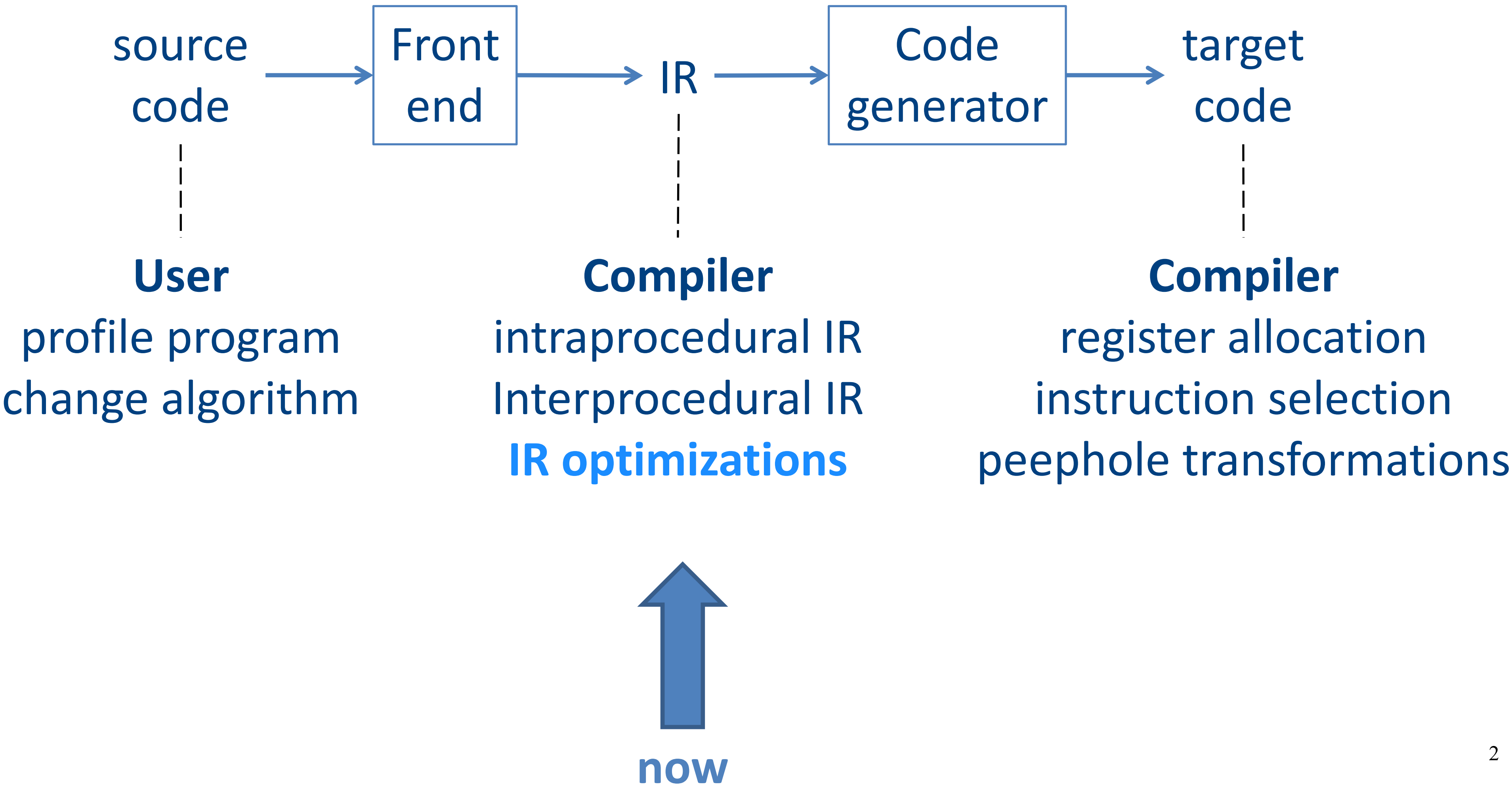
Compilation

Lecture 9

Optimizations

Noam Rinetzky

Optimization points



Program Analysis

- In order to optimize a program, the compiler has to be able to reason about the properties of that program
- An analysis is called **sound** if it never asserts an incorrect fact about a program
- All the analyses we will discuss in this class are sound
  - *(Why?)*

A formalism for IR optimization

- Every phase of the compiler uses some new abstraction:
  - Scanning uses regular expressions
  - Parsing uses CFGs
  - Semantic analysis uses proof systems and symbol tables
  - IR generation uses ASTs
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
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    c = a;
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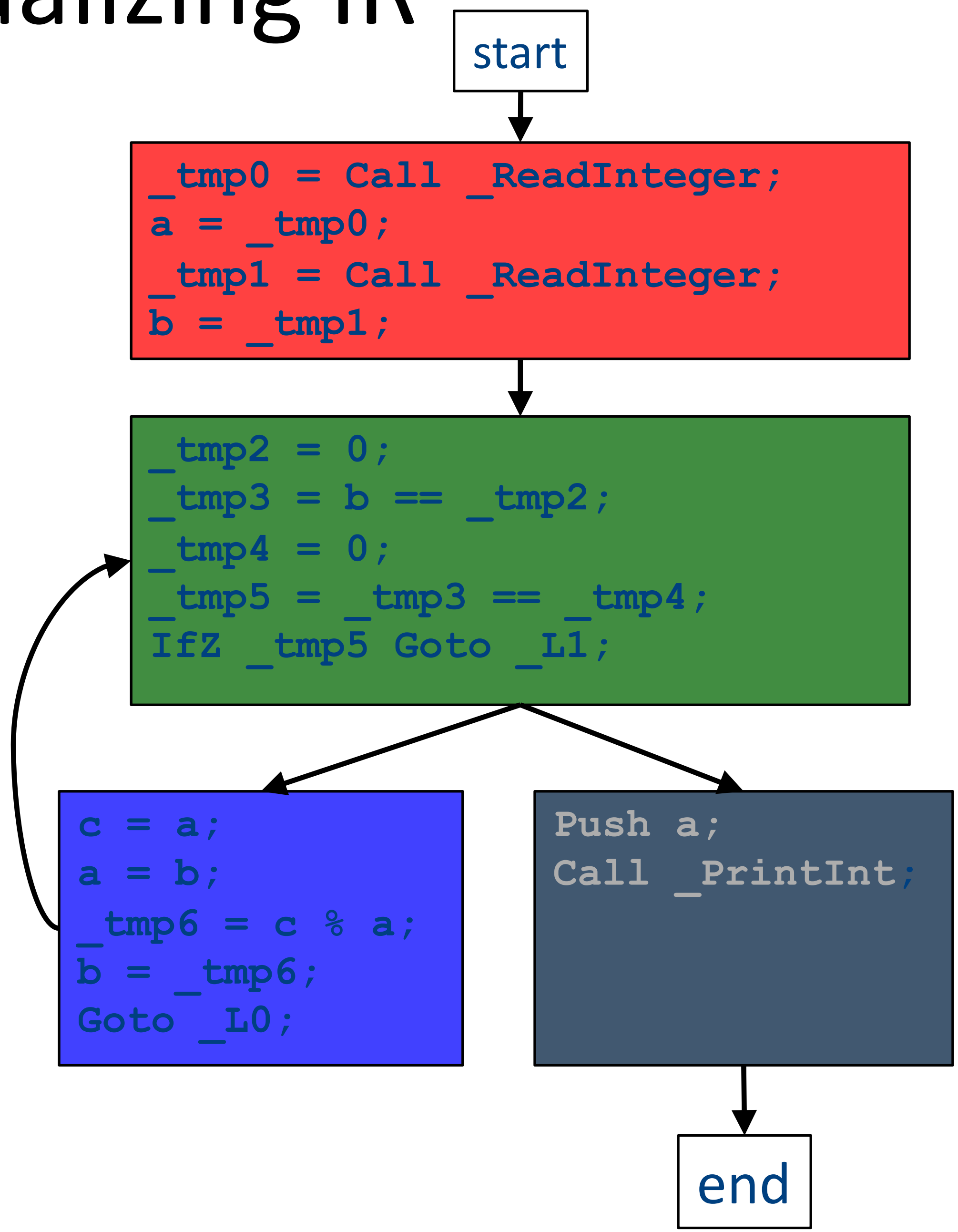


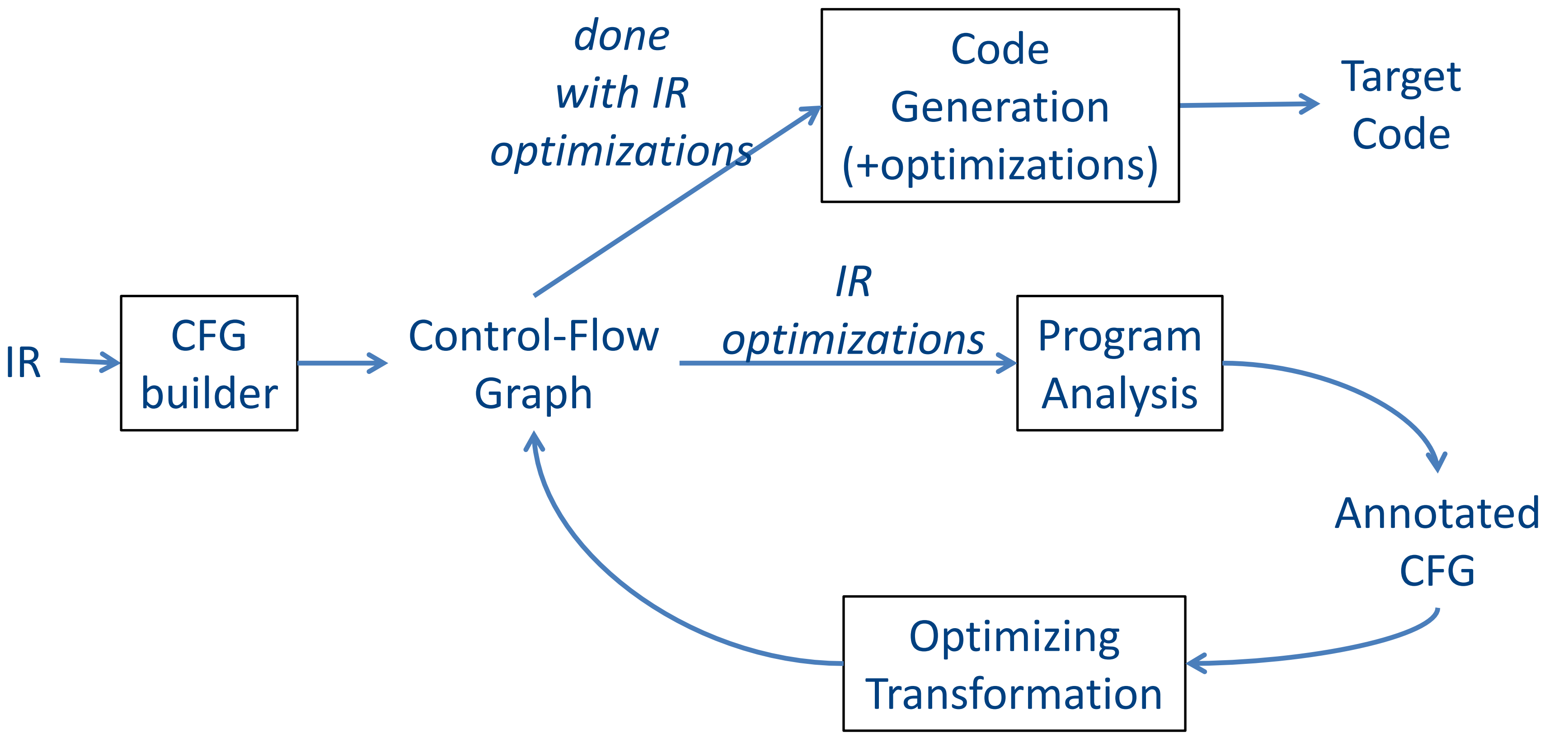
Basic blocks

- A basic block is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence
- Informally, a sequence of instructions that always execute as a group

Control-Flow Graphs

- A control-flow graph (CFG) is a graph of the basic blocks in a function
- The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context free grammar”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block
- There is a dedicated node for the start and end of a function

Optimization path



Types of optimizations

- An optimization is local if it works on just a single basic block
- An optimization is global if it works on an entire control-flow graph
- An optimization is interprocedural if it works across the control-flow graphs of multiple functions
  - We won't talk about this in this course

Local Optimizations

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

Class Object {
  method fn(int);
}

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Explaining the program

Example

For simplicity, ignore Popping return value, parameters etc.

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int a;
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int c;

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Class Object {
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_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Size of Object

Object Class

Explaining the program

Example

```
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int a;
int b;
int c;

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Class Object {
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```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Explaining the program

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Object x;
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Explaining the program

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```

Points to ObjectC

Start of fn

Explaining the program

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b
  …
  v2 = a op b
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b [or: v1 = a]
  …
  v2 = a op b [or: v2 = a]
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b [or: v1 = a]
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common subexpression elimination

```
Object x;
int a;
int b;
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```

Copy Propagation

- If we have a variable assignment
  v1 = v2
  then as long as v1 and v2 are not reassigned, we can rewrite expressions of the form
  a = … v1 …
  as
  a = … v2 …
  provided that such a rewrite is legal

Copy Propagation

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```

Is this transformation OK?
What do we need to know?

Copy Propagation

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_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Dead Code Elimination

- An assignment to a variable v is called dead if the value of that assignment is never read anywhere
- Dead code elimination removes dead assignments from IR
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
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Dead Code Elimination

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_tmp7 = *(ObjectC);
Push c;
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Call _tmp7;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new
Object;
a = 4;
c = a + b;
x.fn(a + b);
```

values never read

values never read

```
_tmp0 = 4;
Push _tmp0;
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_tmp2 = ObjectC;
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Call _tmp7;
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Dead Code Elimination

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Object x;
int a;
int b;
int c;

x = new
Object;
a = 4;
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x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;

*(_tmp1) = ObjectC;




_tmp4 = _tmp0 + b;
c = _tmp4;



_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Applying local optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization
- Common subexpression elimination eliminates unnecessary statements
- Copy propagation helps identify dead code
- Dead code elimination removes statements that are no longer needed
- To get maximum effect, we may have to apply these optimizations numerous times

Applying local optimizations example

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying local optimizations example

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Common sub-expression elimination

Applying local optimizations example

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

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b = a * a;
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Which optimization should we apply here?

Copy propagation

Applying local optimizations example

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Which optimization should we apply here?

Common sub-expression elimination (again)

Other types of local optimizations

- Arithmetic Simplification
  - Replace “hard” operations with easier ones
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- Constant Folding
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite `x = 4 * 5;` as `x = 20;`

Optimizations and analyses

- Most optimizations are only possible given some analysis of the program's behavior
- In order to implement an optimization, we will talk about the corresponding program analyses

Available expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the available expressions in a program
- An expression is called available if some variable in the program holds the value of that expression
- In common subexpression elimination, we replace an available expression by the variable holding its value
- In copy propagation, we replace the use of a variable by the available expression it holds

Finding available expressions

- Initially, no expressions are available
- Whenever we execute a statement
  a = b *op* c:
  - Any expression holding **a** is invalidated
  - The expression **a = b *op* c** becomes available
- **Idea:** Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable

Available expressions example

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
 { b = x, d = x, e = a + b }
f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {  }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = d;
 { b = x, d = a + b, e = a + b }
d = b;
 { b = x, d = x, e = a + b }
f = e;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {  }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
 { b = x, d = x, e = a + b }
f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```

Live variables

- The analysis corresponding to dead code elimination is called liveness analysis
- A variable is live at a point in a program if later in the program its value will be read before it is written to again
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables

Computing live variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order
- Initially, some small set of values are known to be live (which ones depends on the particular program)
- When we see the statement a = b op c:
  - Just before the statement, a is not alive, since its value is about to be overwritten
  - Just before the statement, both b and c are alive, since we're about to read their values
  - (what if we have a = a + b?)

Liveness analysis

```
 { b }
a = b;
 { a, b }
c = a;
 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }
f = e;
 { b, d } - given
```

Which statements are dead?

Dead Code Elimination

```
 { b }
a = b;
 { a, b }
c = a;
 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }
f = e;
 { b, d }
```

Which statements are dead?

Dead Code Elimination

```
 { b }
a = b;
 { a, b }

 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }

 { b, d }
```

Liveness analysis II

```
 { b }
a = b;


 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Liveness analysis II

```
 { b }
a = b;


 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;

 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;



 { a, b }
d = a + b;
 { a, b, d }

 { a, b }
d = a;
 { b, d }
```

Liveness analysis III

```
 { b }
a = b;

 { a, b }
d = a + b;

 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;

 { a, b }
d = a + b;

 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;



 { a, b }




 { a, b }
d = a;
 { b, d }
```

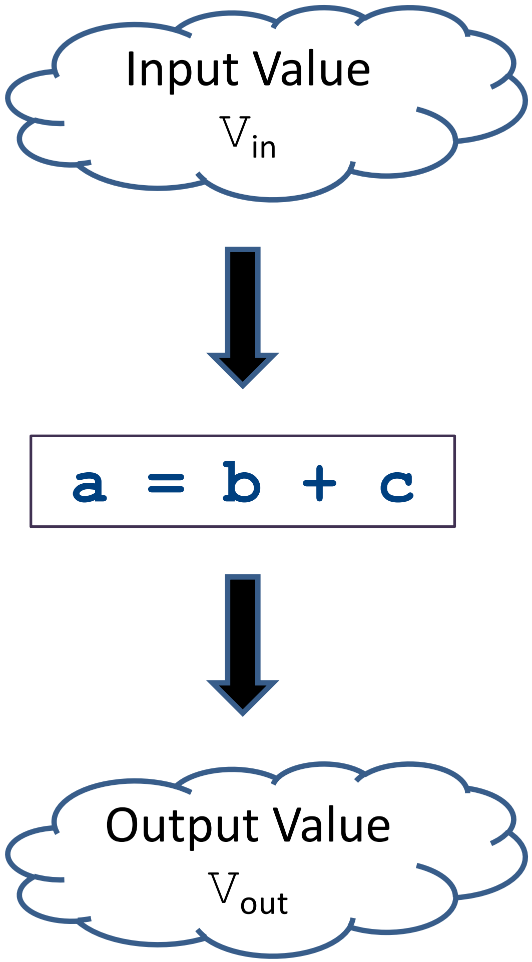
Dead code elimination

```
a = b;
```

If we further apply copy propagation this statement can be eliminated too

```
d = a;
```

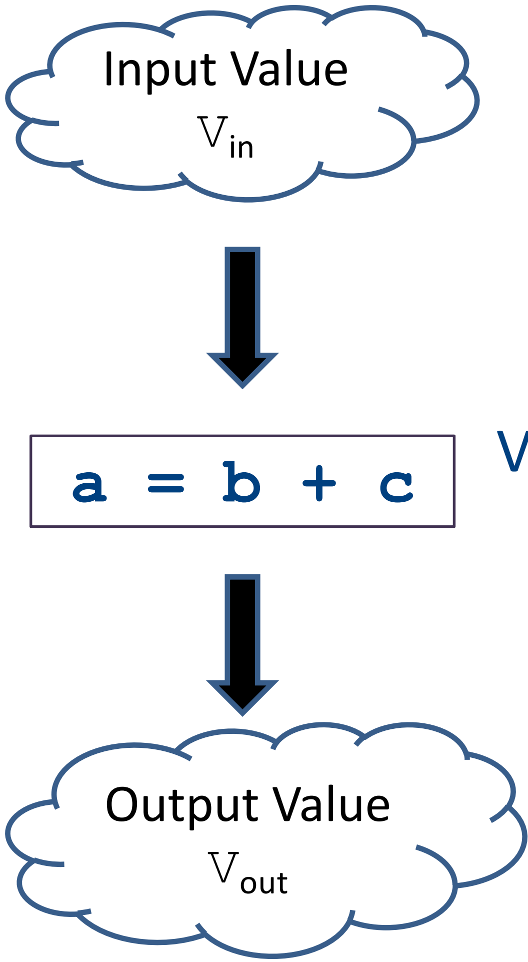
Formalizing local analyses



Transfer Function

$$V_{out} = f_{\mathtt{a=b+c}}(V_{in})$$

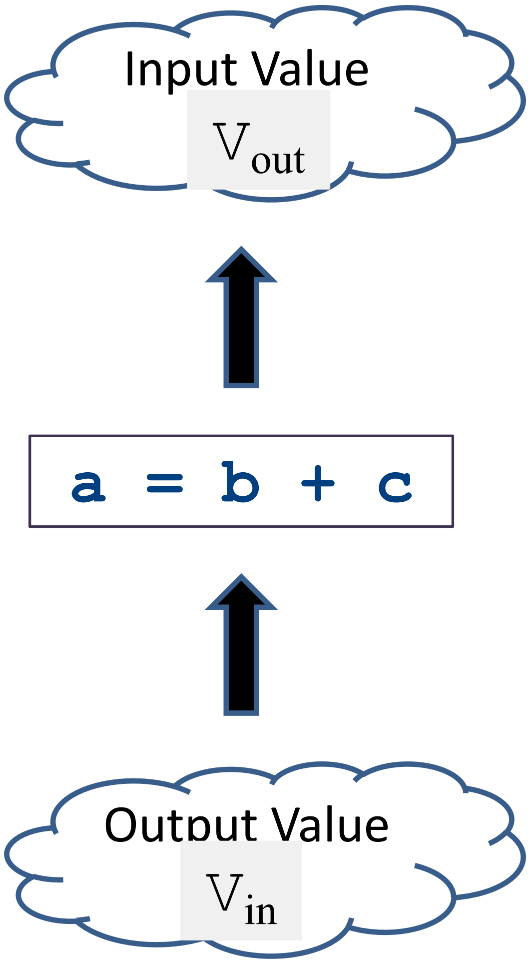
Available Expressions



$V_{out} = (V_{in} \setminus \{e \mid e \text{ contains } \mathbf{a}\}) \cup \{a=b+c\}$

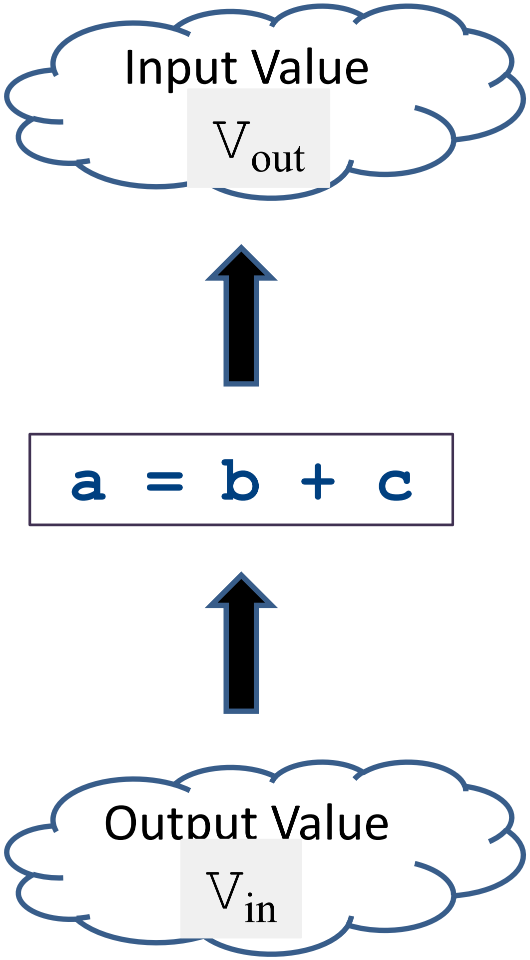
Expressions of the forms
`a`=… and x=…**`a`**…

Live Variables



$$V_{in} = (V_{out} \setminus \{\texttt{a}\}) \cup \{\texttt{b},\texttt{c}\}$$

Live Variables



$$V_{in} = (V_{out} \setminus \{\mathtt{a}\}) \cup \{\mathtt{b,c}\}$$

Information for a local analysis

- What direction are we going?
  - Sometimes forward (available expressions)
  - Sometimes backward (liveness analysis)
- How do we update information after processing a statement?
  - What are the new semantics?
  - What information do we know initially?

Formalizing local analyses

- Define an analysis of a basic block as a quadruple (D, V, F, I) where
  - **D** is a direction (forwards or backwards)
  - **V** is a set of values the program can have at any point
  - **F** is a family of transfer functions defining the meaning of any expression as a function f : **V** → **V**
  - **I** is the initial information at the top (or bottom) of a basic block

Available Expressions

- **Direction:** Forward
- **Values:** Sets of expressions assigned to variables
- **Transfer functions:** Given a set of variable assignments V and statement a = b + c:
  - Remove from V any expression containing a as a subexpression
  - Add to V the expression a = b + c
  - Formally: $\mathbb{V}_{out} = (\mathbb{V}_{in} \setminus \{e \mid e \text{ contains } \mathbf{a}\}) \cup \{a = b + c\}$
- **Initial value:** Empty set of expressions

Liveness Analysis

- **Direction:** Backward
- **Values:** Sets of variables
- **Transfer functions:** Given a set of variable assignments V and statement a = b + c:
- Remove a from V (any previous value of a is now dead.)
- Add b and c to V (any previous value of b or c is now live.)
- Formally: $\mathbb{V}_{in} = (\mathbb{V}_{out} \setminus \{\mathbf{a}\}) \cup \{\mathbf{b}, \mathbf{c}\}$
- **Initial value:** Depends on semantics of language
  - E.g., function arguments and return values (pushes)
  - Result of local analysis of other blocks as part of a global analysis

Running local analyses

- Given an analysis **(D, V, F, I)** for a basic block
- Assume that **D** is "forward;" analogous for the reverse case
- Initially, set OUT[**entry**] to **I**
- For each statement **s**, in order:
  - Set IN[**s**] to OUT[**prev**], where **prev** is the previous statement
  - Set OUT[**s**] to f_s(IN[**s**]), where f_s is the transfer function for statement **s**

Global Optimizations

High-level goals

- Generalize analysis mechanism
  - Reuse common ingredients for many analyses
  - Reuse proofs of correctness
- Generalize from basic blocks to entire CFGs
  - Go from local optimizations to global optimizations

Global analysis

- A global analysis is an analysis that works on a control-flow graph as a whole
- Substantially more powerful than a local analysis
  - (Why?)
- Substantially more complicated than a local analysis
  - (Why?)

Local vs. global analysis

- Many of the optimizations from local analysis can still be applied globally
  - Common sub-expression elimination
  - Copy propagation
  - Dead code elimination
- Certain optimizations are possible in global analysis that aren't possible locally:
  - e.g. code motion: Moving code from one basic block into another to avoid computing values unnecessarily
- Example global optimizations:
  - Global constant propagation
  - Partial redundancy elimination

Loop invariant code motion example

