

# Tentative Schedule

6/3	introduction
13/3	javascript
20/3	Haskel
27/3	No class
3/4	Operational Semantics
17/4	Denotational Semantics
24/4	Axiomatic Semantics
2/5	Exception and continuation
8/5, 15/5, 22/5	Type Systems
29/5, 5/6, 12/6	Concurrency
19/6	Domain Specific Languages
22/6	Summary class

# Operational Semantics

**Mooly Sagiv**

**Semantics with Applications**

**Chapter 2**

**H. Nielson and F. Nielson**

**[http://www.daimi.au.dk/~bra8130/Wiley\\_book/wiley.html](http://www.daimi.au.dk/~bra8130/Wiley_book/wiley.html)**

**The Formal Semantics of Programming Languages**

**An Introduction**

**Glynn Winskel**

# Syntax vs. Semantics

- ◆ The pattern of formation of sentences or phrases in a language

- ◆ Examples

- Regular expressions
- Context free grammars

- ◆ The study or science of meaning in language

- ◆ Examples

- Interpreter
- Compiler
- Better mechanisms will be given today

# Benefits of Formal Semantics

- ◆ Programming language design
  - hard- to-define= hard-to-implement=hard-to-use
- ◆ Programming language implementation
- ◆ Programming language understanding
- ◆ Program correctness
- ◆ Program equivalence
- ◆ Compiler Correctness
- ◆ Automatic generation of interpreter
- ◆ But probably not
  - Automatic compiler generation

# Alternative Formal Semantics

## ◆ Operational Semantics

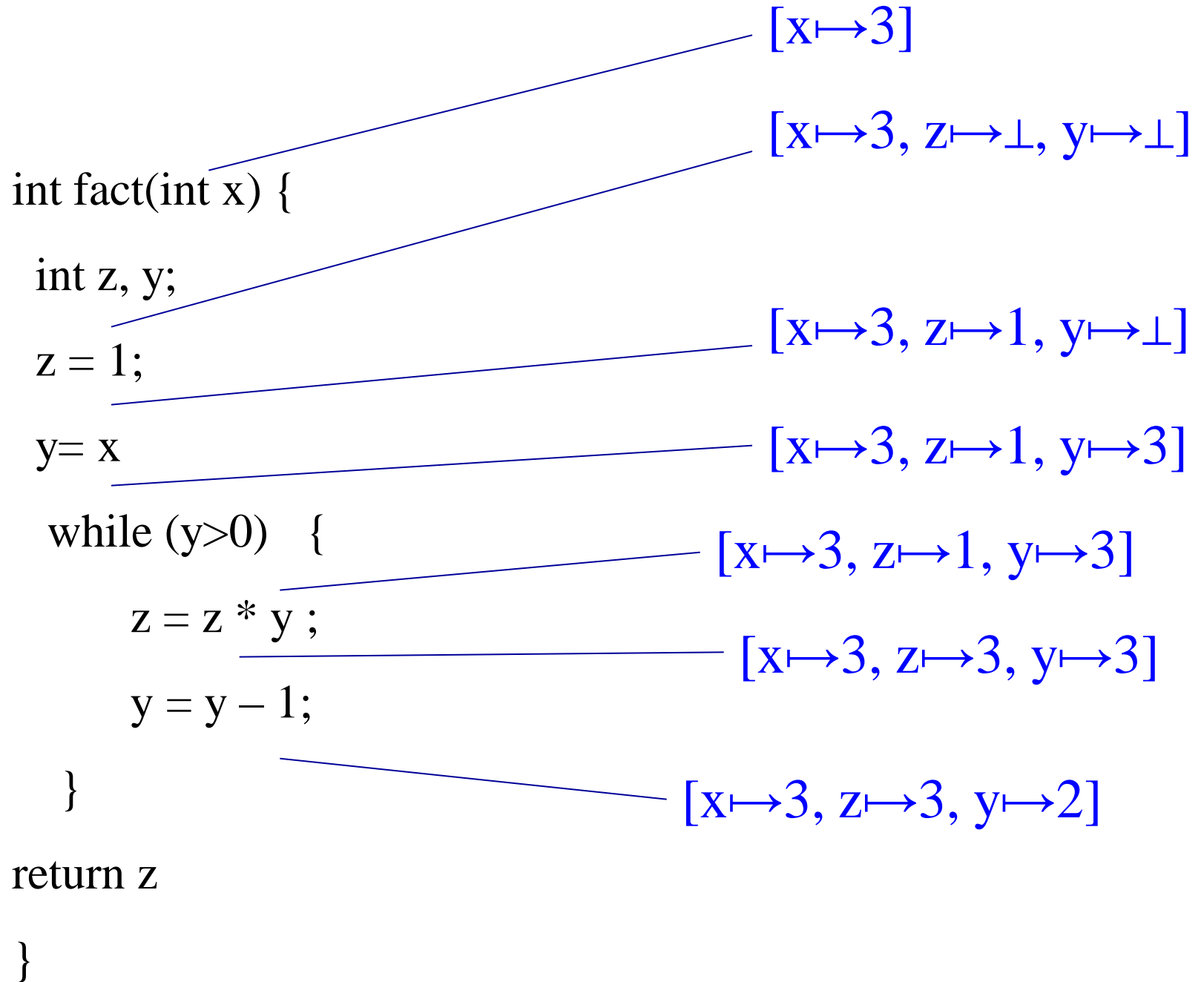
- The meaning of the program is described “operationally”
- Natural Large Step Operational Semantics
- Structural Small Step Operational Semantics

