_2)>$
- Define
  - $L_1[\![st]\!]^{\#}: L_1 \to L_1$
  - $L_2[\![st]\!]^{\#}: L_2 \to L_2$
- How to define $L_1 \times L_2\ [\![st]\!]^{\#}: L_1 \times L_2 \to L_1 \times L_2$
  - Preserve soundness
  - Preserve relative optimality (induced)

Relational Attribute Method

- A Galois connection
  $(P(C_1), \alpha_1, \gamma_1, P(L_1))$ where $\beta_1: C_1 \rightarrow L_1$
  - $\alpha_1(X) = \cup\{\beta_1(c) \mid c \in X\}$
- A Galois connection
  $(P(C_2), \alpha_2, \gamma_2, P(L_2))$ where $\beta_2: C_2 \rightarrow L_2$
- $\alpha_2(X) = \cup\{\beta_2(c) \mid c \in X\}$
- A Galois connection $(P(C_1 \times C_2), \alpha, \gamma, P(L_1 \times L_2))$
  - $\alpha(<X_1, X_2>) = \{<\beta_1(c_1), \beta_2(c_2)> \mid c_1 \in X_1, c_2 \in X_2\}$
  - $\gamma(<Y_1, Y_2>) = \{<c_1, c_2> \mid \beta_1(c_1) \in Y_1 \ \ \beta_2(c_2) \in Y_2\}$
- But how about transformers?

Semantic Reduction

- Consider a Galois connection
  (C, α, γ, A)
- An operation op: $A \rightarrow A$ is a semantic reduction if
  - For all $a \in A$: $op(a) \sqsubseteq a$ and $\gamma(op(a)) = \gamma(a)$

Conclusions(1)

- Good static analysis =
  - Precise enough (for the client)
  - Efficient enough
- Good static analysis
  - Good domain
    - » Abstract non-important details
    - » Represent relevant concrete information
    - » Precise and efficient abstract meaning of abstract interpreters
    - » Efficient join implementation
    - » Small height or widening

Conclusions(2)

- The Theory of Static Analysis is well founded
  - Abstraction
  - Soundness
  - Chaotic iterations
  - Elimination methods
  - Modular methods
- Weak Parts
  - Transformations
  - Predictable approximations
  - User defined abstractions
  - System