```
while (t < 120) {
  z = z + x - y;
}
```

```
w = x - y;
while (t < 120) {
  z = z + w;
}
```

value of expression x – y is not changed by loop body

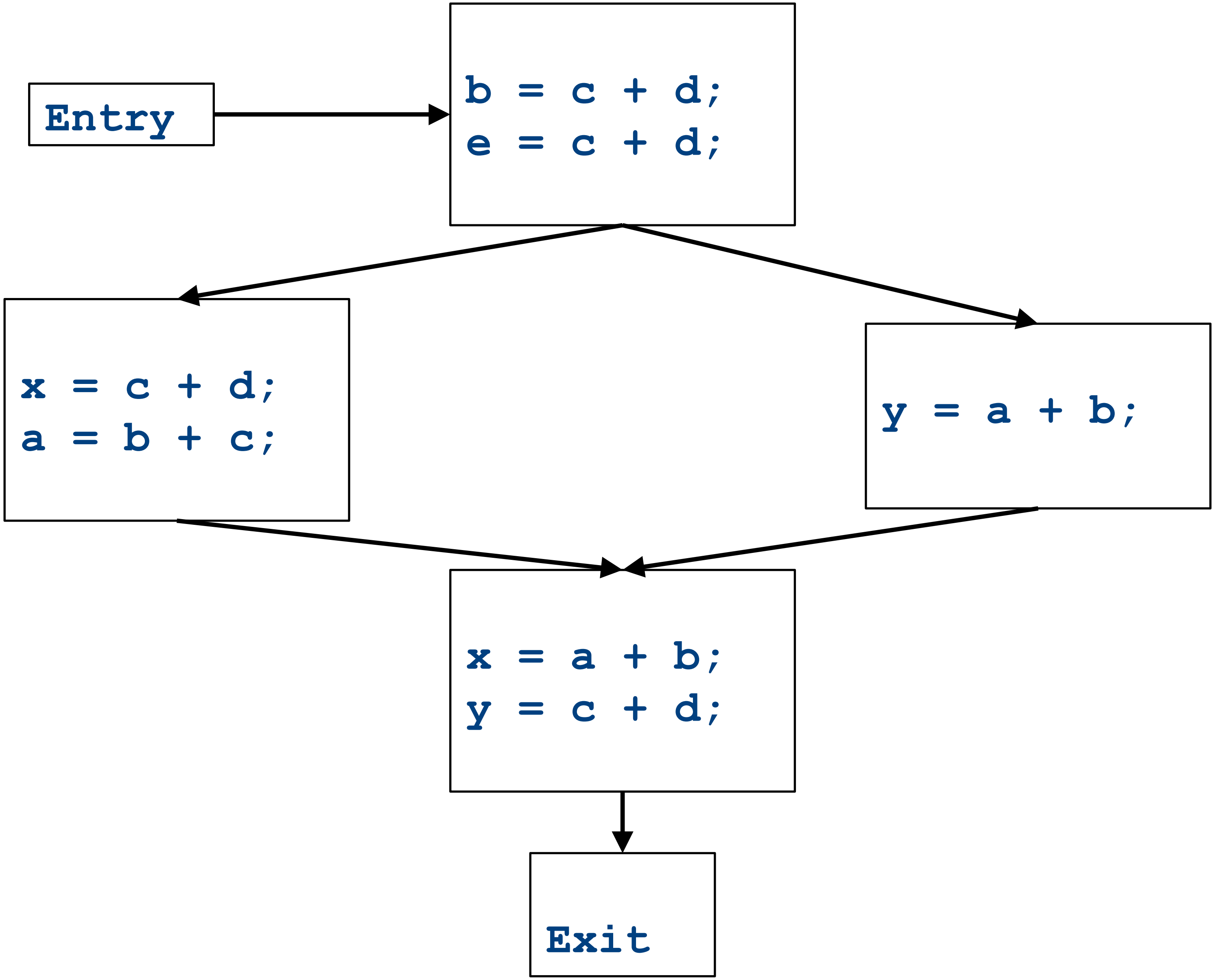
Why global analysis is hard

- Need to be able to handle multiple predecessors/successors for a basic block
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value (but the analysis still needs to terminate!)
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it

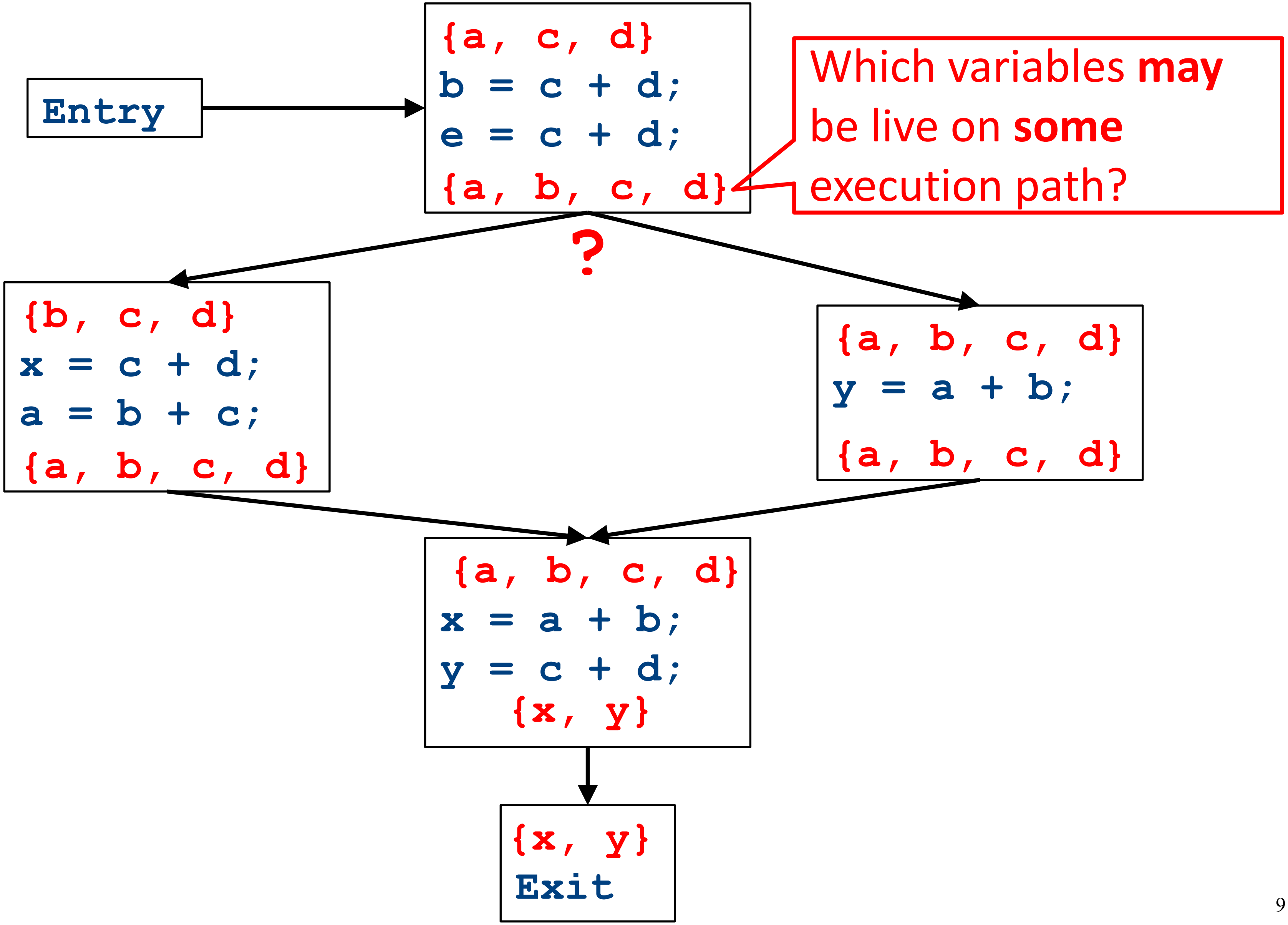
Global dead code elimination

- Local dead code elimination needed to know what variables were live on exit from a basic block
- This information can only be computed as part of a global analysis
- How do we modify our liveness analysis to handle a CFG?

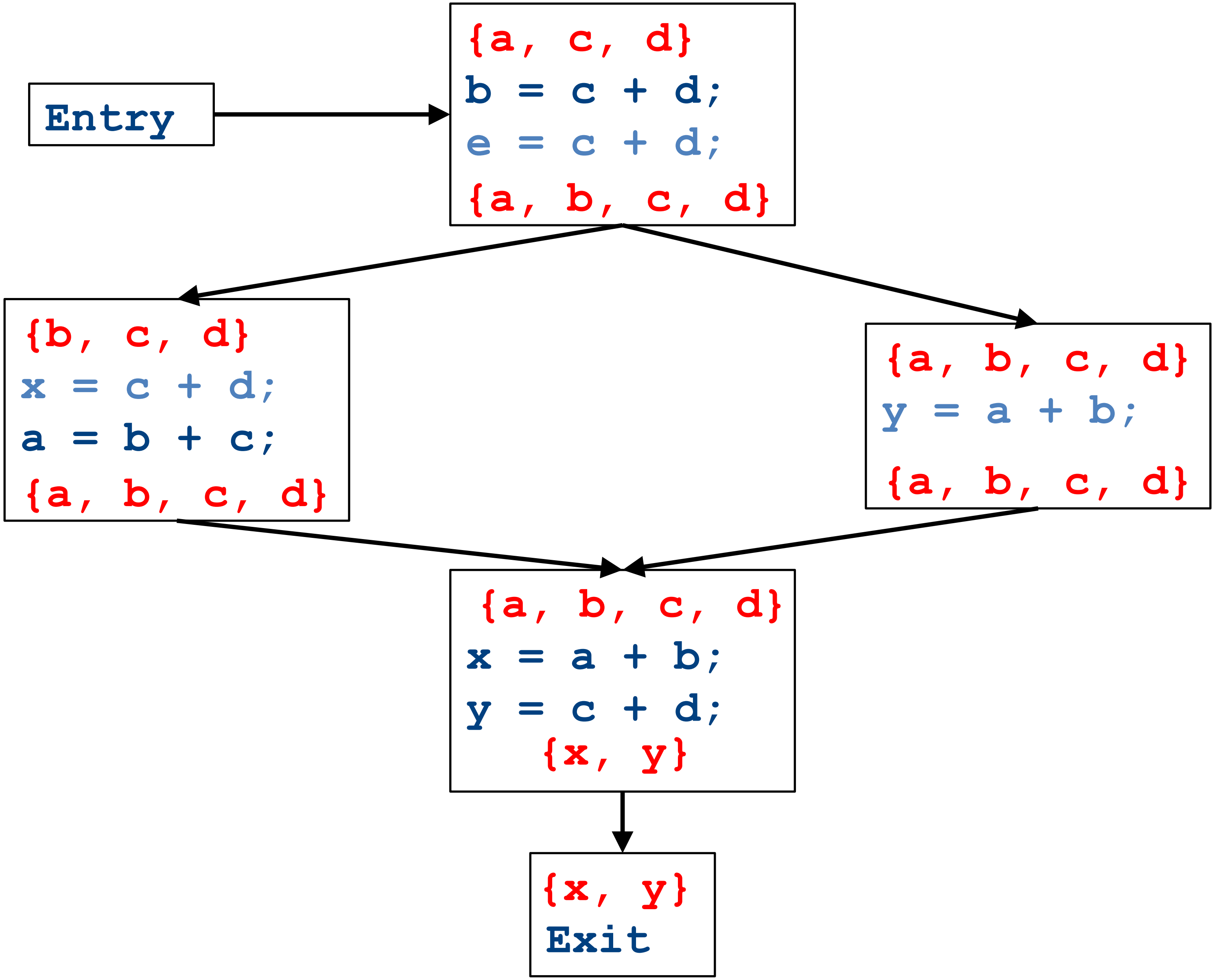
CFGs without loops



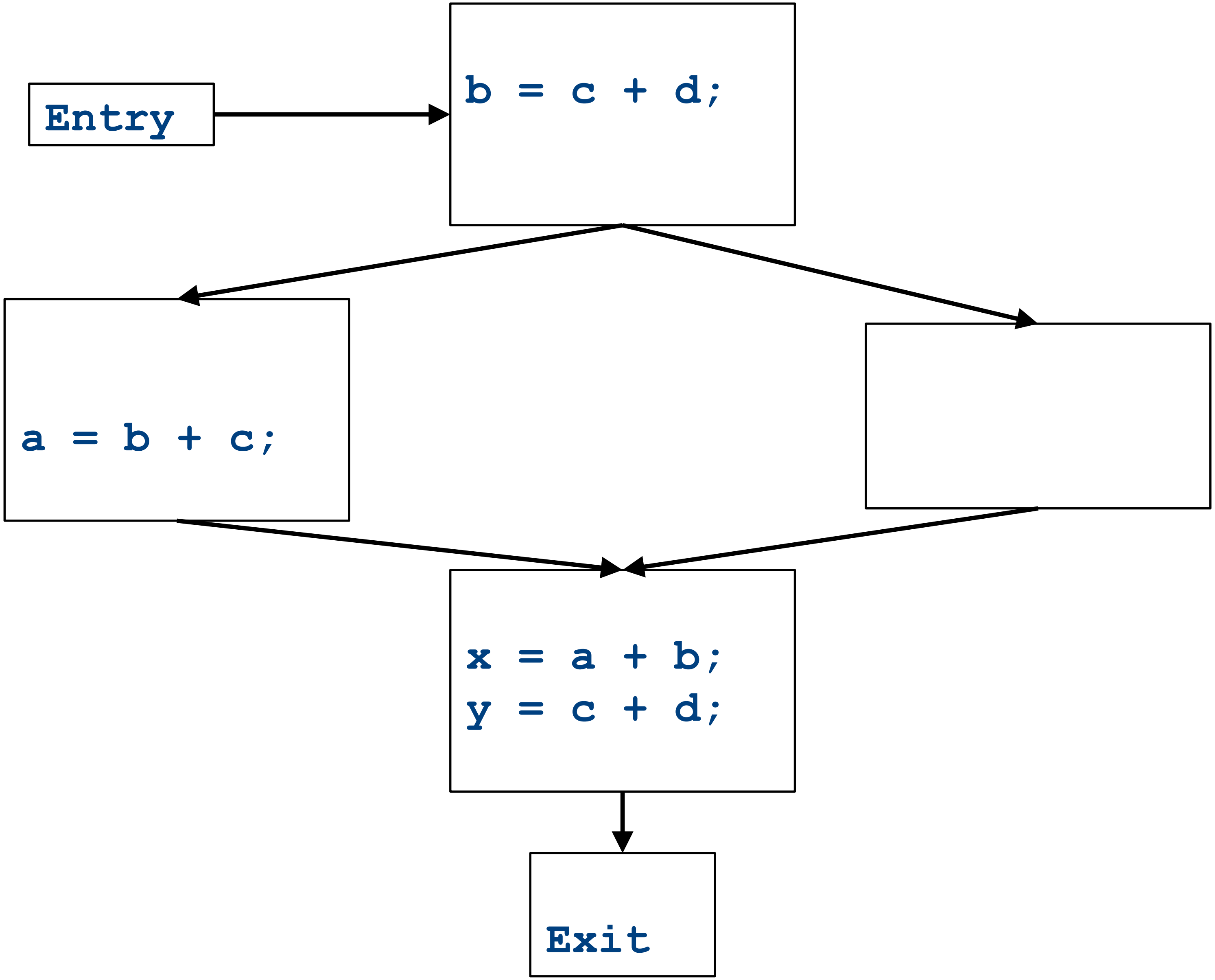
CFGs without loops



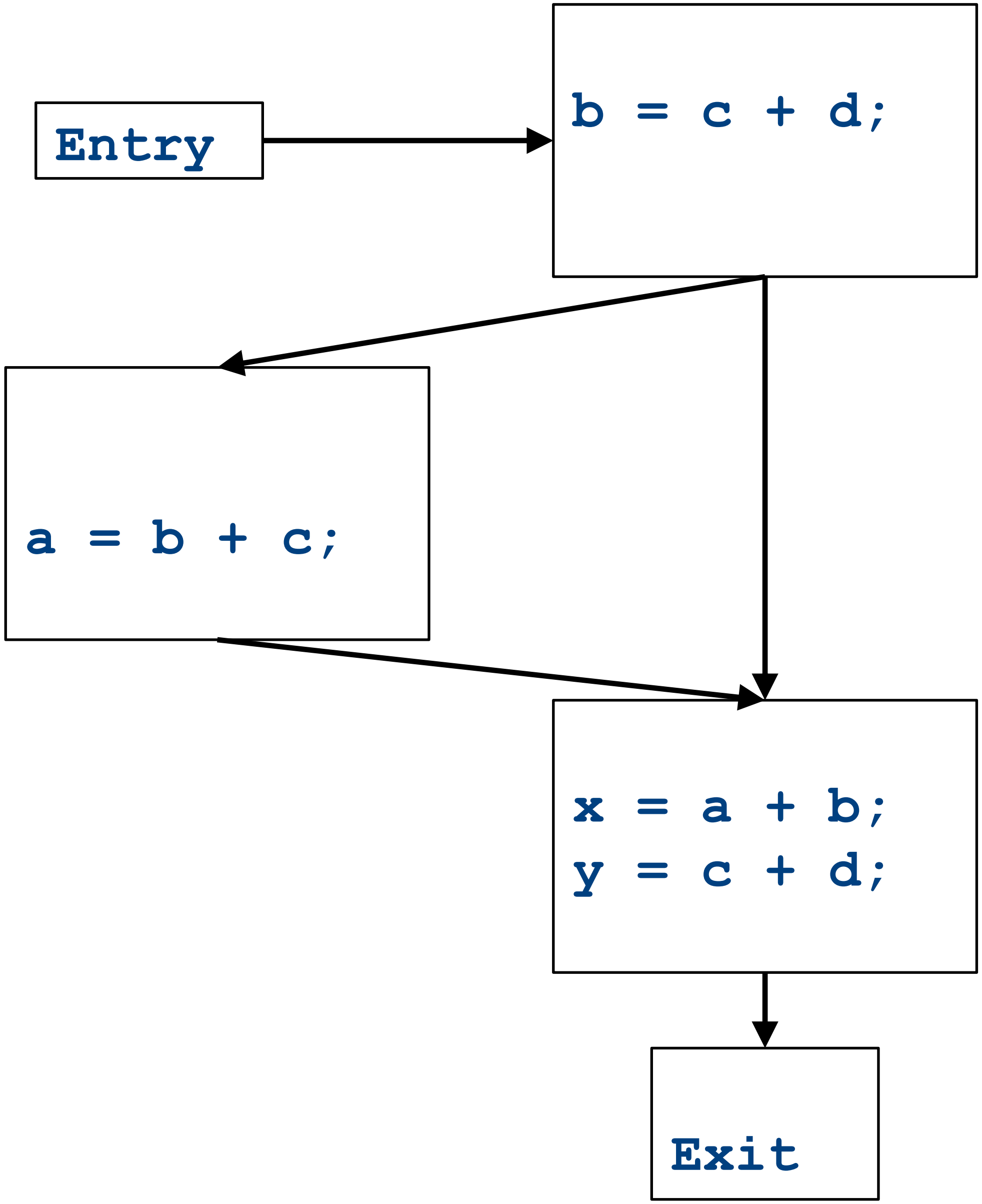
CFGs without loops



CFGs without loops



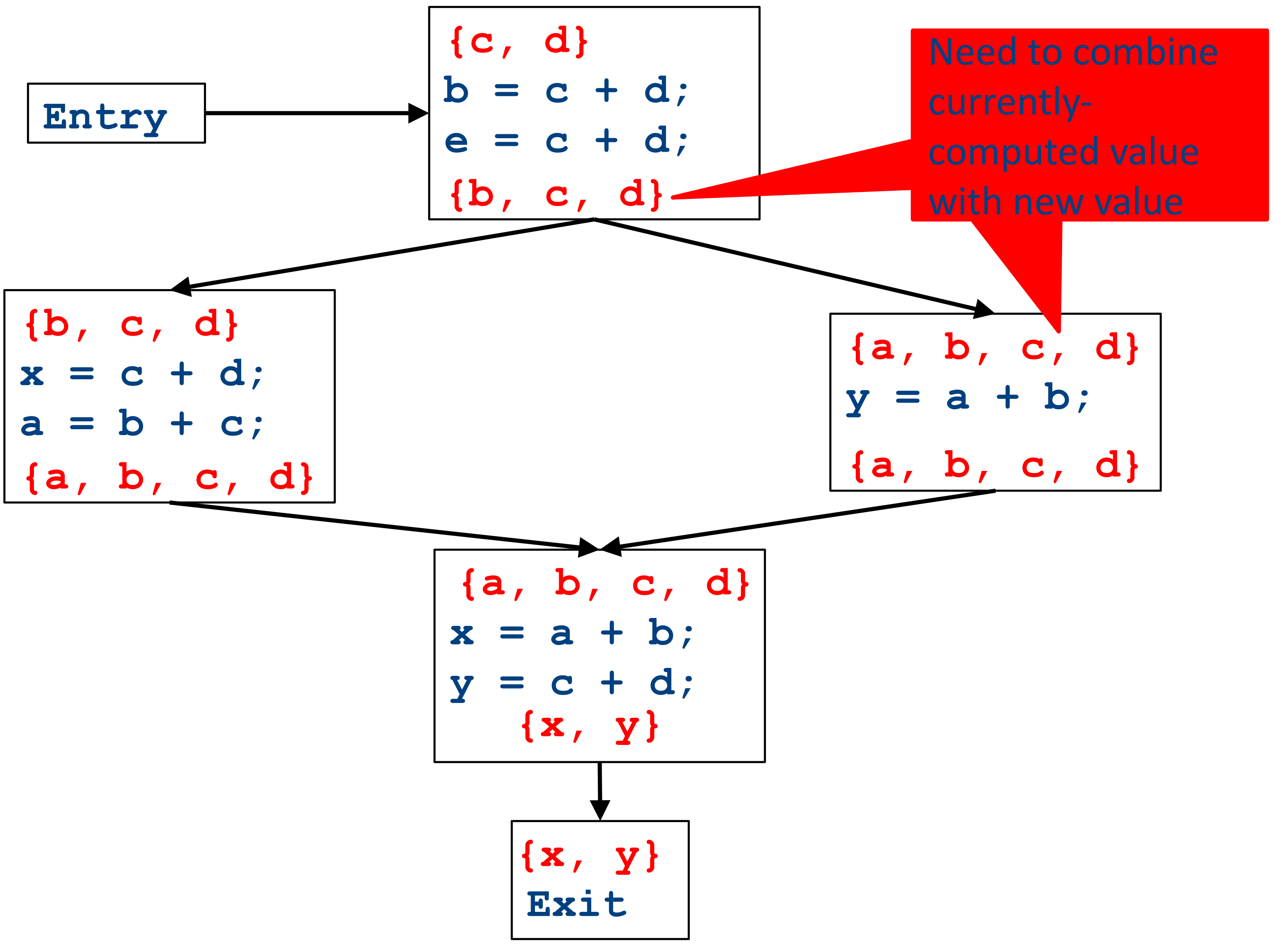
CFGs without loops



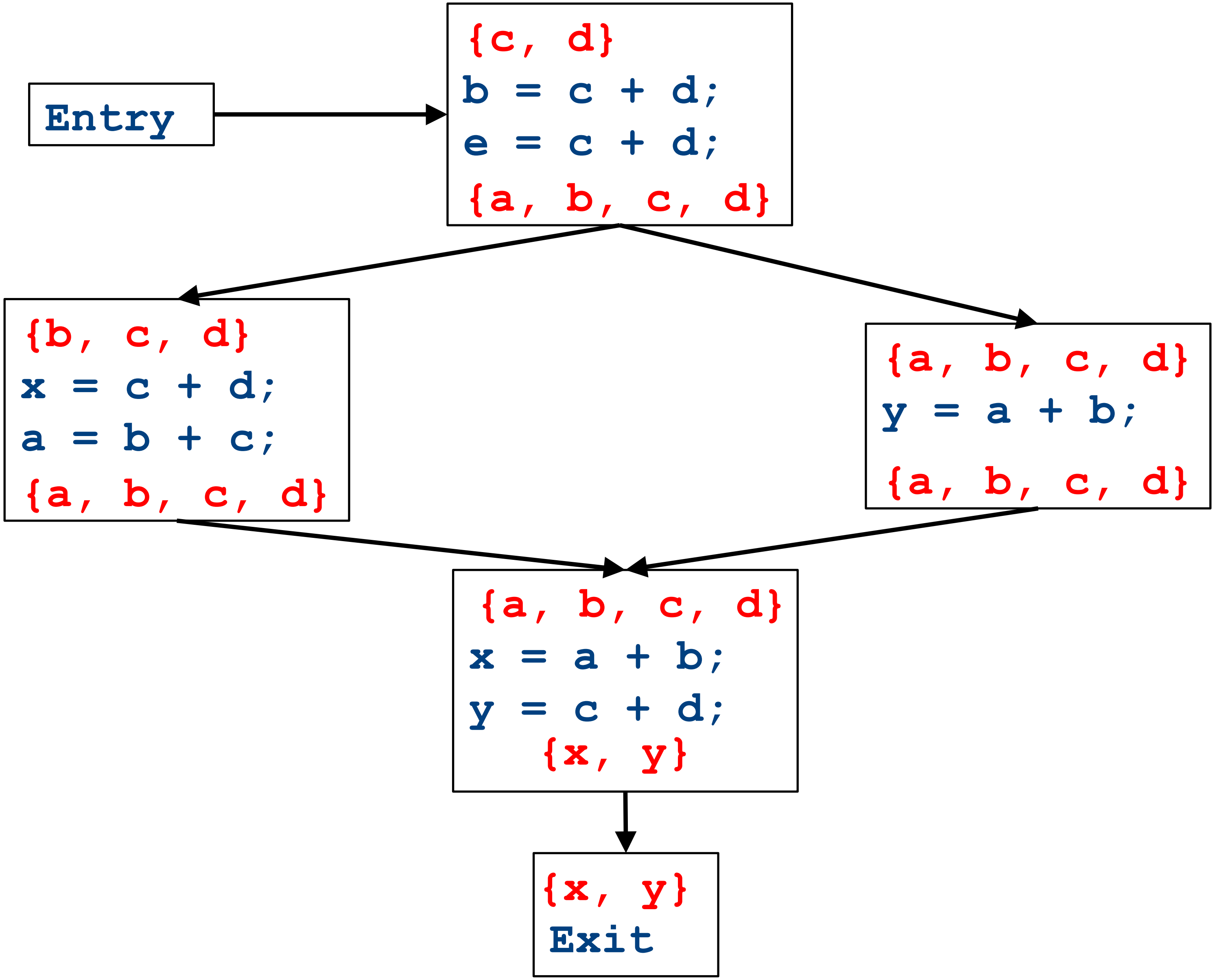
Major changes – part 1

- In a local analysis, each statement has exactly one predecessor
- In a global analysis, each statement may have **multiple** predecessors
- A global analysis must have some means of **combining information** from all predecessors of a basic block

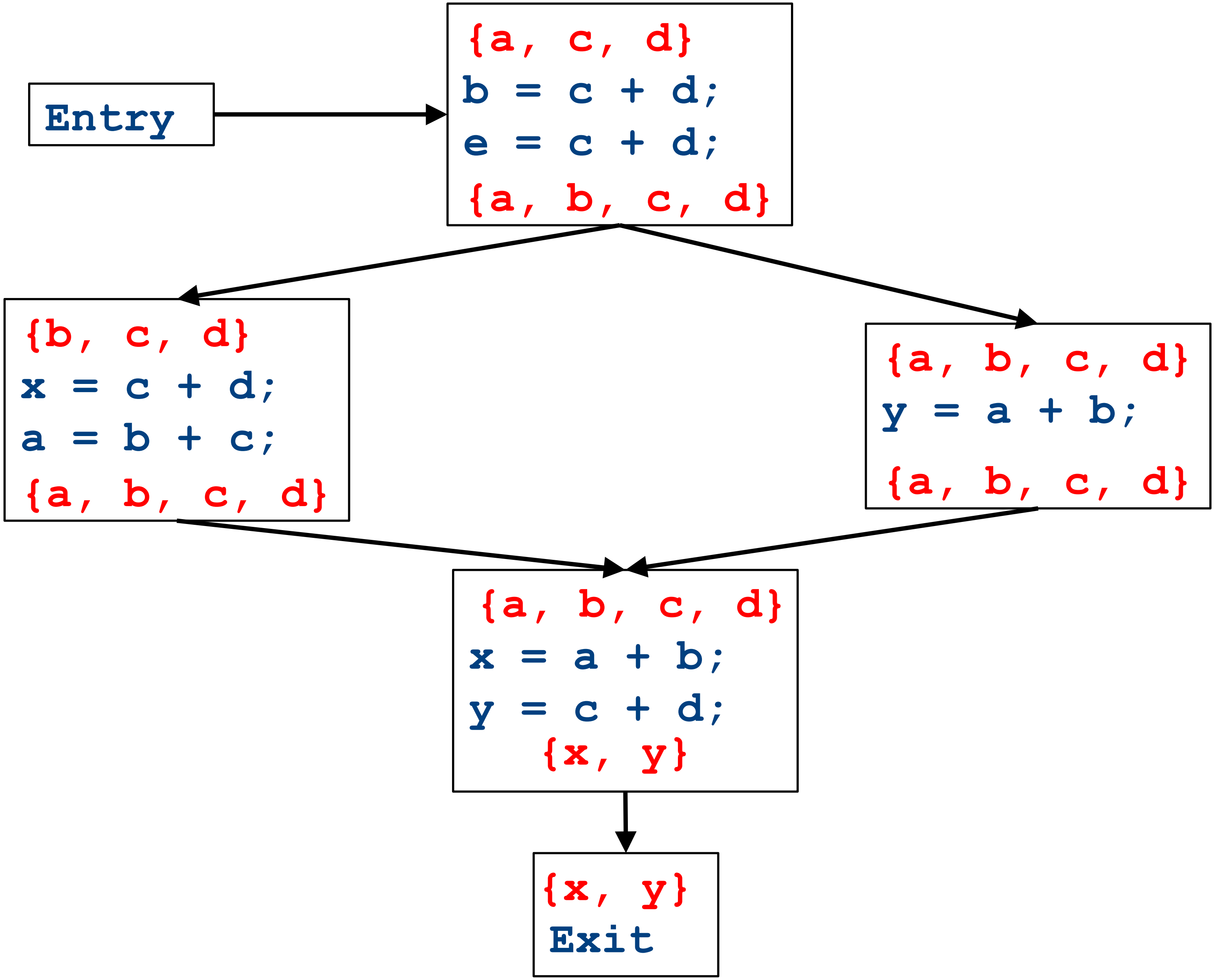
CFGs without loops



CFGs without loops



CFGs without loops

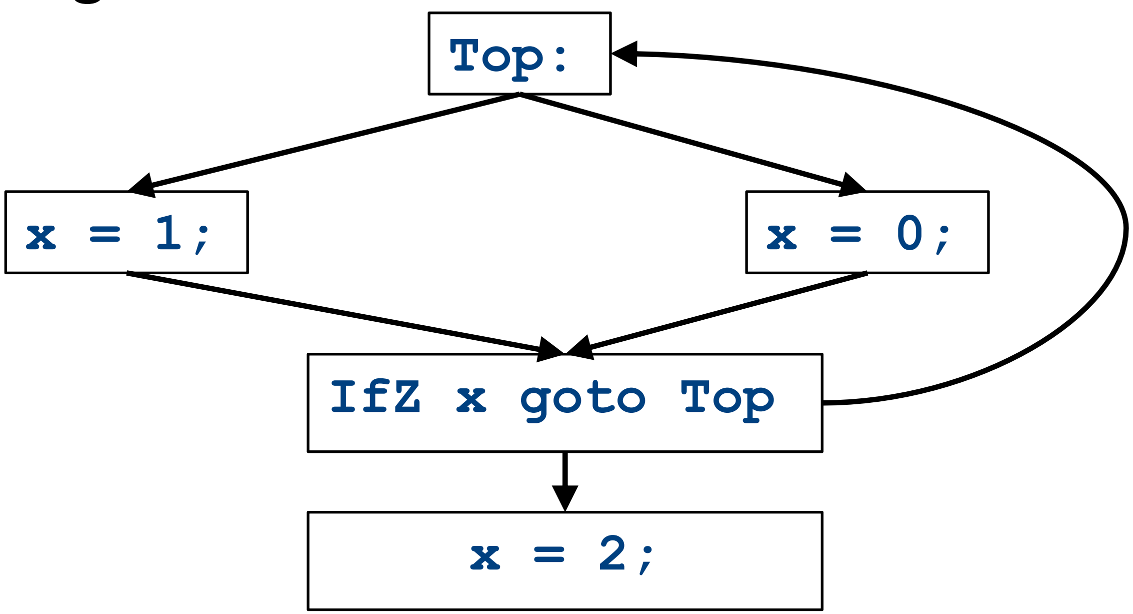


Major changes – part 2

- In a local analysis, there is only one possible path through a basic block
- In a global analysis, there may be **many** paths through a CFG
- May need to recompute values multiple times as more information becomes available
- Need to be careful when doing this not to loop infinitely!
  - (More on that later)
- Can order of computation affect result?

CFGs with loops

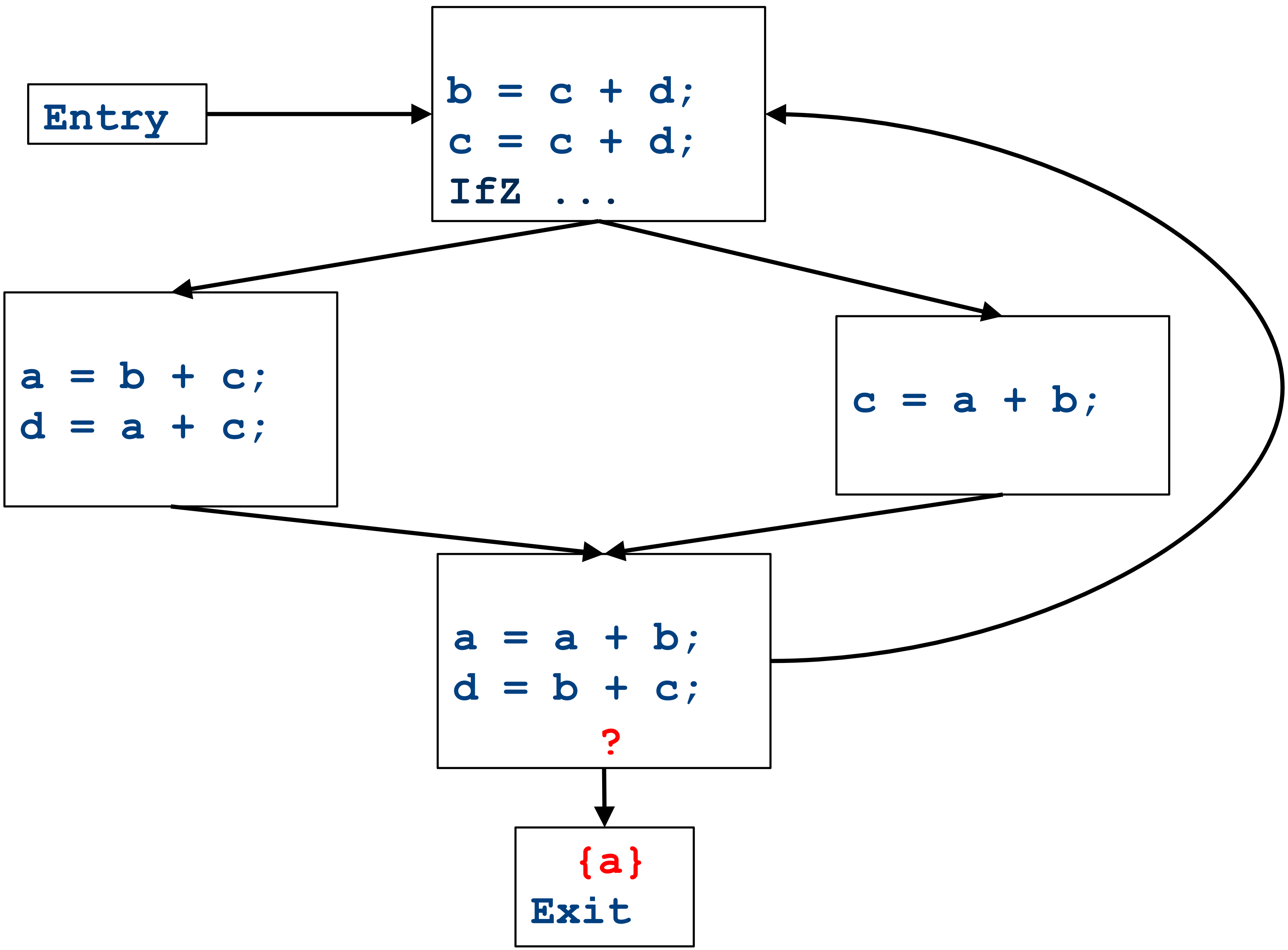
- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths
- When we add loops into the picture, this is no longer true
- Not all possible loops in a CFG can be realized in the actual program



CFGs with loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths
- When we add loops into the picture, this is no longer true
- Not all possible loops in a CFG can be realized in the actual program
- **Sound approximation:** Assume that every possible path through the CFG corresponds to a valid execution
  - Includes all realizable paths, but some additional paths as well
  - May make our analysis less precise (but still sound)
  - Makes the analysis feasible; we'll see how later

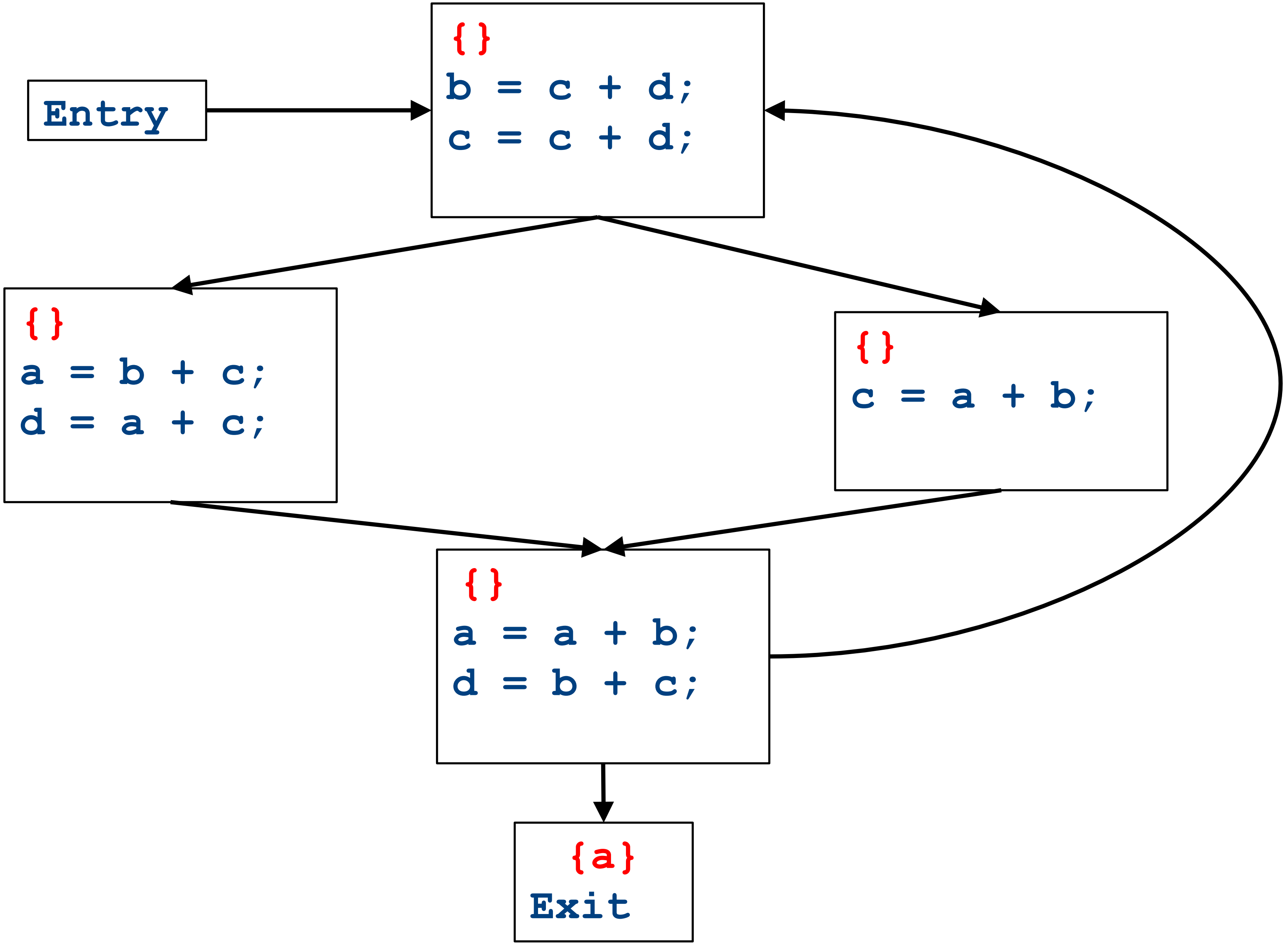
CFGs with loops



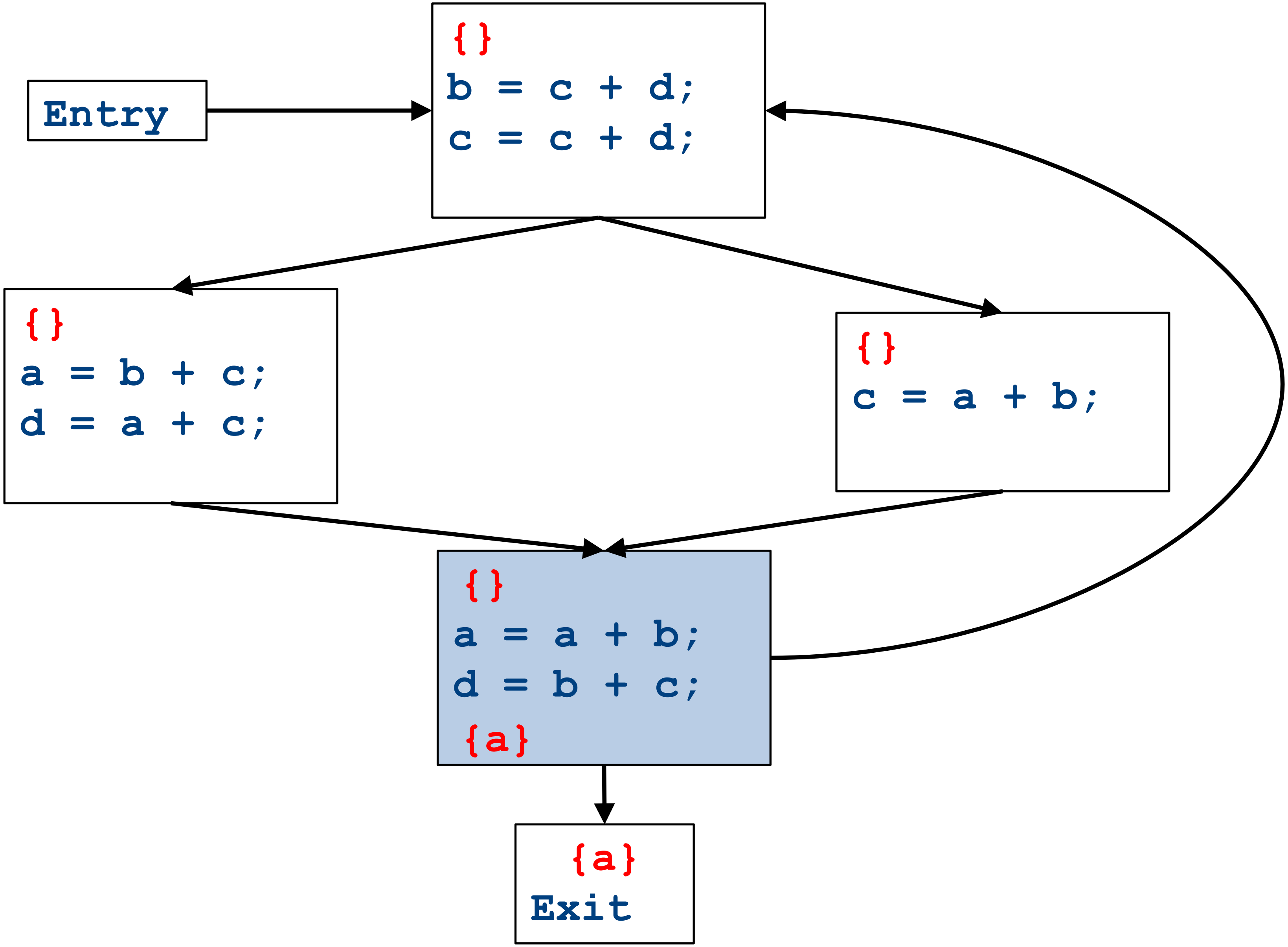
Major changes – part 3

- In a local analysis, there is always a well defined “first” statement to begin processing
- In a global analysis with loops, every basic block might depend on every other basic block
- To fix this, we need to assign initial values to all of the blocks in the CFG

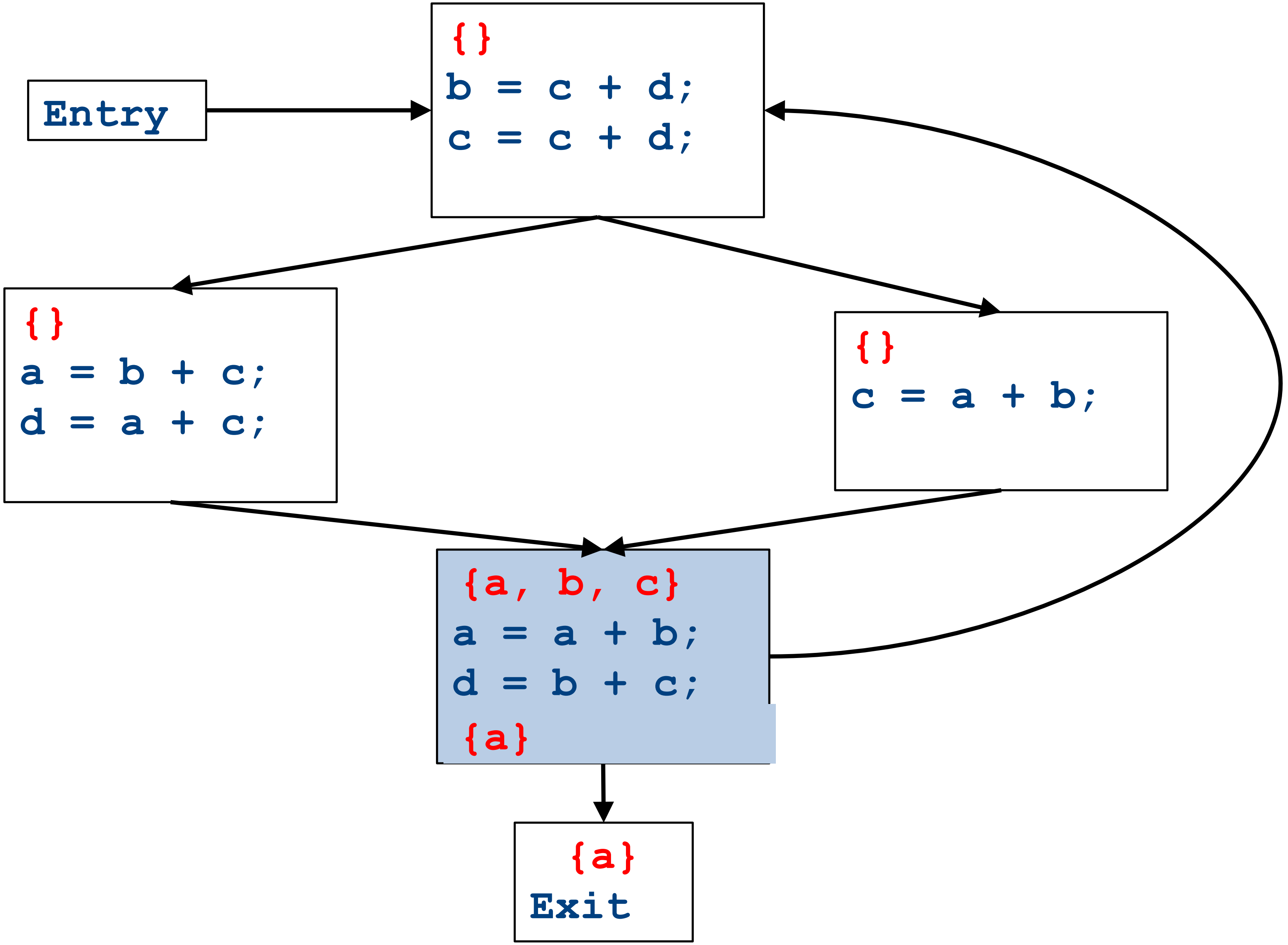
CFGs with loops - initialization



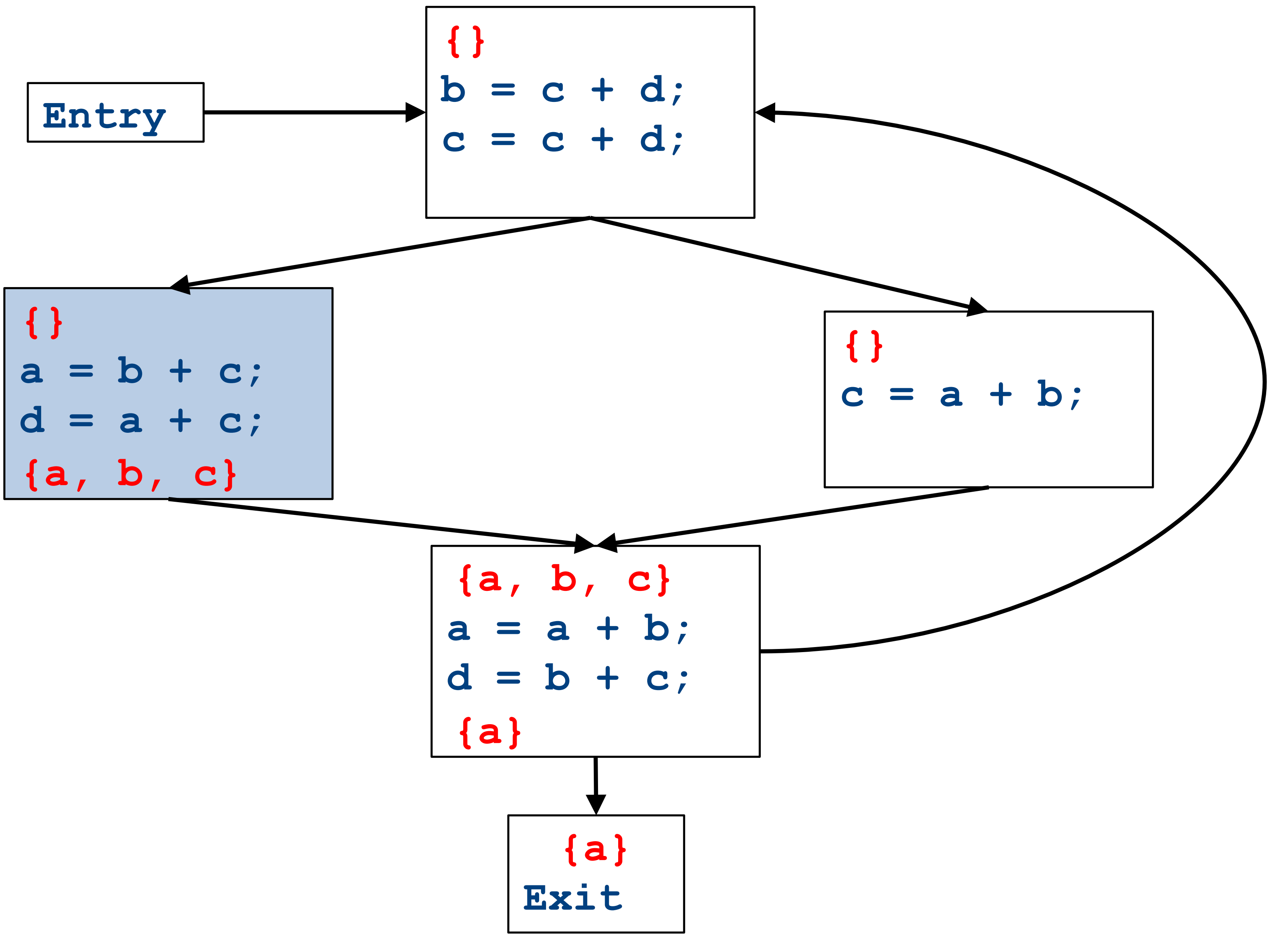
CFGs with loops - iteration



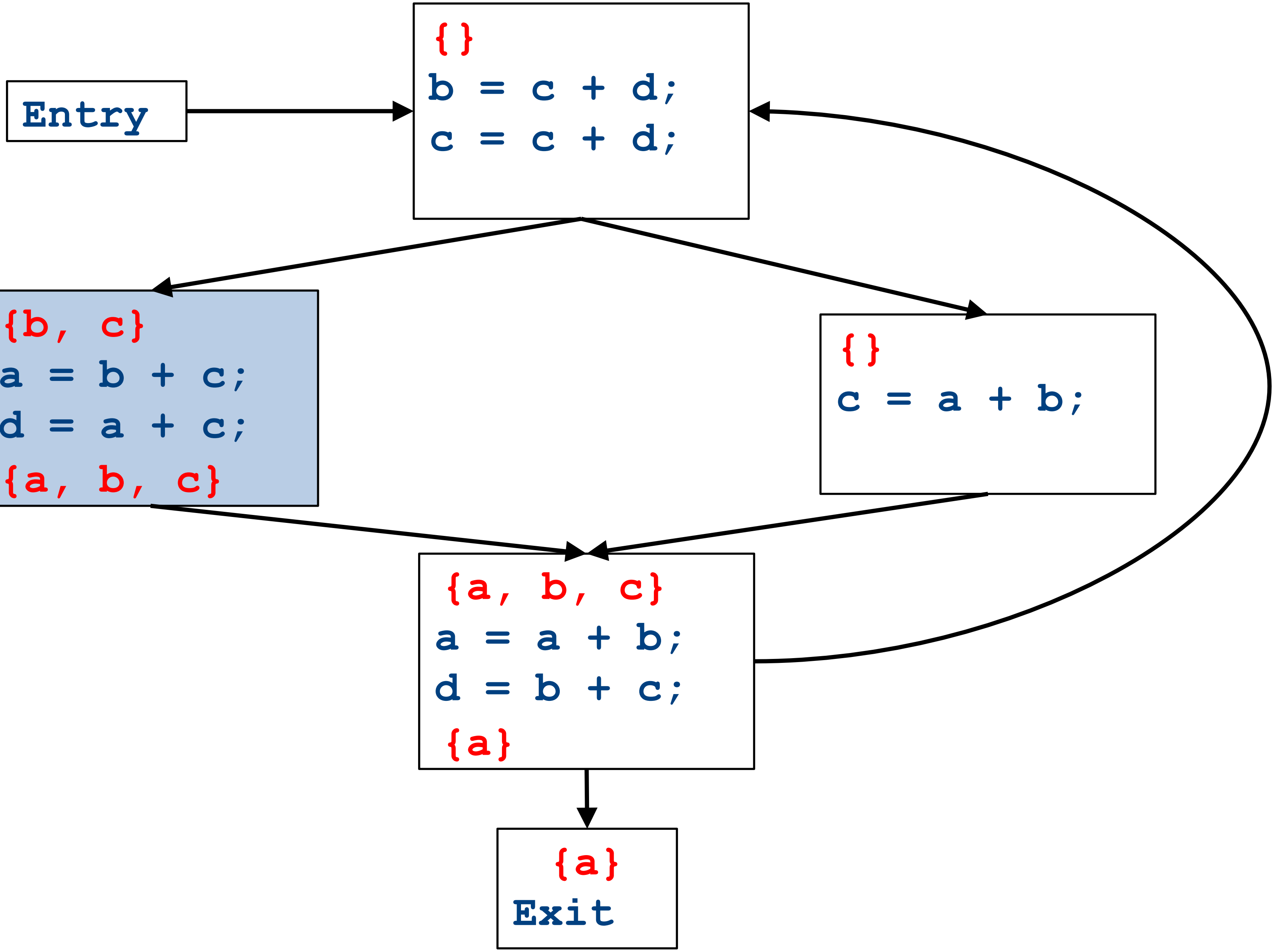
CFGs with loops - iteration



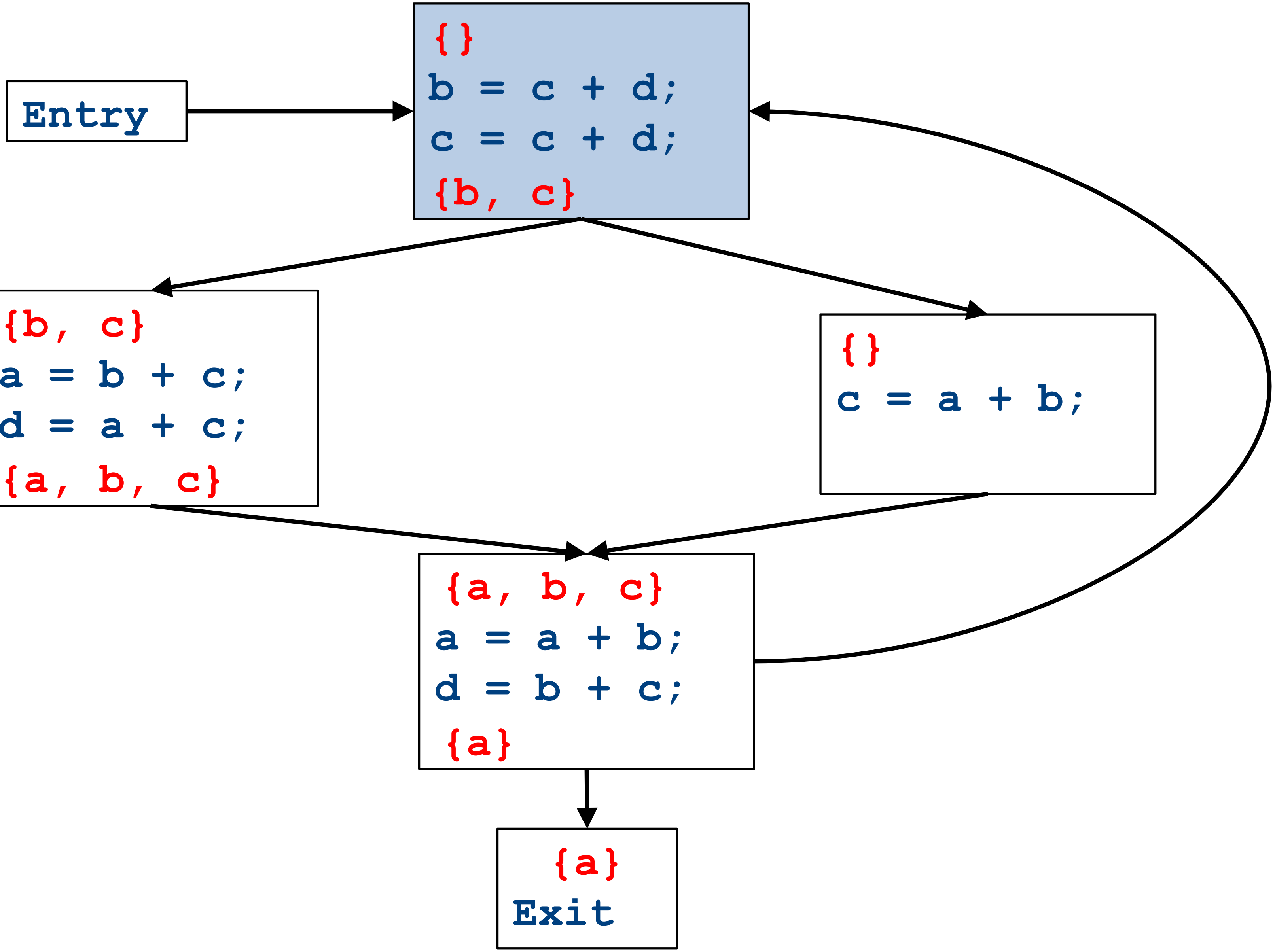
CFGs with loops - iteration



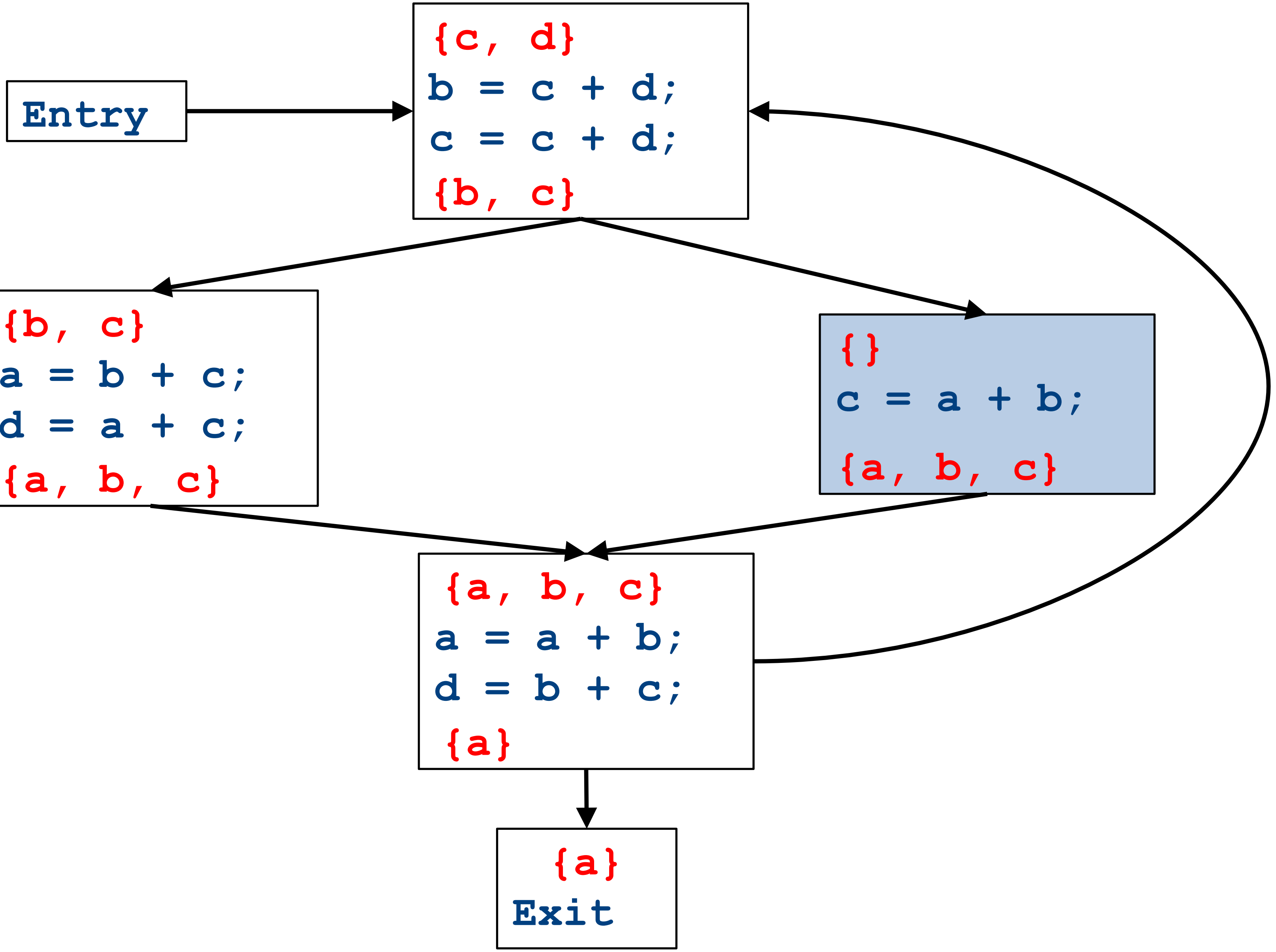
CFGs with loops - iteration



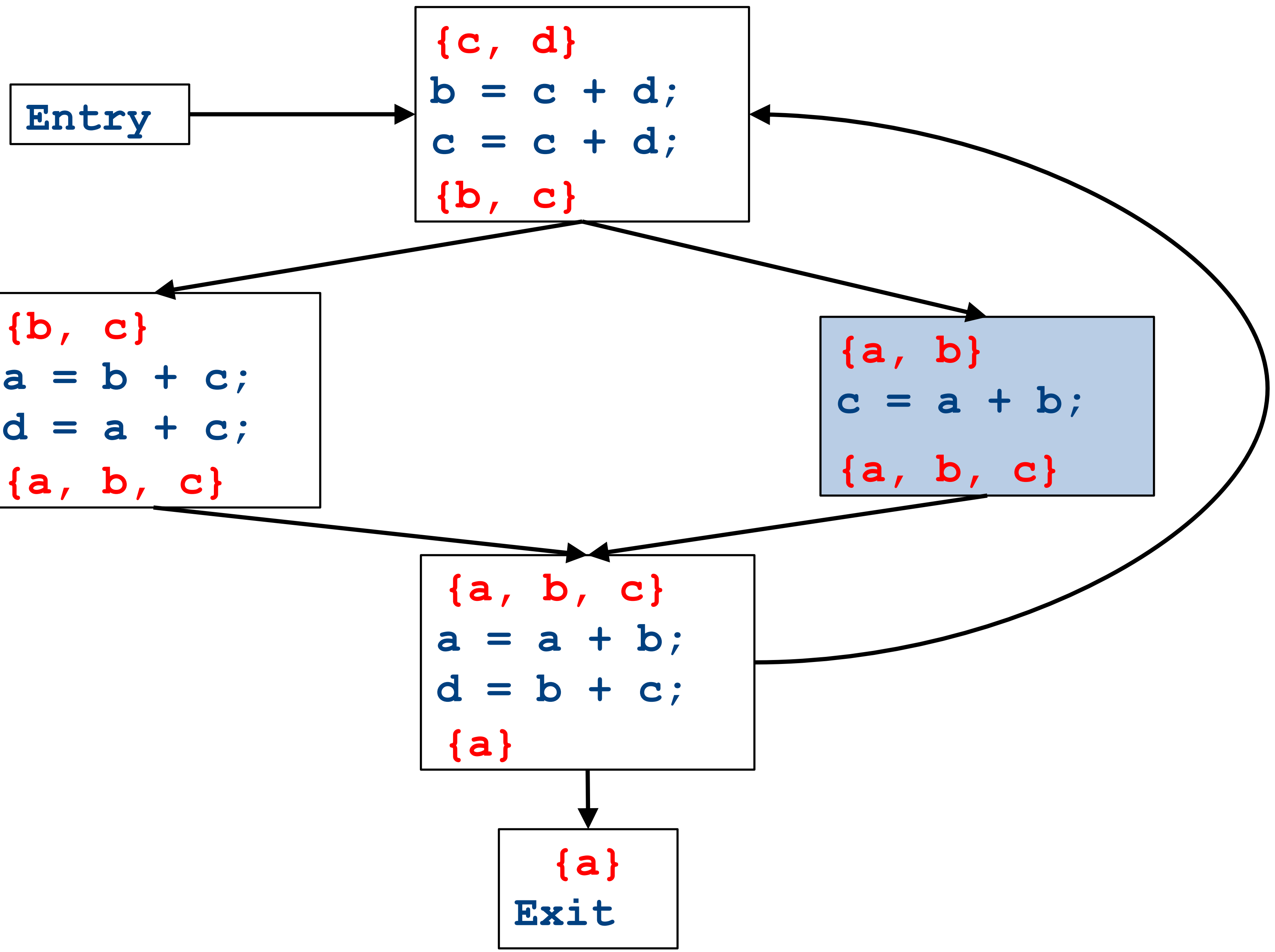
CFGs with loops - iteration



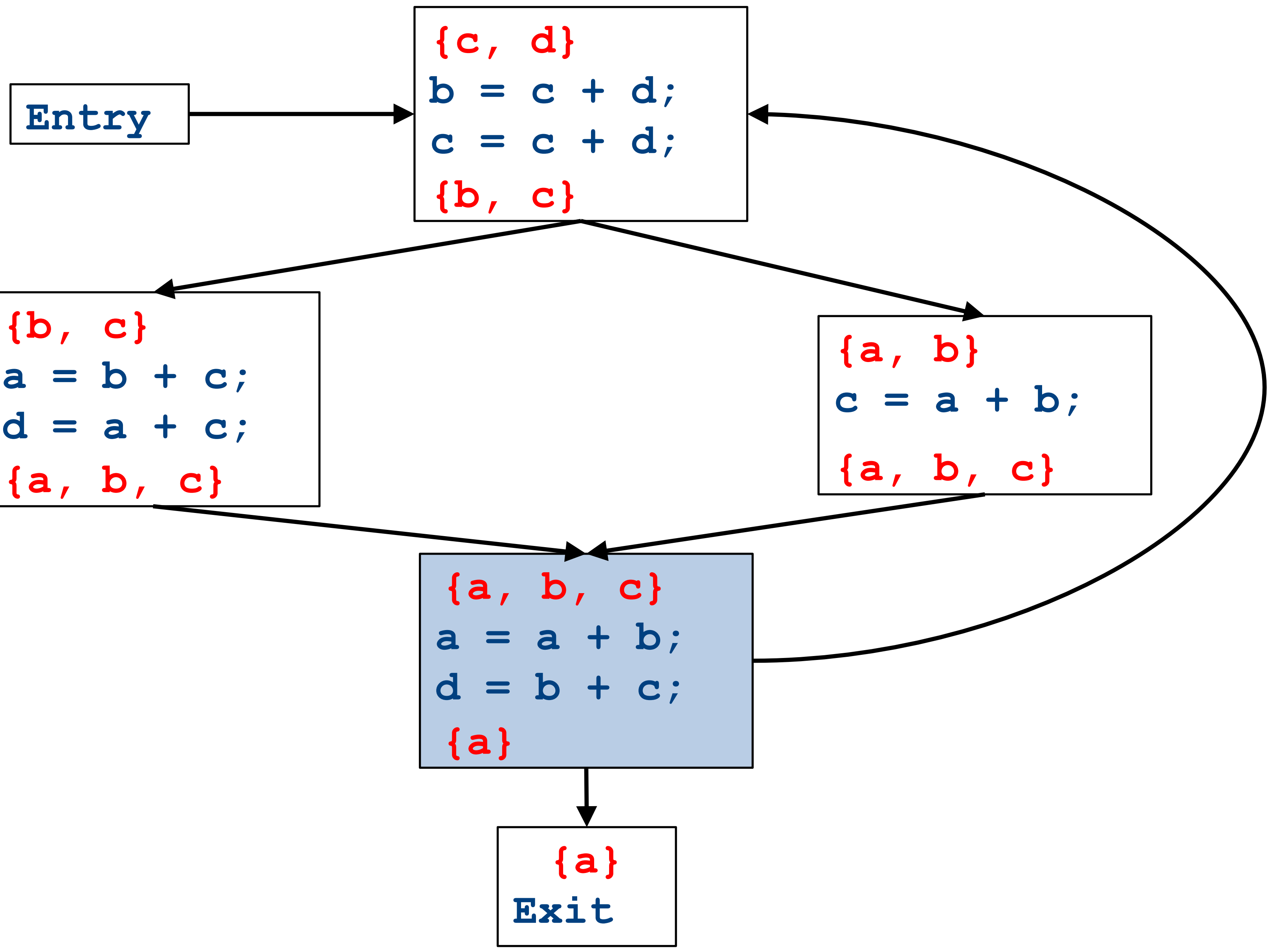
CFGs with loops - iteration



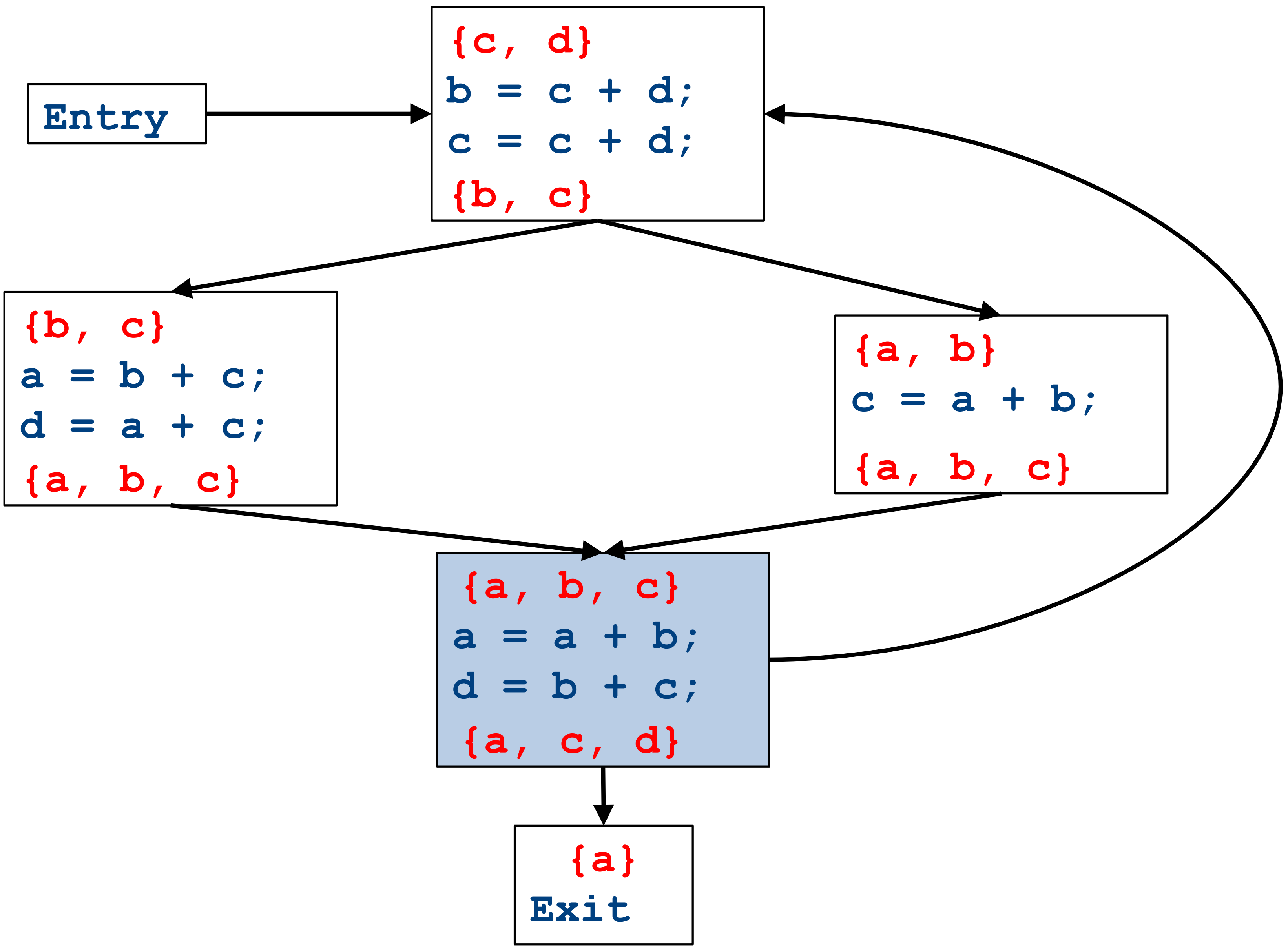
CFGs with loops - iteration



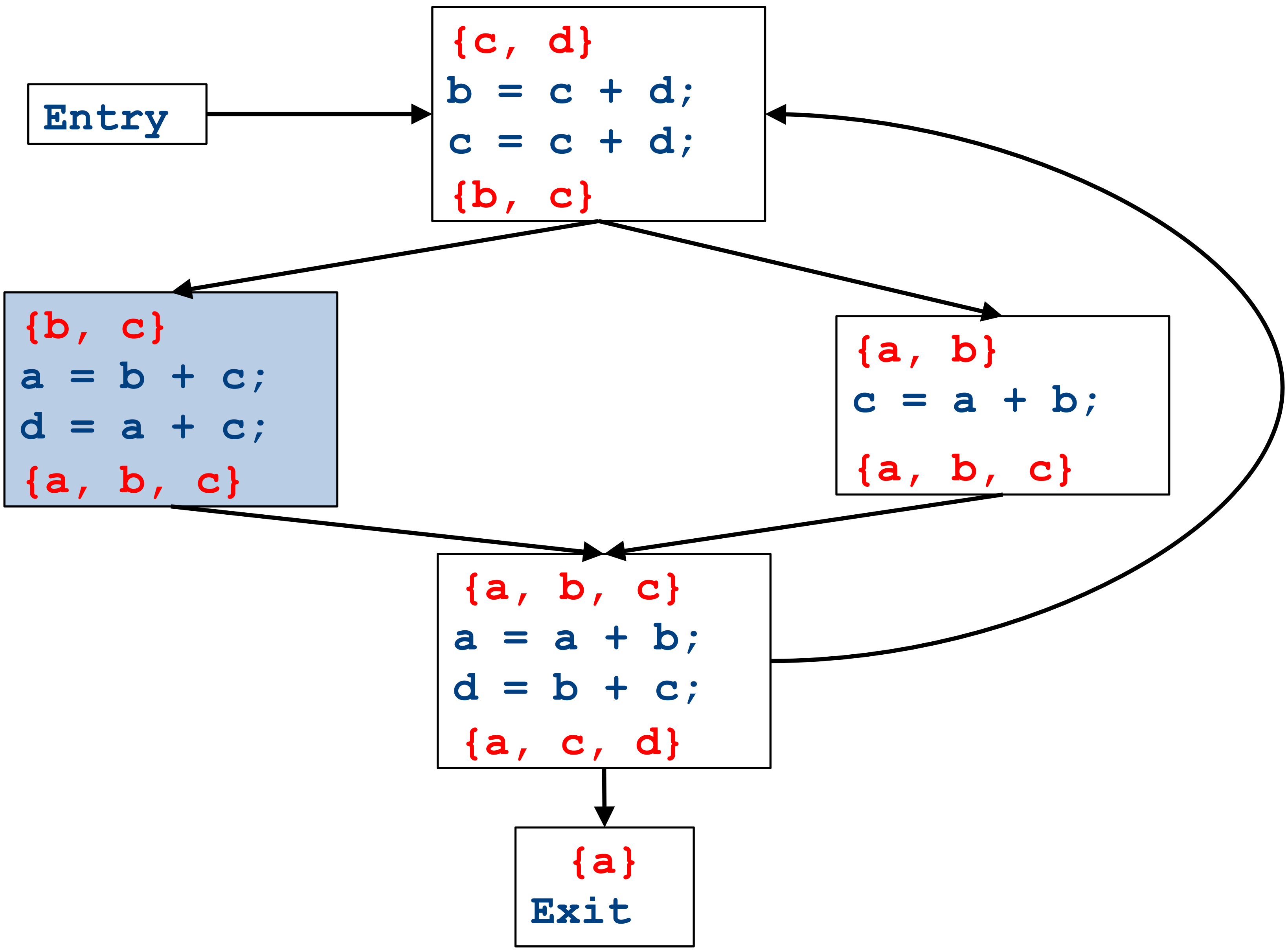
CFGs with loops - iteration



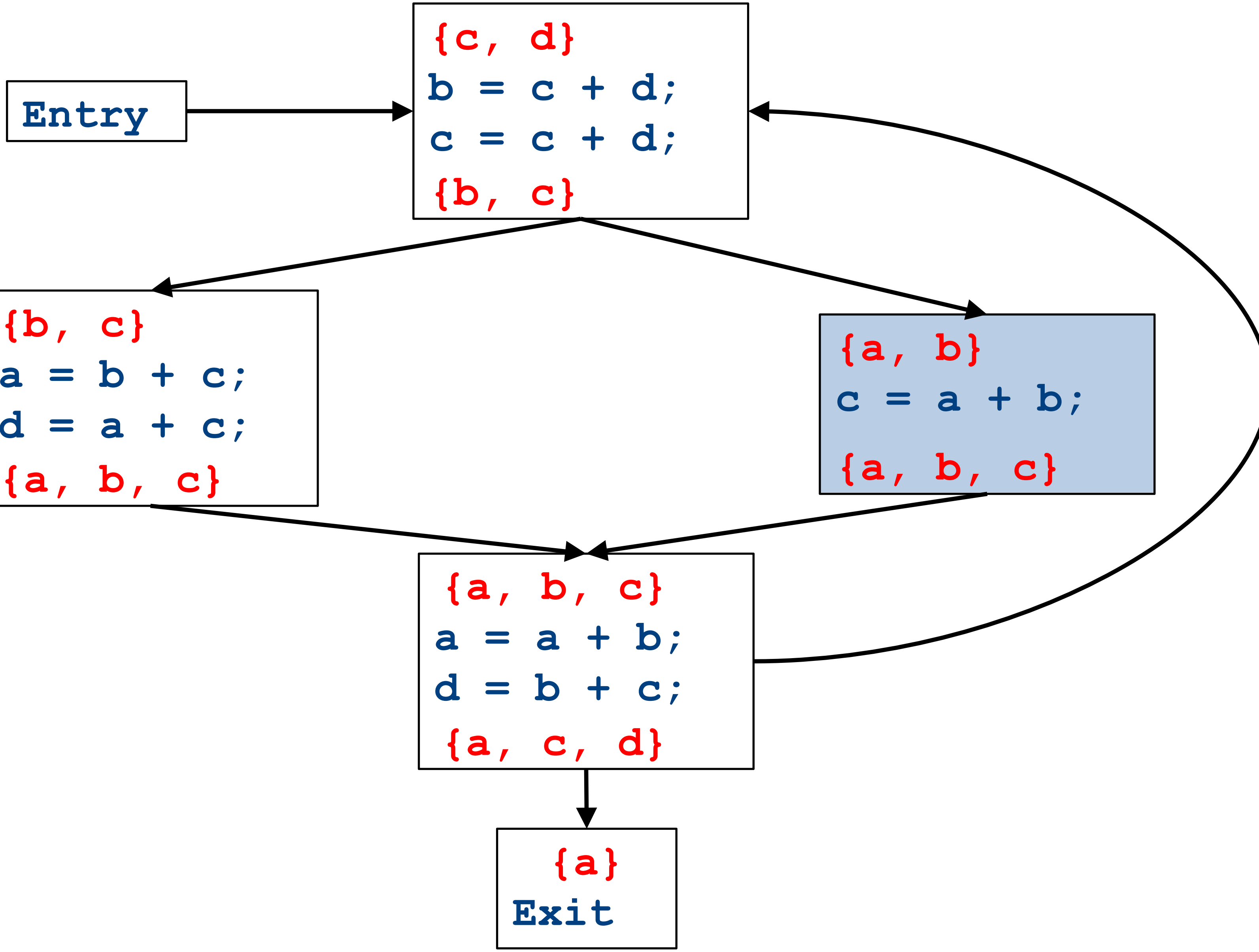
CFGs with loops - iteration



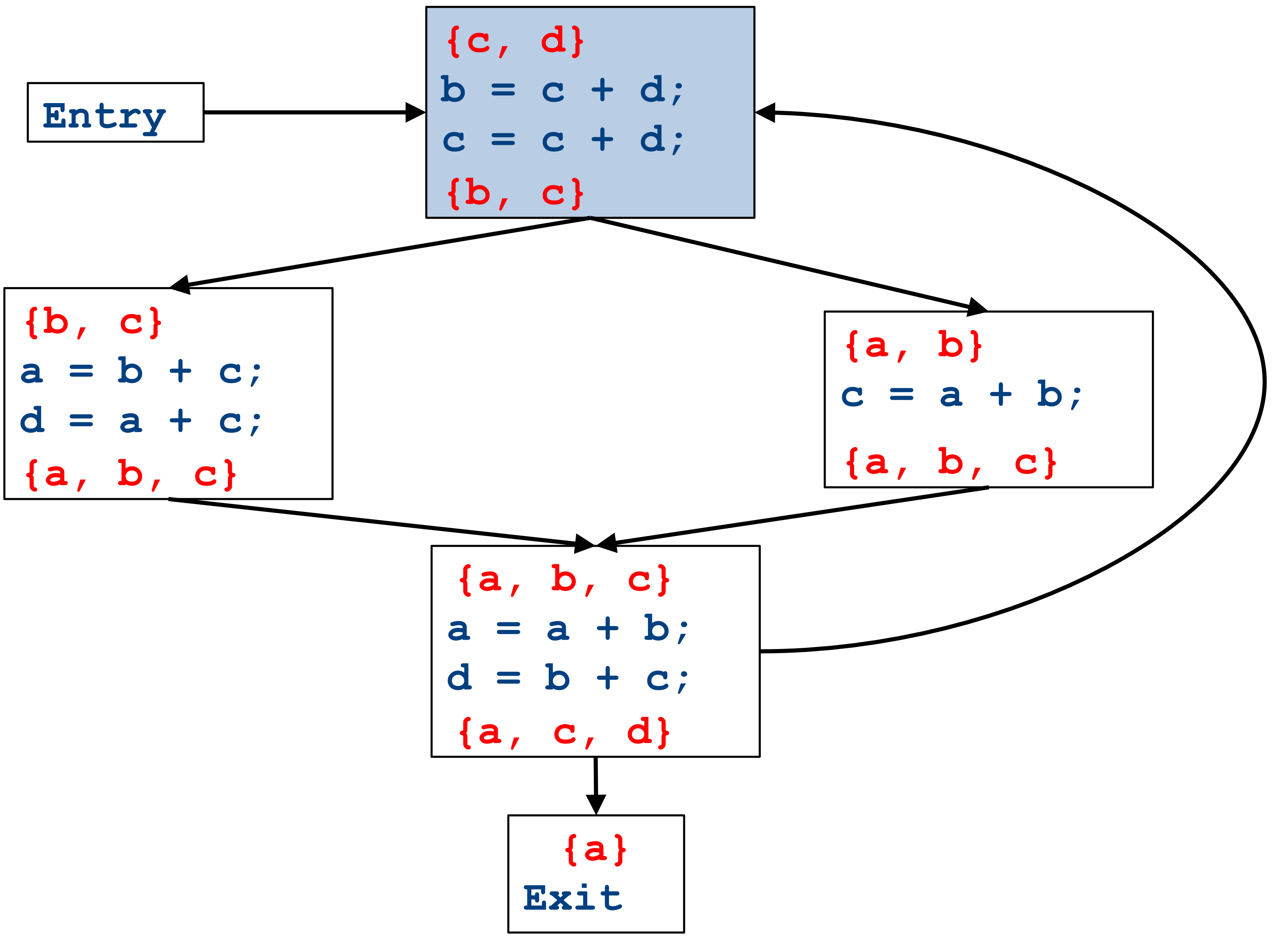
CFGs with loops - iteration



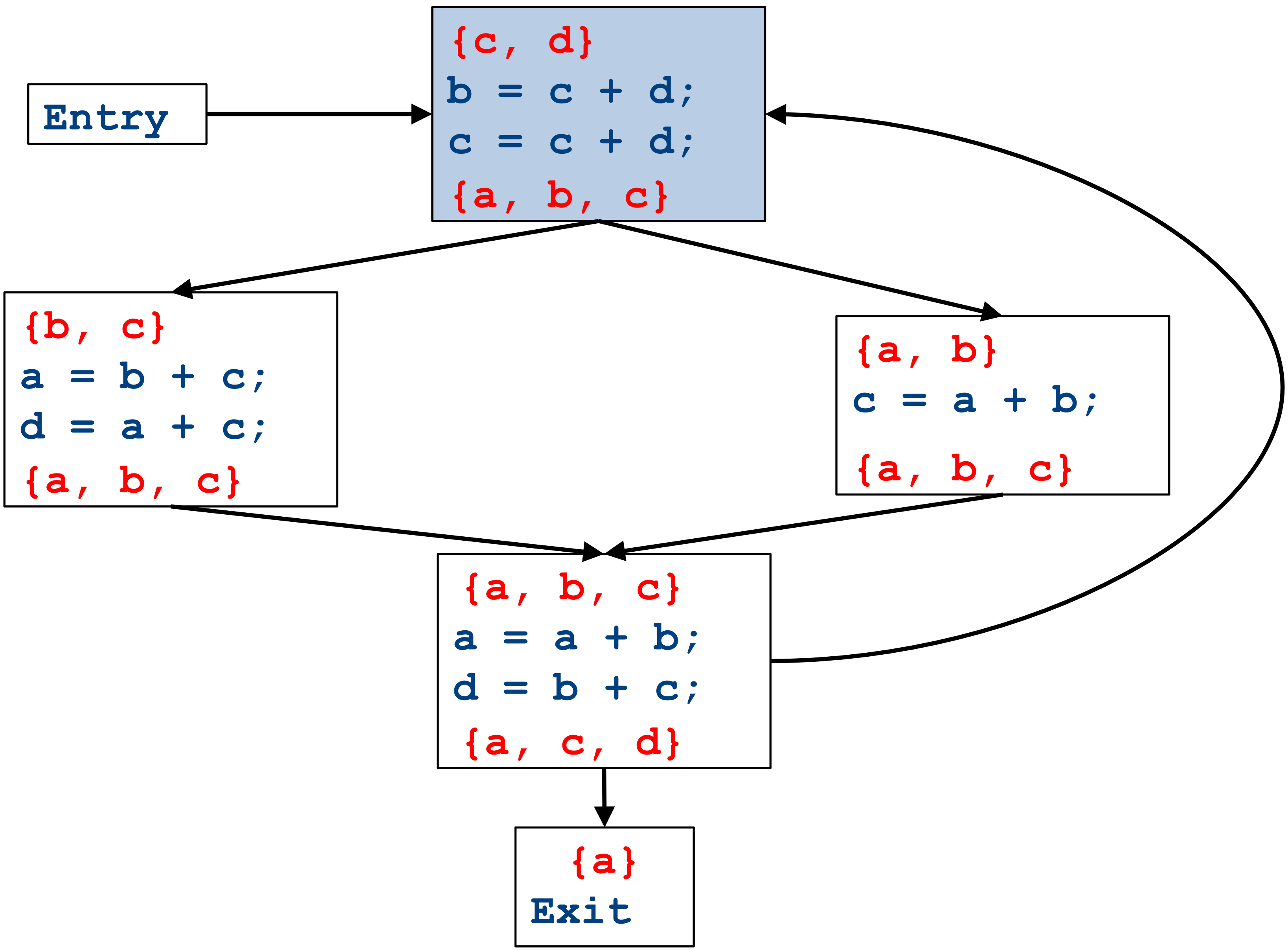
CFGs with loops - iteration



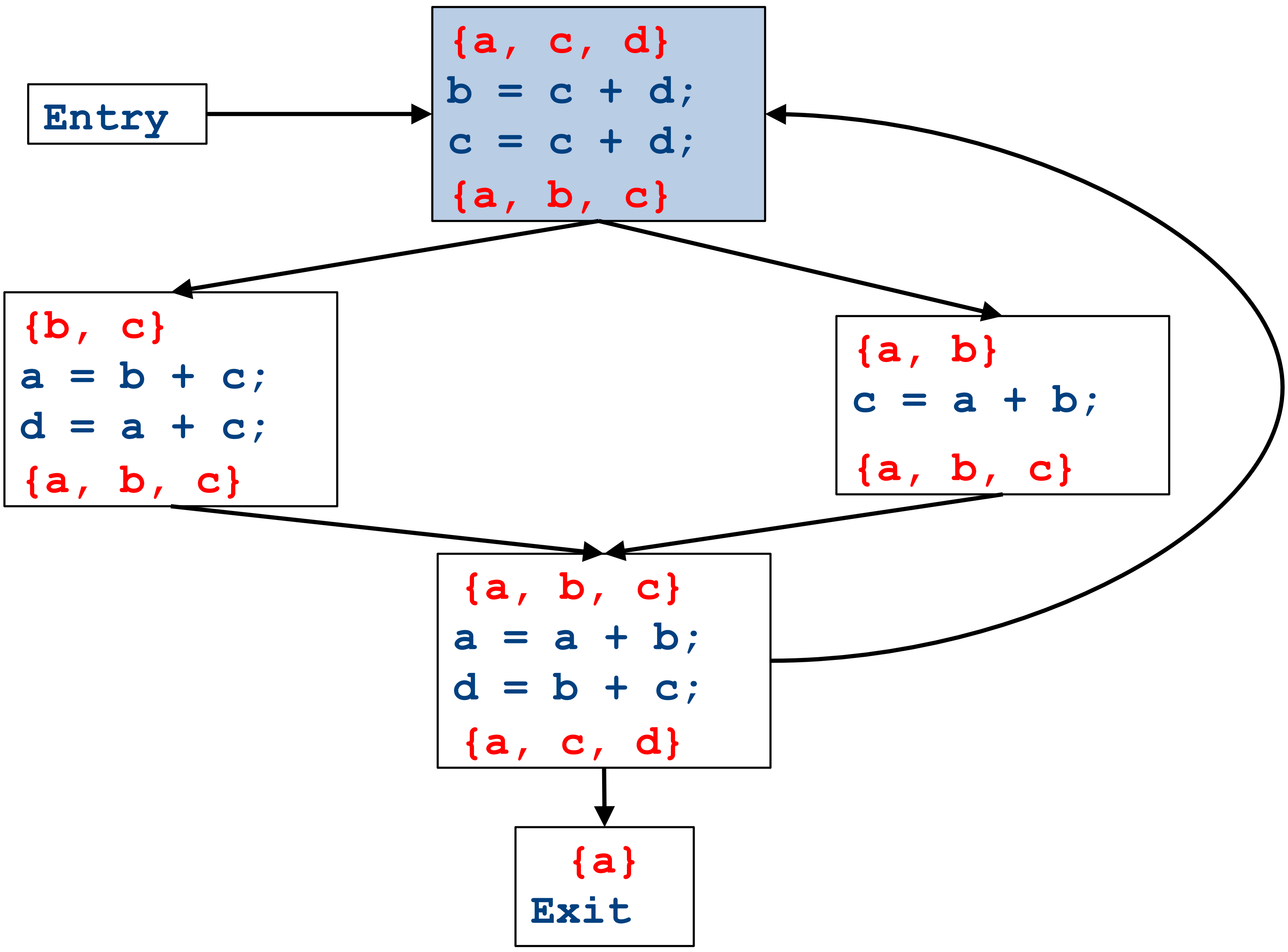
CFGs with loops - iteration



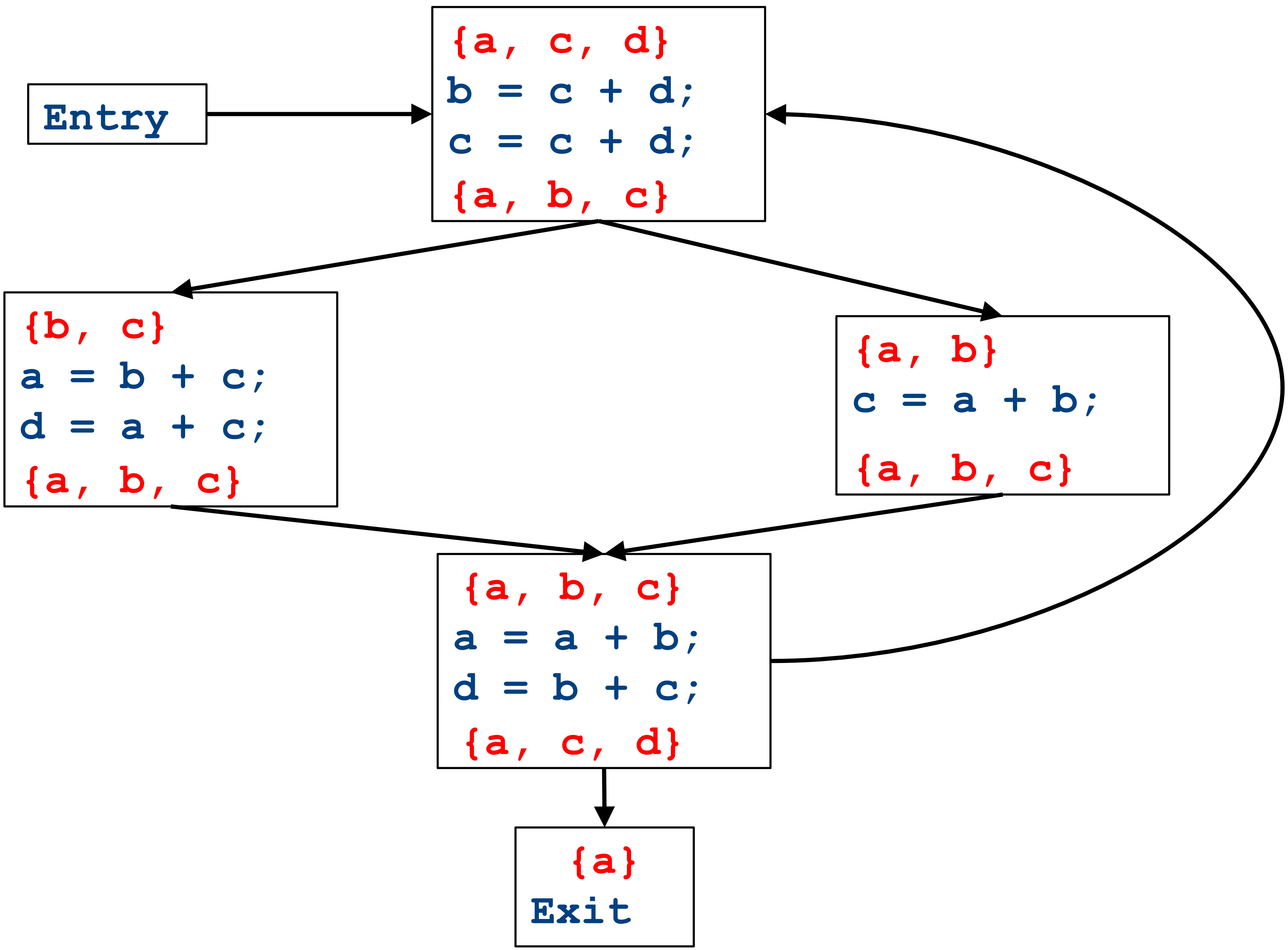
CFGs with loops - iteration



CFGs with loops - iteration



CFGs with loops - iteration



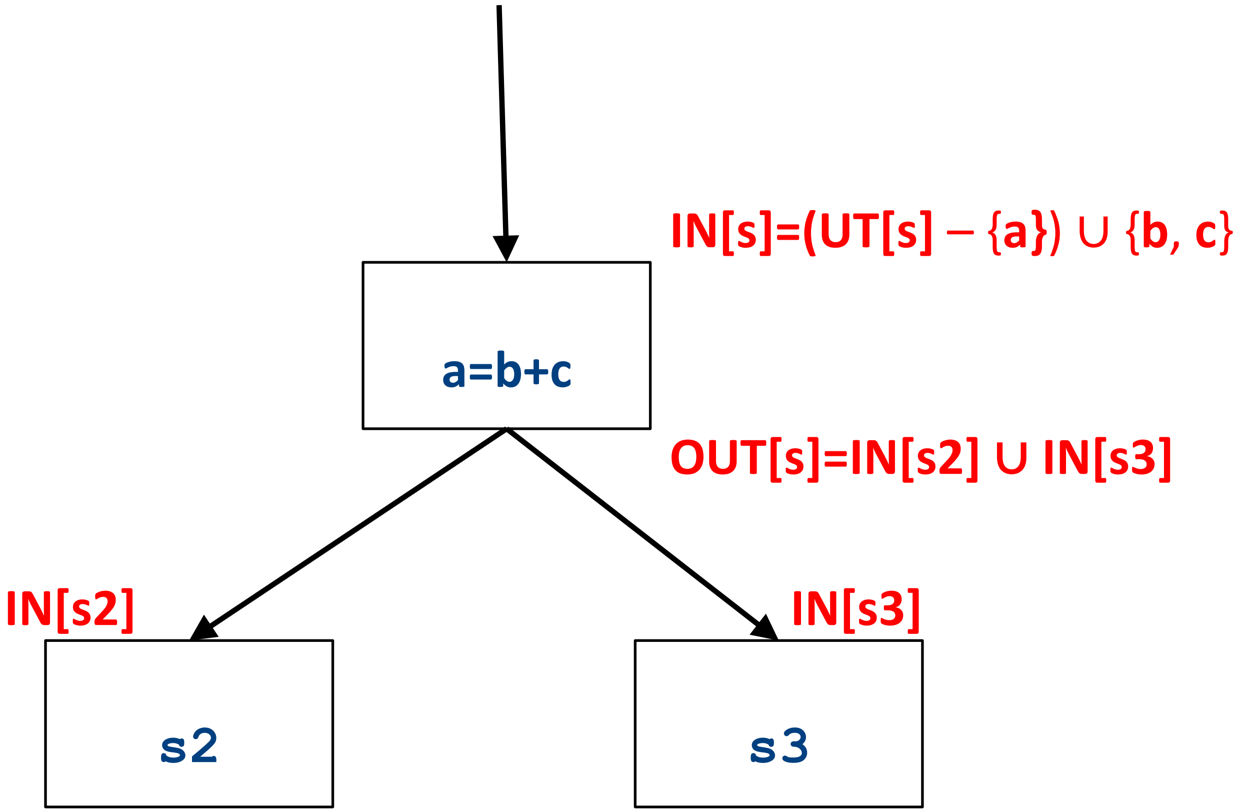
Summary of differences