## ◆ Denotational Semantics

- The meaning of the program is an input/output relation
- Mathematically challenging but complicated

## ◆ Axiomatic Semantics

- The meaning of the program are observed properties



```

int fact(int x) {
    int z, y;
    z = 1;
    y = x
    while (y > 0) {
        z = z * y;
        y = y - 1;
    }
    return z
}

```

$[x \mapsto 3, z \mapsto 3, y \mapsto 2]$   
 $[x \mapsto 3, z \mapsto 3, y \mapsto 2]$   
 $[x \mapsto 3, z \mapsto 6, y \mapsto 2]$   
 $[x \mapsto 3, z \mapsto 6, y \mapsto 1]$

```

int fact(int x) {
    int z, y;
    z = 1;
    y = x
    while (y > 0) {
        z = z * y;
        y = y - 1;
    }
    return z
}

```

$[x \mapsto 3, z \mapsto 6, y \mapsto 1]$   
 $[x \mapsto 3, z \mapsto 6, y \mapsto 1]$   
 $[x \mapsto 3, z \mapsto 6, y \mapsto 1]$   
 $[x \mapsto 3, z \mapsto 6, y \mapsto 0]$



```

int fact(int x) {
    int z, y;
    z = 1;
    y = x [x ↦ 3, z ↦ 6, y ↦ 0]
    while (y > 0) {
        z = z * y;
        y = y - 1;
    }
    return z [x ↦ 3, z ↦ 6, y ↦ 0]
}

```

```
int fact(int x) {  
    int z, y;  
    z = 1;  
    y = x; [x ↦ 3, z ↦ 6, y ↦ 0]  
    while (y > 0) {  
        z = z * y;  
        y = y - 1;  
    }  
    return 6 [x ↦ 3, z ↦ 6, y ↦ 0]  
}
```

# Denotational Semantics

```
int fact(int x) {
```

```
  int z, y;
```

```
  z = 1;
```

```
  y = x ;
```

$f = \lambda x. \text{if } x = 0 \text{ then } 1 \text{ else } x * f(x - 1)$

```
  while (y > 0) {
```

```
    z = z * y ;
```

```
    y = y - 1;
```

```
  }
```

```
  return z;
```

```
}
```

# Axiomatic Semantics

{x=n}

```
int fact(int x) { int z, y;
```

```
z = 1;
```

{x=n ∧ z=1}

```
y = x
```

{x=n ∧ z=1 ∧ y=n}

```
while
```

```
{x=n ∧ y ≥ 0 ∧ z=n! / y!}
```

```
(y>0) {
```

```
{x=n ∧ y > 0 ∧ z=n! / y!}
```

```
z = z * y ;
```

```
{x=n ∧ y>0 ∧ z=n!/(y-1)!}
```

```
y = y - 1;
```

```
{x=n ∧ y ≥ 0 ∧ z=n!/y!}
```

```
} return z} {x=n ∧ z=n!}
```

# Operational Semantics

Natural Large Step Semantics

# Operational Semantics of Arithmetic Expressions

$Aexp \rightarrow$  | number

|  $Axp$  PLUS  $Aexp$

|  $Aexp$  MINUS  $Aexp$

|  $Aexp$  MUL  $Aexp$

| UMINUS  $Aexp$

$A[\ ]: Aexp \rightarrow Z$

$$A[n] = \text{val}(n)$$

$$A[e_1 \text{ PLUS } e_2] = A[e_1] + A[e_2]$$

$$A[e_1 \text{ MINUS } e_2] = A[e_1] - A[e_2]$$

$$A[e_1 \text{ MUL } e_2] = A[e_1] * A[e_2]$$

$$A[\text{UMINUS } e] = -A[e]$$

# Handling Variables

Aexp  $\rightarrow$  | number  
| variable  
| Aexp PLUS Aexp  
| Aexp MINUS Aexp  
| Aexp MUL Aexp  
| UMINUS Exp

