

Assignment 4 - Computational Geometry (0368-4211)

Due: January 19, 2009

Problem 1

(a) Let P be a given set of n points in d dimensions, and let C denote the convex hull of P . Design an $O(n^2)$ -time algorithm that determines all the vertices of C . (**Hint:** Solve n linear-programming problems, one for each point in P , where the variables are the coefficients of an appropriate hyperplane.) (**Note:** This is a weaker problem than the one of computing the complete structure of C , which, in higher dimensions, can have a much larger complexity.)

(b) Extend the solution of (a) to find, in $O(n^{j+2})$ time, all j -dimensional faces of the convex hull of P ($j = 0$ in part (a)).

Problem 2

(a) Let T be a set of n triangles (in general position, but possibly intersecting) in the plane. Using duality, express the property that a line ℓ intersects a triangle $\Delta = abc$ in T . Use this representation to determine whether there exists a line which *separates* T , i.e., a line ℓ that does not meet any triangle of T and each side of ℓ fully contains at least one triangle.

(b) Do the same for a set D of n disks in the plane, using duality, as above.

(c) **Bonus, worth extra 10%:** Solve (a) and (b) without duality, directly in the primal plane.

Problem 3

Let e_1, \dots, e_n be n horizontal segments in the plane. Find, in linear time, a line that intersects all the segments and has the largest possible slope, or determine that there is no such line.

Problem 4

Use duality to solve the following problems. Let P be a set of n points in the plane (in general position). Let L_1 be the set of all lines ℓ such that ℓ passes through at least one point of P and there is exactly one point of P lying below ℓ .

- (a) Describe the structure of L_1 in the dual plane.
- (b) Using an analysis either in the primal or in the dual plane, show that the combinatorial complexity of L_1 , properly defined, is $O(n)$.

Problem 5

Let P and Q be two convex polytopes in three dimensions, with m and n facets, respectively.

- (a) Assume that P and Q are disjoint. Find, in $O(m + n)$ time, the *vertical distance* between P and Q . This is the amount by which one of them has to be translated up or down (in the z -direction) until it touches the other polytope. (**Hint:** Show that the problem is equivalent to finding two parallel planes supporting P and Q on different sides, with *maximal* vertical distance between them. Then solve the latter problem using linear programming, where the variables are the coefficients of the planes; there are *four* variables!) Note that the distance can be infinite (when?). You have to detect such cases too.
- (b) Assume now that P and Q intersect. Find, in linear time (similar to (a)), the *vertical separation* between P and Q . This is the smallest amount by which one of them has to be translated up or down (in the z -direction) until their interiors become disjoint (and they touch each other).