- Need to be able to handle multiple predecessors/successors for a basic block
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value
  - But the analysis still needs to terminate!
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it

Global liveness analysis

- Initially, set IN[**s**] = { } for each statement **s**
- Set IN[**exit**] to the set of variables known to be live on exit (language-specific knowledge)
- Repeat until no changes occur:
  - For each statement **s** of the form **a = b + c**, in any order you'd like:
    - Set OUT[**s**] to set union of IN[**p**] for each successor **p** of **s**
    - Set IN[**s**] to (OUT[**s**] – **a**) ∪ {**b**, **c**}.
- Yet another fixed-point iteration!

Global liveness analysis



Why does this work?

- To show correctness, we need to show that
  - The algorithm eventually terminates, and
  - When it terminates, it has a sound answer
- Termination argument:
  - Once a variable is discovered to be live during some point of the analysis, it always stays live
  - Only finitely many variables and finitely many places where a variable can become live
- Soundness argument (sketch):
  - Each individual rule, applied to some set, correctly updates liveness in that set
  - When computing the union of the set of live variables, a variable is only live if it was live on some path leaving the statement

Abstract Interpretation

- Theoretical foundations of program analysis

- Cousot and Cousot 1977

- Abstract meaning of programs
  - Executed at compile time

Another view of local optimization

- In local optimization, we want to reason about some property of the runtime behavior of the program
- Could we run the program and just watch what happens?
- **Idea:** Redefine the semantics of our programming language to give us information about our analysis

Properties of local analysis

- The only way to find out what a program will actually do is to run it
- Problems:
  - The program might not terminate
  - The program might have some behavior we didn't see when we ran it on a particular input
- However, this is not a problem inside a basic block
  - Basic blocks contain no loops
  - There is only one path through the basic block

Assigning new semantics

- Example: Available Expressions
- Redefine the statement **a = b + c to** mean "**a now holds the value of b + c,** and any variable holding the value **a is** now invalid"
- Run the program assuming these new semantics
- Treat the optimizer as an interpreter for these new semantics

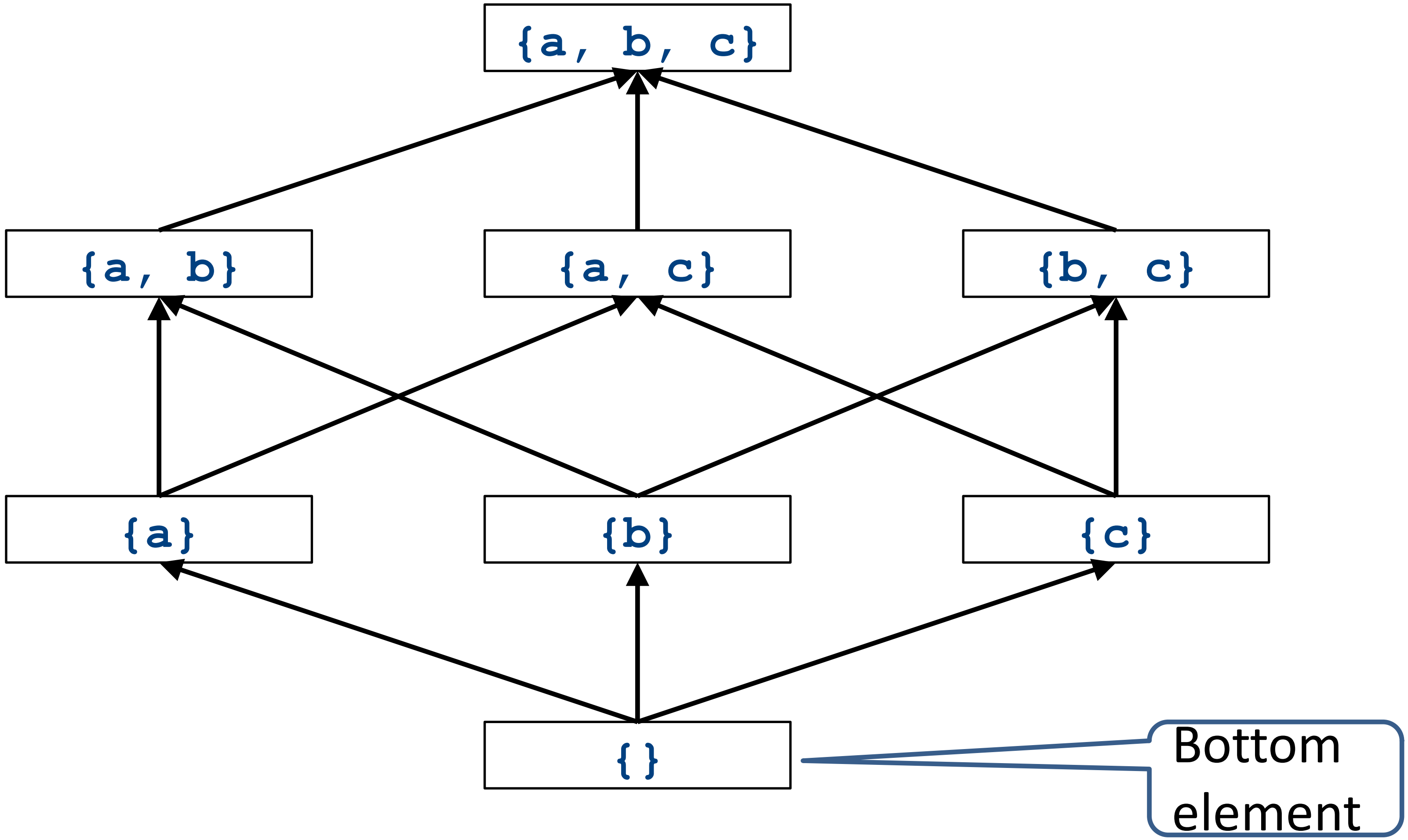
Theory to the rescue

- Building up all of the machinery to design this analysis was tricky
- The key ideas, however, are mostly independent of the analysis:
  - We need to be able to compute functions describing the behavior of each statement
  - We need to be able to merge several subcomputations together
  - We need an initial value for all of the basic blocks
- There is a beautiful formalism that captures many of these properties

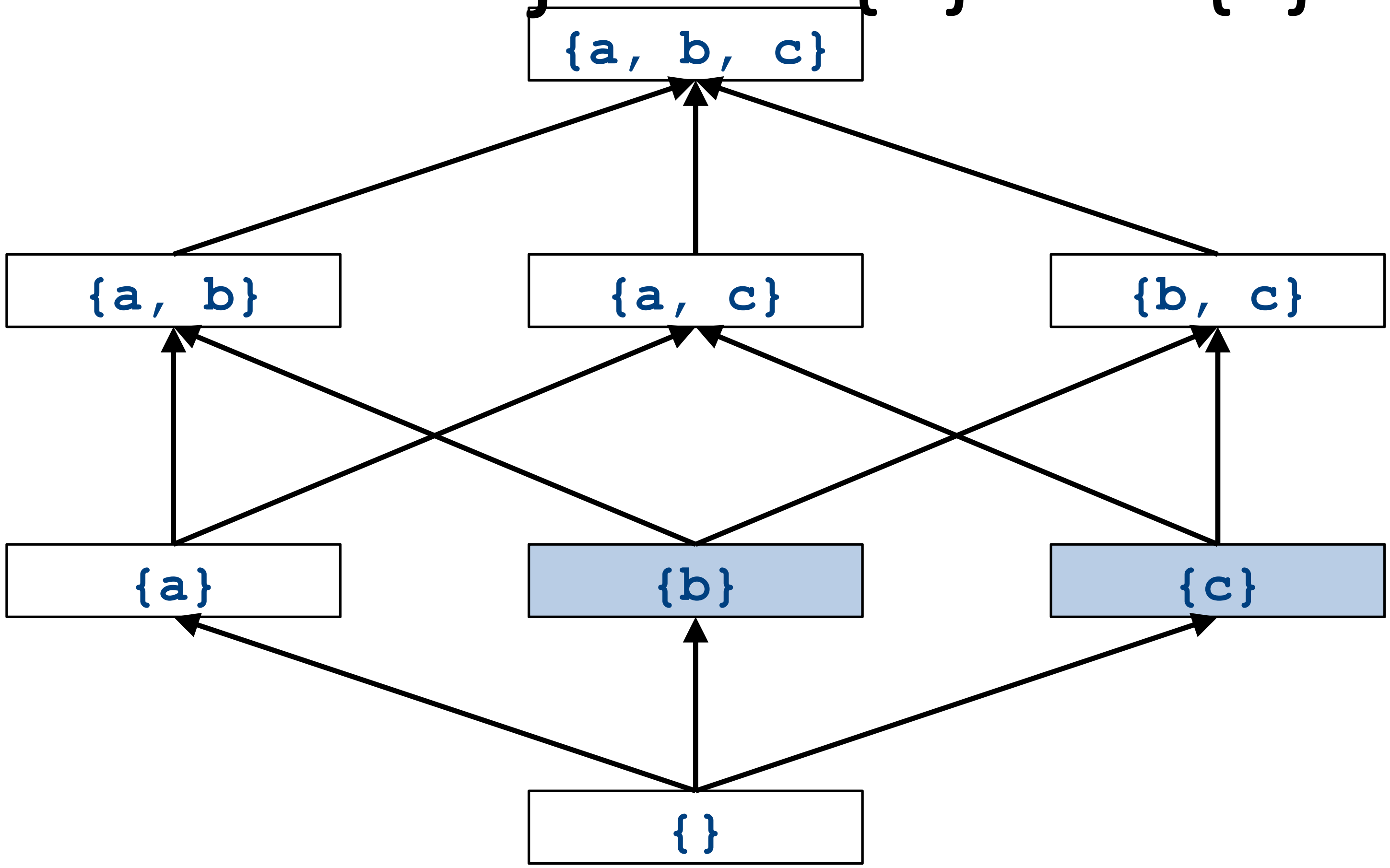
Join semilattices

- A join semilattice is a ordering defined on a set of elements
- Any two elements have some join that is the smallest element larger than both elements
- There is a unique bottom element, which is smaller than all other elements
- Intuitively:
  - The join of two elements represents combining information from two elements by an overapproximation
- The bottom element represents “no information yet” or “the least conservative possible answer”

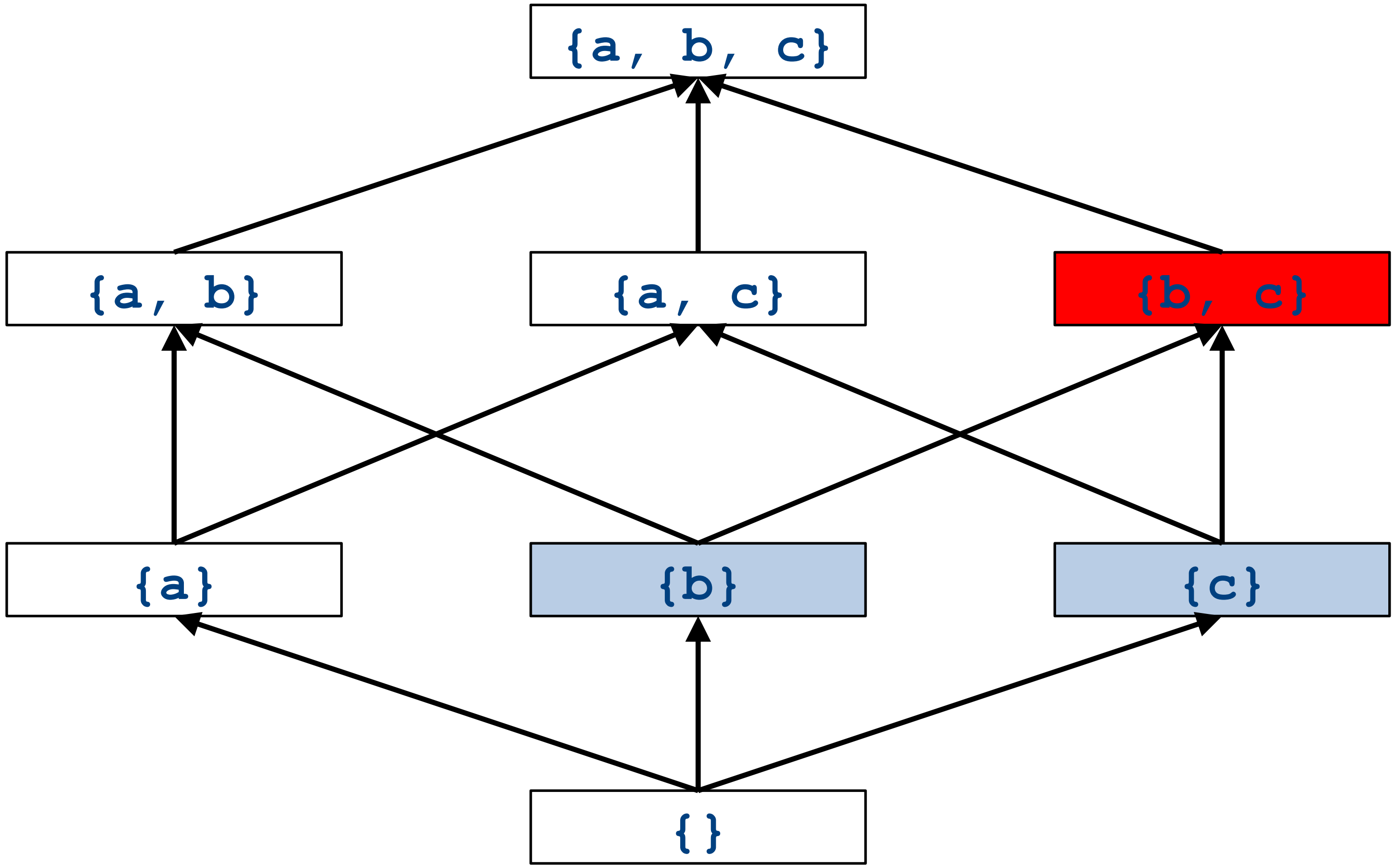
Join semilattice for liveness



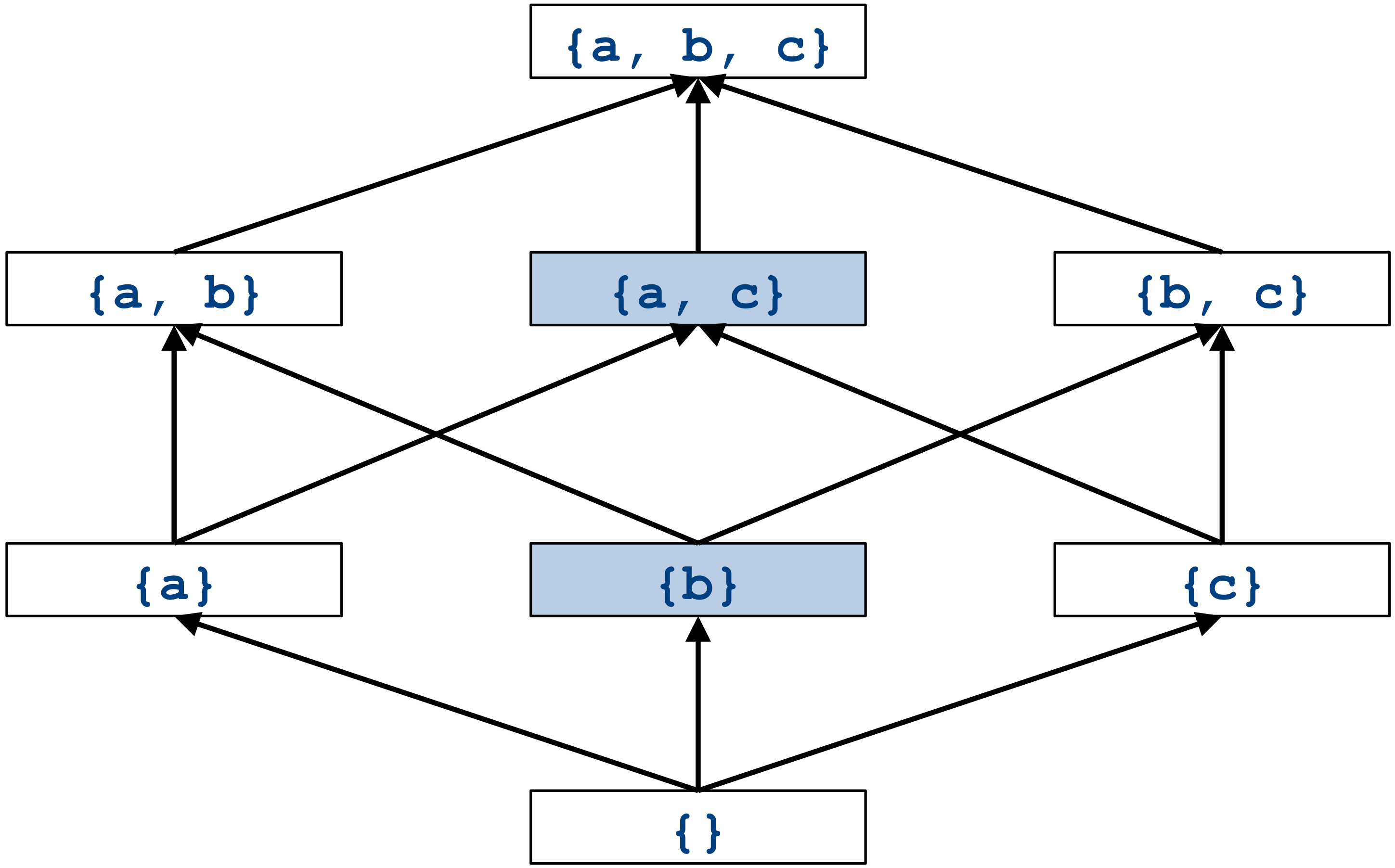
What is the join of {b} and {c}?



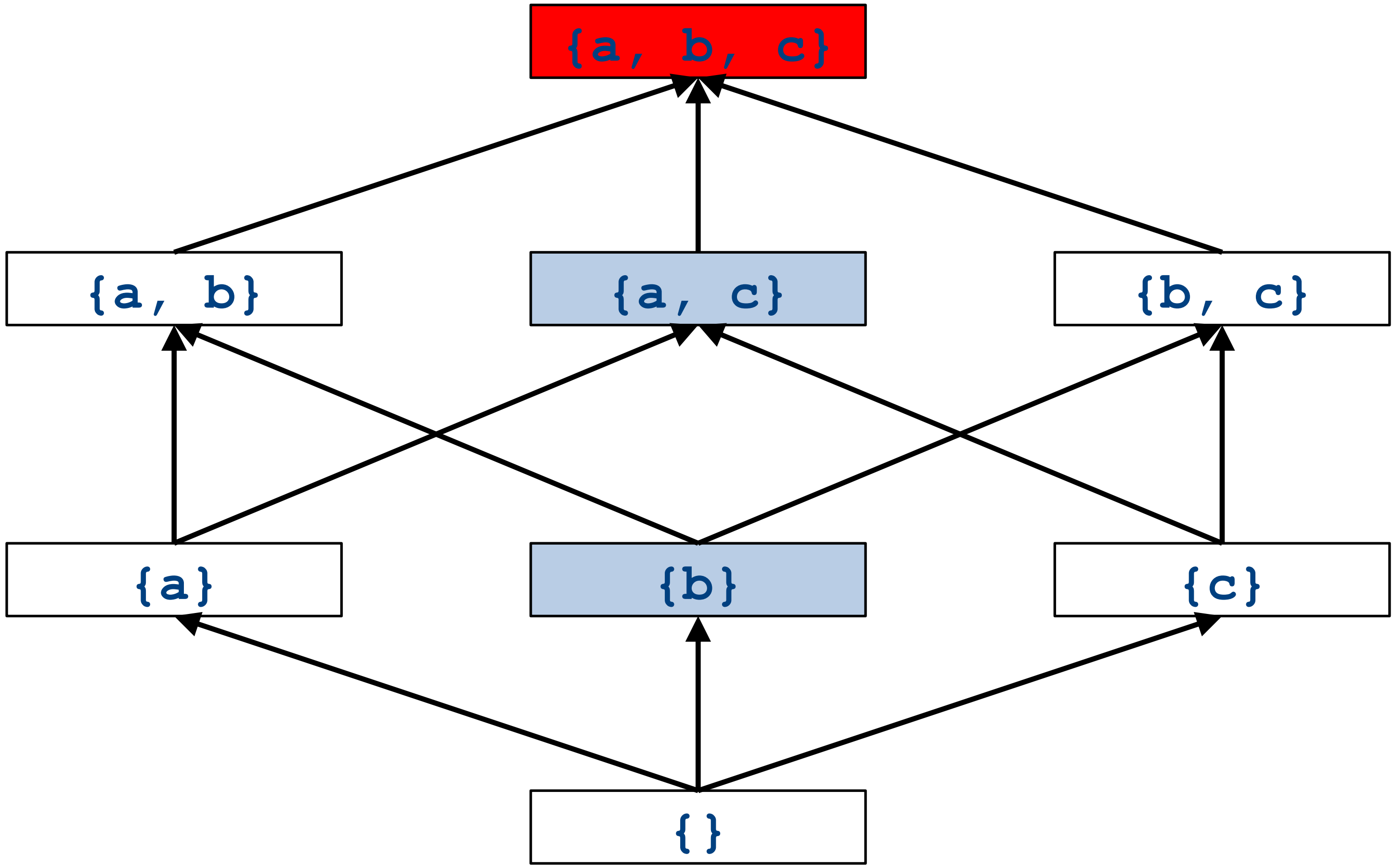
What is the join of {b} and {c}?



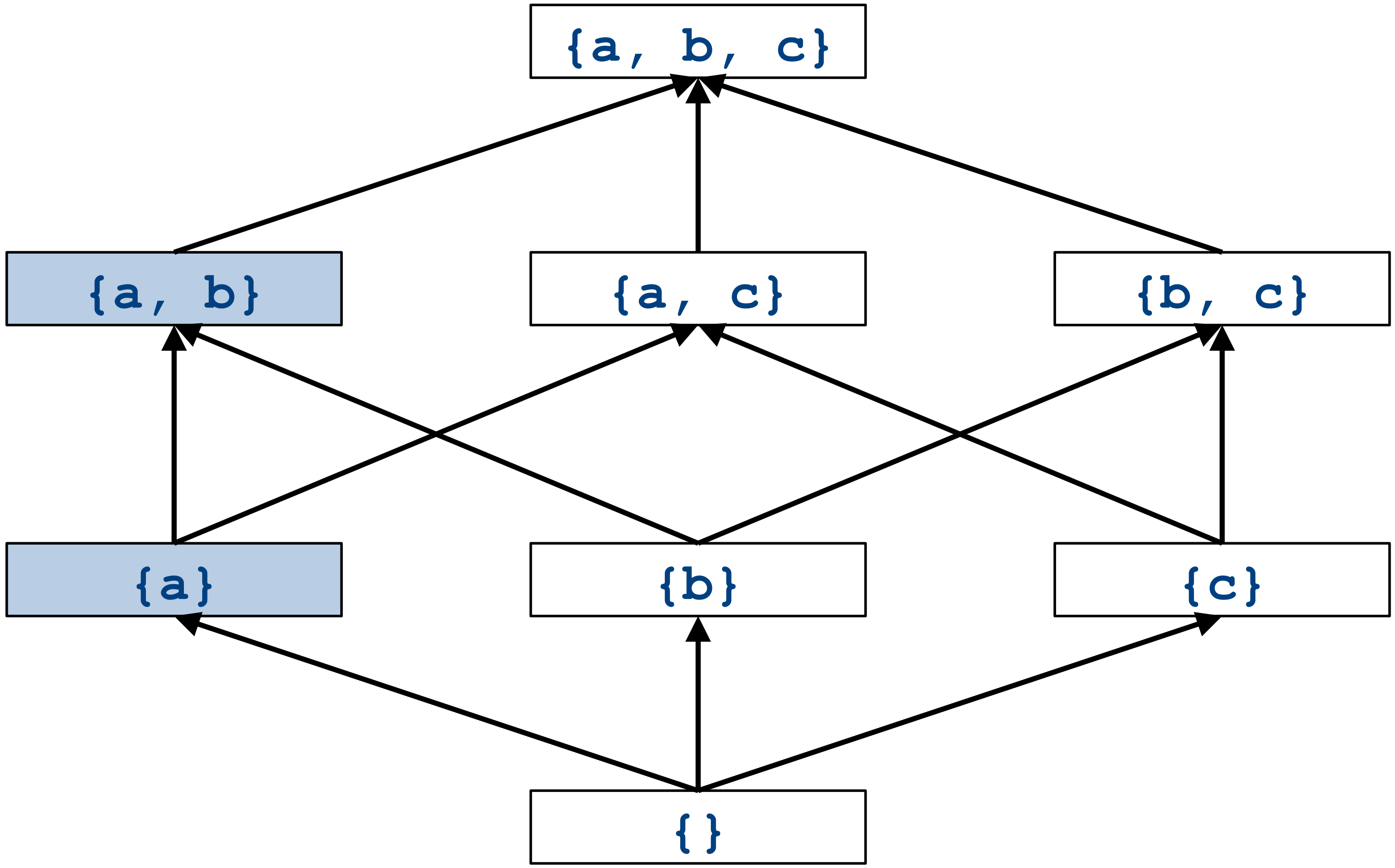
What is the join of {b} and {a,c}?



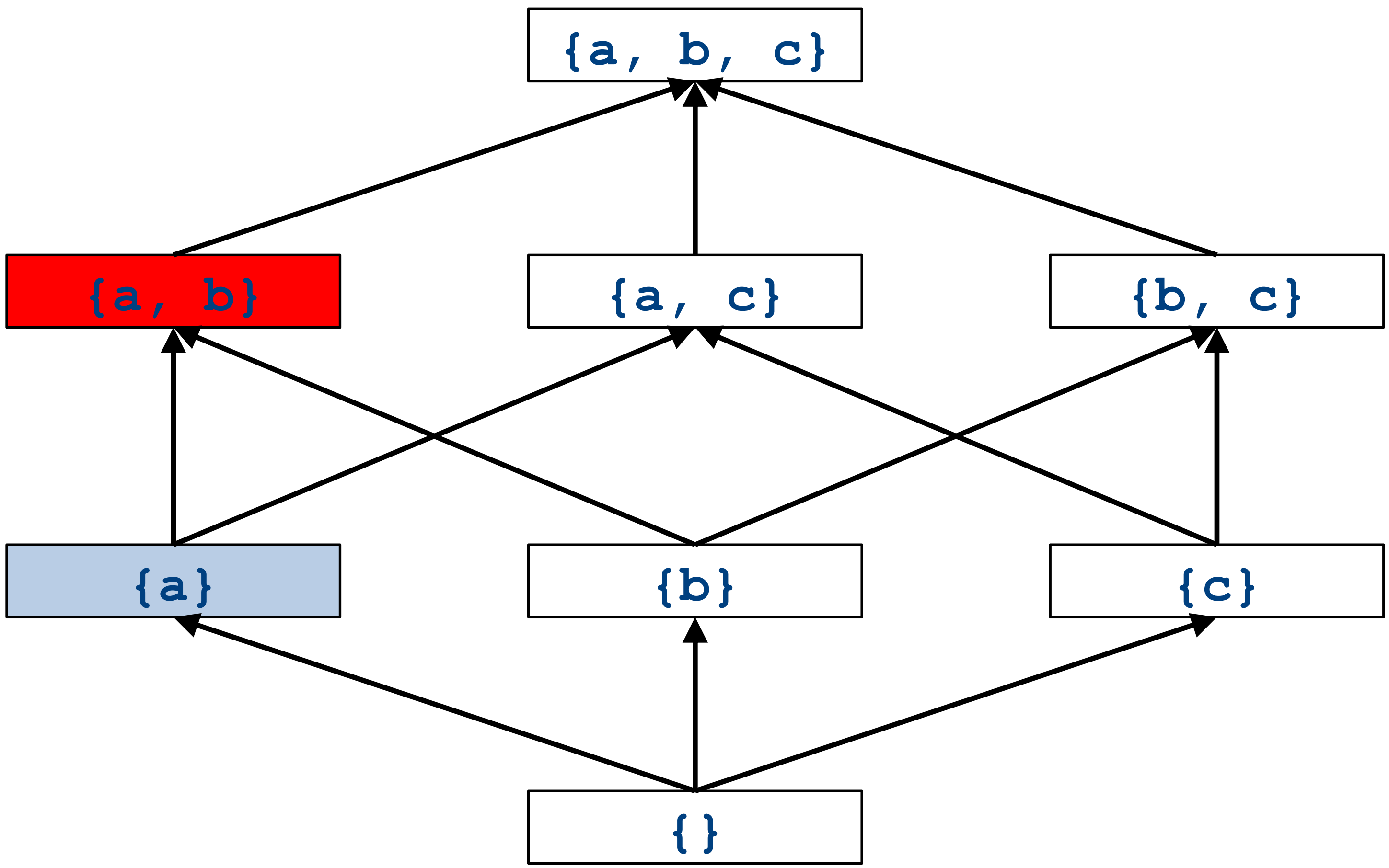
What is the join of {b} and {a,c}?



What is the join of {a} and {a,b}?



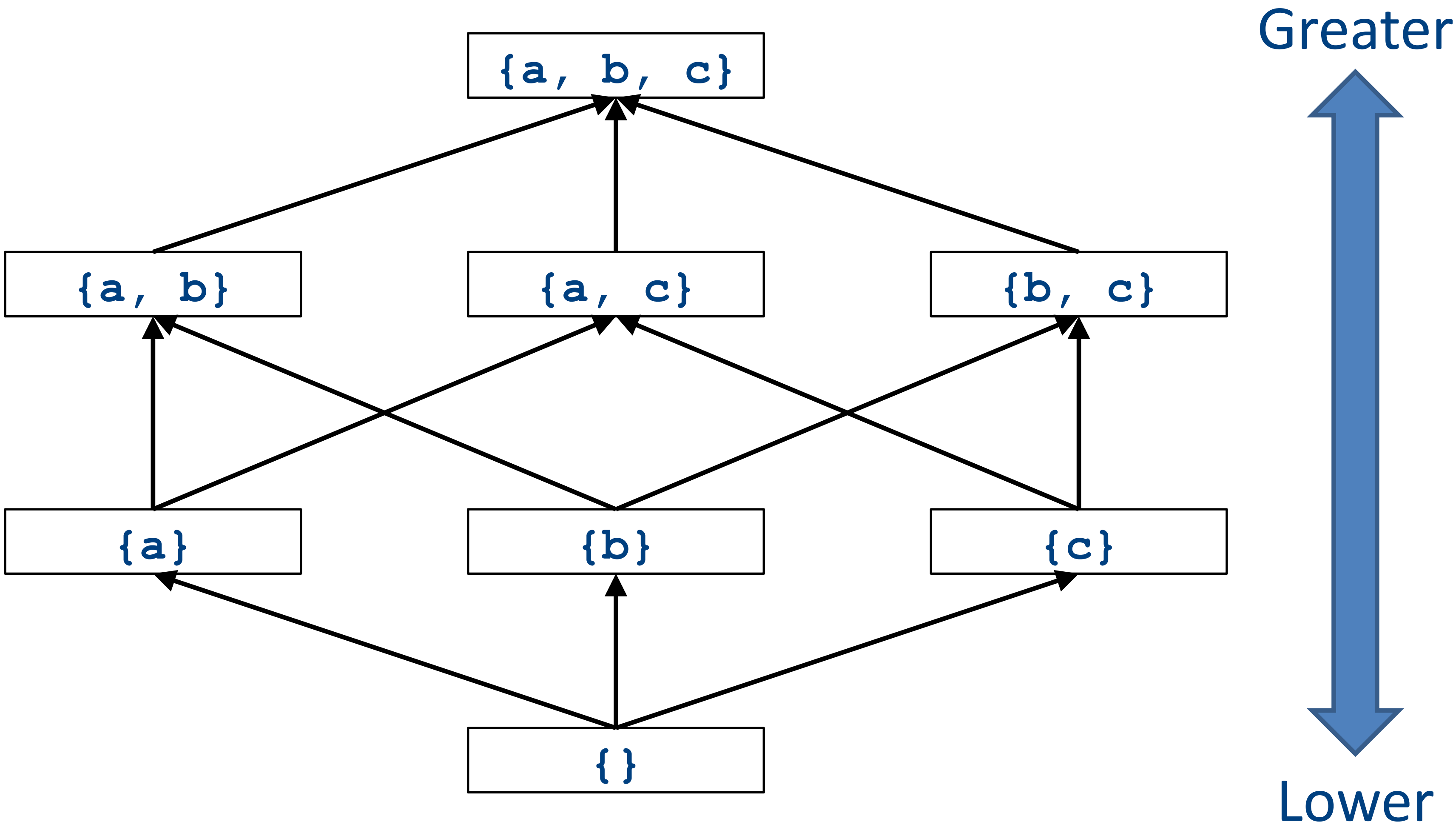
What is the join of {a} and {a,b}?



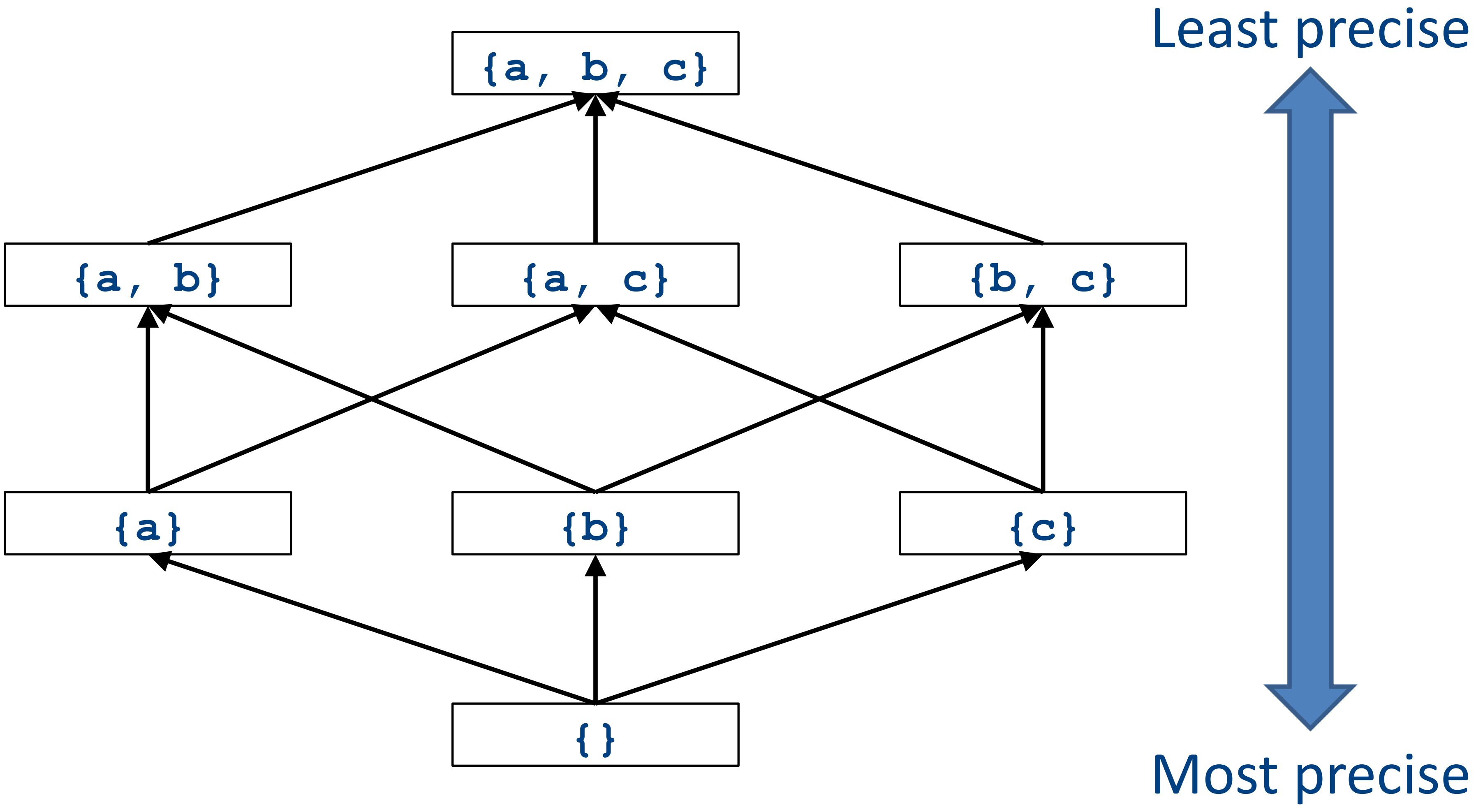
Formal definitions

- A join semilattice is a pair (V, ⊔), where
- V is a domain of elements