- ◆ Need the notions of states
- ◆ States  $\text{State} = \text{Var} \rightarrow Z$
- ◆ Lookup in a state  $s$ :  $s \ x$
- ◆ Update of a state  $s$ :  $s \ [ \ x \mapsto 5 ]$

# Example State Manipulations

- ◆  $[x \mapsto 1, y \mapsto 7, z \mapsto 16] y =$
- ◆  $[x \mapsto 1, y \mapsto 7, z \mapsto 16] t =$
- ◆  $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5] =$
- ◆  $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5] x =$
- ◆  $[x \mapsto 1, y \mapsto 7, z \mapsto 16][x \mapsto 5] y =$



# Semantics of arithmetic expressions

- ◆ Assume that arithmetic expressions are side-effect free
- ◆  $A \llbracket \text{Aexp} \rrbracket : \text{State} \rightarrow \mathbb{Z}$
- ◆ Defined by induction on the syntax tree
  - $A \llbracket n \rrbracket s = n$
  - $A \llbracket x \rrbracket s = s \ x$
  - $A \llbracket e_1 \text{ PLUS } e_2 \rrbracket s = A \llbracket e_1 \rrbracket s + A \llbracket e_2 \rrbracket s$
  - $A \llbracket e_1 \text{ MUL } e_2 \rrbracket s = A \llbracket e_1 \rrbracket s * A \llbracket e_2 \rrbracket s$
  - $A \llbracket \text{UMINUS } e \rrbracket s = -A \llbracket e \rrbracket s$
- ◆ Compositional
- ◆ Properties can be proved by structural induction

# Semantics of Boolean expressions

- ◆ Assume that Boolean expressions are side-effect free
- ◆  $T = \{ff, tt\}$
- ◆  $B \llbracket \text{Bexp} \rrbracket : \text{State} \rightarrow T$
- ◆ Defined by induction on the syntax tree
  - $B \llbracket \text{true} \rrbracket s = tt$
  - $B \llbracket \text{false} \rrbracket s = ff$
  - $B \llbracket e_1 = e_2 \rrbracket s = \begin{cases} tt & \text{if } A \llbracket e_1 \rrbracket s = A \llbracket e_2 \rrbracket s \\ ff & \text{if } A \llbracket e_1 \rrbracket s \neq A \llbracket e_2 \rrbracket s \end{cases}$
  - $B \llbracket e_1 \wedge e_2 \rrbracket s = \begin{cases} tt & \text{if } B \llbracket e_1 \rrbracket s = tt \text{ and } B \llbracket e_2 \rrbracket s = tt \\ ff & \text{if } B \llbracket e_1 \rrbracket s = ff \text{ or } B \llbracket e_2 \rrbracket s = ff \end{cases}$
  - $B \llbracket e_1 \geq e_2 \rrbracket s =$

# The **While** Programming Language

- ◆ Abstract syntax

$S ::= x := a \mid \mathbf{skip} \mid S_1 ; S_2 \mid \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \mid$   
 $\mathbf{while} \ b \ \mathbf{do} \ S$

- ◆ Use parentheses for precedence

- ◆ Informal Semantics

- **skip** behaves like no-operation
- Import meaning of arithmetic and Boolean operations

# Example While Program

$y := 1;$

while  $\neg(x=1)$  do (

$y := y * x;$

$x := x - 1$

)

# General Notations

## ◆ Syntactic categories

- Var the set of program variables
- Aexp the set of arithmetic expressions
- Bexp the set of Boolean expressions
- Stm set of program statements

## ◆ Semantic categories

- Natural values  $N = \{0, 1, 2, \dots\}$
- Truth values  $T = \{ff, tt\}$
- States  $State = Var \rightarrow N$
- Lookup in a state  $s: s \ x$
- Update of a state  $s: s \ [ \ x \mapsto 5 ]$

# Natural Operational Semantics

- ◆ Describe the “overall” effect of program constructs
- ◆ Ignores non terminating computations

# Natural Semantics

## ◆ Notations

- $\langle S, s \rangle$  - the program statement  $S$  is executed on input state  $s$
- $s$  representing a terminal (final) state

## ◆ For every statement $S$ , write meaning rules

$$\langle S, i \rangle \rightarrow o$$

“If the statement  $S$  is executed on an input state  $i$ , it terminates and yields an output state  $o$ ”

