Assignment 1 - Computational Geometry (0368-4211)

Due: December 1, 2008

Problem 1

Let E be a set of n segments in the plane, so that all the $2n$ endpoints of the segments are distinct and lie in convex position. Let P be the set of these endpoints, and let C denote the convex hull of P.

Assume that C is given. Give a linear-time algorithm to determine whether any pair of segments in E intersect each other.

Problem 2

Let P and Q be two convex polygons with m and n edges, respectively, in general position.

(a) Give an algorithm, with $O(m+n)$ running time, to test whether P and Q intersect each other.

(b) If P and Q are disjoint, find, in $O(m+n)$ time, the pair of points $p \in P$, $q \in Q$ (not necessarily vertices!), at smallest distance from each other.

(c) Give an algorithm, with $O(m+n)$ running time, for finding the pair of points $p \in P$, $q \in Q$, at largest distance from each other. (Here P and Q need not be disjoint.)

Problem 3

Let P and Q be two convex polygons with m and n edges, respectively, in general position. The Minkowski sum $P \oplus Q$ is the set $\{x + y \mid x \in P, y \in Q\}$. Show how to compute $P \oplus Q$ through the following steps:

(a) Show that $P \oplus Q$ is a convex polygon.

(b) Let u be a direction, and let $p \in P$, $q \in Q$ be the vertices such that the supporting line to P with outward normal direction u touches P at p, and the supporting line to Q with outward normal direction u touches Q at q. Then $p + q$ is a vertex of $P \oplus Q$, and is touched by the line which supports $P \oplus Q$ and has outward normal direction u. Show the converse direction too.

(c) Use the normal diagrams of P and Q to find the vertices of $P \oplus Q$. How many vertices does $P \oplus Q$ have?

Problem 4

(a) Let π be an *x-monotone* polygonal path $p_1 p_2 \cdots p_n$ with n vertices; that is, $x_{p_1} < x_{p_2} < \cdots < x_{p_n}$. Show how to compute $CH(\pi)$ in $O(n)$ time.

(b) Let π be a polygonal path $p_1 p_2 \cdots p_n$ with n vertices, of which k are *locally x-extreme* vertices; namely, vertices p_j such that both adjacent edges go from p_j to the left or both go to the right. Show how to compute $CH(\pi)$ in $O(n \log k)$ time.

Problem 5

Show that computing the convex hull of n points in the plane requires $\Omega(n \log n)$ time, by reduction from sorting (assuming that sorting n numbers is known to require $\Omega(n \log n)$ time in the model of computation under consideration).