- ⊔ is a join operator that is
  - commutative: $x \sqcup y = y \sqcup x$
  - associative: $(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)$
  - idempotent: $x \sqcup x = x$
- If $x \sqcup y = z$, we say that z is the join or (**l**east **u**pper **b**ound) of x and y
- Every join semilattice has a bottom element denoted ⊥ such that $\perp \sqcup x = x$ for all x

Join semilattices and ordering



Join semilattices and ordering



Join semilattices and orderings

- Every join semilattice (V, $\sqcup$) induces an ordering relationship $\sqsubseteq$ over its elements
- Define $x \sqsubseteq y$ iff $x \sqcup y = y$
- Need to prove
  - Reflexivity: $x \sqsubseteq x$
  - Antisymmetry: If $x \sqsubseteq y$ and $y \sqsubseteq x$, then $x = y$
  - Transitivity: If $x \sqsubseteq y$ and $y \sqsubseteq z$, then $x \sqsubseteq z$

An example join semilattice

- The set of natural numbers and the **max** function
- Idempotent
  - **max**{a, a} = a
- Commutative
  - **max**{a, b} = **max**{b, a}
- Associative
  - **max**{a, **max**{b, c}} = **max**{**max**{a, b}, c}
- Bottom element is 0:
  - **max**{0, a} = a
- What is the ordering over these elements?

A join semilattice for liveness

- Sets of live variables and the set union operation
- Idempotent:
  - $x \cup x = x$
- Commutative:
  - $x \cup y = y \cup x$
- Associative:
  - $(x \cup y) \cup z = x \cup (y \cup z)$
- Bottom element:
  - The empty set: $\emptyset \cup x = x$
- What is the ordering over these elements?

Semilattices and program analysis

- Semilattices naturally solve many of the problems we encounter in global analysis
- How do we combine information from multiple basic blocks?
- What value do we give to basic blocks we haven't seen yet?
- How do we know that the algorithm always terminates?

Semilattices and program analysis

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A general framework

- A global analysis is a tuple $(\mathtt{D}, \mathtt{V}, \sqsubseteq, \mathtt{F}, \mathtt{I})$, where
  - $\mathtt{D}$ is a direction (forward or backward)
    - The order to visit statements within a basic block, not the order in which to visit the basic blocks
  - $\mathtt{V}$ is a set of values
  - $\sqcup$ is a join operator over those values
  - $\mathtt{F}$ is a set of transfer functions $f : \mathbf{V} \rightarrow \mathbf{V}$
  - $\mathtt{I}$ is an initial value
- The only difference from local analysis is the introduction of the join operator

Running global analyses

- Assume that ($\mathbb{D}$, $\mathbb{V}$, $\sqcup$, $\mathbb{F}$, $\mathbb{I}$) is a forward analysis
- Set OUT[**s**] = $\perp$ for all statements **s**
- Set OUT[**entry**] = $\mathbb{I}$
- Repeat until no values change:
  - For each statement **s** with predecessors $\mathbf{p_1}$, $\mathbf{p_2}$, … , $\mathbf{p_n}$:
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] $\sqcup$ OUT[$\mathbf{p_2}$] $\sqcup$ … $\sqcup$ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s (IN[**s**])
- The order of this iteration does not matter
  - This is sometimes called chaotic iteration

For comparison

- Set OUT[**s**] = ⊥ for all statements **s**
- Set OUT[**entry**] = **I**
- Repeat until no values change:
  - For each statement **s** with predecessors $\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_n}$:
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] ⊔ OUT[$\mathbf{p_2}$] ⊔ … ⊔ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s (IN[**s**])

- Set IN[**s**] = {} for all statements **s**
- Set OUT[**exit**] = the set of variables known to be live on exit
- Repeat until no values change:
  - For each statement **s** of the form **a=b+c**:
    - Set OUT[**s**] = set union of IN[**x**] for each successor **x** of **s**
    - Set IN[**s**] = (OUT[**s**]-{a}) ∪ {b,c}

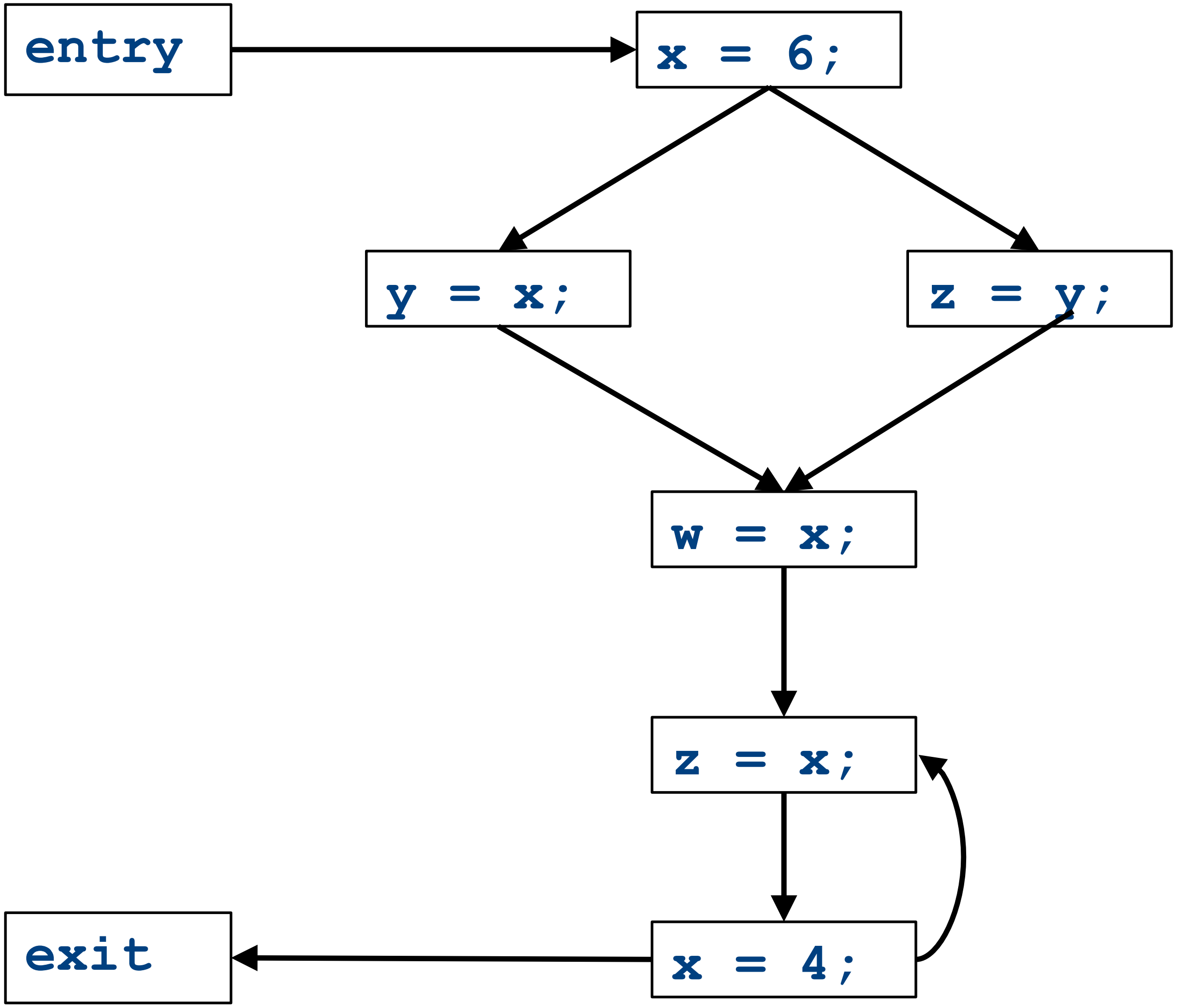
The dataflow framework

- This form of analysis is called the dataflow framework
- Can be used to easily prove an analysis is sound
- With certain restrictions, can be used to prove that an analysis eventually terminates
  - Again, more on that later

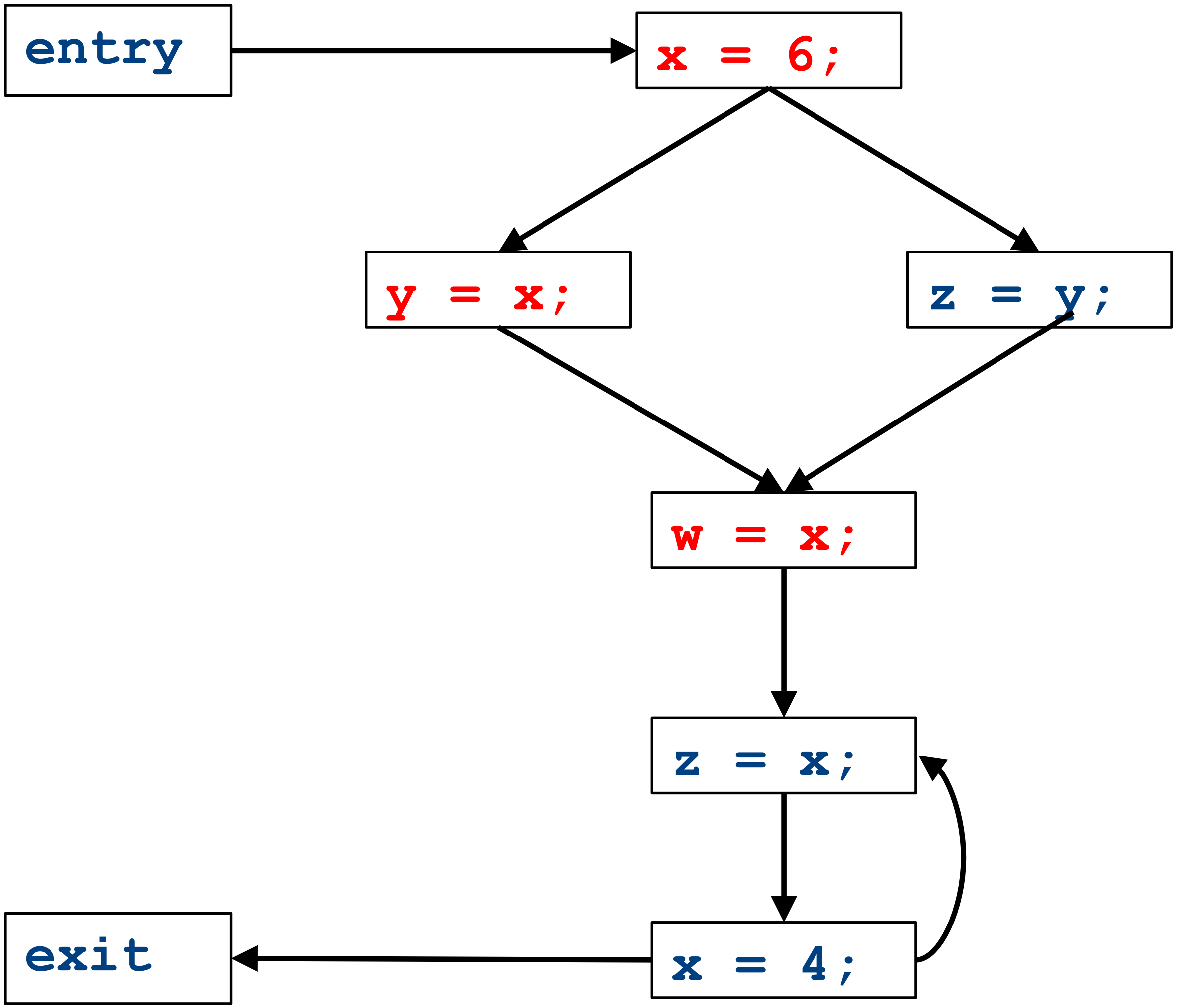
Global constant propagation