- ◆ The meaning of a program  $P$  on an input state  $s$  is the set of outputs states  $o$  such that  $\langle P, i \rangle \rightarrow o$
- ◆ The meaning of compound statements is defined using the meaning immediate constituent statements
- ◆ Inductive definitions
- ◆ Notice that  $\rightarrow$  means large-step here in contrast to the first lecture where  $\rightarrow$  means small-step

# Natural Semantics for While

$$[\text{ass}_{\text{ns}}] \langle x := a, s \rangle \rightarrow s[x \mapsto \mathbf{A}[[a]]s]$$

axioms

$$[\text{skip}_{\text{ns}}] \langle \mathbf{skip}, s \rangle \rightarrow s$$

$$[\text{comp}_{\text{ns}}] \frac{\langle S_1, s \rangle \rightarrow s', \langle S_2, s' \rangle \rightarrow s''}{\langle S_1; S_2, s \rangle \rightarrow s''}$$

rules

$$\langle S_1; S_2, s \rangle \rightarrow s''$$

$$[\text{if}^{\text{tt}}_{\text{ns}}] \langle S_1, s \rangle \rightarrow s'$$

$$\frac{\langle S_1, s \rangle \rightarrow s'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s \rangle \rightarrow s'}$$

if  $\mathbf{B}[[b]]s = \text{tt}$

$$[\text{if}^{\text{ff}}_{\text{ns}}] \langle S_2, s \rangle \rightarrow s'$$

$$\frac{\langle S_2, s \rangle \rightarrow s'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, s \rangle \rightarrow s'}$$

if  $\mathbf{B}[[b]]s = \text{ff}$



# Natural Semantics for While (More rules)

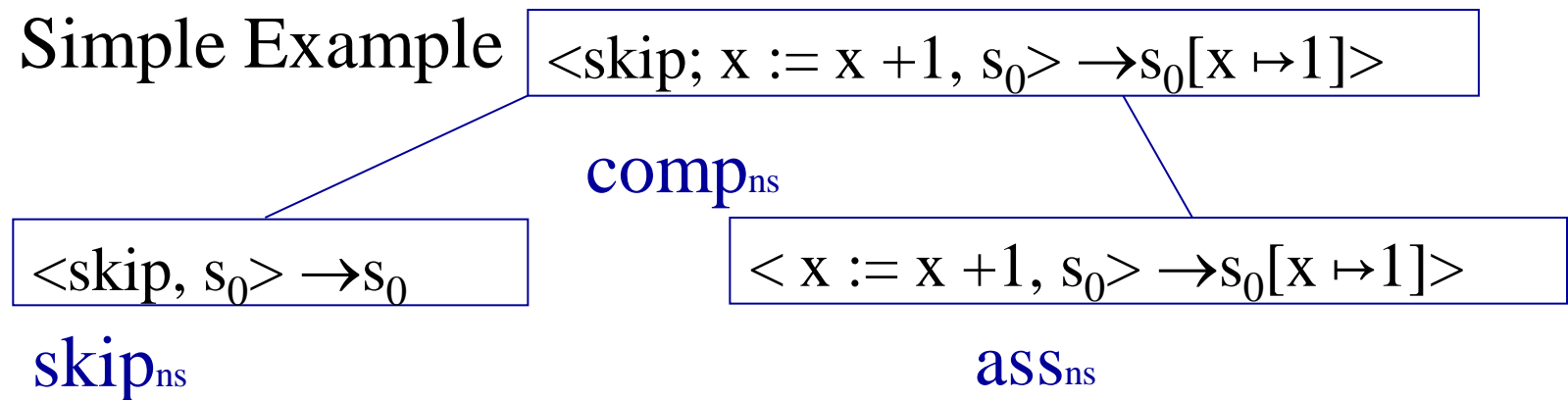
$$\frac{[\text{while}_{\text{ns}}^{\text{ff}}] \quad \langle \text{while } b \text{ do } S, s \rangle \rightarrow s}{\text{if } \mathbf{B}[[b]]s = \text{ff}}$$

$$\frac{[\text{while}_{\text{ns}}^{\text{tt}}] \quad \langle S, s \rangle \rightarrow s', \langle \text{while } b \text{ do } S, s' \rangle \rightarrow s''}{\langle \text{while } b \text{ do } S, s \rangle \rightarrow s''} \quad \text{if } \mathbf{B}[[b]]s = \text{tt}$$

# A Derivation Tree

- ◆ A “proof” that  $\langle S, s \rangle \rightarrow s'$
- ◆ The root of tree is  $\langle S, s \rangle \rightarrow s'$
- ◆ Leaves are instances of axioms
- ◆ Internal nodes rules
  - Immediate children match rule premises

