# Advanced Topics in Computational and Combinatorial Geometry

# Assignment 3 (short answers and hints)

# Problem 1

1. There are faces in the arrangement of the lines. Each triple of faces generate a different . Thus there are in total: .
2. Any such triple must be vertices of the same face, so the number is , over all faces of , where number of vertices of . This can be written as
3. Use the Clarkson-Shor’s technique – straightforward.

# Problem 2

Observation: if there is an interval *I* of length *k* that does not contain any point of *R*, then there is an interval *I’* of this kind of length *k/2* starting at position that is multiple of *k/2* (there are *2n/k* such intervals). Thus it is enough to upper bound the probability that at least one of the *2n/k* intervals of length *k/2* does not contain a point of *R*.

*Prob(there is an "empty" interval of length k* ***at any position****) ≤
≤ Prob(there is one of the 2n/k intervals of length k/2* ***at fixed positions*** *that is empty) ≤
≤ 2n/k Prob(there is an empty interval of length k/2* ***at fixed position****) ≤
≤*

Substitute .

# Problem 3

Very similar to the construction and proof we used in class. One has to go through the steps and modify them to the special scenario here.

# Problem 4

1. The cost of inserting a line , at step , is proportional to the complexity of the zone of in the arrangement of the first inserted lines, which is , for a total of .
2. Same as in (a): Each face is x-monotone: has a top boundary and a bottom boundary. To insert a new graph "into" a current face , we traverse the top and bottom boundaries from left to right and look for intersections. Again, the cost of inserting is proportional to the complexity of its zone, which now is . Summing over , we get .
3. The simples (roundabout) way is to extend each graph to a fully defined graph by two steep rays from its endpoints (as in class), construct the arrangement of the s, and then delete from it the rays.