- Constant propagation is an optimization that replaces each variable that is known to be a constant value with that constant
- An elegant example of the dataflow framework

Global constant propagation



Global constant propagation



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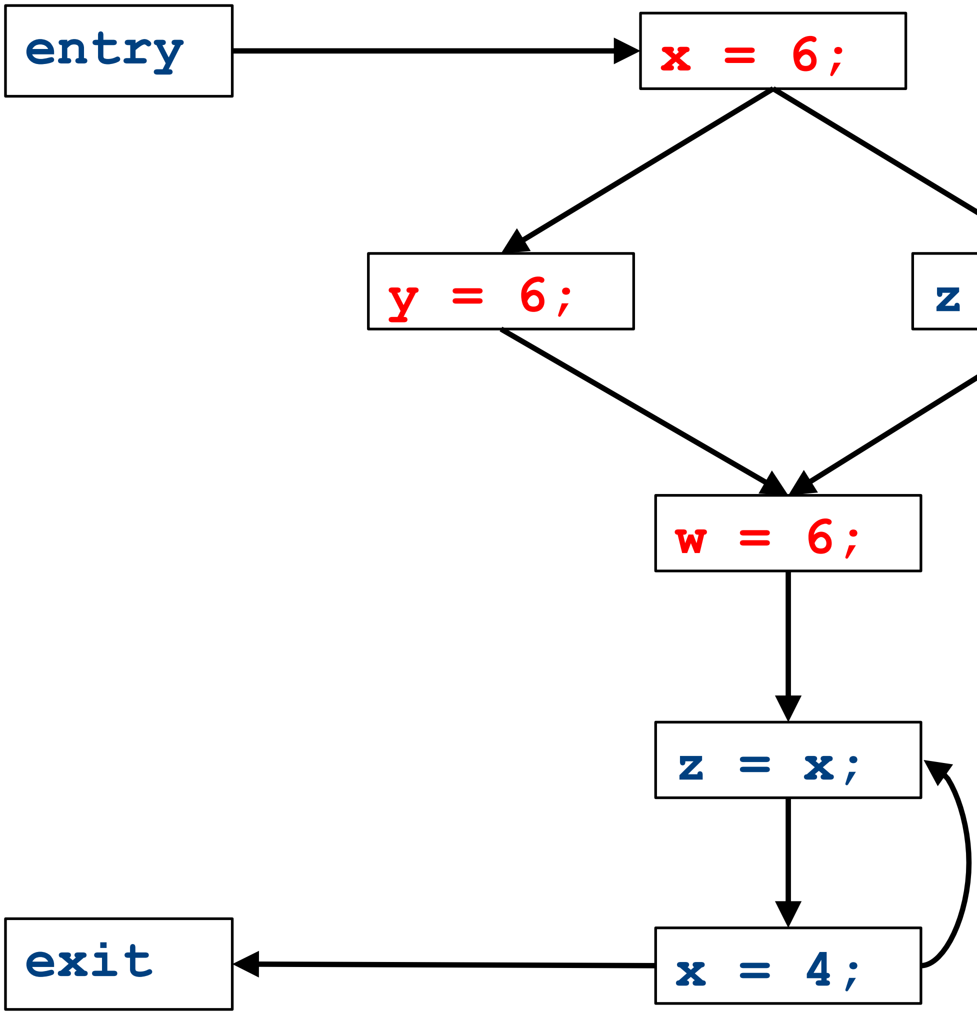


Constant propagation analysis

- In order to do a constant propagation, we need to track what values might be assigned to a variable at each program point
- Every variable will either
  - Never have a value assigned to it,
  - Have a single constant value assigned to it,
  - Have two or more constant values assigned to it, or
  - Have a known non-constant value.
  - Our analysis will propagate this information throughout a CFG to identify locations where a value is constant

Properties of constant propagation

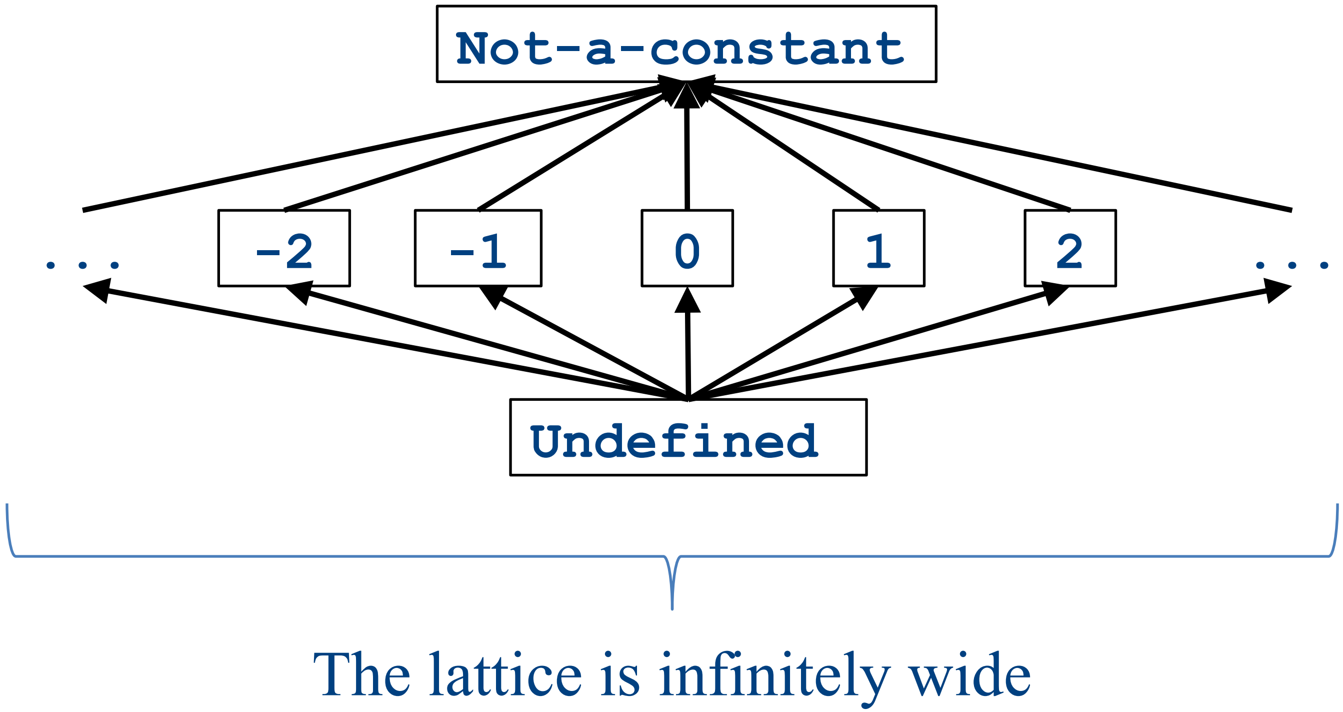
- For now, consider just some single variable **x**
- At each point in the program, we know one of three things about the value of **x**:
  - **x** is definitely not a constant, since it's been assigned two values or assigned a value that we know isn't a constant
  - **x** is definitely a constant and has value **k**
  - We have never seen a value for **x**
- Note that the first and last of these are **not** the same!
  - The first one means that there may be a way for **x** to have multiple values
  - The last one means that **x** never had a value at all

Defining a join operator

- The join of any two different constants is **Not-a-Constant**
  - (If the variable might have two different values on entry to a statement, it cannot be a constant)
- The join of **Not a Constant** and any other value is **Not-a-Constant**
  - (If on some path the value is known not to be a constant, then on entry to a statement its value can't possibly be a constant)
- The join of **Undefined** and any other value is that other value
  - (If **x** has no value on some path and does have a value on some other path, we can just pretend it always had the assigned value)

A semilattice for constant propagation

- One possible semilattice for this analysis is shown here (for each variable):



A semilattice for constant propagation

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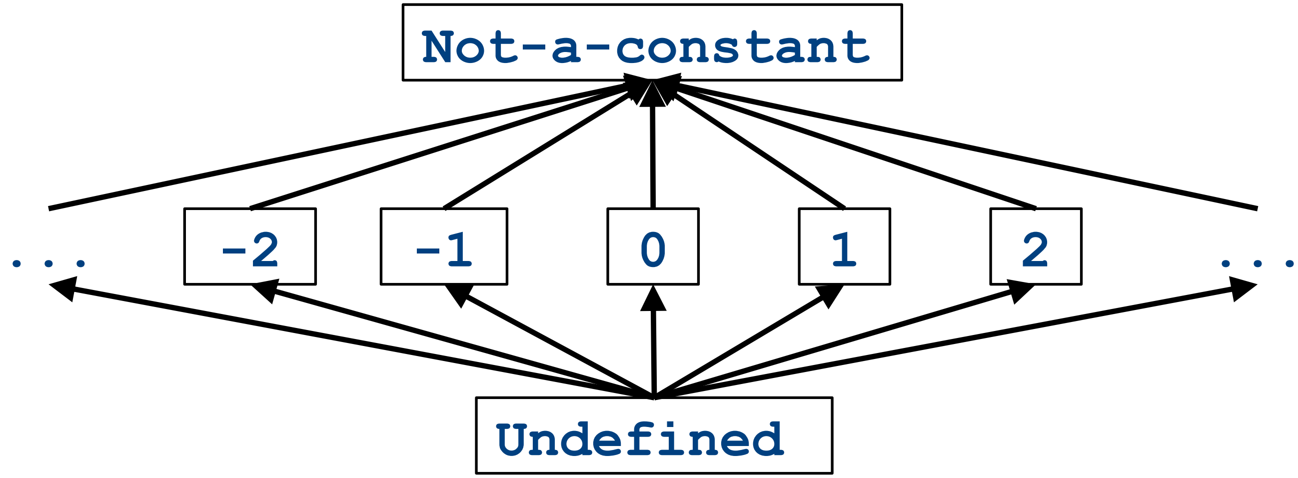
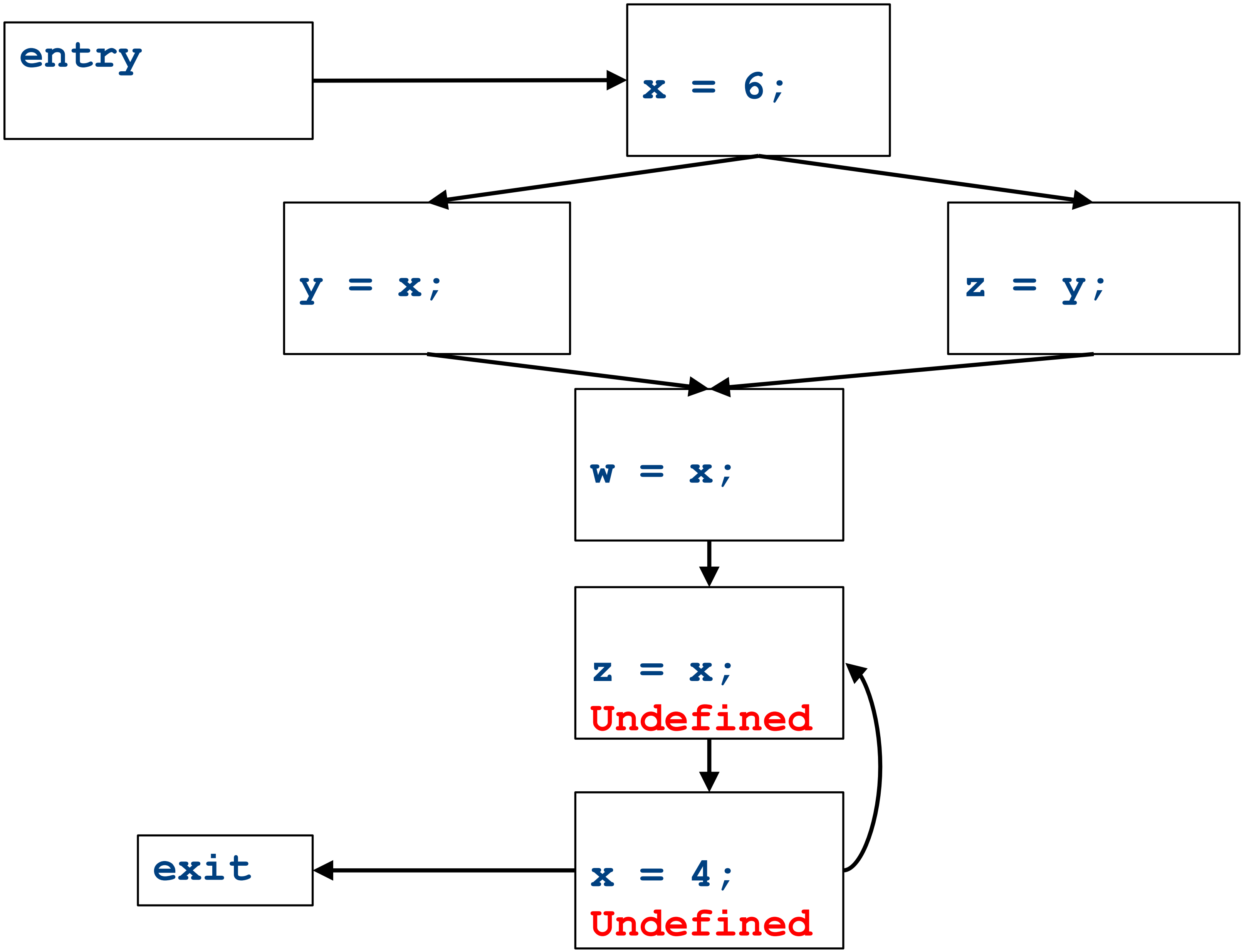
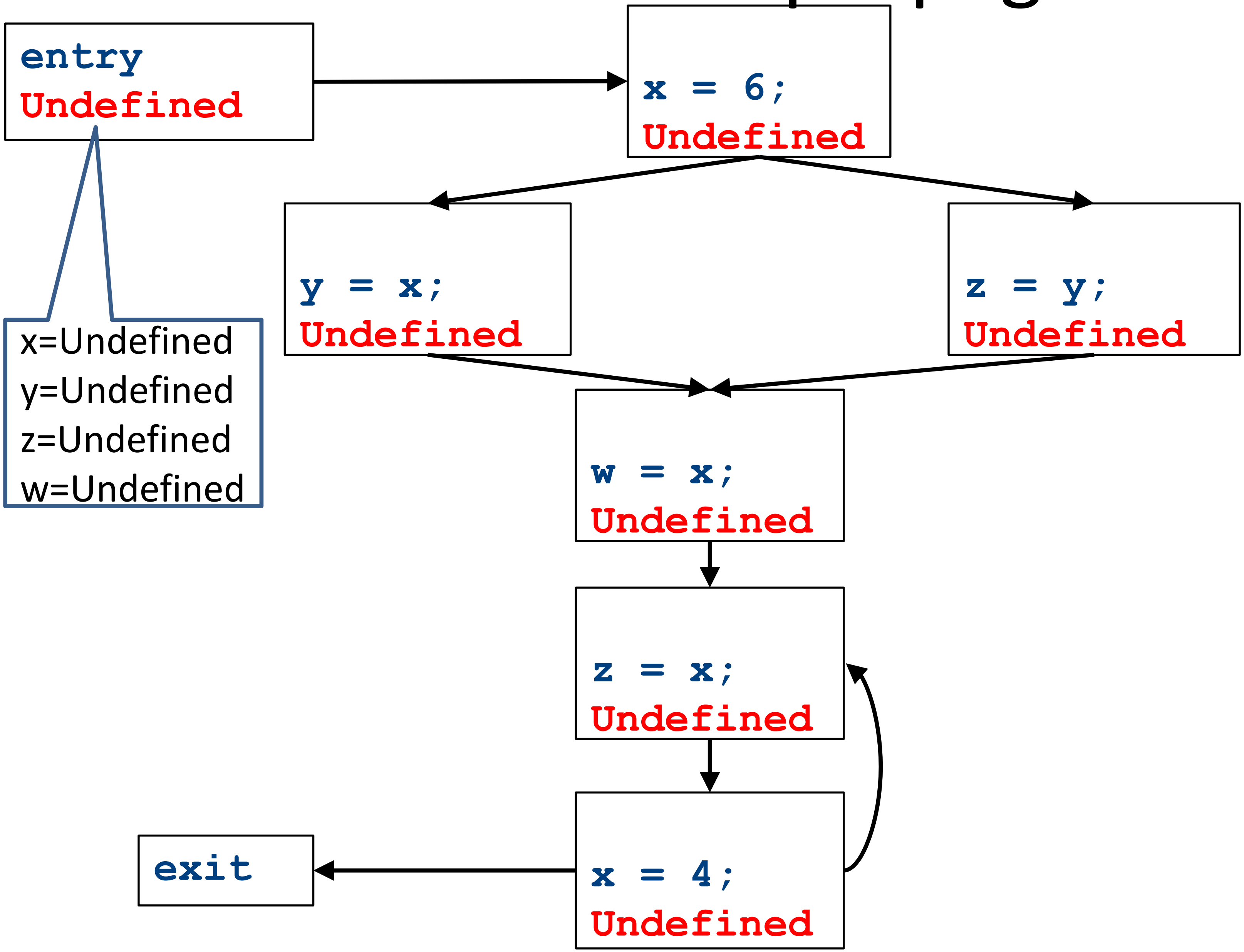


- Note:
  - The join of any two different constants is **Not-a-Constant**
  - The join of **Not a Constant** and any other value is **Not-a-Constant**
  - The join of **Undefined** and any other value is that other value

Global constant propagation



Global constant propagation



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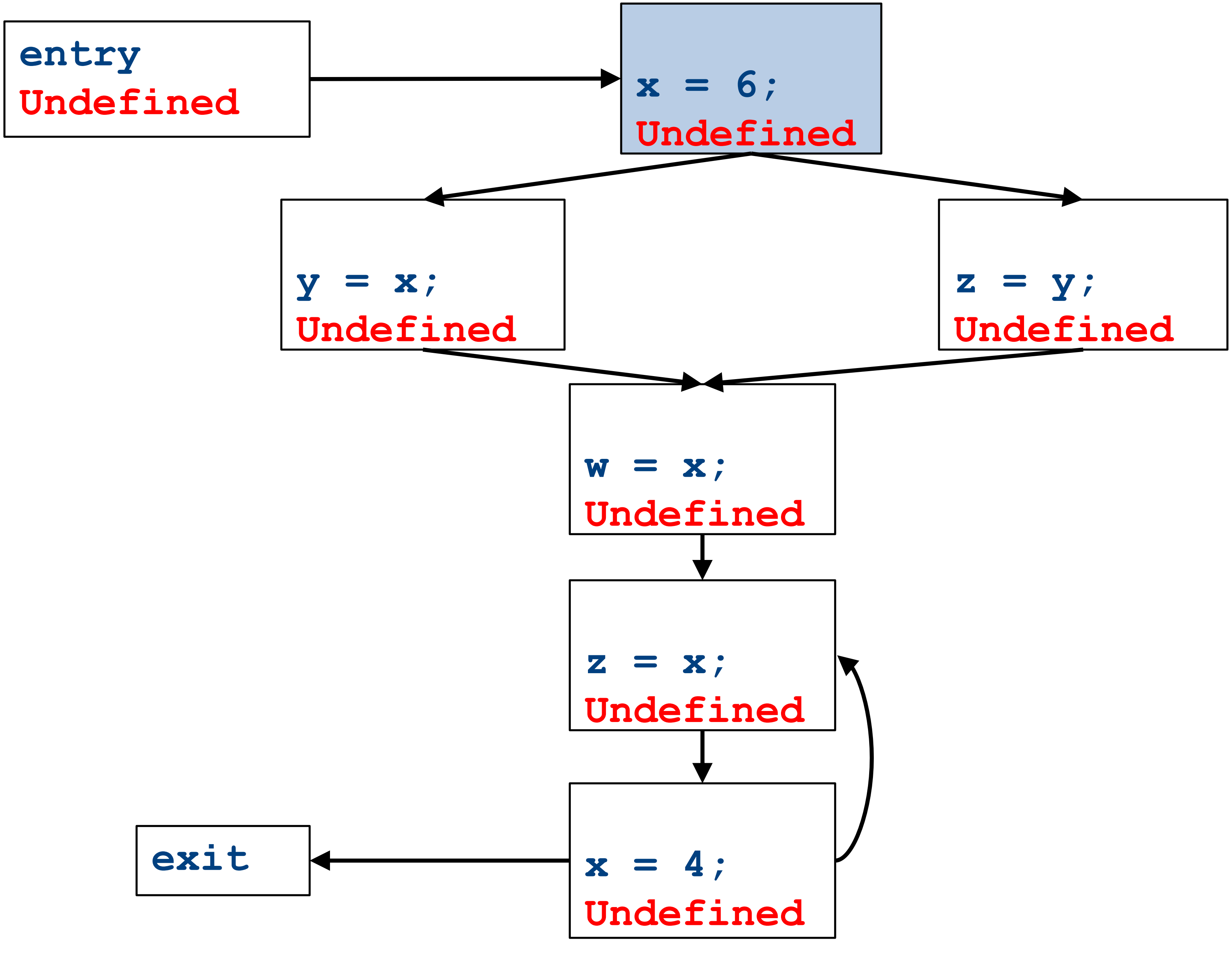

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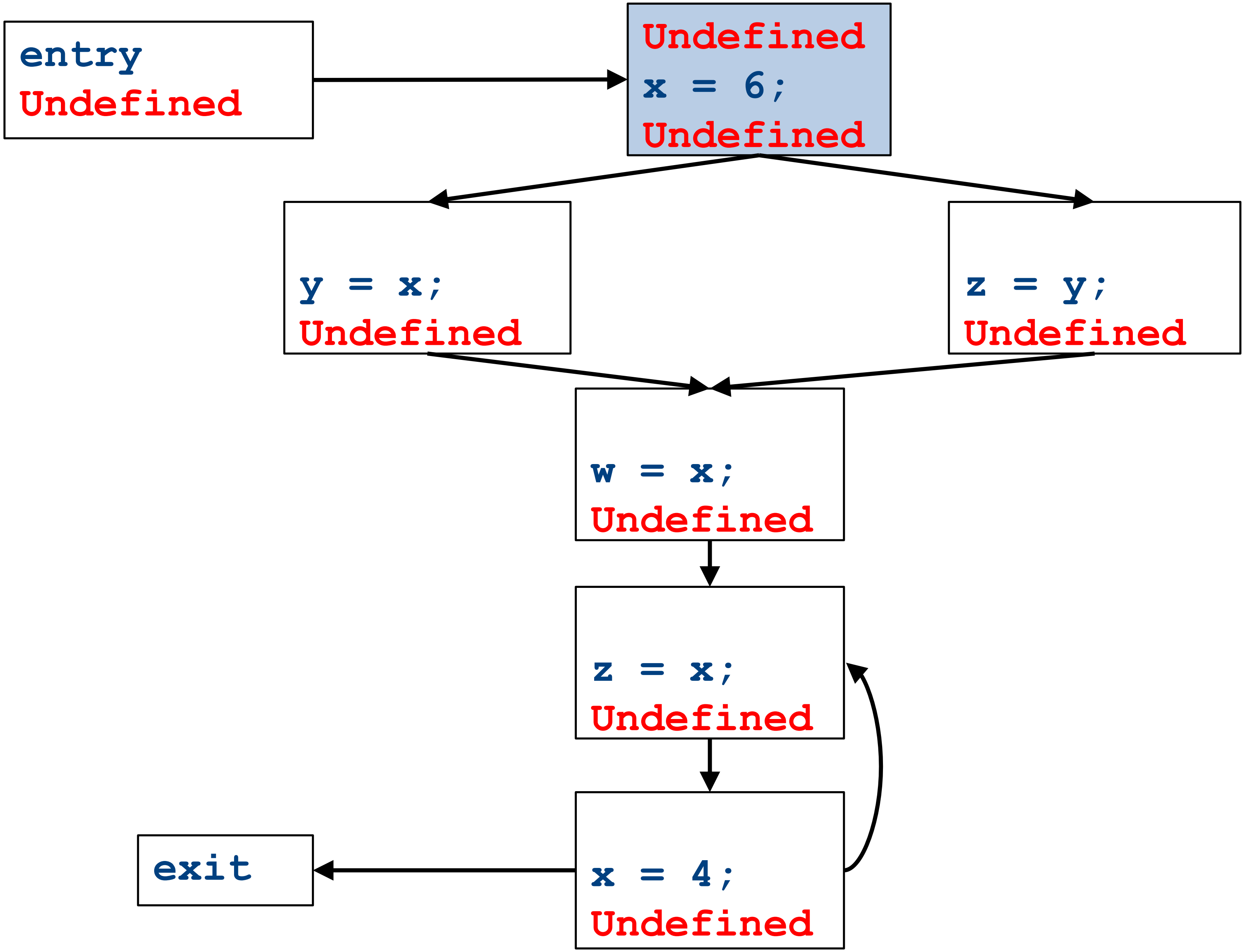

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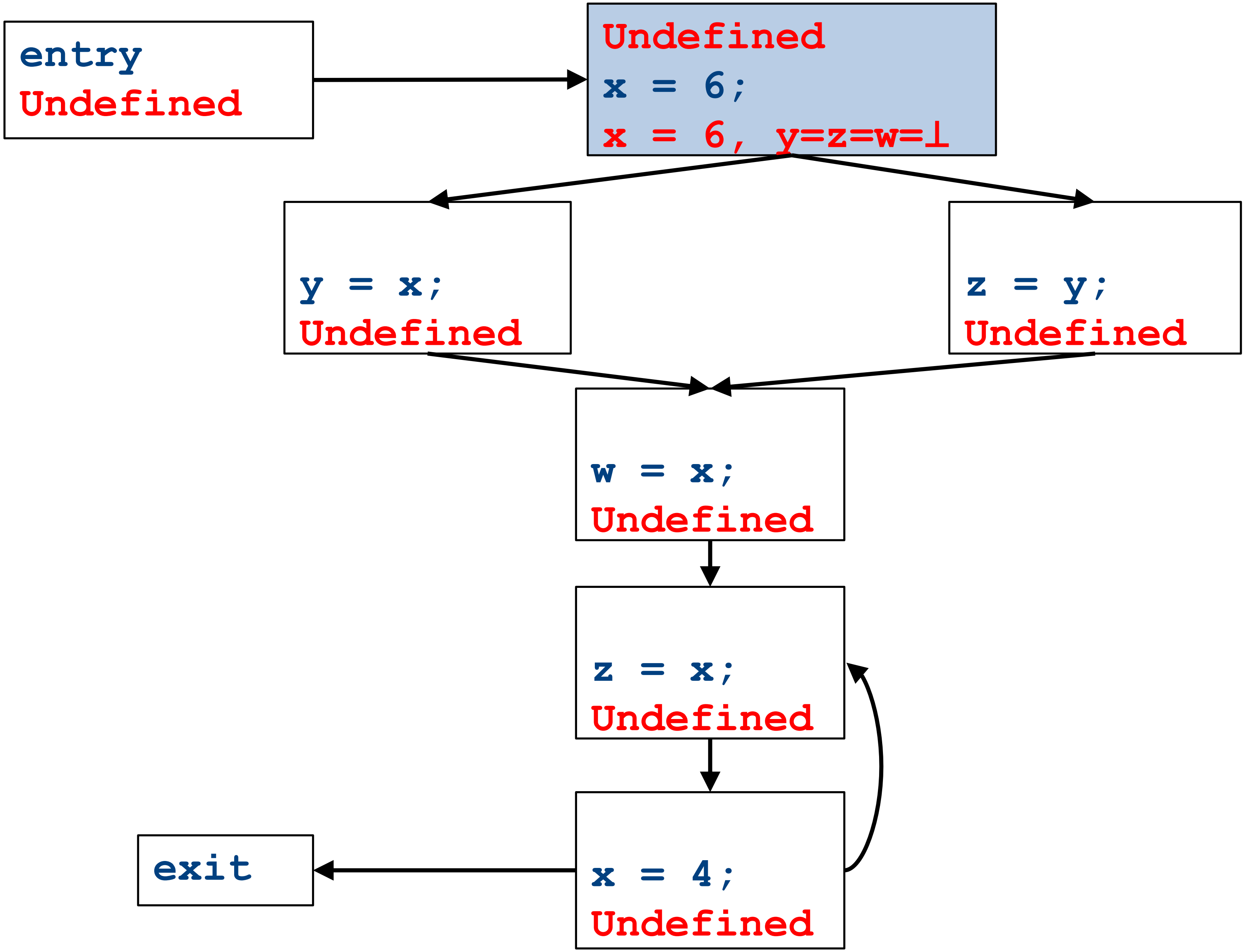

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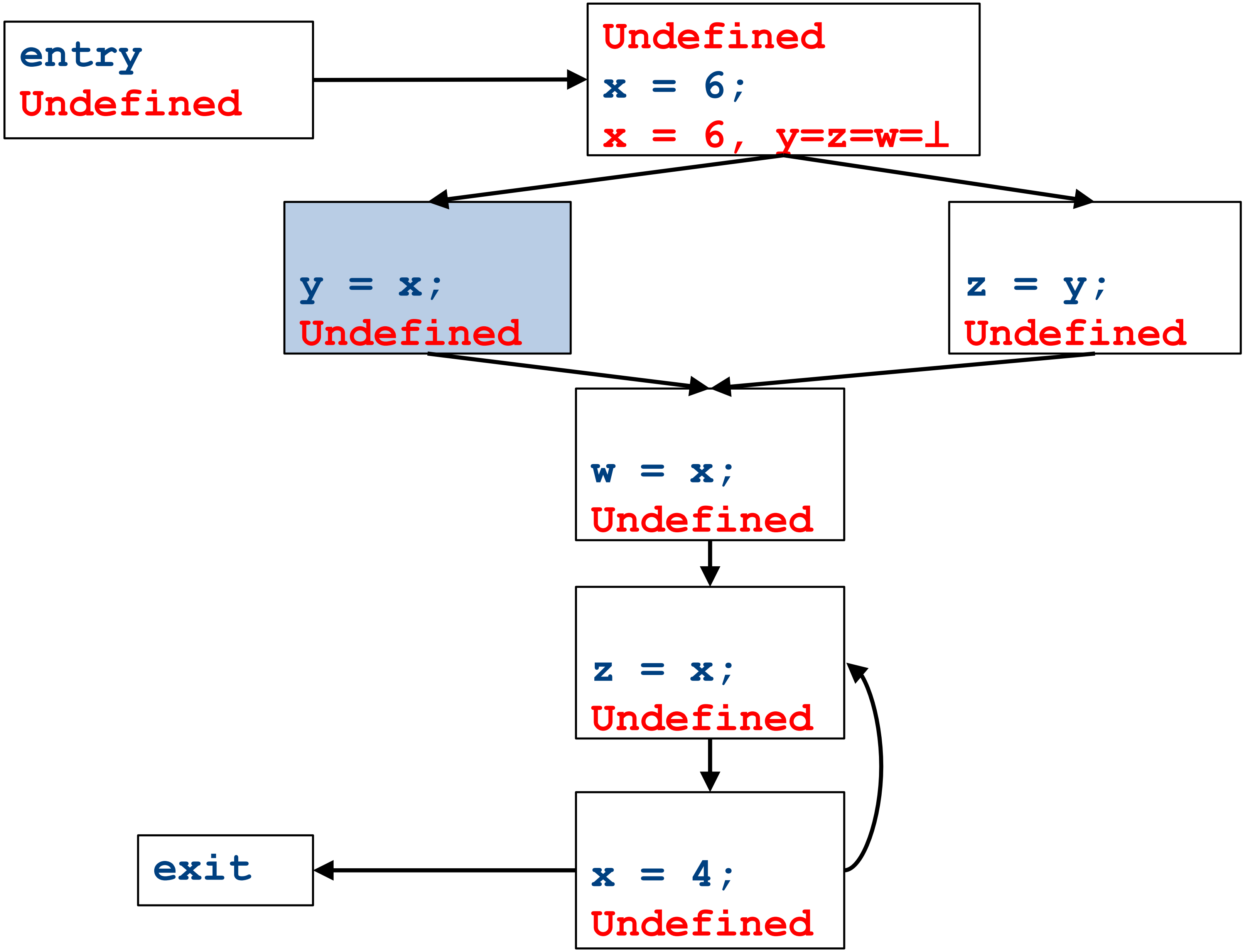

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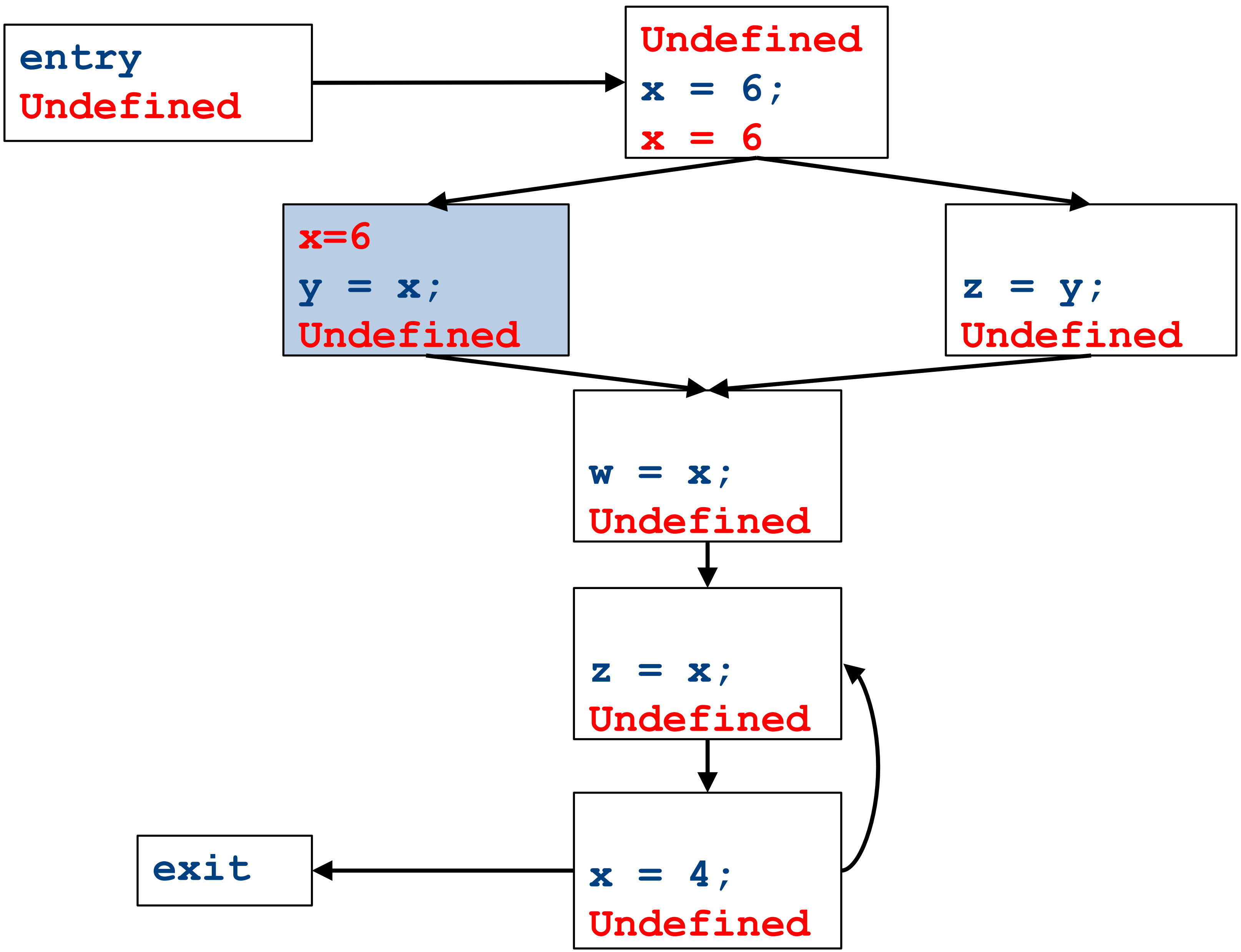

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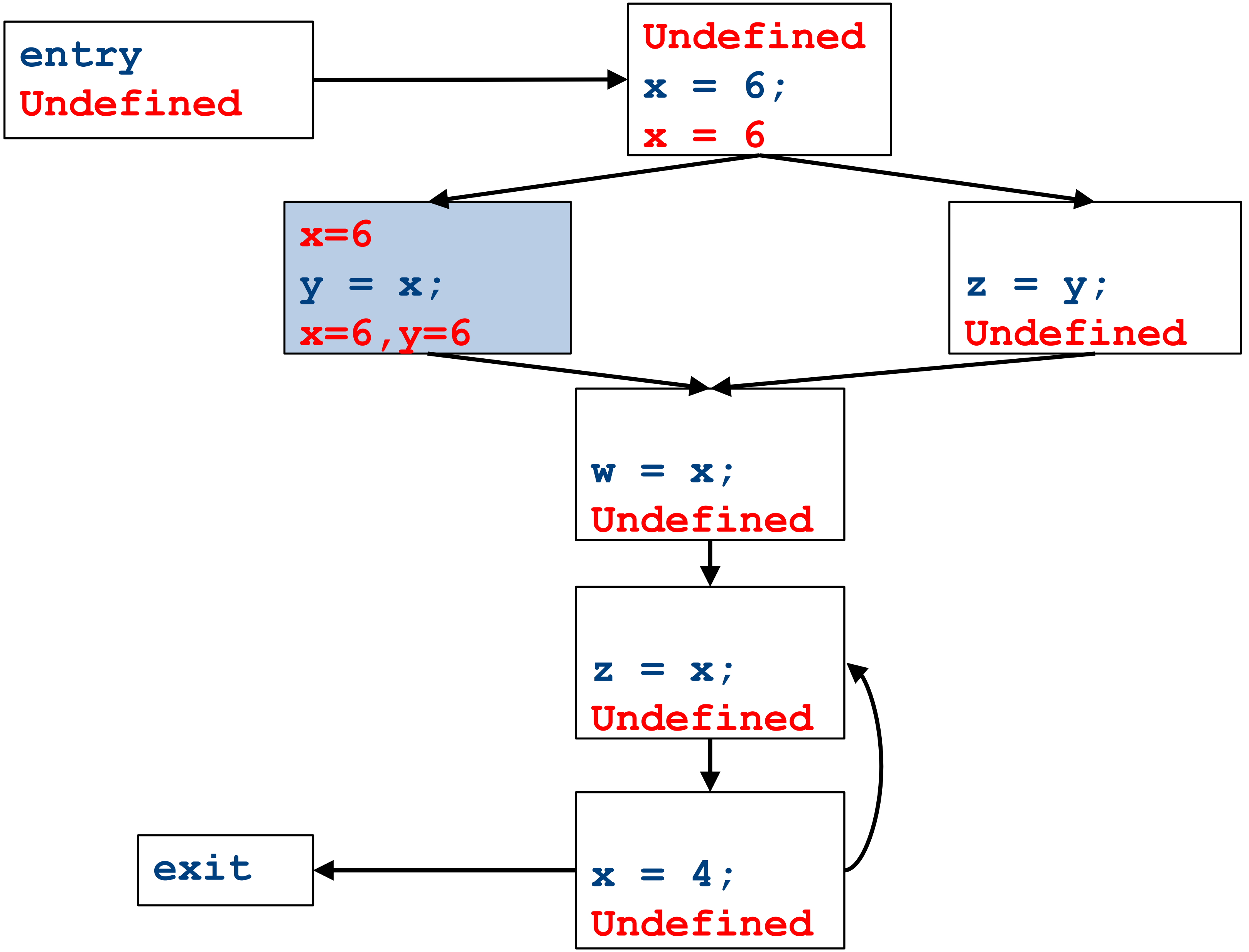

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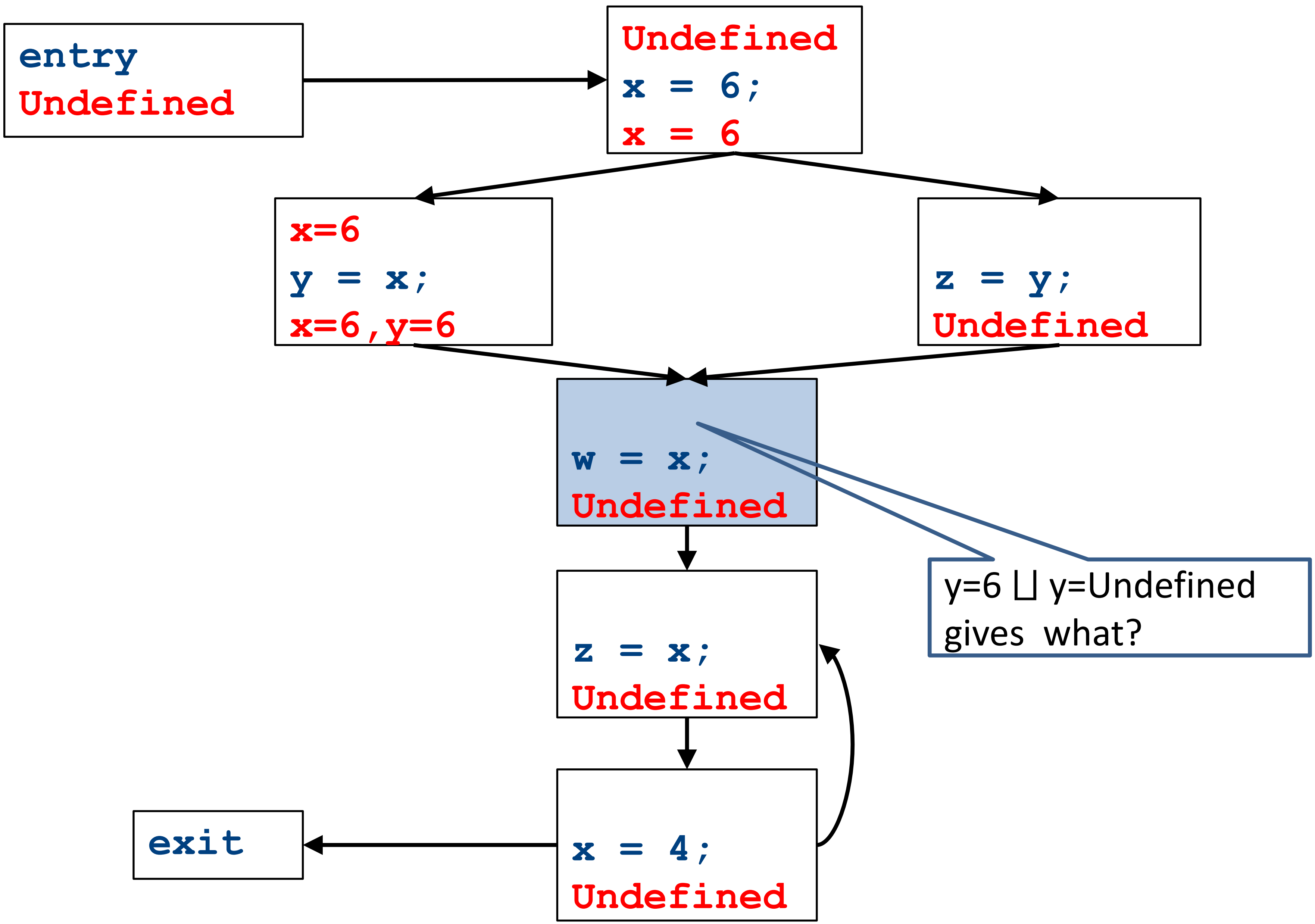

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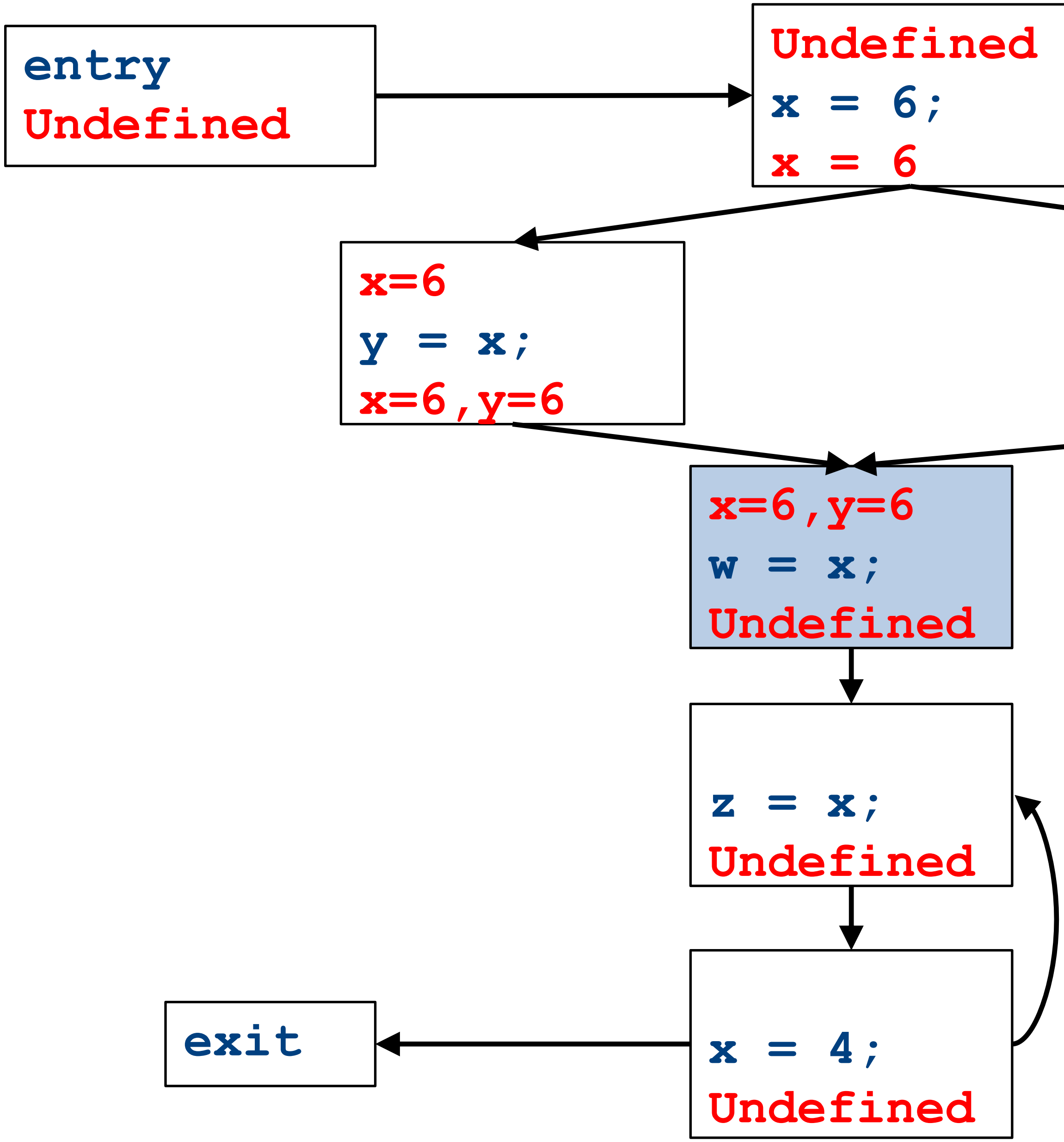

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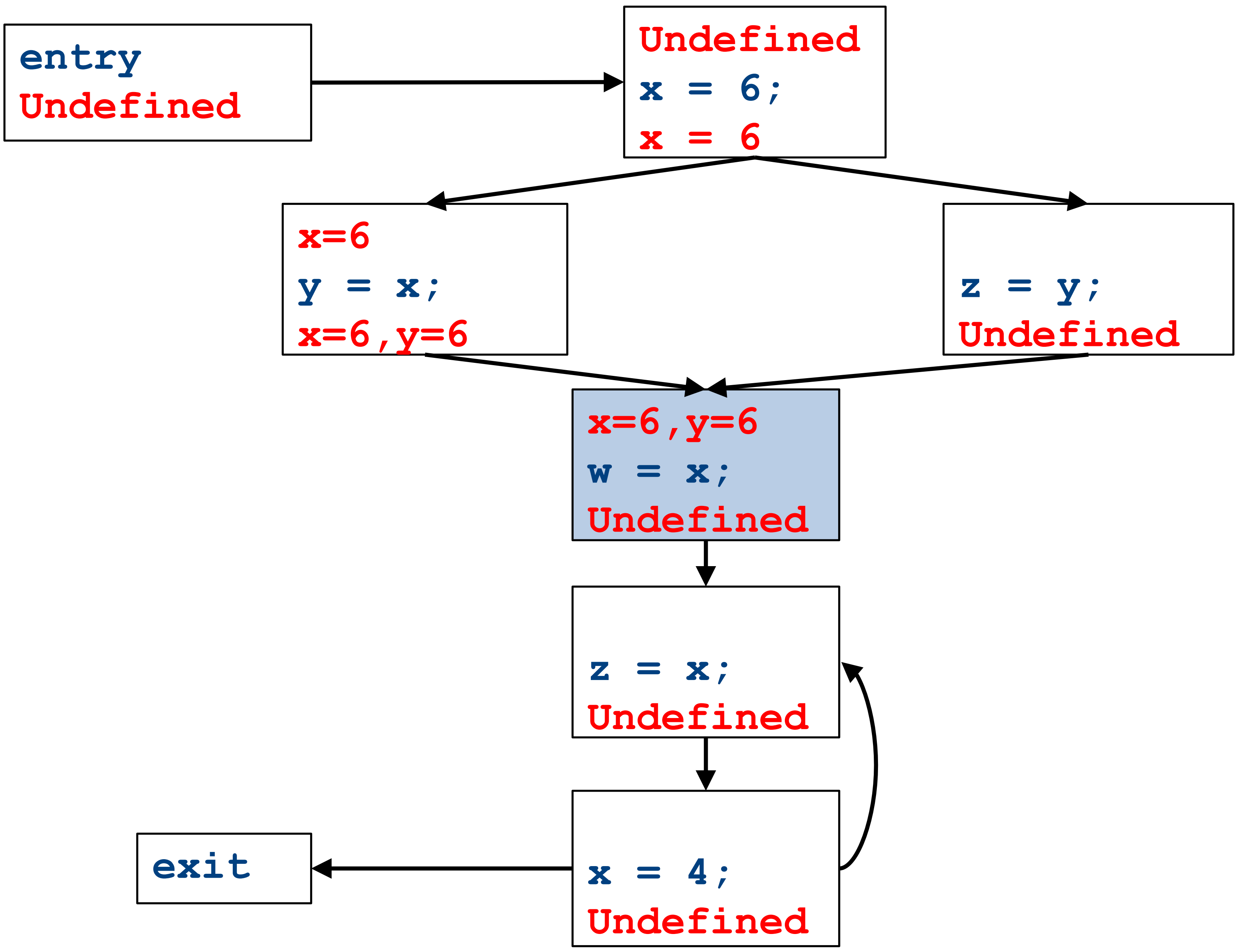

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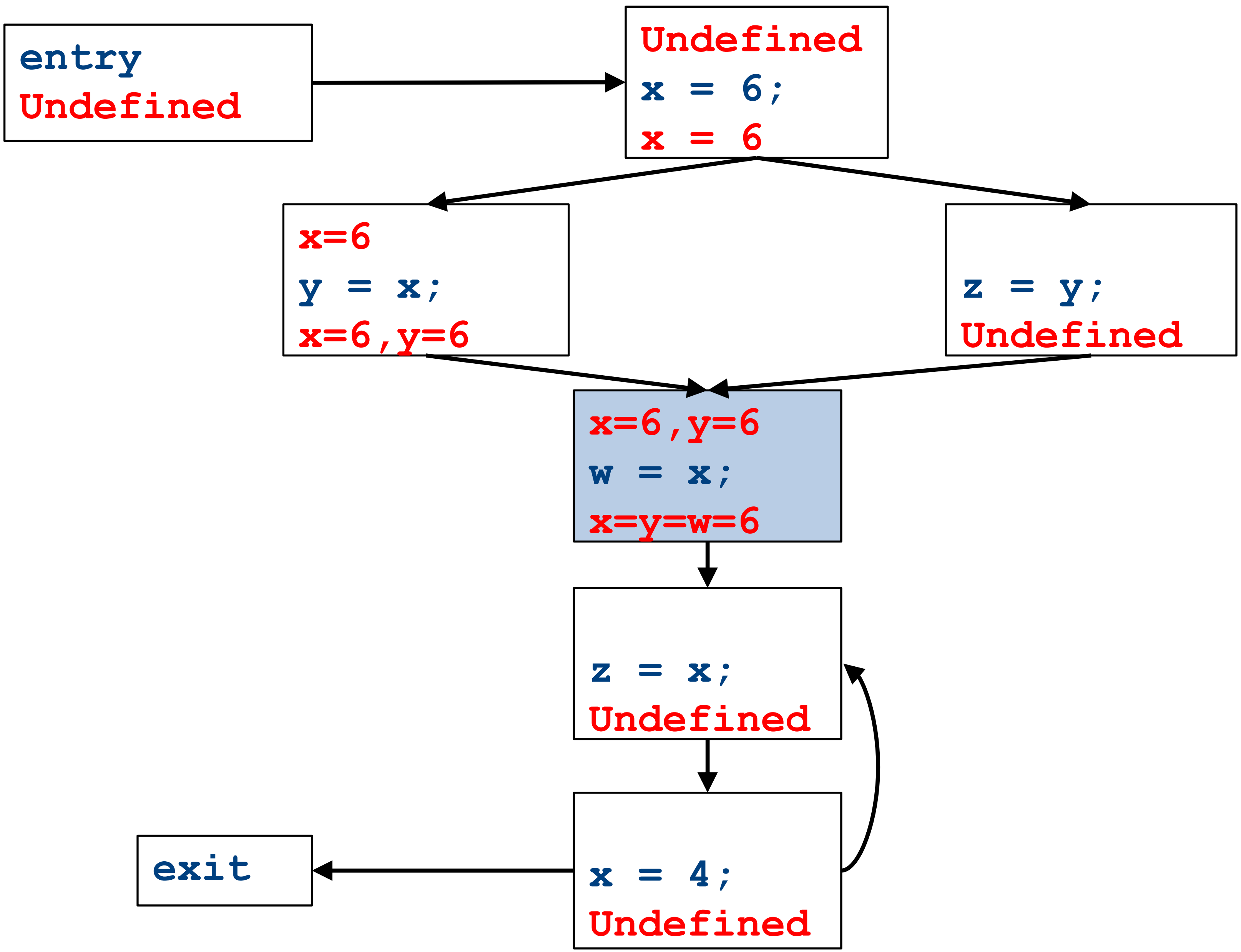

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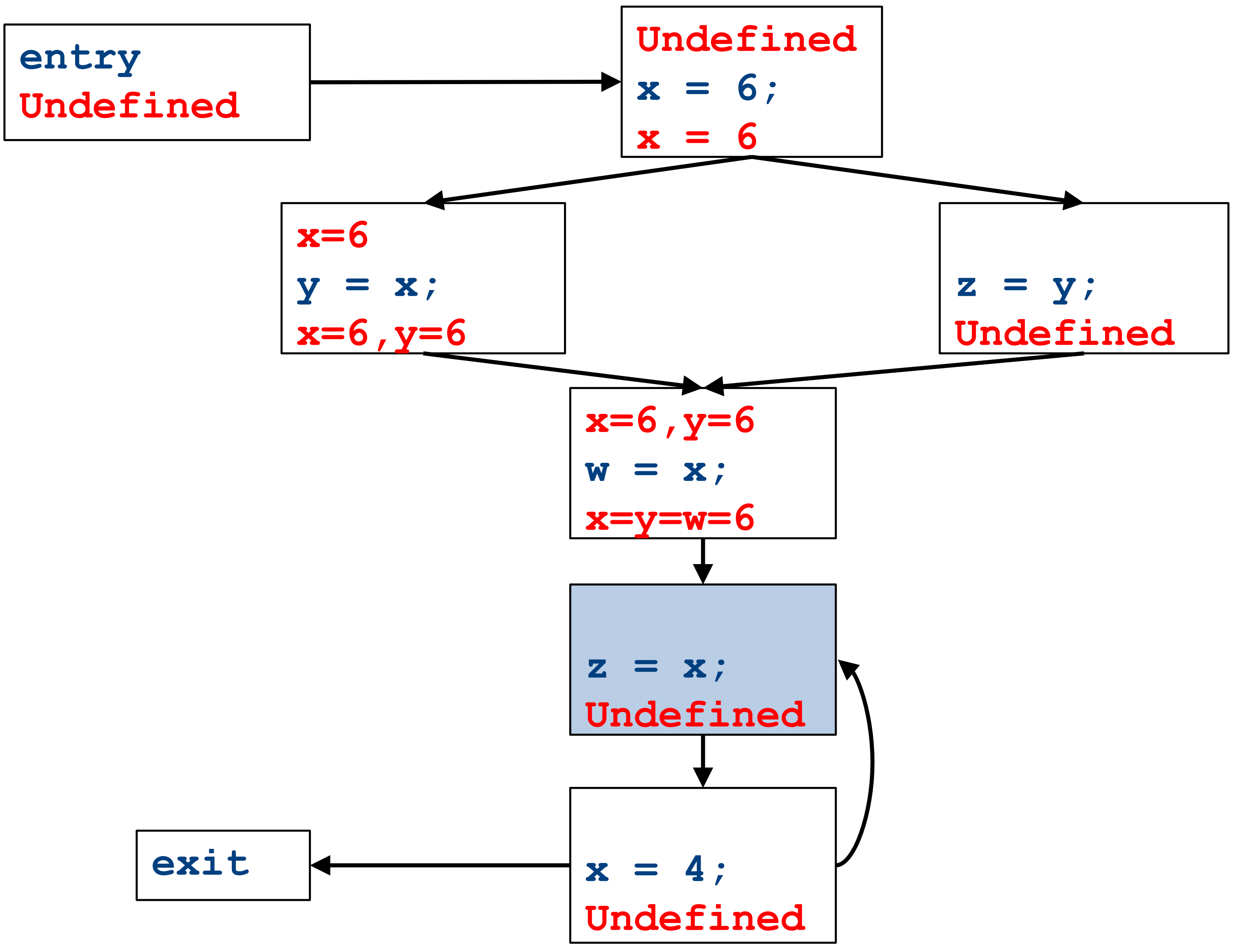

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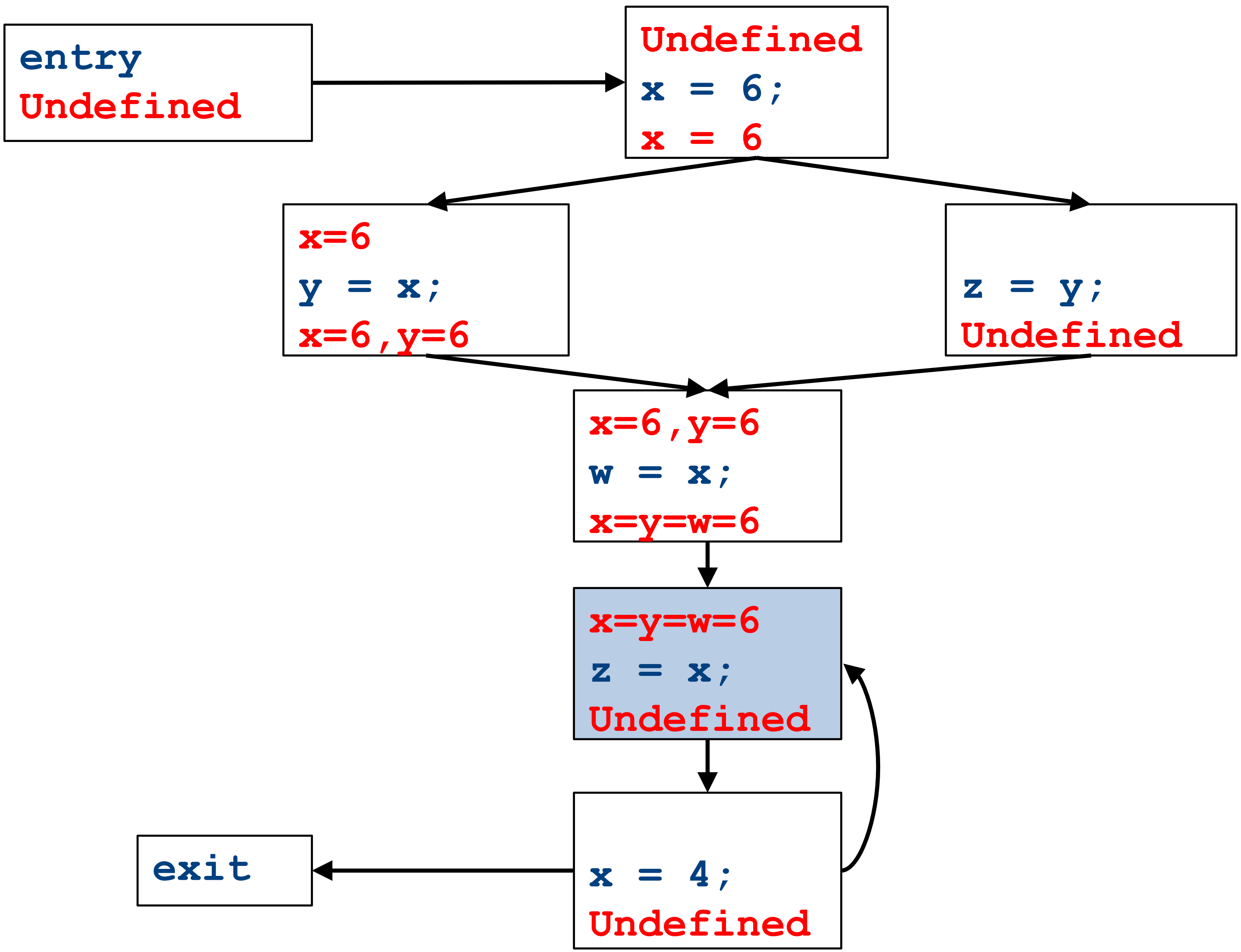

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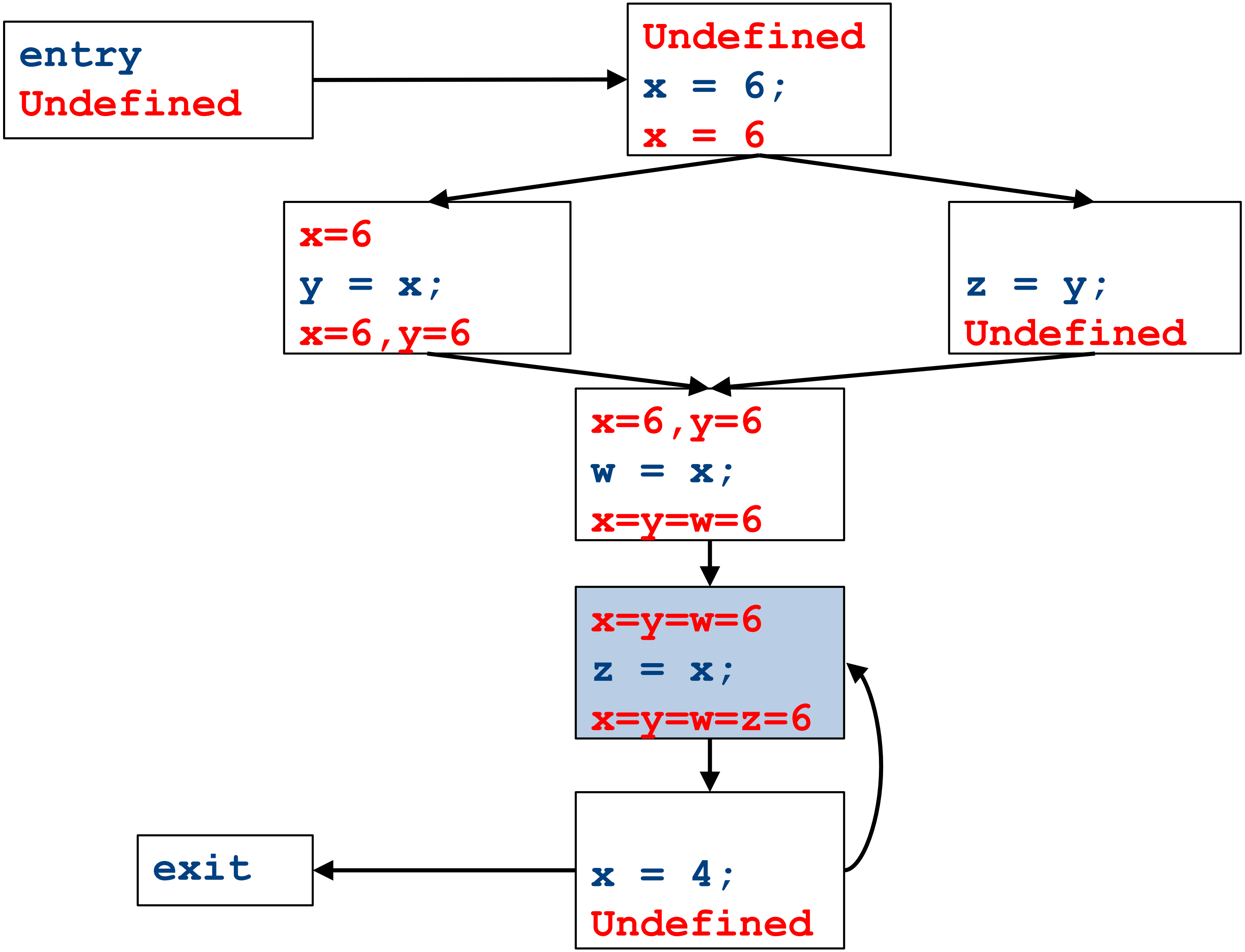

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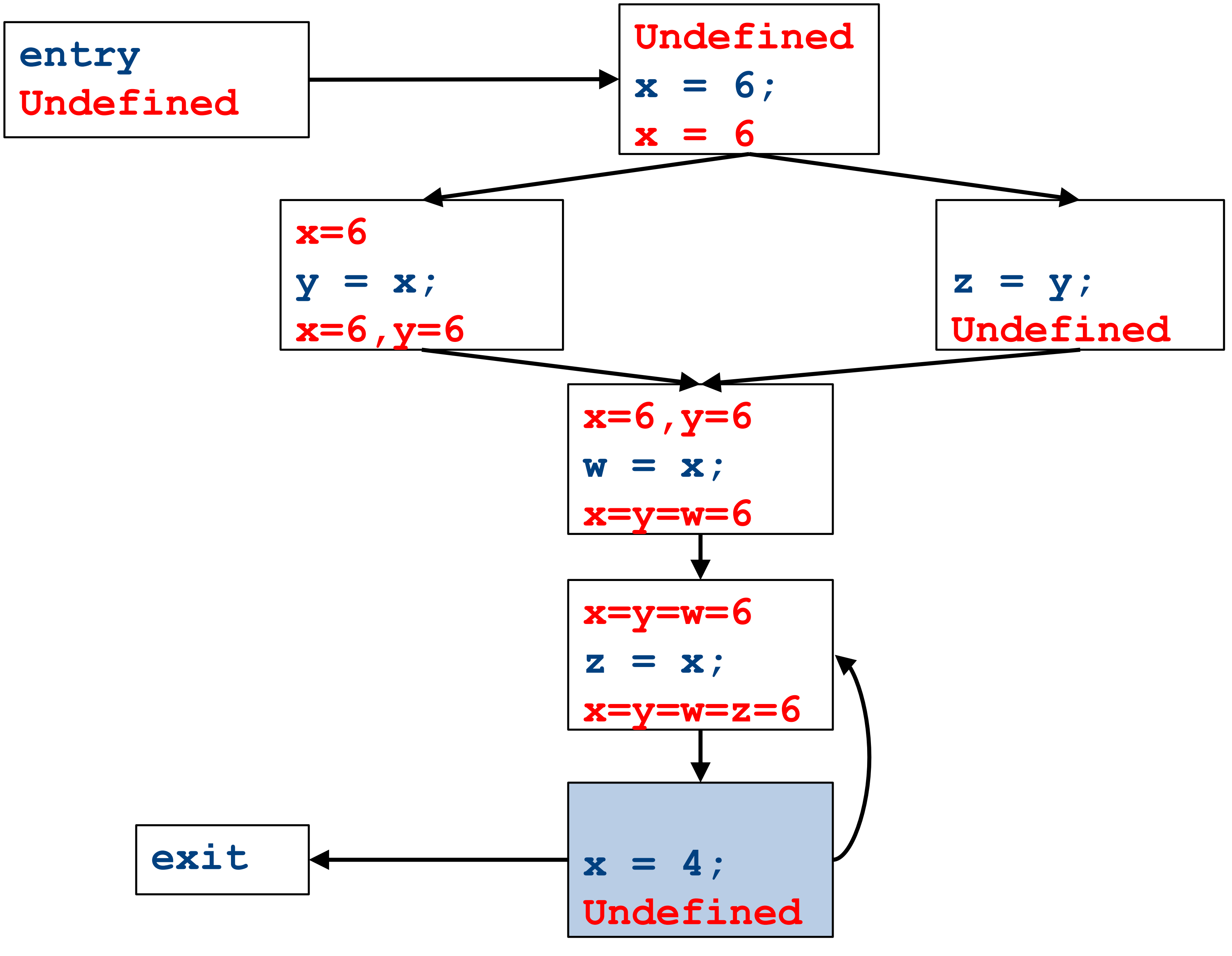

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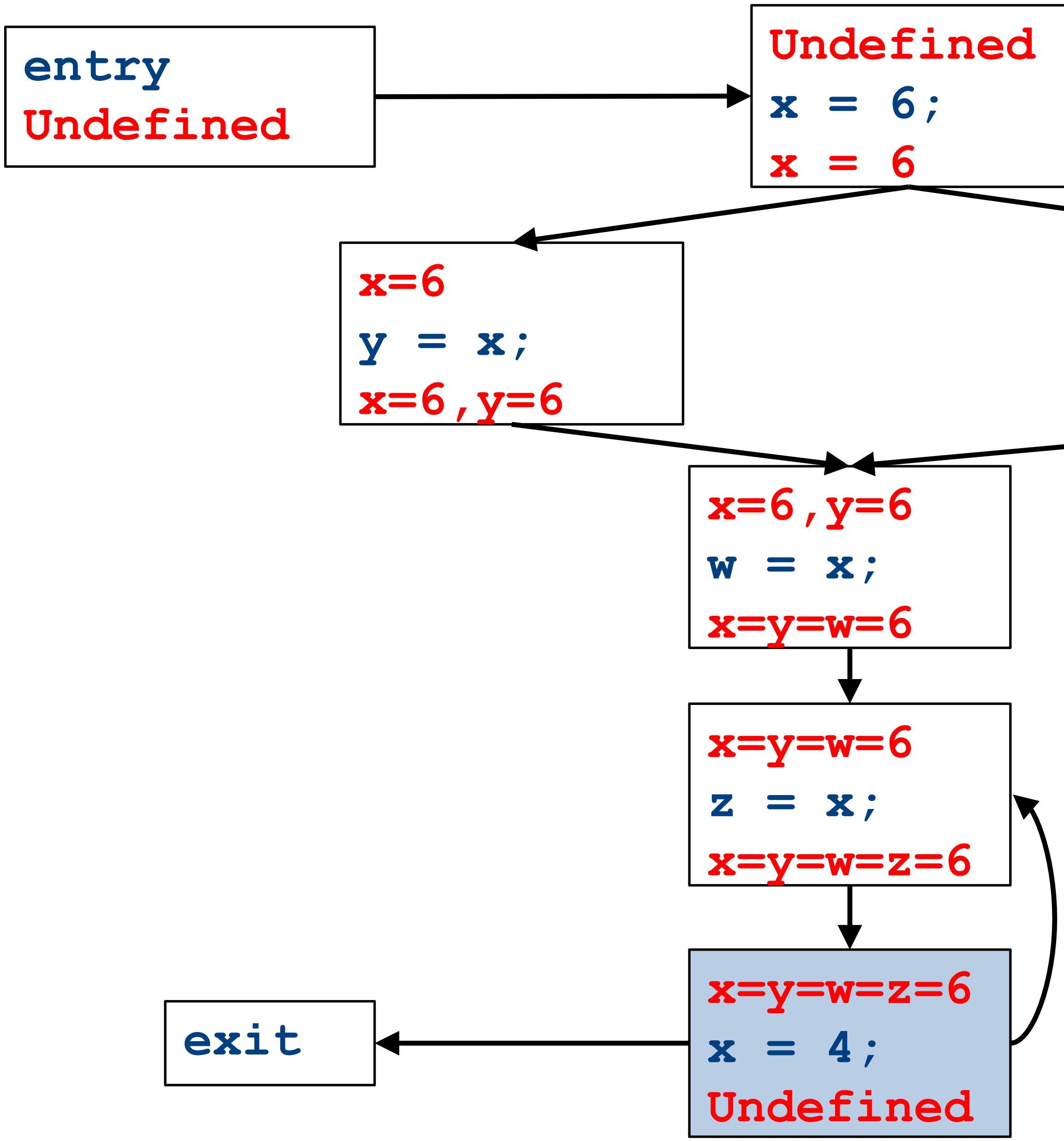