## ◆ Simple Example



# An Example Derivation Tree

$\langle (x := x+1; y := x+1); z := y \rangle, s_0 \rangle \rightarrow s_0[x \mapsto 1][y \mapsto 2][z \mapsto 2]$

$\text{comp}_{\text{ns}}$

$\langle x := x+1; y := x+1, s_0 \rangle \rightarrow s_0[x \mapsto 1][y \mapsto 2]$

$\langle z := y, s_0[x \mapsto 1][y \mapsto 2] \rangle \rightarrow s_0[x \mapsto 1][y \mapsto 2][z \mapsto 2]$

$\text{comp}_{\text{ns}}$

$\langle x := x+1; s_0 \rangle \rightarrow s_0[x \mapsto 1]$

$\langle y := x+1, s_0[x \mapsto 1] \rangle \rightarrow s_0[x \mapsto 1][y \mapsto 2]$

$\text{ass}_{\text{ns}}$

$\text{ass}_{\text{ns}}$

# Top Down Evaluation of Derivation Trees

- ◆ Given a program  $S$  and an input state  $s$
- ◆ Find an output state  $s'$  such that  
 $\langle S, s \rangle \rightarrow s'$
- ◆ Start with the root and repeatedly apply rules until the axioms are reached
- ◆ Inspect different alternatives in order
- ◆ In While  $s'$  and the derivation tree is unique

# Example of Top Down Tree Construction

- ◆ Input state  $s$  such that  $s \cdot x = 2$
- ◆ Factorial program

$$\langle y := 1; \text{while } \neg(x=1) \text{ do } (y := y * x; x := x - 1), s \rangle \rightarrow s[y \mapsto 2][x \mapsto 1] \quad \triangleright$$

$\text{comp}_{\text{ns}}$

$$\langle W, s[y \mapsto 1] \rangle \rightarrow s[y \mapsto 2][x \mapsto 1] \quad \triangleright$$

$$\langle y := 1, s \rangle \rightarrow s[y \mapsto 1]$$

$\text{ass}_{\text{ns}}$

$\text{while}_{\text{ns}}^{\text{tt}}$

$$\langle W, s[y \mapsto 2][x \mapsto 1] \rangle \rightarrow s[y \mapsto 2][x \mapsto 1] \quad \triangleright$$

$\text{while}_{\text{ns}}^{\text{ff}}$

$$\langle (y := y * x; x := x - 1, s[y \mapsto 1]) \rangle \rightarrow s[y \mapsto 2][x \mapsto 1] \quad \triangleright$$

$\text{comp}_{\text{ns}}$

$$\langle y := y * x; s[y \mapsto 1] \rangle \rightarrow s[y \mapsto 2]$$

$\text{ass}_{\text{ns}}$

$$\langle x := x - 1, s[y \mapsto 2] \rangle \rightarrow s[y \mapsto 2][x \mapsto 1] \quad \triangleright$$

$\text{ass}_{\text{ns}}$

# Semantic Equivalence

- ◆  $S_1$  and  $S_2$  are **semantically equivalent** if for all  $s$  and  $s'$   
 $\langle S_1, s \rangle \rightarrow s'$  if and only if  $\langle S_2, s \rangle \rightarrow s'$
- ◆ Simple example  
“while b do S”  
is semantically equivalent to:  
“if b then (S ; while b do S) else skip”

# Deterministic Semantics for While

## (Theorem 2.9, page 39)

- ◆ If  $\langle S, s \rangle \rightarrow s_1$  and  $\langle S, s \rangle \rightarrow s_2$  then  $s_1 = s_2$
- ◆ The proof uses induction on the shape of derivation trees
  - Prove that the property holds for all simple derivation trees by showing it holds for axioms
  - Prove that the property holds for all composite trees:
    - » For each rule assume that the property holds for its premises (induction hypothesis) and prove it holds for the conclusion of the rule

# The Semantic Function $S_{ns}$

- ◆ The meaning of a statement  $S$  is defined as a partial function from **State** to **State**
- ◆  $S_{ns}: \mathbf{Stm} \rightarrow (\mathbf{State} \hookrightarrow \mathbf{State})$
- ◆  $S_{ns} \llbracket S \rrbracket s = s'$  if  $\langle S, s \rangle \rightarrow s'$  and otherwise  $S_{ns} \llbracket S \rrbracket s$  is undefined
- ◆ Examples
  - $S_{ns} \llbracket \text{skip} \rrbracket s = s$
  - $S_{ns} \llbracket x := 1 \rrbracket s = s [x \mapsto 1]$
  - $S_{ns} \llbracket \text{while true do skip} \rrbracket s = \text{undefined}$



# Structural Operational Semantics

- ◆ Emphasizes the individual steps
- ◆ For every statement  $S$ , write meaning rules  $\langle S, i \rangle \Rightarrow \gamma$   
“If the **first** step of executing the statement  $S$  on an input state  $i$  leads to  $\gamma$ ”
- ◆ Two possibilities for  $\gamma$ 
  - $\gamma = \langle S', s' \rangle$  The execution of  $S$  is not completed,  $S'$  is the remaining computation which need to be performed on  $s'$
  - $\gamma = o$  The execution of  $S$  has terminated with a final state  $o$
  - $\gamma$  is a stuck configuration when there are no transitions
- ◆ The meaning of a program  $P$  on an input state  $s$  is the set of final states that can be executed in arbitrary finite steps
- ◆  $\Rightarrow$  means small step as  $\rightarrow$  in the first lecture

# Structural Semantics for While

$$[\text{ass}_{\text{sos}}] \langle x := a, s \rangle \Rightarrow s[x \mapsto \mathbf{A}[[a]]s]$$

axioms

$$[\text{skip}_{\text{sos}}] \langle \mathbf{skip}, s \rangle \Rightarrow s$$

$$[\text{comp}^1_{\text{sos}}] \langle S_1, s \rangle \Rightarrow \langle S'_1, s' \rangle$$

---

rules

$$\langle S_1; S_2, s \rangle \Rightarrow \langle S'_1; S_2, s' \rangle$$

$$[\text{comp}^2_{\text{sos}}] \langle S_1, s \rangle \Rightarrow s'$$

---

$$\langle S_1; S_2, s \rangle \Rightarrow \langle S_2, s' \rangle$$

# Structural Semantics for While if construct

$[if_{sos}^{tt}] \langle \text{if } b \text{ then } S_1 \text{ else } S_2, s \rangle \Rightarrow \langle S_1, s \rangle$  if  $\mathbf{B}[[b]]s=tt$

$[if_{os}^{ff}] \langle \text{if } b \text{ then } S_1 \text{ else } S_2, s \rangle \Rightarrow \langle S_2, s \rangle$  if  $\mathbf{B}[[b]]s=ff$

# Structural Semantics for While while construct

$[\text{while}_{\text{sos}}]$   $\langle \text{while } b \text{ do } S, s \rangle \Rightarrow$   
 $\langle \text{if } b \text{ then } (S; \text{while } b \text{ do } S) \text{ else skip}, s \rangle$

# Derivation Sequences

- ◆ A finite derivation sequence starting at  $\langle S, s \rangle$

$\gamma_0, \gamma_1, \gamma_2 \dots, \gamma_k$  such that

–  $\gamma_0 = \langle S, s \rangle$

–  $\gamma_i \Rightarrow \gamma_{i+1}$

–  $\gamma_k$  is either stuck configuration or a final state

- ◆ An infinite derivation sequence starting at  $\langle S, s \rangle$

$\gamma_0, \gamma_1, \gamma_2 \dots$  such that

–  $\gamma_0 = \langle S, s \rangle$

–  $\gamma_i \Rightarrow \gamma_{i+1}$

- ◆  $\gamma_0 \Rightarrow^i \gamma_i$  in  $i$  steps

- ◆  $\gamma_0 \Rightarrow^* \gamma_i$  in finite number of steps

- ◆ For each step there is a derivation tree

# Example

◆ Let  $s_0$  such that

$$s_0 x = 5$$

and

$$s_0 y = 7$$

◆  $S = (z := x; x := y); y := z$

# Factorial Program

◆ Input state  $s$  such that  $s.x = 3$

$y := 1; \text{ while } \neg(x=1) \text{ do } (y := y * x; x := x - 1)$

$\langle y := 1; W, s \rangle$

$\Rightarrow \langle W, s[y \mapsto 1] \rangle$

$\Rightarrow \langle \text{if } \neg \neg(x=1) \text{ then skip else } ((y := y * x; x := x - 1); W), s[y \mapsto 1] \rangle$

$\Rightarrow \langle ((y := y * x; x := x - 1); W), s[y \mapsto 1] \rangle$

$\Rightarrow \langle (x := x - 1; W), s[y \mapsto 3] \rangle$

$\Rightarrow \langle W, s[y \mapsto 3][x \mapsto 2] \rangle$

$\Rightarrow \langle \text{if } \neg \neg(x=1) \text{ then skip else } ((y := y * x; x := x - 1); W), s[y \mapsto 3][x \mapsto 2] \rangle$

$\Rightarrow \langle ((y := y * x; x := x - 1); W), s[y \mapsto 3][x \mapsto 2] \rangle$