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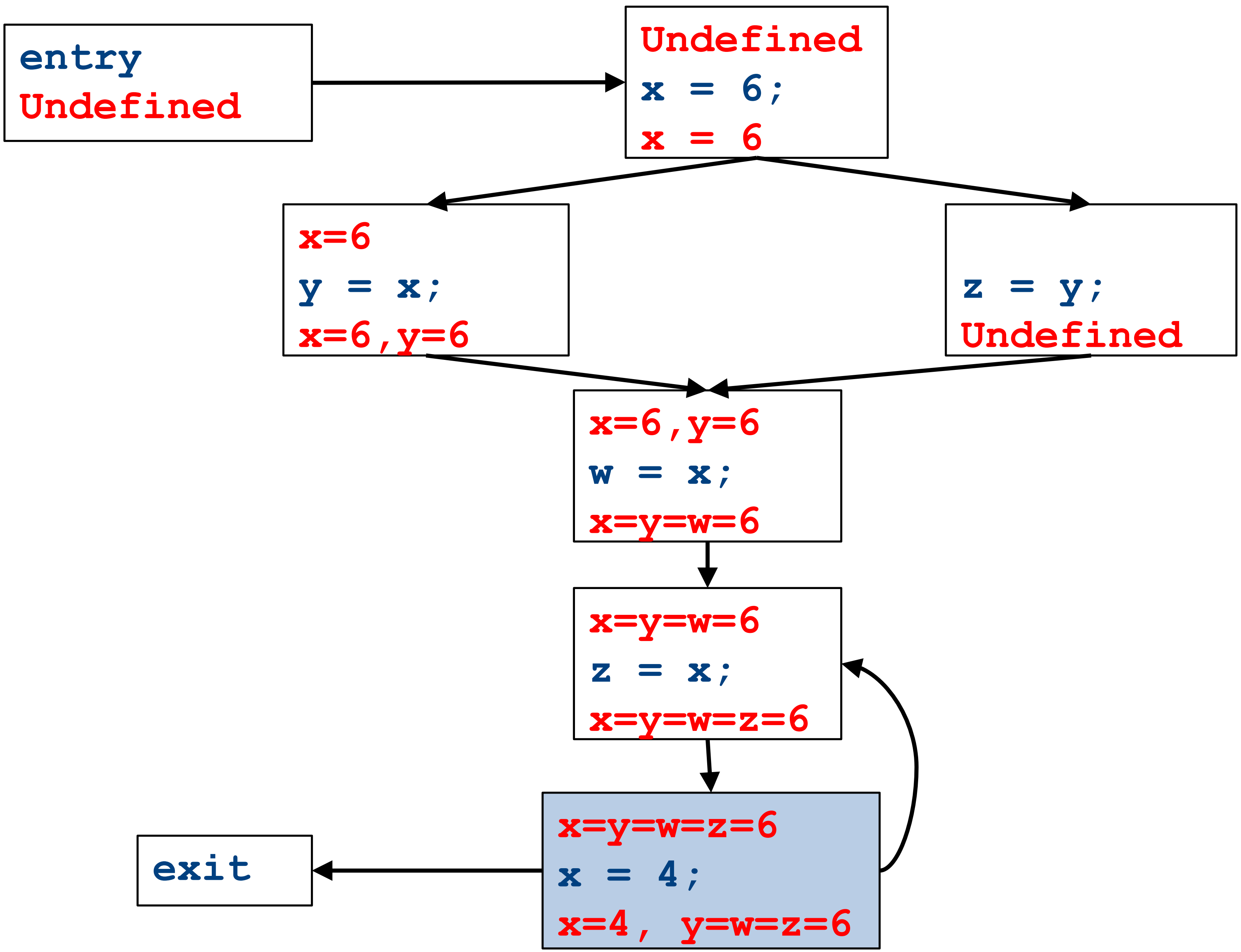

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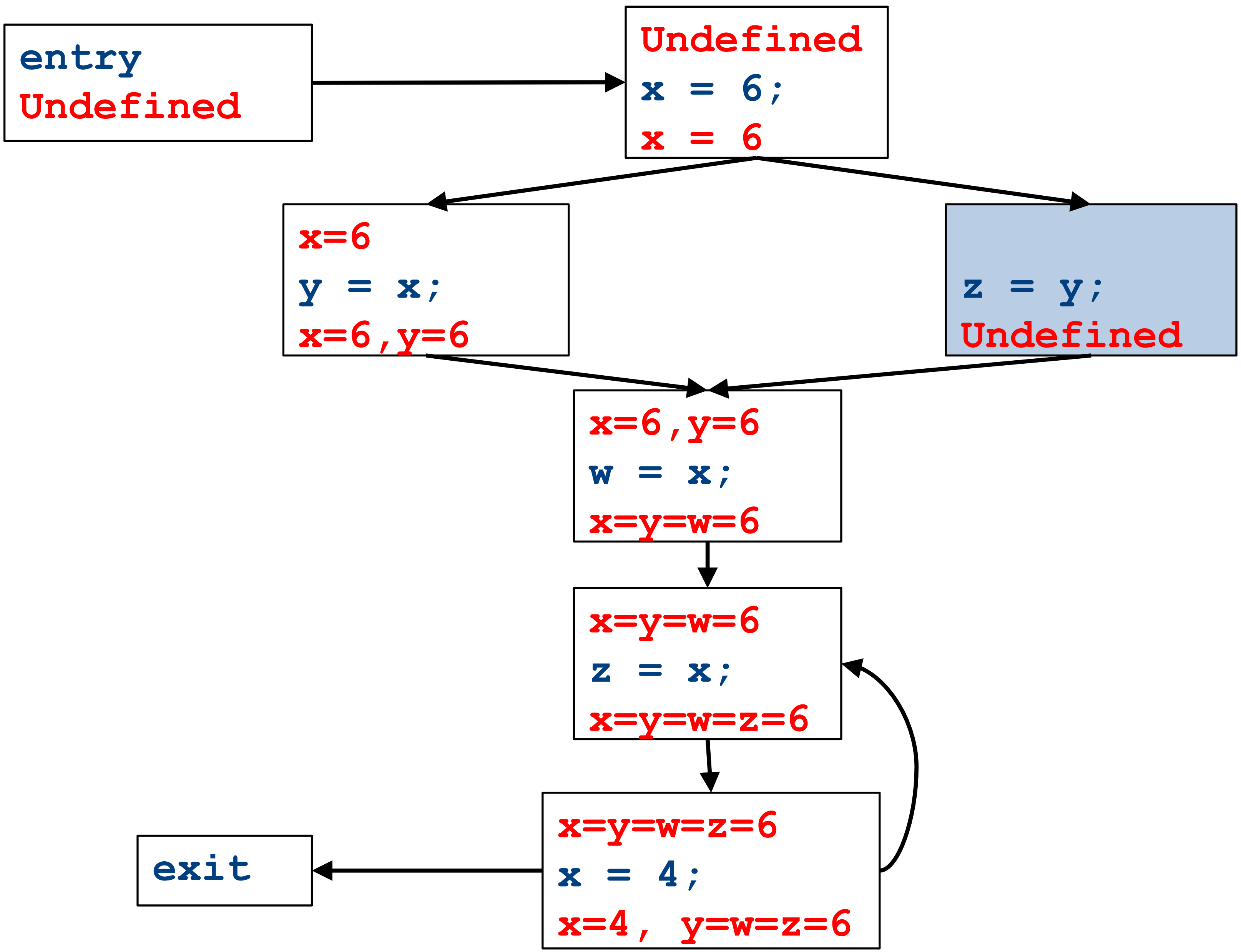

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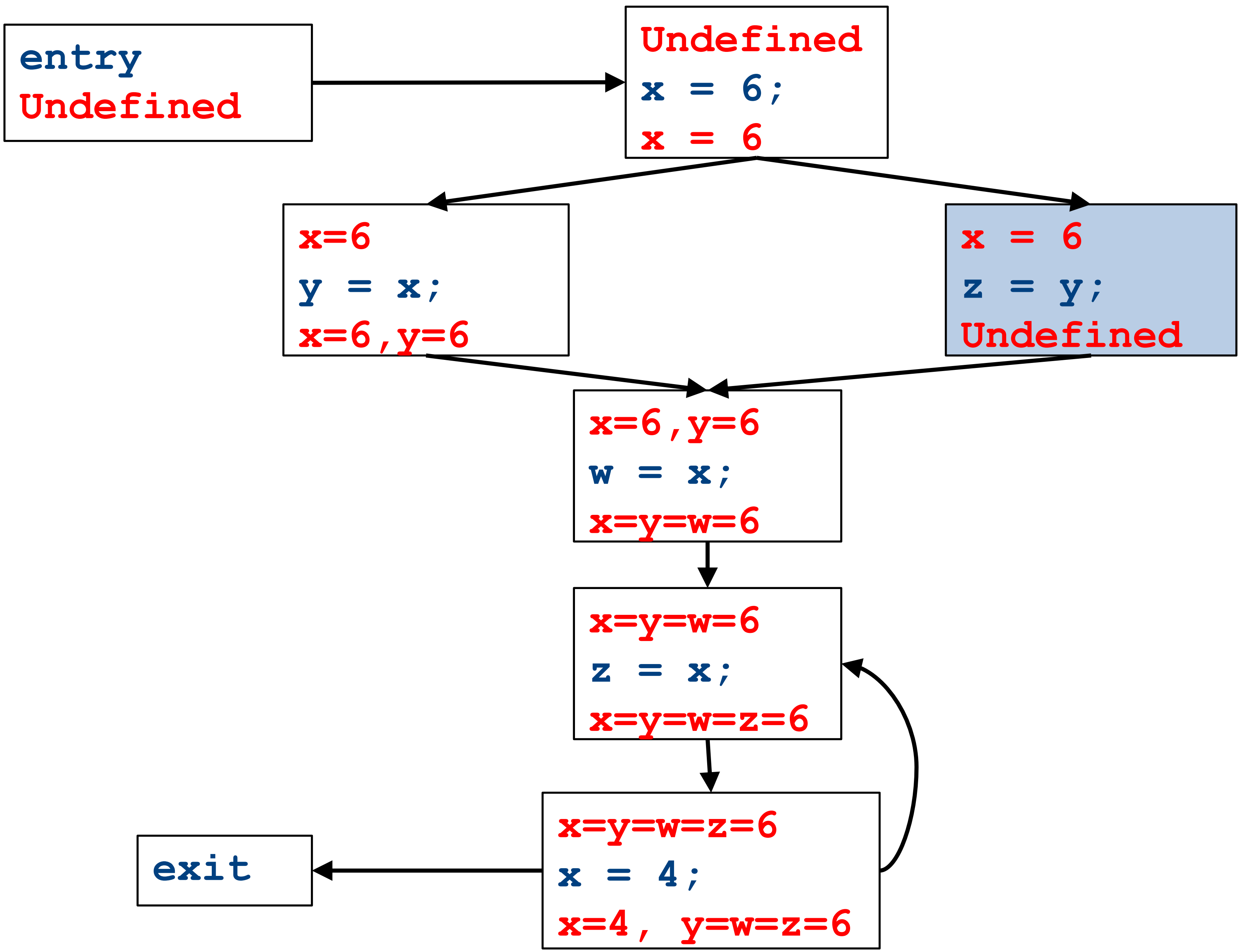

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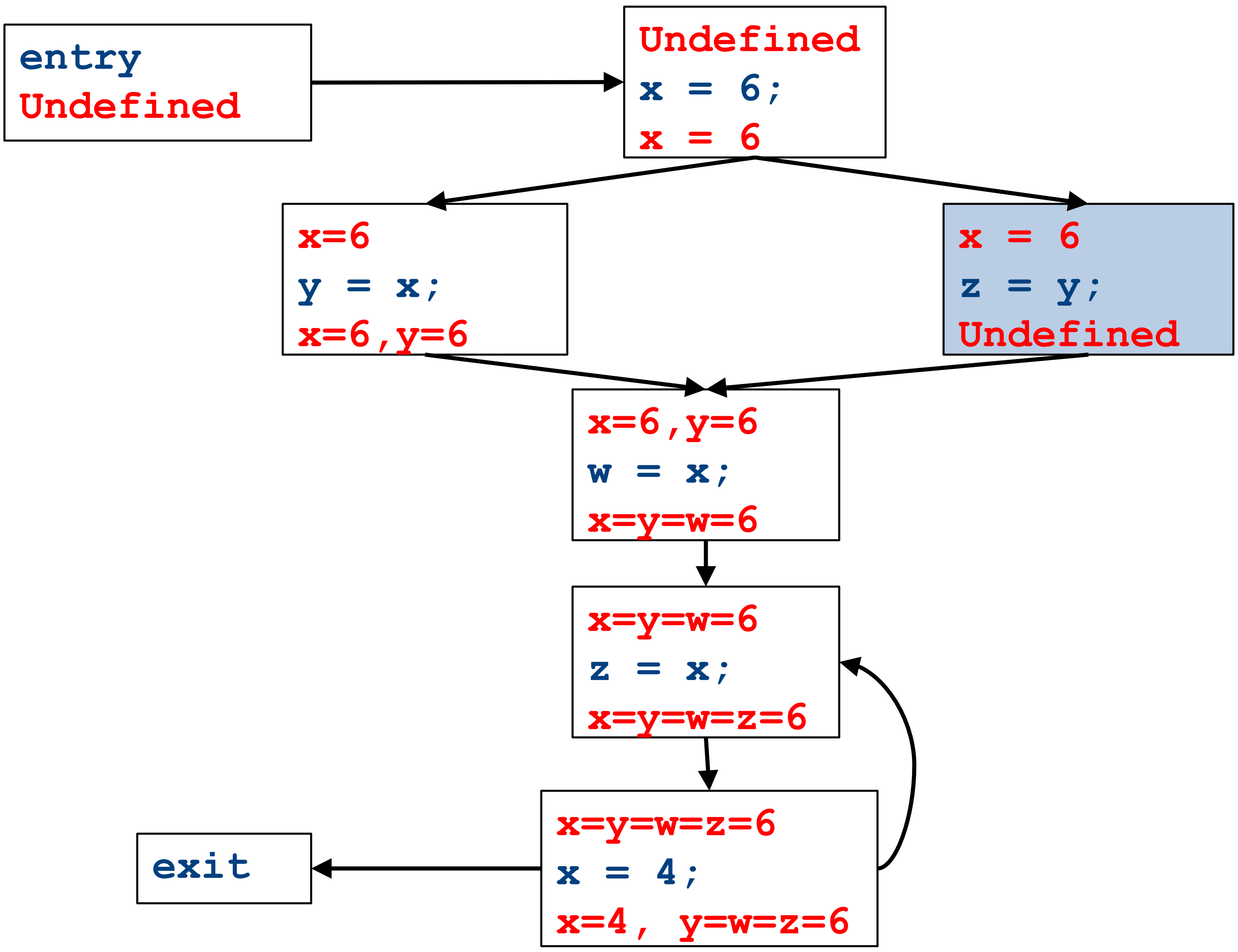

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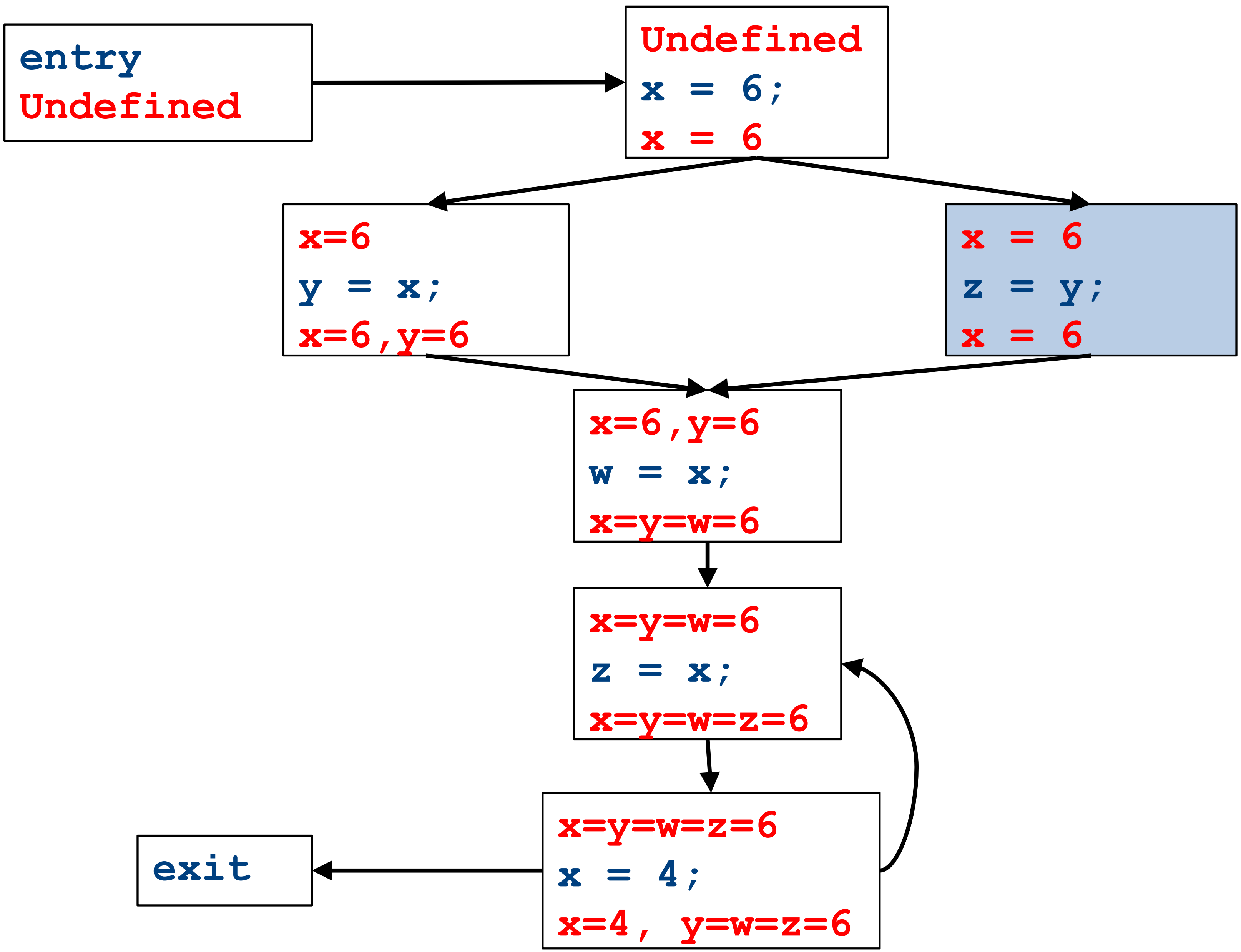

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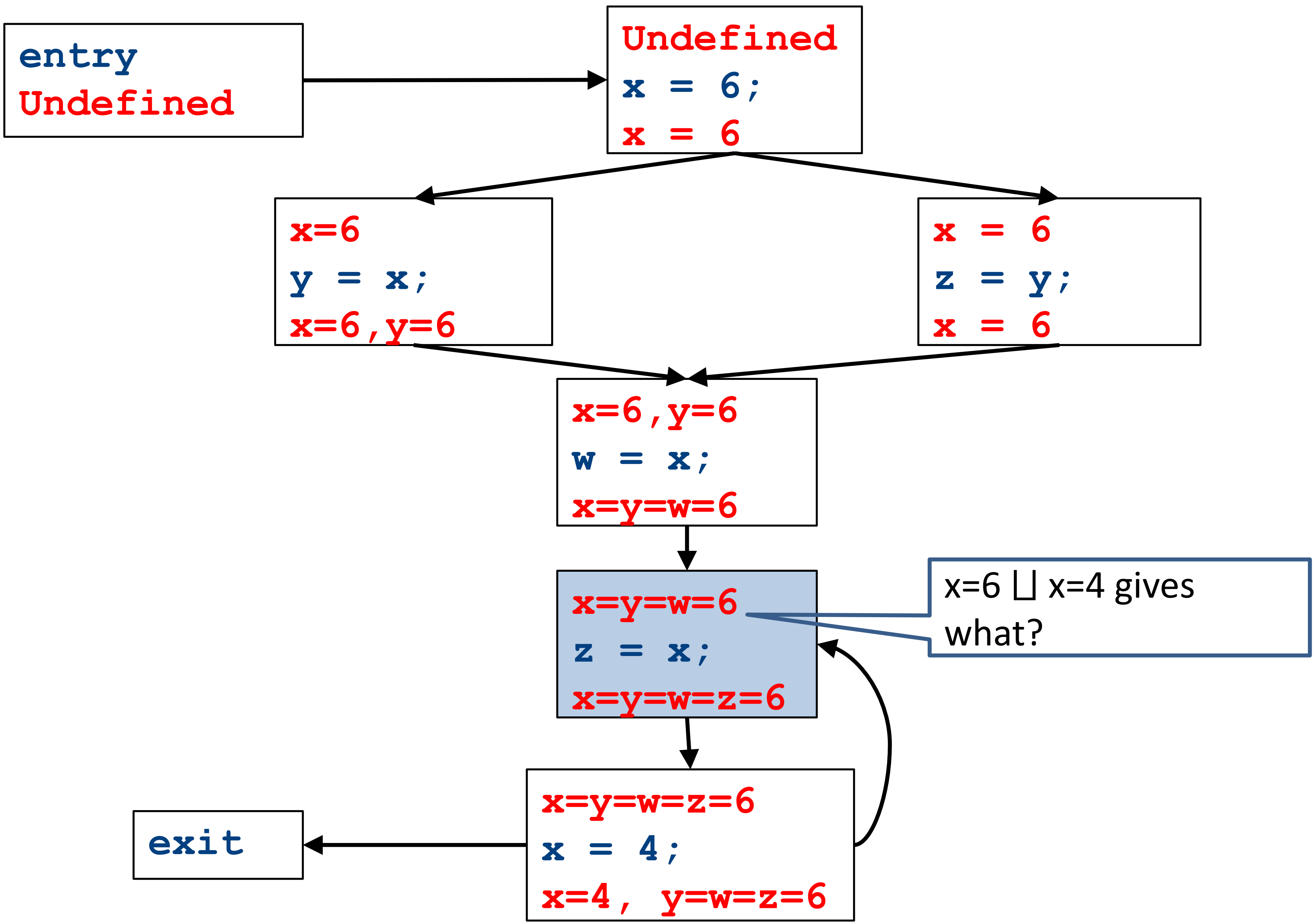

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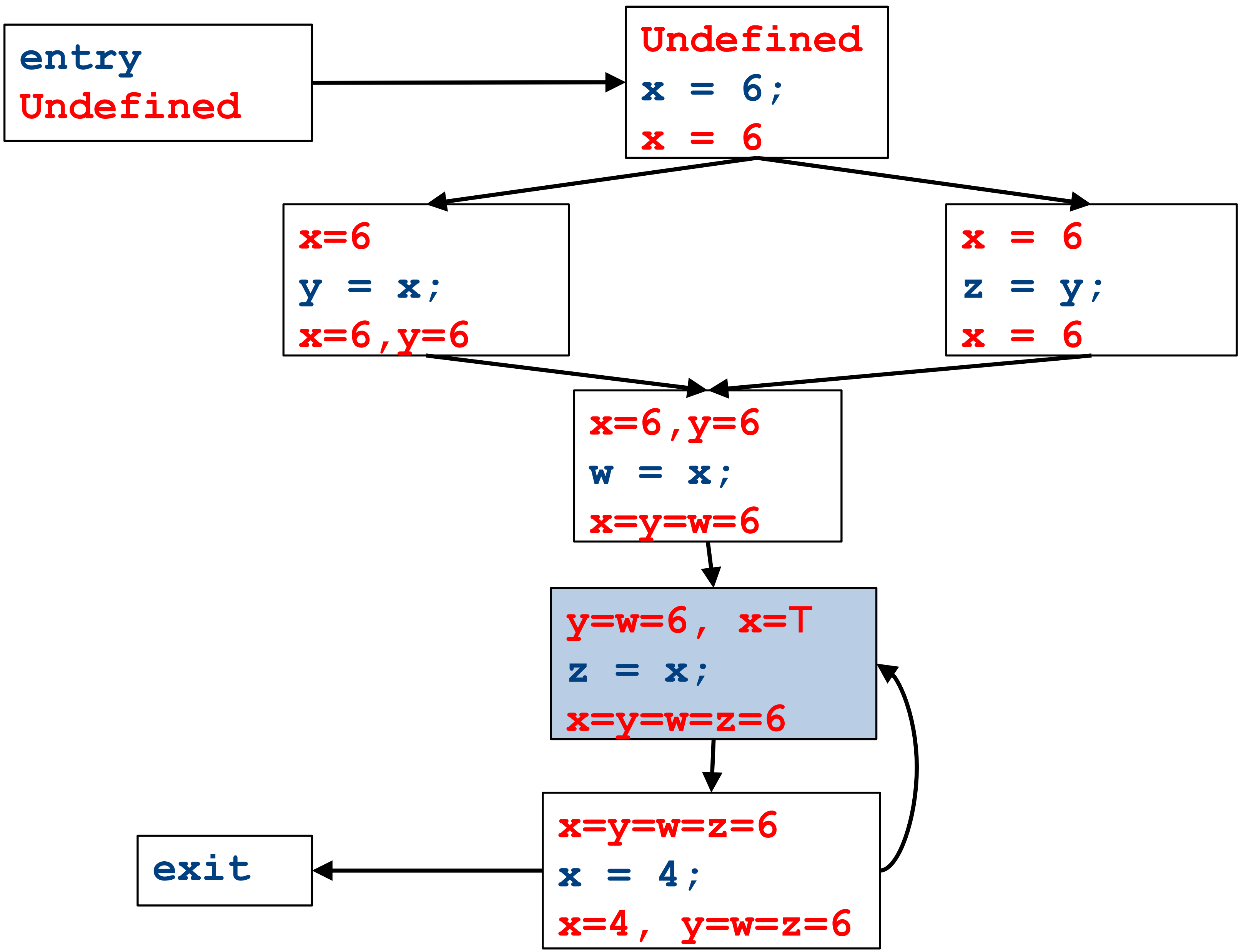

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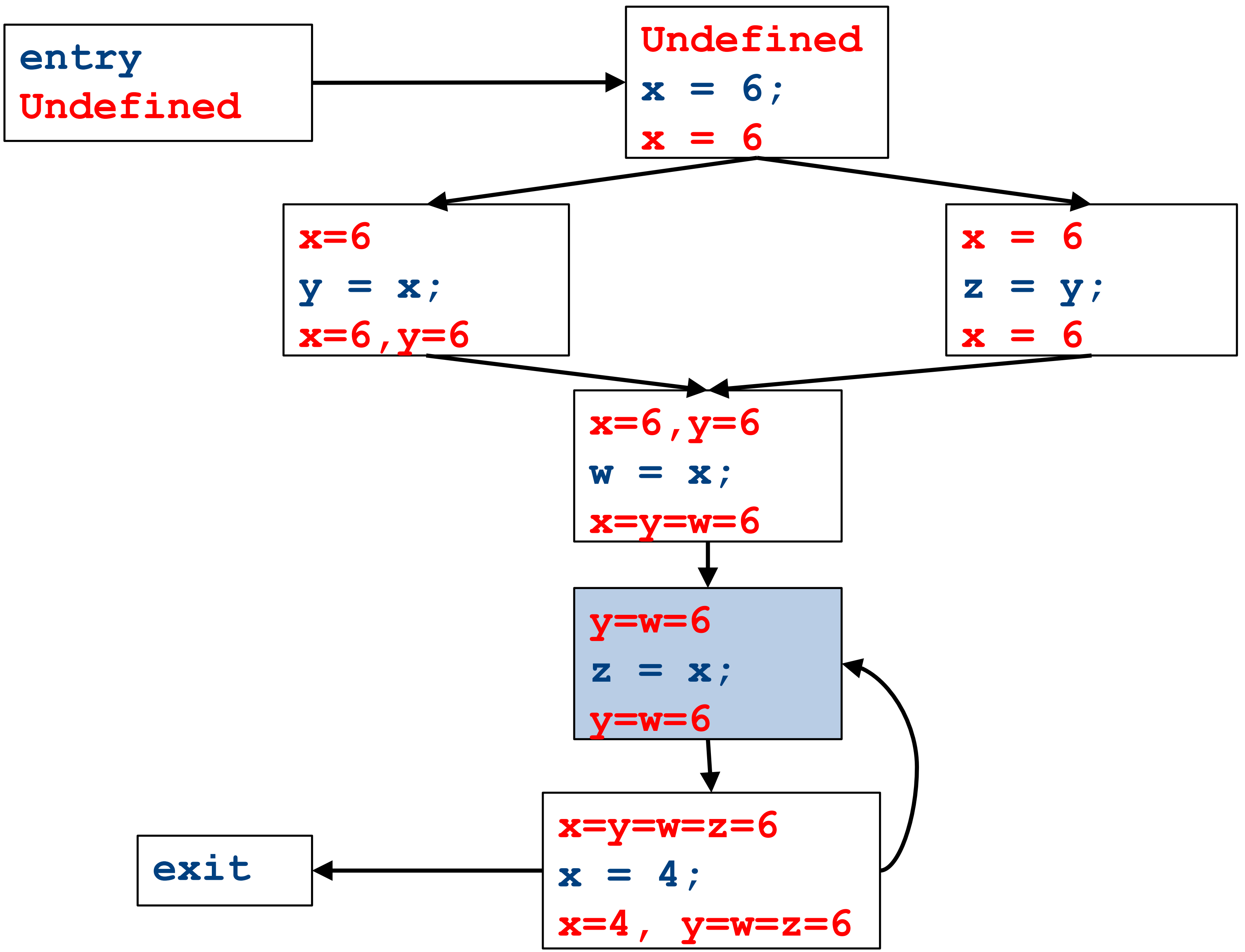

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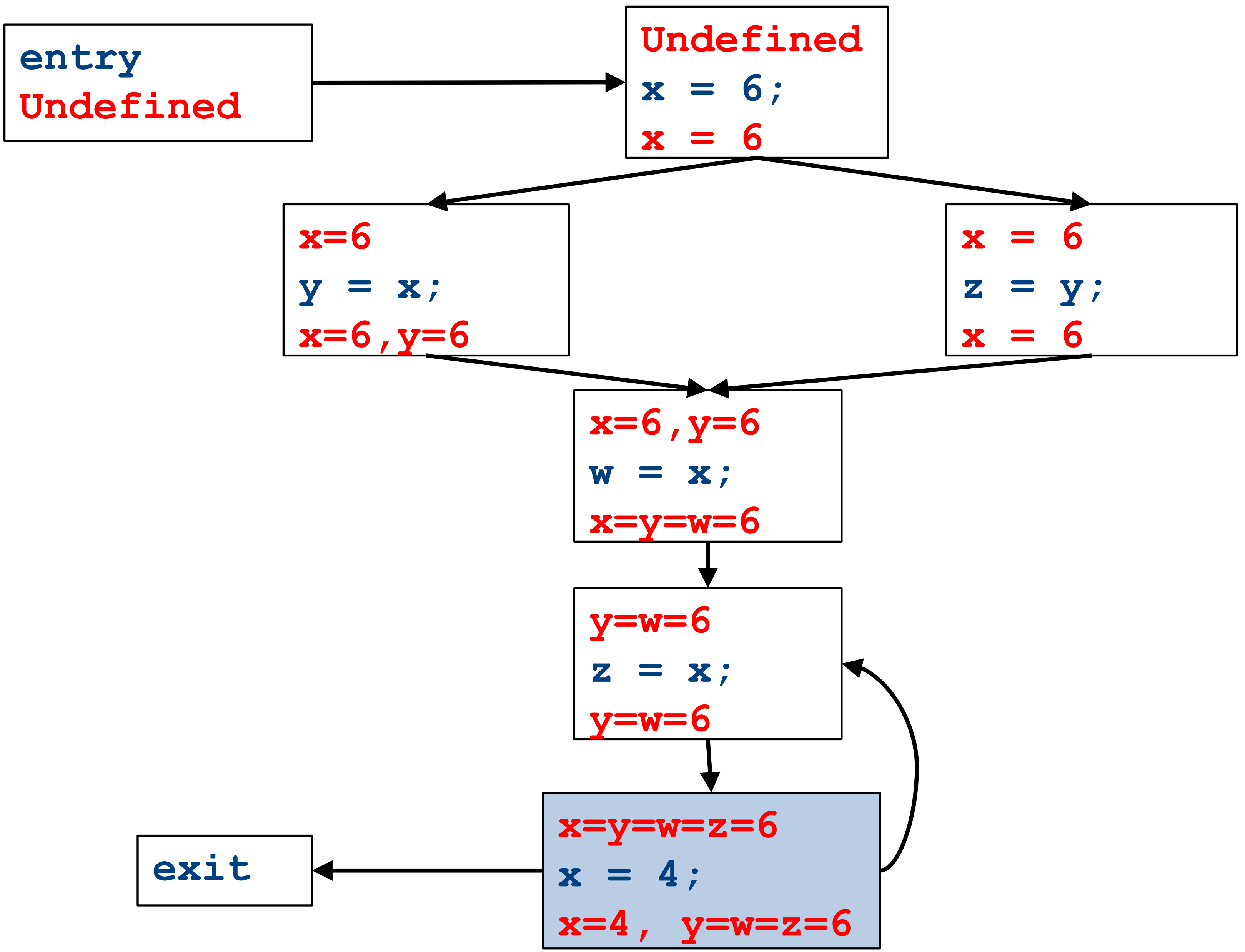

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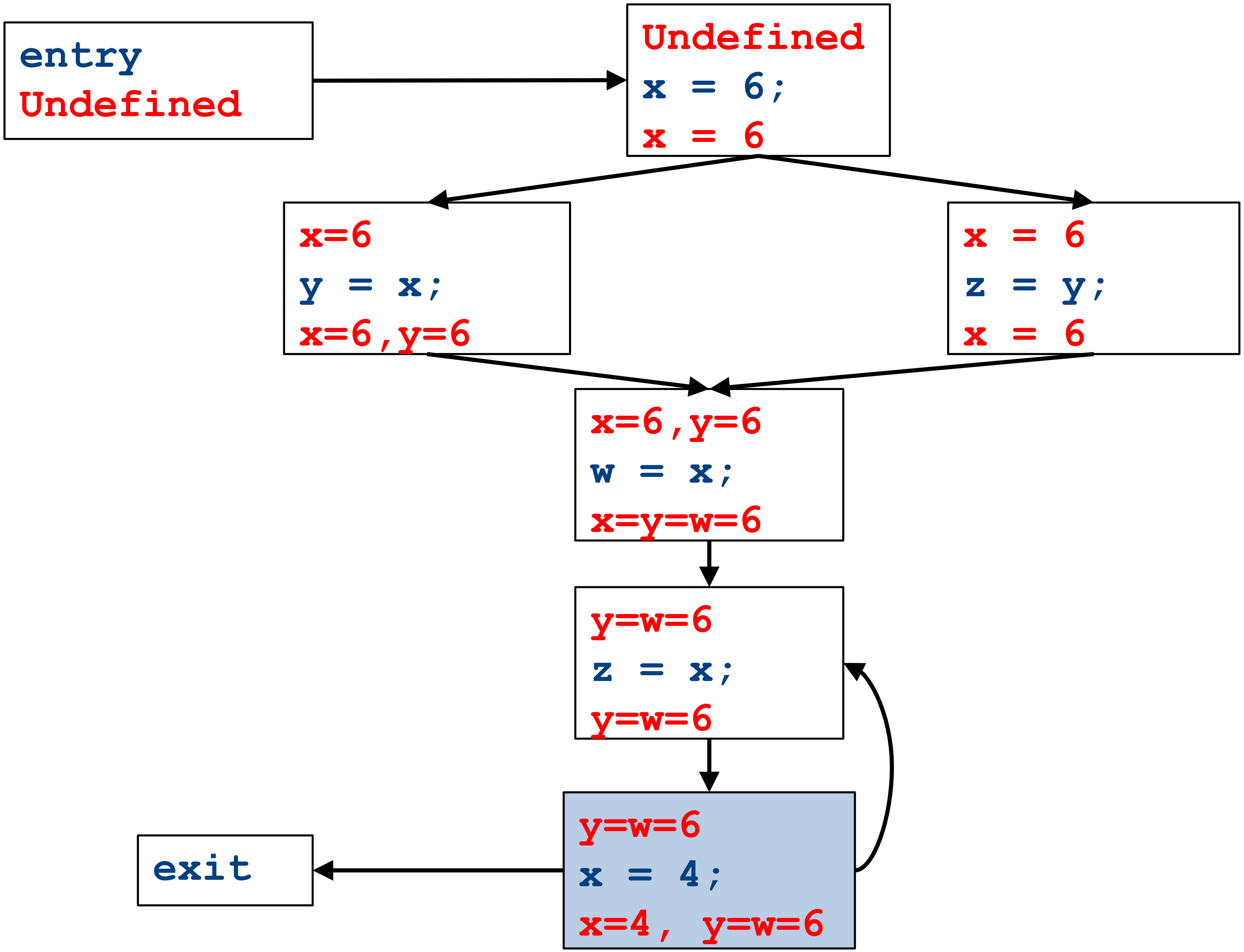

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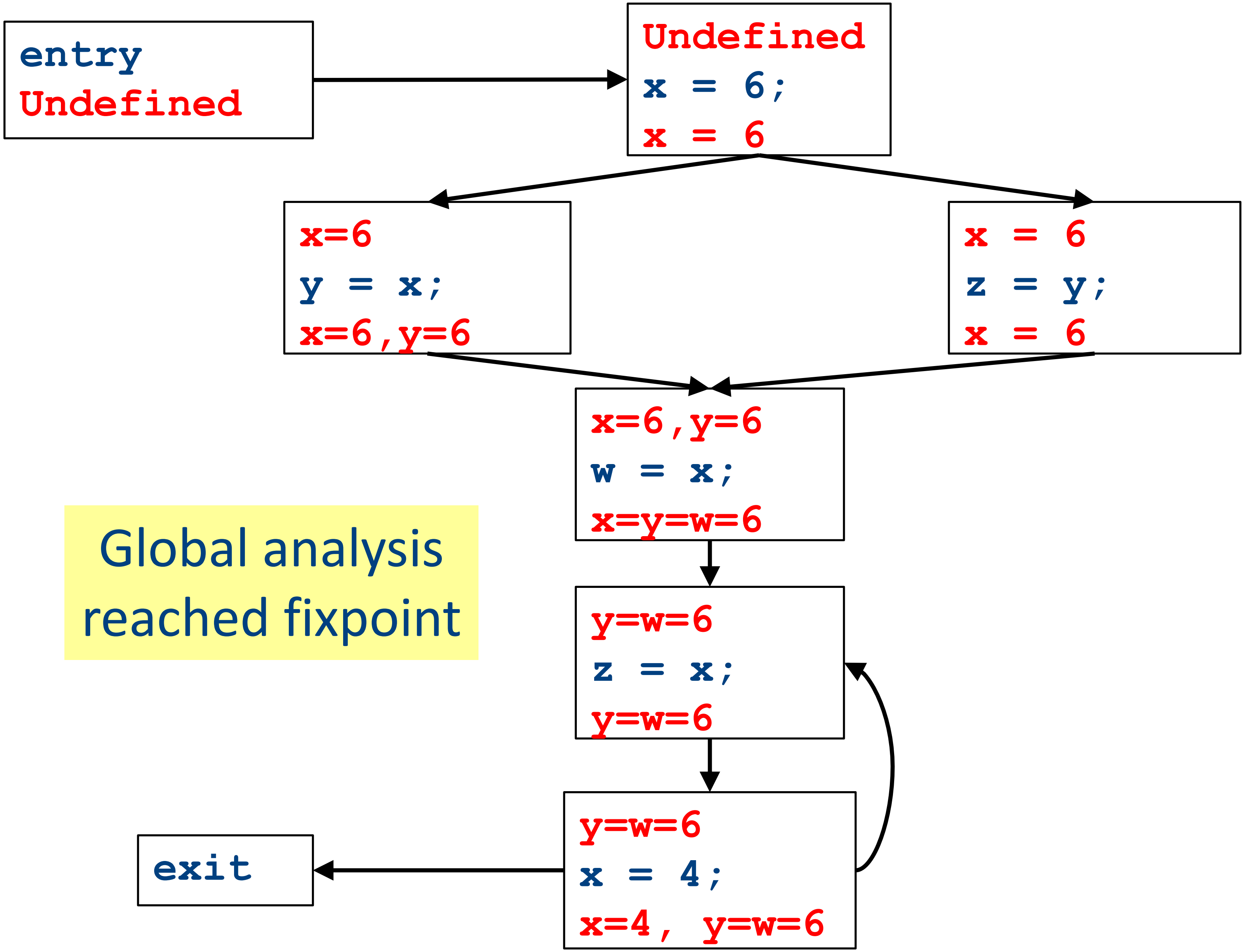
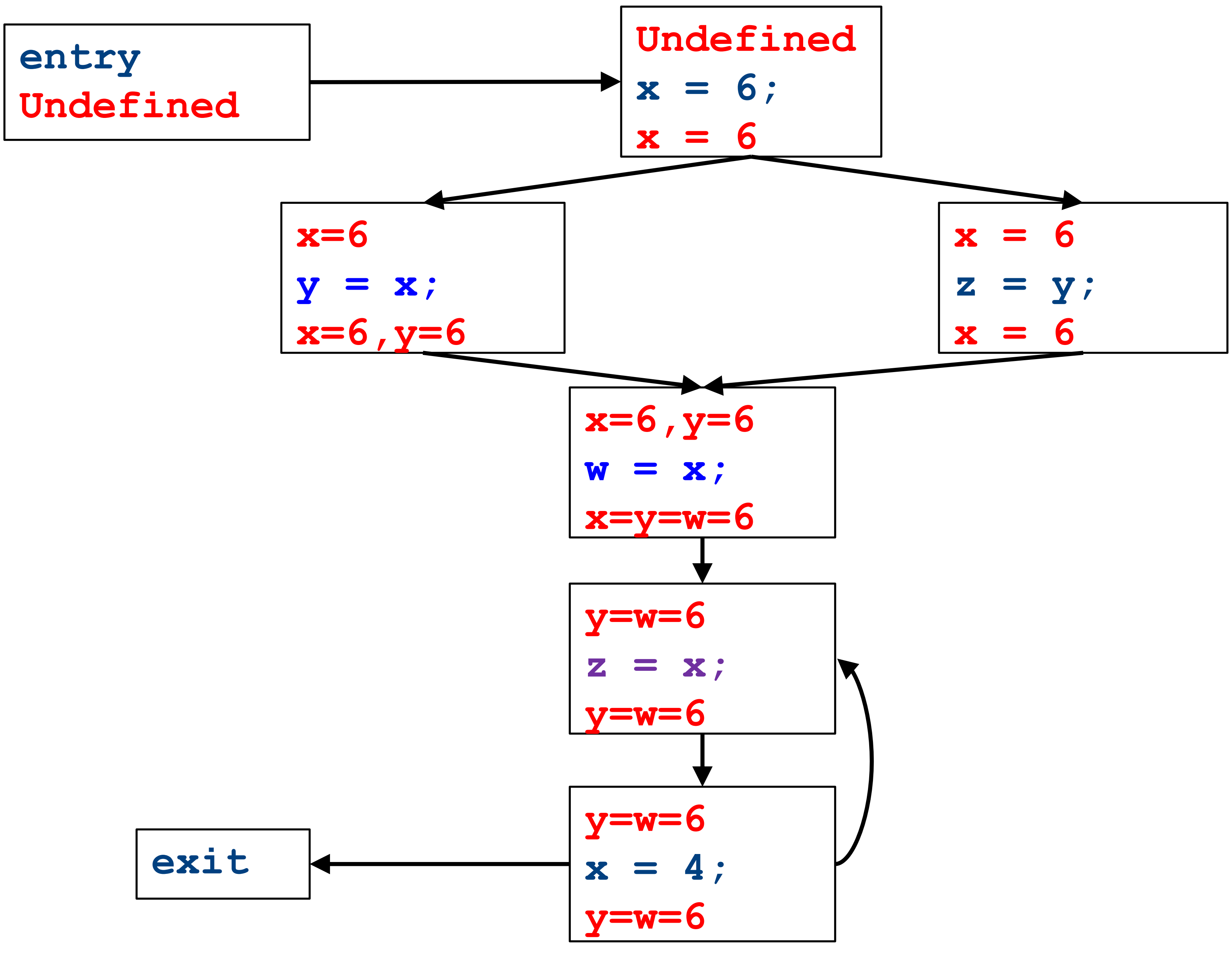


Global analysis reached fixpoint

Global constant propagation



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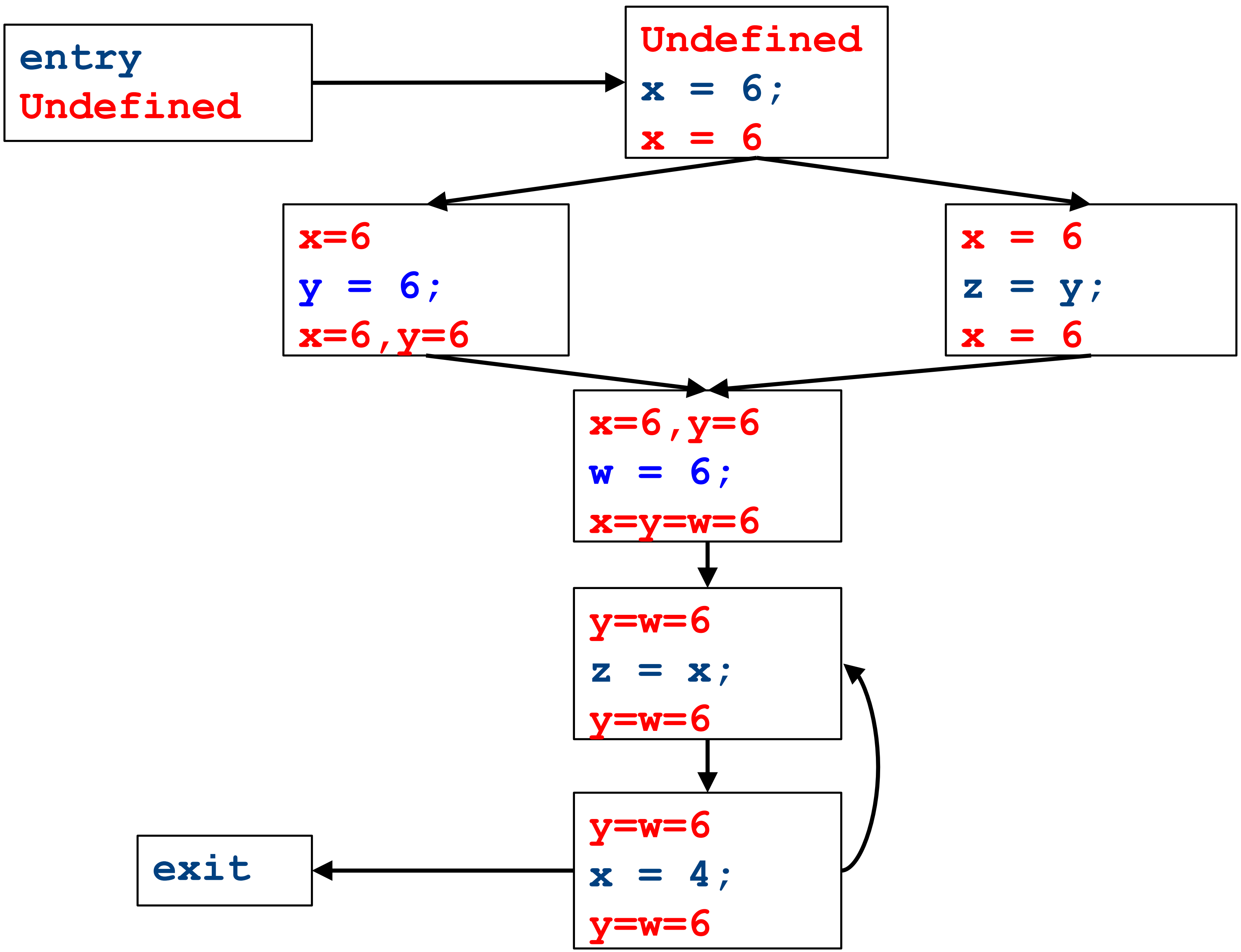

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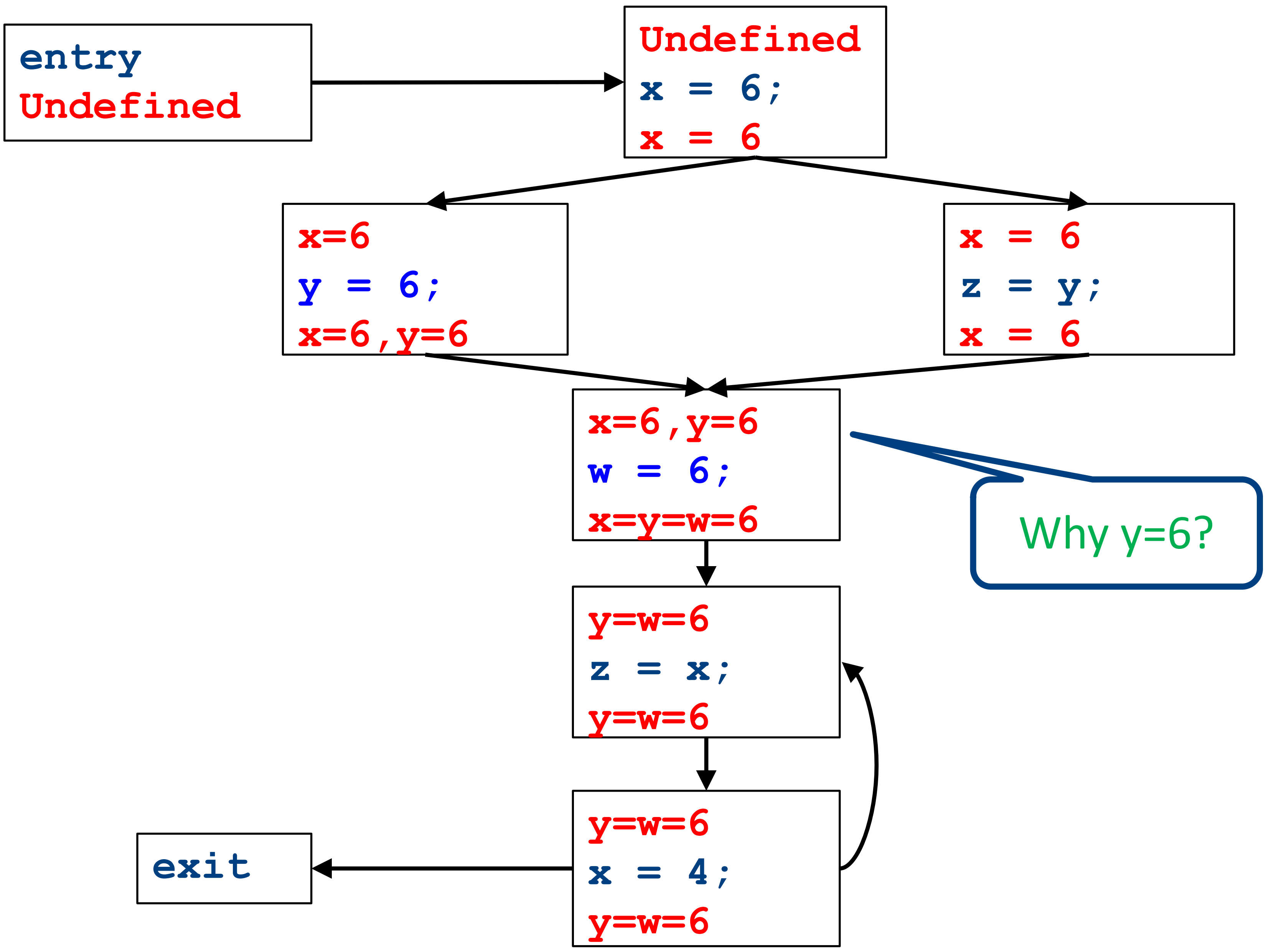


Dataflow for constant propagation

- Direction: **Forward**
- Semilattice: Vars→ {Undefined, 0, 1, -1, 2, -2, …, Not-a-Constant}
  - Join mapping for variables point-wise
    $\{x \mapsto 1, y \mapsto 1, z \mapsto 1\} \sqcup \{x \mapsto 1, y \mapsto 2, z \mapsto \text{Not-a-Constant}\} =$
    $\{x \mapsto 1, y \mapsto \text{Not-a-Constant}, z \mapsto \text{Not-a-Constant}\}$
- Transfer functions:
  - $f_{\mathtt{x=k}}(V) = V|_{x \mapsto k}$ *(update V by mapping x to k)*
  - $f_{\mathtt{x=a+b}}(V) = V|_{x \mapsto \text{Not-a-Constant}}$ *(assign Not-a-Constant)*
- Initial value: **x is Undefined**
  - (When might we use some other value?)

Proving termination

- Our algorithm for running these analyses continuously loops until no changes are detected
- Given this, how do we know the analyses will eventually terminate?
  - In general, **we don‘t**

Terminates?

Liveness Analysis

- A variable is live at a point in a program if later in the program its value will be read before it is written to again

Join semilattice definition

- A join semilattice is a pair $(V, \sqcup)$, where
- V is a domain of elements
- $\sqcup$ is a join operator that is
  - commutative: $x \sqcup y = y \sqcup x$
  - associative: $(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)$
  - idempotent: $x \sqcup x = x$
- If $x \sqcup y = z$, we say that z is the join or (**L**east **U**pper **B**ound) of x and y
- Every join semilattice has a bottom element denoted $\bot$ such that $\bot \sqcup x = x$ for all x

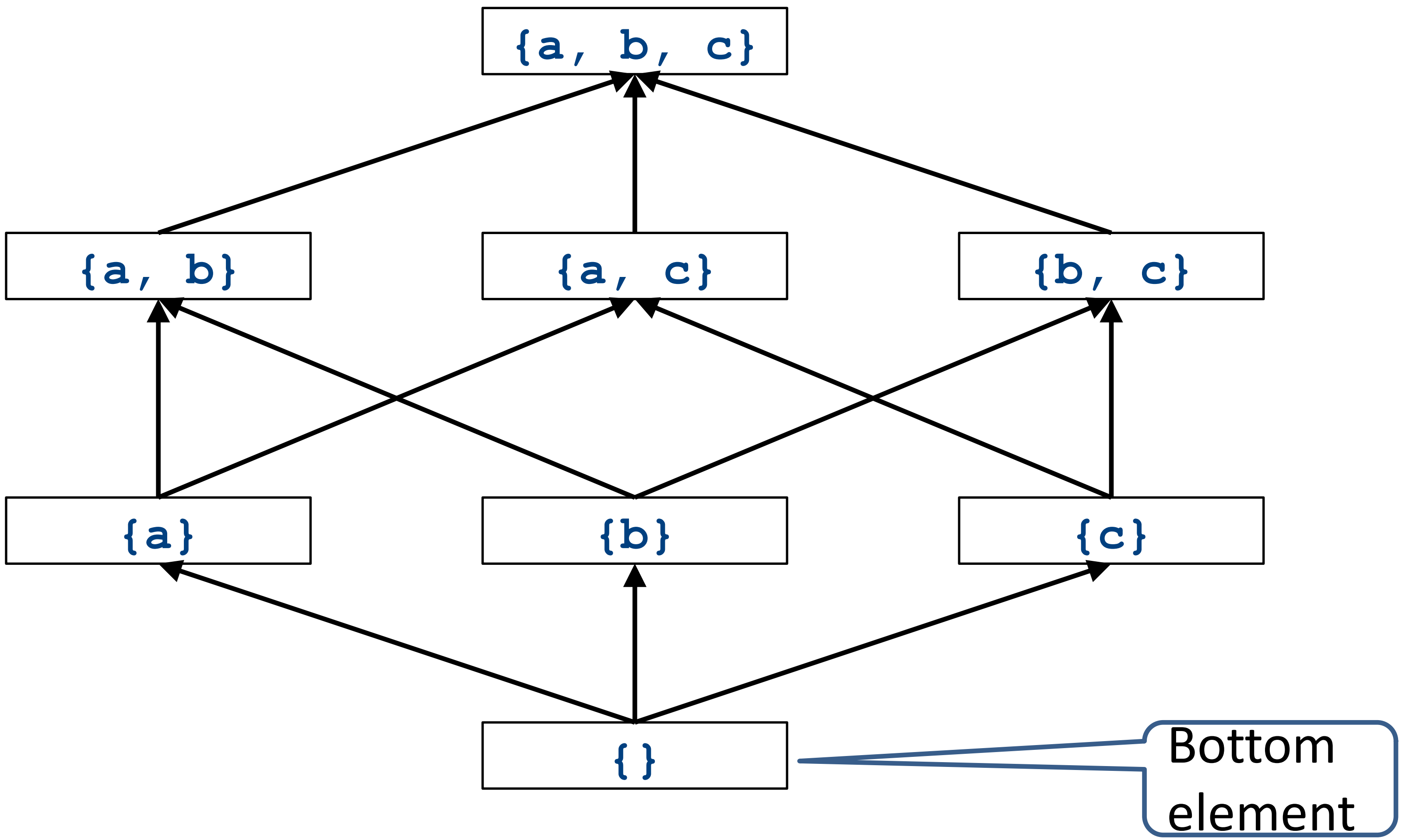
Partial ordering induced by join

- Every join semilattice $(V, \sqcup)$ induces an ordering relationship $\sqsubseteq$ over its elements
- Define $x \sqsubseteq y$ iff $x \sqcup y = y$
- Need to prove
  - Reflexivity: $x \sqsubseteq x$
  - Antisymmetry: If $x \sqsubseteq y$ and $y \sqsubseteq x$, then $x = y$
  - Transitivity: If $x \sqsubseteq y$ and $y \sqsubseteq z$, then $x \sqsubseteq z$

A join semilattice for liveness

- Sets of live variables and the set union operation
- Idempotent:
  - $x \cup x = x$
- Commutative:
  - $x \cup y = y \cup x$
- Associative:
  - $(x \cup y) \cup z = x \cup (y \cup z)$
- Bottom element:
  - The empty set: $\emptyset \cup x = x$
- Ordering over elements = subset relation

Join semilattice example for liveness



Dataflow framework

- A global analysis is a tuple (D, V, ⊔, F, I), where
  - D is a direction (forward or backward)
    - The order to visit statements within a basic block, **NOT** the order in which to visit the basic blocks
  - V is a set of values (sometimes called domain)
  - ⊔ is a join operator over those values
  - F is a set of transfer functions f_s : **V → V** (for every statement s)
  - I is an initial value

Running global analyses

- Assume that $(\mathbb{D}, \mathbb{V}, \sqcup, \mathbb{F}, \mathbb{I})$ is a forward analysis