$\Rightarrow \langle (x := x - 1; W), s[y \mapsto 6][x \mapsto 2] \rangle$

$\Rightarrow \langle W, s[y \mapsto 6][x \mapsto 1] \rangle$

$\Rightarrow \langle \text{if } \neg \neg(x=1) \text{ then skip else } ((y := y * x; x := x - 1); W), s[y \mapsto 6][x \mapsto 1] \rangle$

$\Rightarrow \langle \text{skip}, s[y \mapsto 6][x \mapsto 1] \rangle \Rightarrow s[y \mapsto 6][x \mapsto 1]$

# Program Termination

- ◆ Given a statement  $S$  and input  $s$ 
  - $S$  **terminates** on  $s$  if there exists a finite derivation sequence starting at  $\langle S, s \rangle$
  - $S$  **terminates successfully** on  $s$  if there exists a finite derivation sequence starting at  $\langle S, s \rangle$  leading to a final state
  - $S$  **loops** on  $s$  if there exists an infinite derivation sequence starting at  $\langle S, s \rangle$



# Properties of the Semantics

- ◆  $S_1$  and  $S_2$  are **semantically equivalent** if:
  - for all  $s$  and  $\gamma$  which is either final or stuck  $\langle S_1, s \rangle \Rightarrow^* \gamma$  if and only if  $\langle S_2, s \rangle \Rightarrow^* \gamma$
  - there is an infinite derivation sequence starting at  $\langle S_1, s \rangle$  if and only if there is an infinite derivation sequence starting at  $\langle S_2, s \rangle$
- ◆ **Deterministic**
  - If  $\langle S, s \rangle \Rightarrow^* s_1$  and  $\langle S, s \rangle \Rightarrow^* s_2$  then  $s_1 = s_2$
- ◆ The execution of  $S_1; S_2$  on an input can be split into two parts:
  - execute  $S_1$  on  $s$  yielding a state  $s'$
  - execute  $S_2$  on  $s'$

# Sequential Composition

- ◆ If  $\langle S_1; S_2, s \rangle \Rightarrow^k s'$  then there exists a state  $s'$  and numbers  $k_1$  and  $k_2$  such that
  - $\langle S_1, s \rangle \Rightarrow^{k_1} s'$
  - $\langle S_2, s' \rangle \Rightarrow^{k_2} s''$
  - and  $k = k_1 + k_2$
- ◆ The proof uses induction on the length of derivation sequences
  - Prove that the property holds for all derivation sequences of length 0
  - Prove that the property holds for all other derivation sequences:
    - » Show that the property holds for sequences of length  $k+1$  using the fact it holds on all sequences of length  $k$  (induction hypothesis)

# The Semantic Function $S_{\text{sos}}$

- ◆ The meaning of a statement  $S$  is defined as a partial function from **State** to **State**
- ◆  $S_{\text{sos}}: \mathbf{Stm} \rightarrow (\mathbf{State} \hookrightarrow \mathbf{State})$
- ◆  $S_{\text{sos}}[[S]]s = s'$  if  $\langle S, s \rangle \Rightarrow^* s'$  and otherwise  $S_{\text{sos}}[[S]]s$  is undefined

# An Equivalence Result

- ◆ For every statement  $S$  of the While language
  - $S_{\text{nat}}[[S]] = S_{\text{sos}}[[S]]$

# Extensions to While

- ◆ Abort statement (like C exit w/o return value)
- ◆ Non-determinism
- ◆ Parallelism
- ◆ Local Variables
- ◆ Procedures
  - Static Scope
  - Dynamic scope

# The **While** Programming Language with **Abort**

- ◆ Abstract syntax

$S ::= x := a \mid \mathbf{skip} \mid S_1 ; S_2 \mid \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \mid$   
 $\mathbf{while} \ b \ \mathbf{do} \ S \mid \mathbf{abort}$

- ◆ **Abort** terminates the execution

- ◆ No new rules are needed in natural and structural operational semantics

- ◆ Statements

- if  $x = 0$  then abort else  $y := y / x$

- skip

- abort

- while true do skip

# Conclusion

- ◆ The natural semantics cannot distinguish between looping and abnormal termination (unless the states are modified)
- ◆ In the structural operational semantics looping is reflected by infinite derivations and abnormal termination is reflected by stuck configuration

# The **While** Programming Language with Non-Determinism

- ◆ Abstract syntax

$S ::= x := a \mid \mathbf{skip} \mid S_1 ; S_2 \mid \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \mid$   
 $\mathbf{while} \ b \ \mathbf{do} \ S \mid S_1 \ \mathbf{or} \ S_2$

- ◆ Either  $S_1$  or  $S_2$  is executed

- ◆ Example

- $x := 1 \ \mathbf{or} \ (x := 2 ; x := x+2)$



# The While Programming Language with Non-Determinism Natural Semantics

$$\frac{[\text{or}_\text{ns}^1] \langle S_1, s \rangle \rightarrow s'}{\langle S_1 \text{ or } S_2, s \rangle \rightarrow s'}$$

$$\frac{[\text{or}_\text{ns}^2] \langle S_2, s \rangle \rightarrow s'}{\langle S_1 \text{ or } S_2, s \rangle \rightarrow s'}$$

# The While Programming Language with Non-Determinism Structural Semantics

# The While Programming Language with Non-Determinism

## Examples

- ◆  $x := 1$  or  $(x := 2 ; x := x+2)$
- ◆  $(\text{while true do skip})$  or  $(x := 2 ; x := x+2)$

# Conclusion

- ◆ In the natural semantics non-determinism will suppress looping if possible (mnemonic)
- ◆ In the structural operational semantics non-determinism does suppress not termination configuration

# The **While** Programming Language with Parallel Constructs

- ◆ Abstract syntax

$S ::= x := a \mid \mathbf{skip} \mid S_1 ; S_2 \mid \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \mid$   
 $\mathbf{while} \ b \ \mathbf{do} \ S \mid S_1 \ \mathbf{par} \ S_2$

- ◆ All the interleaving of  $S_1$  or  $S_2$  are executed

- ◆ Example

- $x := 1 \ \mathbf{par} \ (x := 2 ; x := x+2)$

# The **While** Programming Language with Parallel Constructs Structural Semantics

$$[\text{par}_{\text{sos}}^1] \underline{\langle S_1, s \rangle \Rightarrow \langle S'_1, s' \rangle}$$

$$\langle S_1 \text{ par } S_2, s \rangle \Rightarrow \langle S'_1 \text{ par } S_2, s' \rangle$$

$$[\text{par}_{\text{sos}}^2] \underline{\langle S_1, s \rangle \Rightarrow s'}$$

$$\langle S_1 \text{ par } S_2, s \rangle \Rightarrow \langle S_2, s' \rangle$$

$$[\text{par}_{\text{sos}}^3] \underline{\langle S_2, s \rangle \Rightarrow \langle S'_2, s' \rangle}$$

$$\langle S_1 \text{ par } S_2, s \rangle \Rightarrow \langle S_1 \text{ par } S'_2, s' \rangle$$

$$[\text{par}_{\text{sos}}^4] \underline{\langle S_2, s \rangle \Rightarrow s'}$$

$$\langle S_1 \text{ par } S_2, s \rangle \Rightarrow \langle S_1, s' \rangle$$

# The **While** Programming Language with Parallel Constructs Natural Semantics

# Conclusion

- ◆ In the natural semantics immediate constituent is an atomic entity so we cannot express interleaving of computations
- ◆ In the structural operational semantics we concentrate on small steps so interleaving of computations can be easily expressed



# The **While** Programming Language with local variables and procedures

- ◆ Abstract syntax

$S ::= x := a \mid \mathbf{skip} \mid S_1 ; S_2 \mid \mathbf{if} \ b \ \mathbf{then} \ S_1 \ \mathbf{else} \ S_2 \mid$   
 $\mathbf{while} \ b \ \mathbf{do} \ S \mid$

$\mathbf{begin} \ D_v \ D_p \ S \ \mathbf{end} \mid \mathbf{call} \ p$

$D_v ::= \mathbf{var} \ x := a ; D_v \mid \varepsilon$

$D_p ::= \mathbf{proc} \ p \ \mathbf{is} \ S ; D_p \mid \varepsilon$

# Conclusions Local Variables

- ◆ The natural semantics can “remember” local states
- ◆ Need to introduce stack or heap into state of the structural semantics

# Summary

- ◆ Operational Semantics is useful for:
  - Language Designers
  - Compiler/Interpreter Writer
  - Programmers
- ◆ Natural operational semantics is a useful abstraction
  - Can handle many PL features
  - No stack/ program counter
  - Simple
  - “Mostly” compositional
- ◆ Other abstractions exist

# Further Reading

- ◆ Ankur Taly: Operational Semantics for JavaScript
- ◆ Pietro Cenciarelli?, *Alexander Knapp, Bernhard Reus, and Martin Wirsing*: An Event-Based Structural Operational Semantics of Multi-threaded Java
- Alan Jeffrey and Julian Rathke: Java Jr.: Fully abstract trace semantics for a core Java language