- For every statement s maintain values before - IN[s] - and after - OUT[s]
- Set OUT[**s**] = $\bot$ for all statements **s**
- Set OUT[**entry**] = $\mathbb{I}$
- Repeat until no values change:
  - For each statement **s** with predecessors PRED[s]={$\mathbf{p_1}$, $\mathbf{p_2}$, … , $\mathbf{p_n}$}
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] $\sqcup$ OUT[$\mathbf{p_2}$] $\sqcup$ … $\sqcup$ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s(IN[**s**])
- The order of this iteration does not matter
  - Chaotic iteration

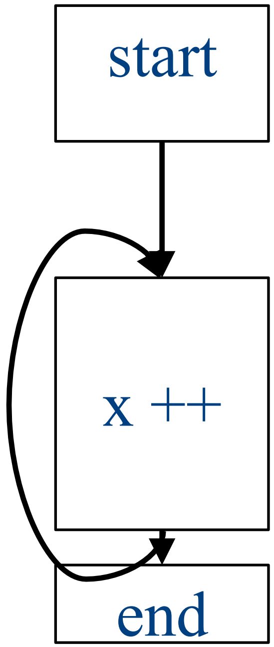
Proving termination

- Our algorithm for running these analyses continuously loops until no changes are detected
- Problem: how do we know the analyses will eventually terminate?

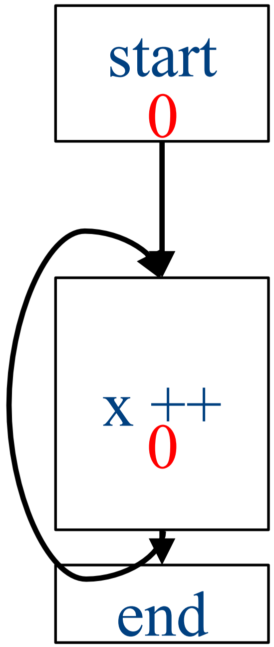
A non-terminating analysis

- The following analysis will loop infinitely on any CFG containing a loop:
- Direction: Forward
- Domain: $\mathbb{N}$
- Join operator: **max**
- Transfer function: $f(n) = n + 1$
- Initial value: 0

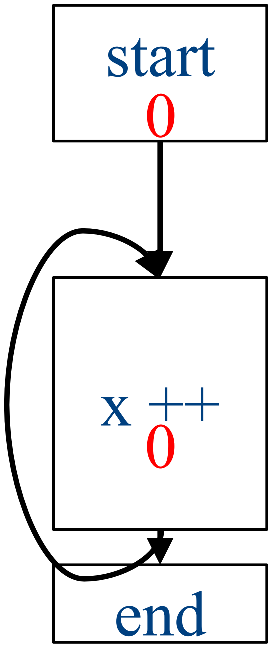
A non-terminating analysis



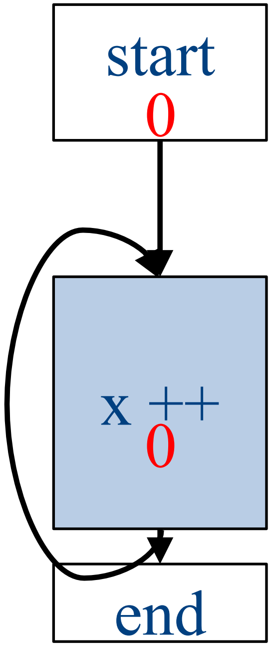
Initialization



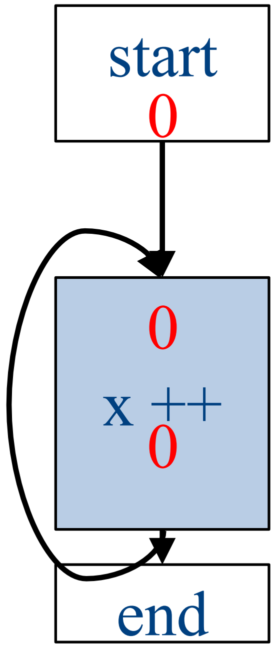
Fixed-point iteration



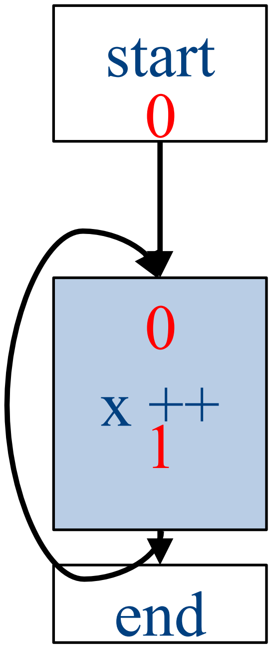
Choose a block



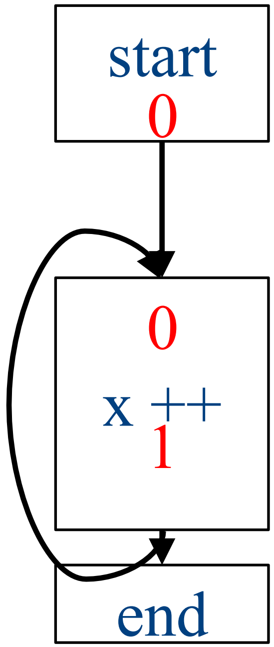
Iteration 1



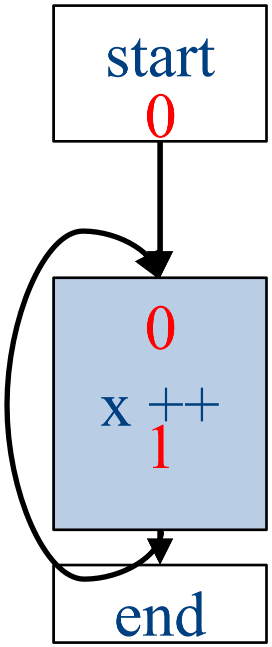
Iteration 1



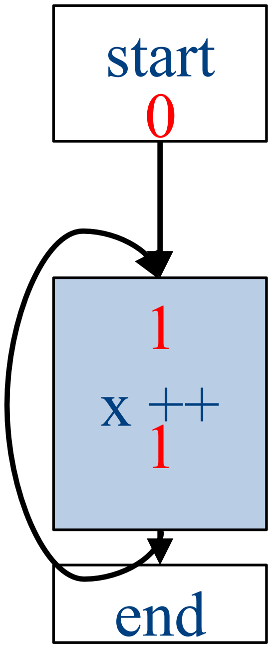
Choose a block



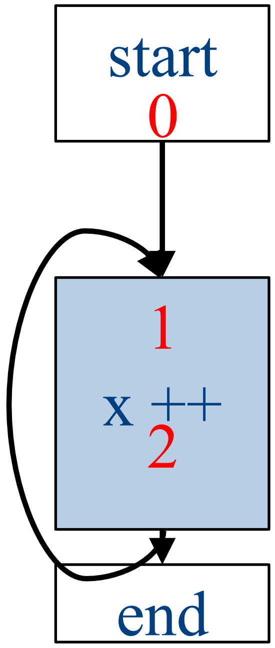
Iteration 2



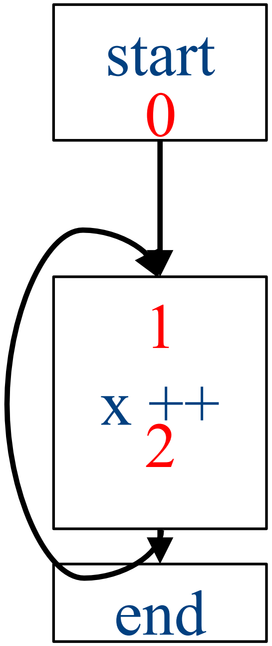
Iteration 2



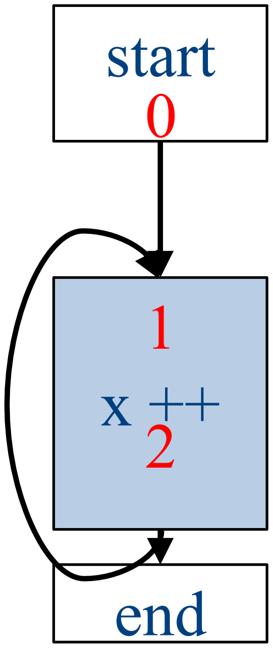
Iteration 2



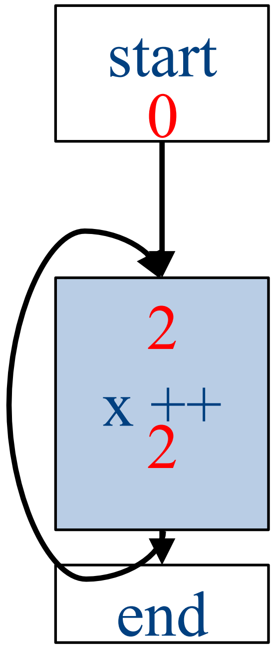
Choose a block



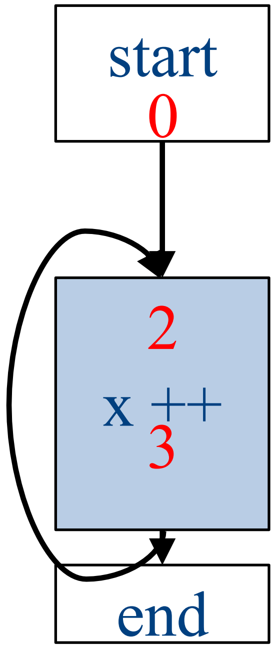
Iteration 3



Iteration 3

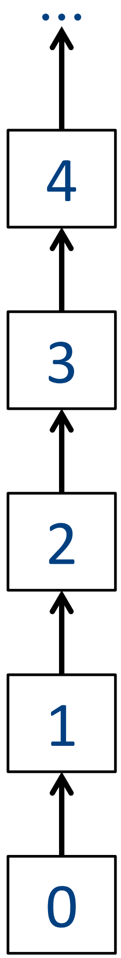


Iteration 3



Why doesn't this terminate?

- Values can increase without bound
- Note that "increase" refers to the lattice ordering, not the ordering on the natural numbers
- The height of a semilattice is the length of the longest increasing sequence in that semilattice
- The dataflow framework is not guaranteed to terminate for semilattices of infinite height
- Note that a semilattice can be infinitely large but have finite height
  – e.g. constant propagation



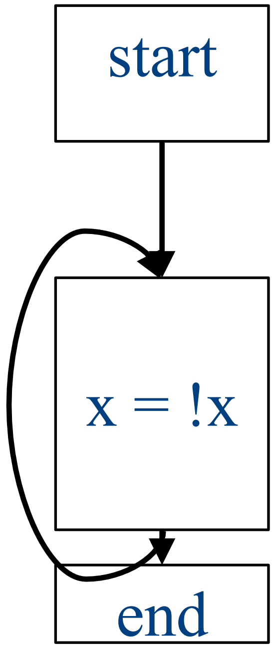
Height of a lattice

- An increasing chain is a sequence of elements $\bot \sqsubseteq a_1 \sqsubseteq a_2 \sqsubseteq \ldots \sqsubseteq a_k$
  - The length of such a chain is k
- The height of a lattice is the length of the maximal increasing chain
- For liveness with *n* program variables:
  - $\{\} \subseteq \{v_1\} \subseteq \{v_1, v_2\} \subseteq \ldots \subseteq \{v_1, \ldots, v_n\}$
- For available expressions it is the number of expressions of the form a=b op c
  - For *n* program variables and *m* operator types: mn^3

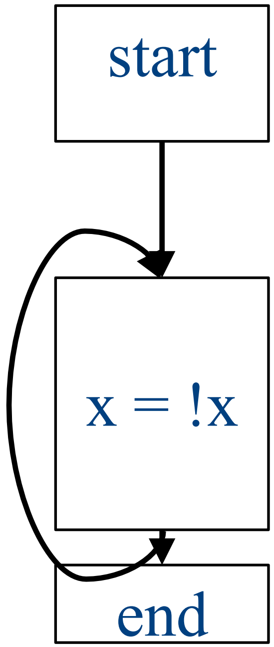
Another non-terminating analysis

- This analysis works on a finite-height semilattice, but will not terminate on certain CFGs:
- Direction: Forward
- Domain: Boolean values `true` and `false`
- Join operator: Logical OR
- Transfer function: Logical NOT
- Initial value: `false`

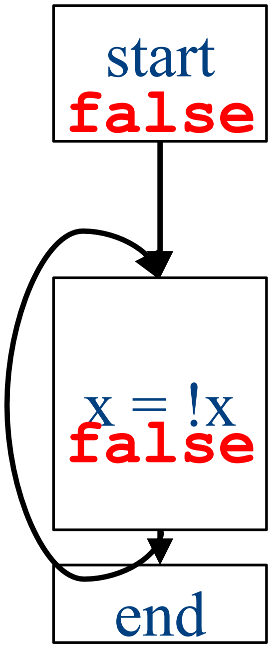
A non-terminating analysis



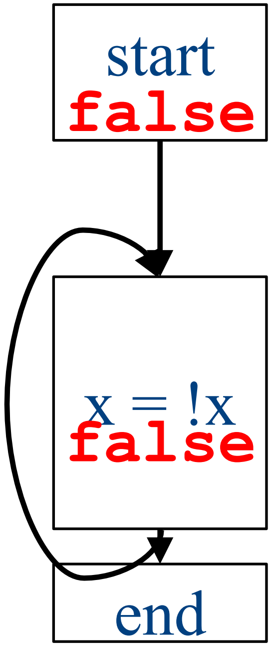
A non-terminating analysis



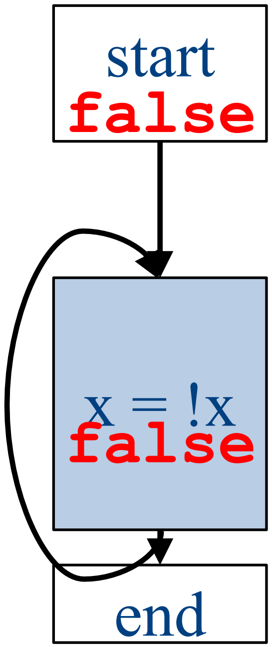
Initialization



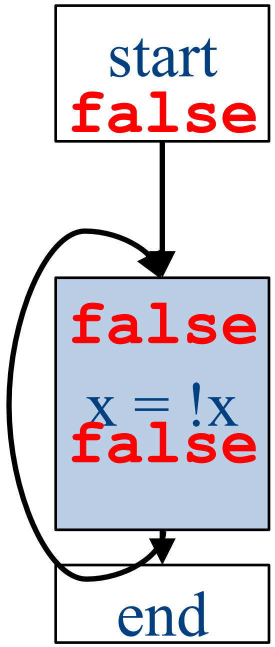
Fixed-point iteration



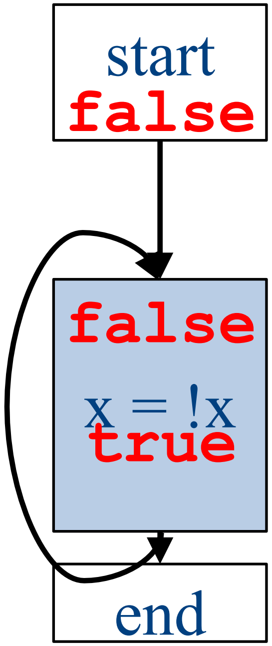
Choose a block



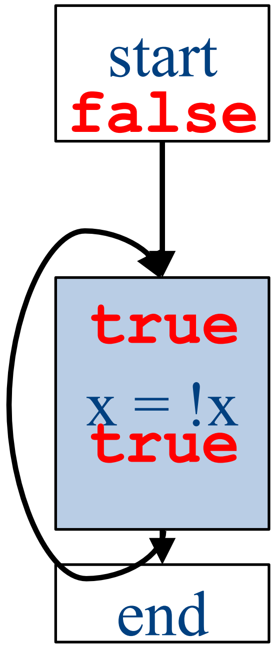
Iteration 1



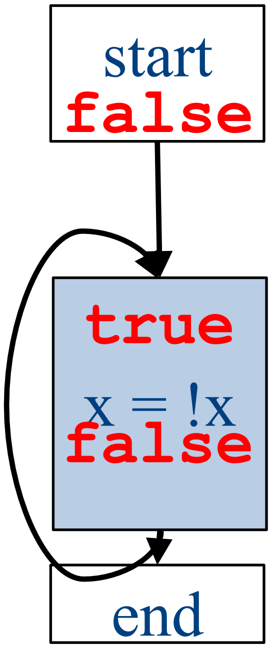
Iteration 1



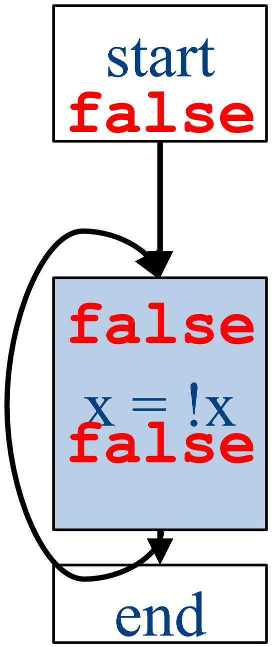
Iteration 2



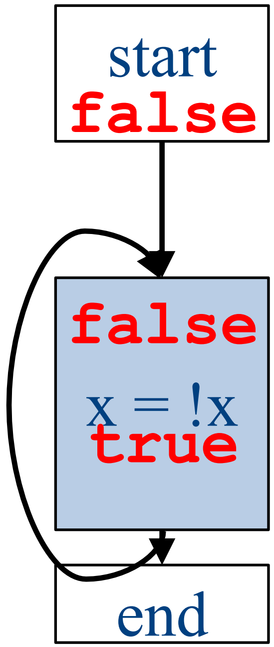
Iteration 2



Iteration 3

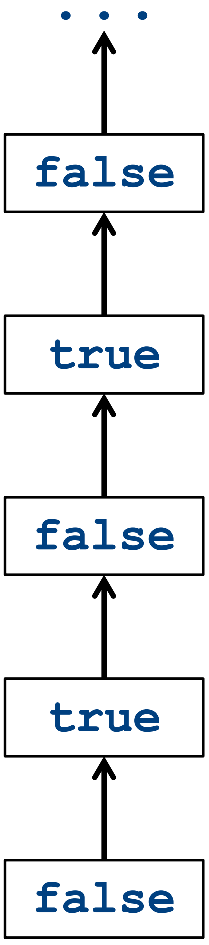


Iteration 3



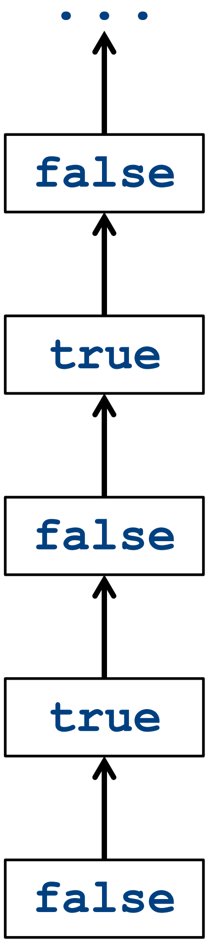
Why doesn't it terminate?

- Values can loop indefinitely
- Intuitively, the join operator keeps pulling values up
- If the transfer function can keep pushing values back down again, then the values might cycle forever



Why doesn't it terminate?

- Values can loop indefinitely
- Intuitively, the join operator keeps pulling values up
- If the transfer function can keep pushing values back down again, then the values might cycle forever
- How can we fix this?



Monotone transfer functions

- A transfer function *f* is monotone iff
  if $x \sqsubseteq y$, then $f(x) \sqsubseteq f(y)$
- Intuitively, if you know less information about a program point, you can't "gain back" more information about that program point
- Many transfer functions are monotone, including those for liveness and constant propagation
- Note: Monotonicity does **not** mean that $x \sqsubseteq f(x)$
  - (This is a different property called extensivity)

Liveness and monotonicity

- A transfer function f is monotone iff

  if $x \sqsubseteq y$, then $f(x) \sqsubseteq f(y)$
- Recall our transfer function for **a = b + c** is
  - $f_{a = b + c}(V) = (V - \{a\}) \cup \{b, c\}$
- Recall that our join operator is set union and induces an ordering relationship

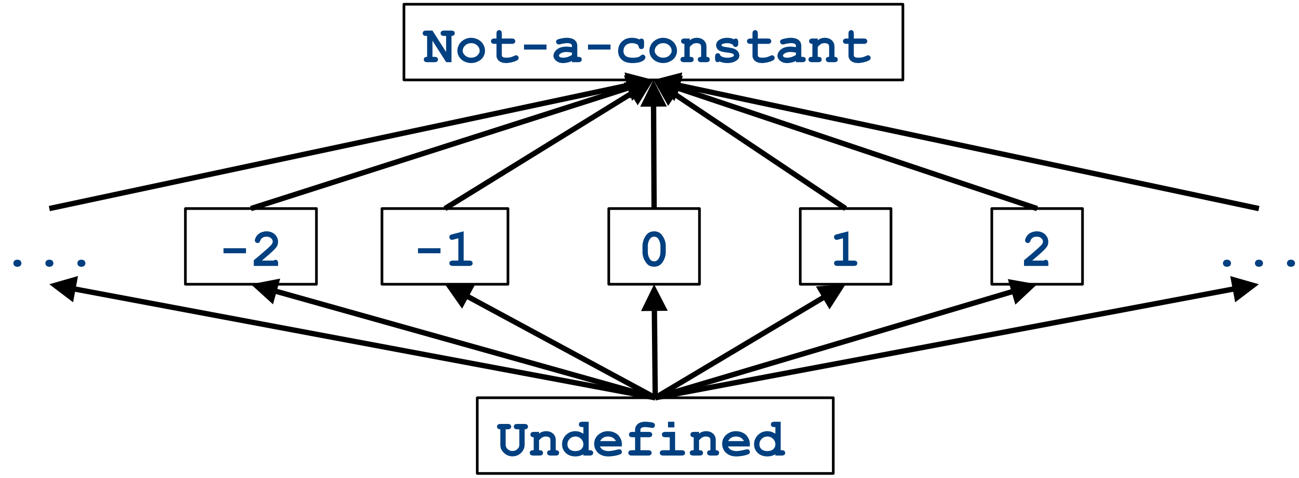
  $X \sqsubseteq Y$ iff $X \subseteq Y$
- Is this monotone?

Is constant propagation monotone?

- A transfer function *f* is monotone iff

  if $x \sqsubseteq y$, then $f(x) \sqsubseteq f(y)$

- Recall our transfer functions
  - $f_{\mathbf{x=k}}(V) = V[x \mapsto k]$ *(update V by mapping x to k)*
  - $f_{\mathbf{x=a+b}}(V) = V[x \mapsto \text{Not-a-Constant}]$ *(assign Not-a-Constant)*
- Is this monotone?



The grand result

- Theorem: A dataflow analysis with a **finite-height semilattice** and family of **monotone transfer functions** always terminates
- Proof sketch:
  - The join operator can only bring values up
  - Transfer functions can never lower values back down below where they were in the past (monotonicity)
  - Values cannot increase indefinitely (finite height)

An "optimality" result

- A transfer function f is distributive if
  $$f(a \sqcup b) = f(a) \sqcup f(\mathrm{b})$$
  for every domain elements a and b
- If all transfer functions are distributive then the fixed-point solution is the solution that would be computed by joining results from all (potentially infinite) control-flow paths
  - Join over all paths
- Optimal if we ignore program conditions

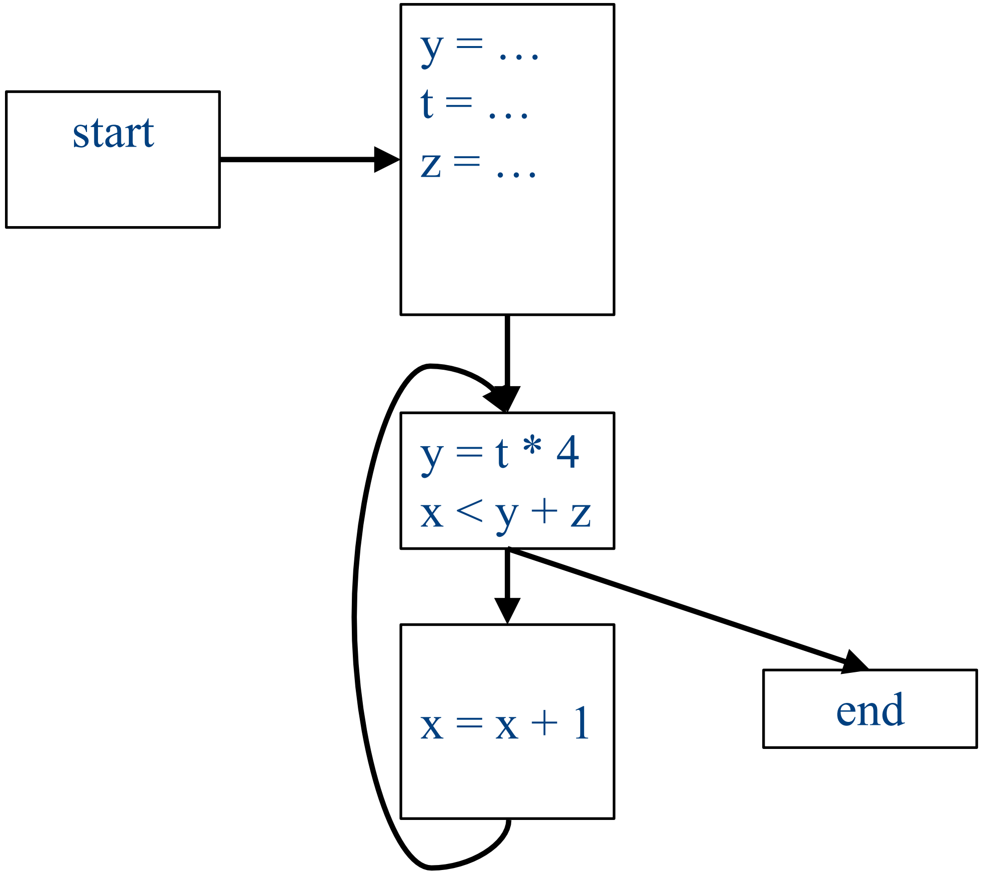
An “optimality” result

- A transfer function *f* is distributive if
  $$f(a \sqcup b) = f(a) \sqcup f(\mathrm{b})$$
  for every domain elements *a* and *b*
- If all transfer functions are distributive then the fixed-point solution is equal to the solution computed by joining results from all (potentially infinite) control-flow paths
  - Join over all paths
- Optimal if we pretend all control-flow paths can be executed by the program
- Which analyses use distributive functions?

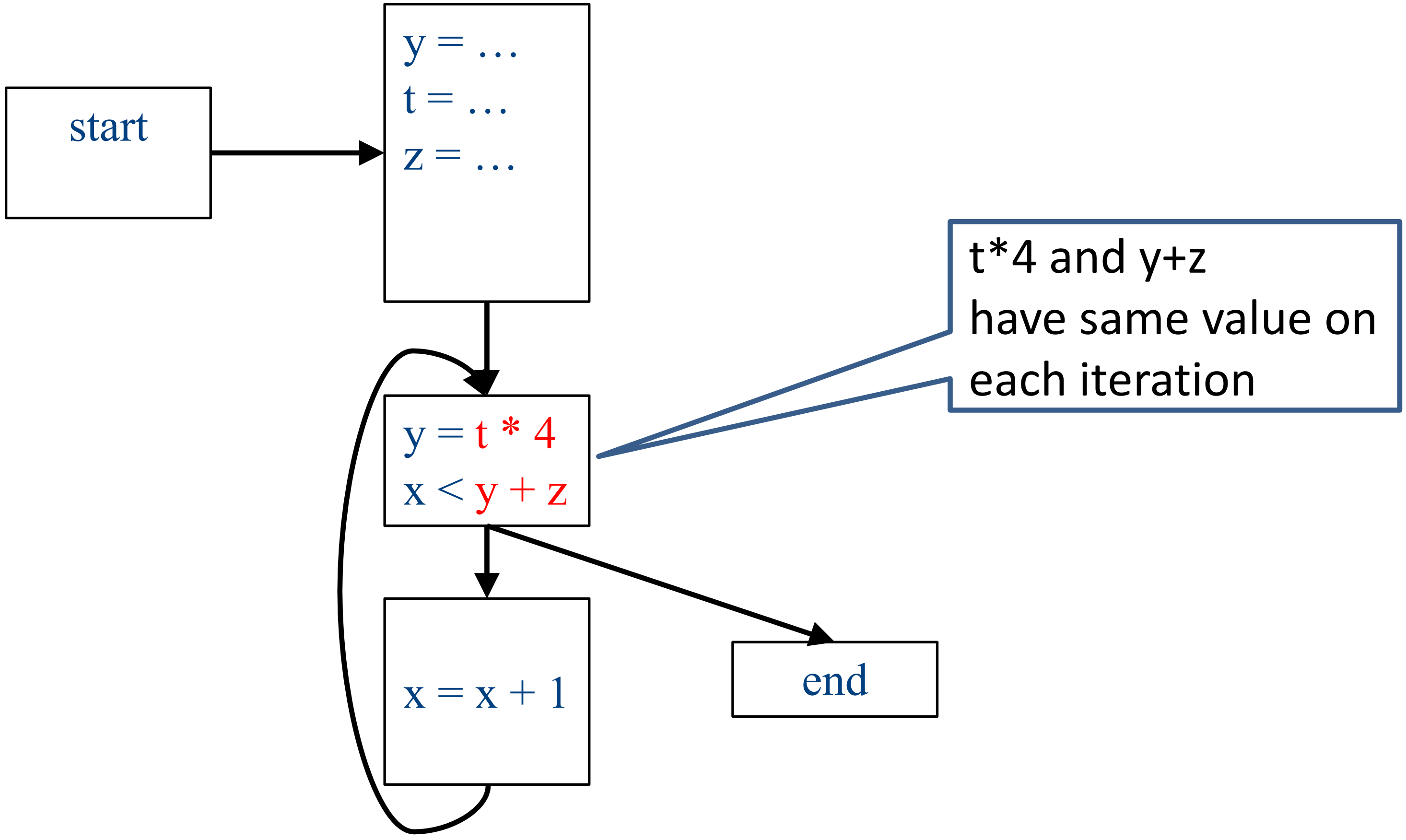
Loop optimizations

- Most of a program's computations are done inside loops
  - Focus optimizations effort on loops
- The optimizations we've seen so far are independent of the control structure
- Some optimizations are specialized to loops
  - Loop-invariant code motion
  - (Strength reduction via induction variables)
- Require another type of analysis to find out where expressions get their values from
  - Reaching definitions
    - (Also useful for improving register allocation)

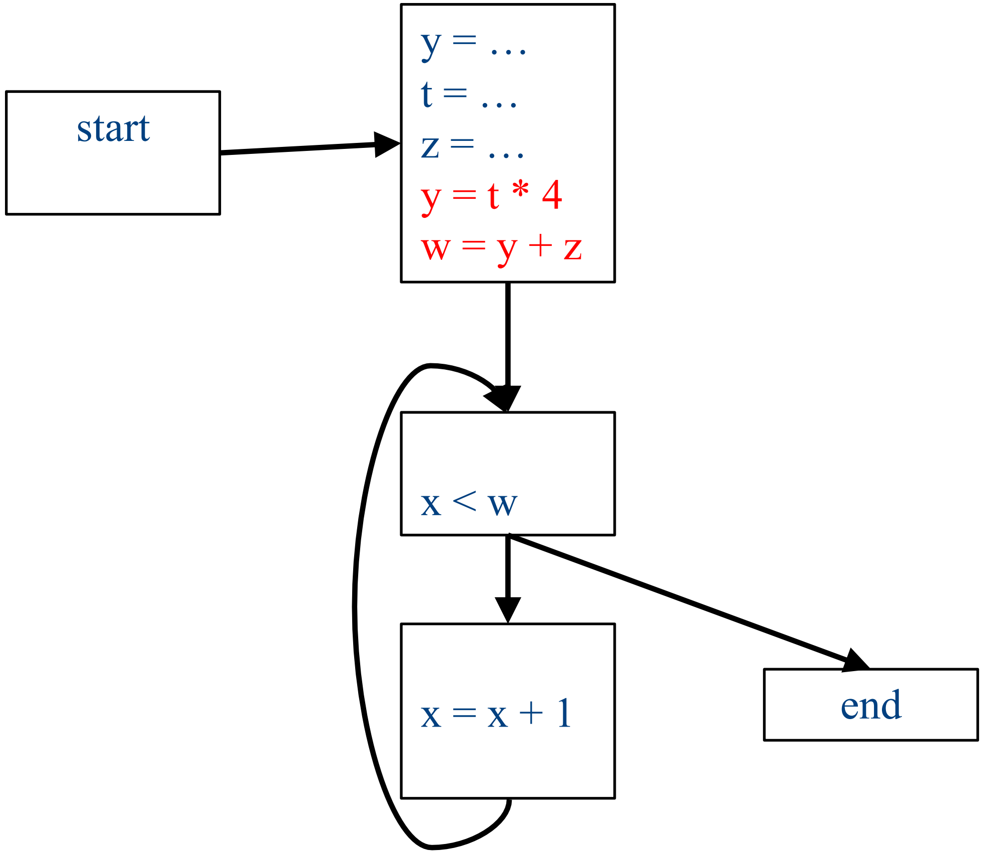
Loop invariant computation



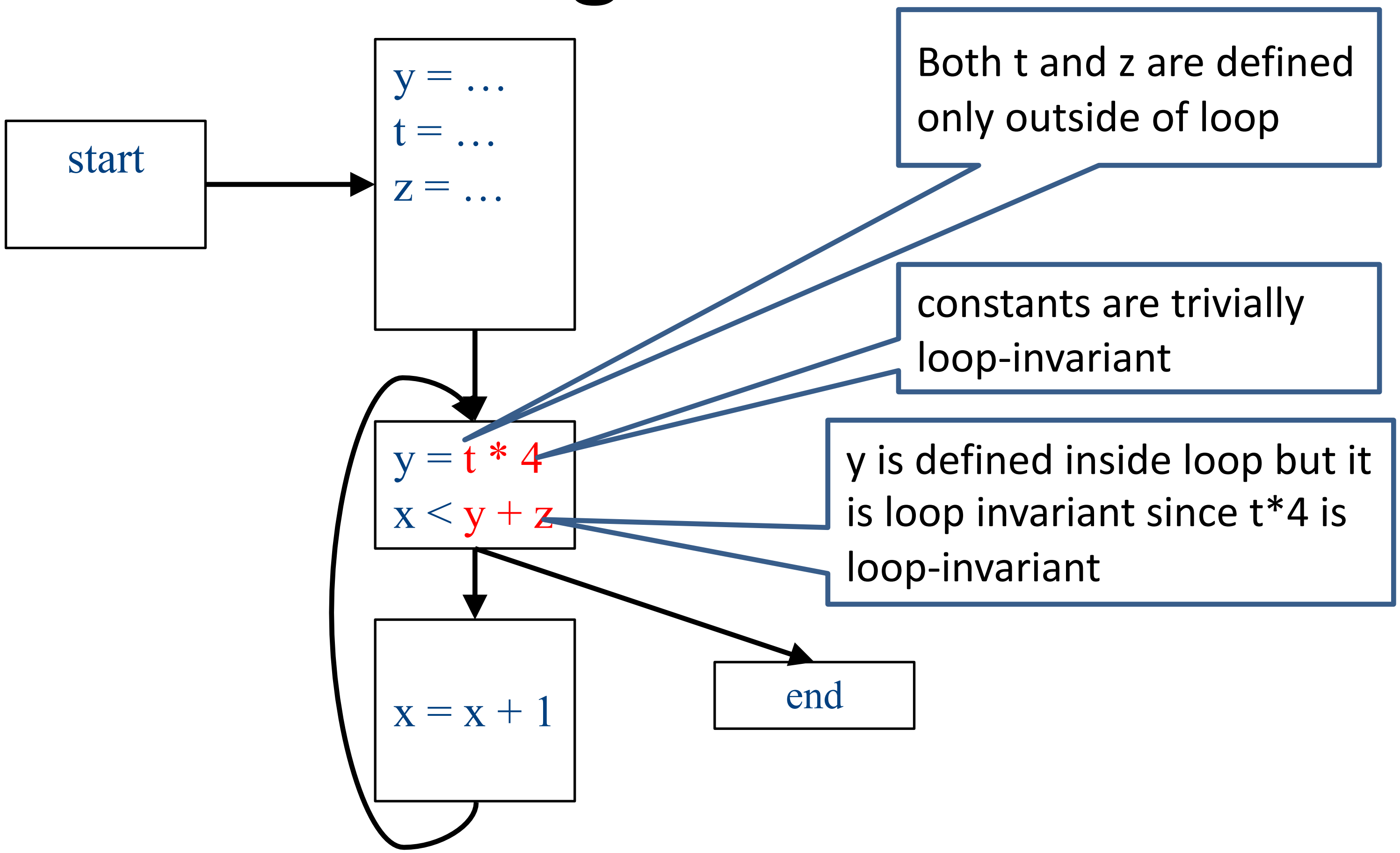
Loop invariant computation



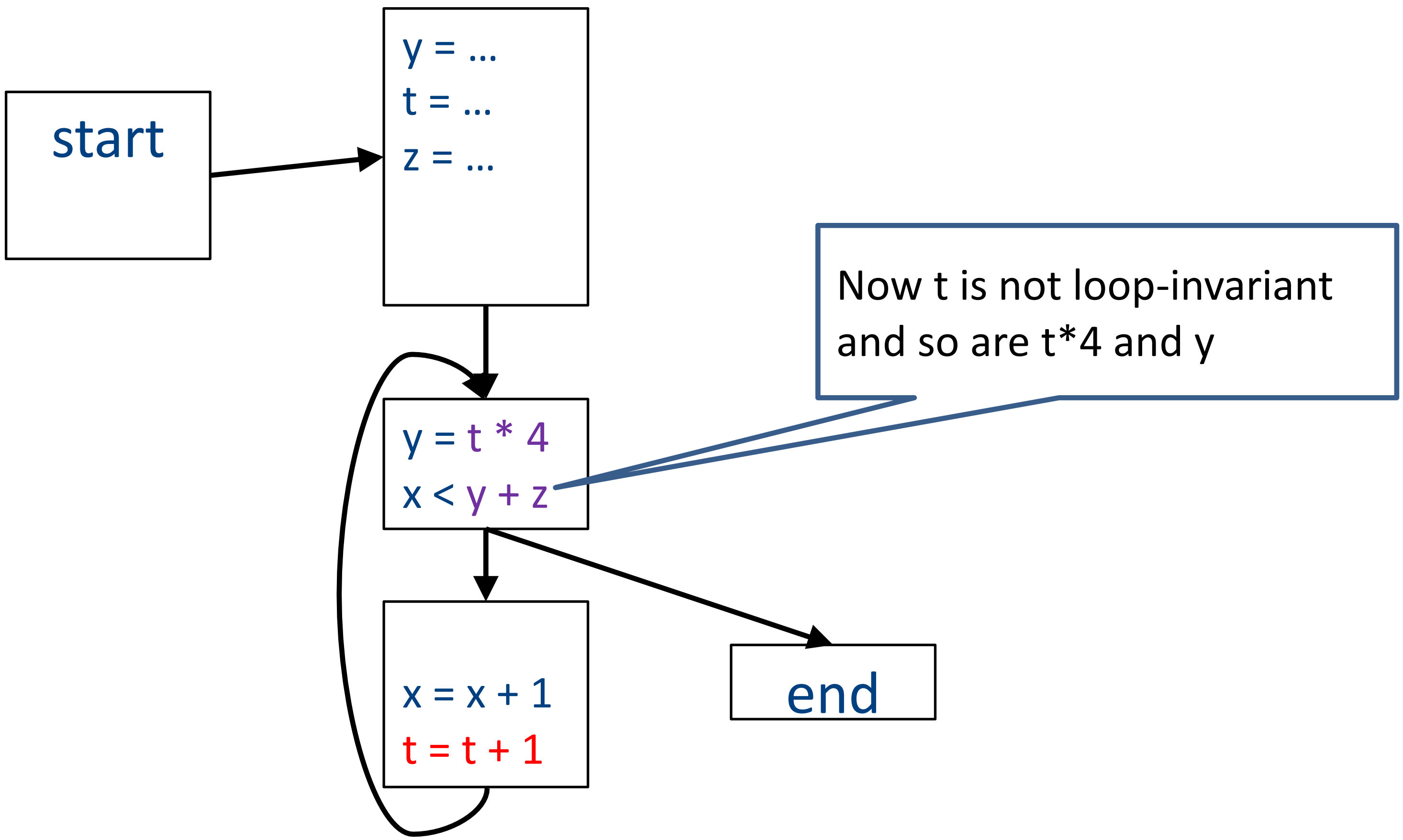
Code hoisting



What reasoning did we use?



What about now?



Loop-invariant code motion

- *d*: t = a_1 op a_2
  - *d* is a program location
- a_1 op a_2 loop-invariant (for a loop L) if computes the same value in each iteration
  - Hard to know in general
- Conservative approximation
  - Each a_i is a constant, or
  - All definitions of a_i that reach *d* are outside L, or
  - Only one definition of of a_i reaches *d*, and is loop-invariant itself
- Transformation: hoist the loop-invariant code outside of the loop

Reaching definitions analysis

- A definition *d*: t = … reaches a program location if there is a path from the definition to the program location, along which the defined variable is never redefined

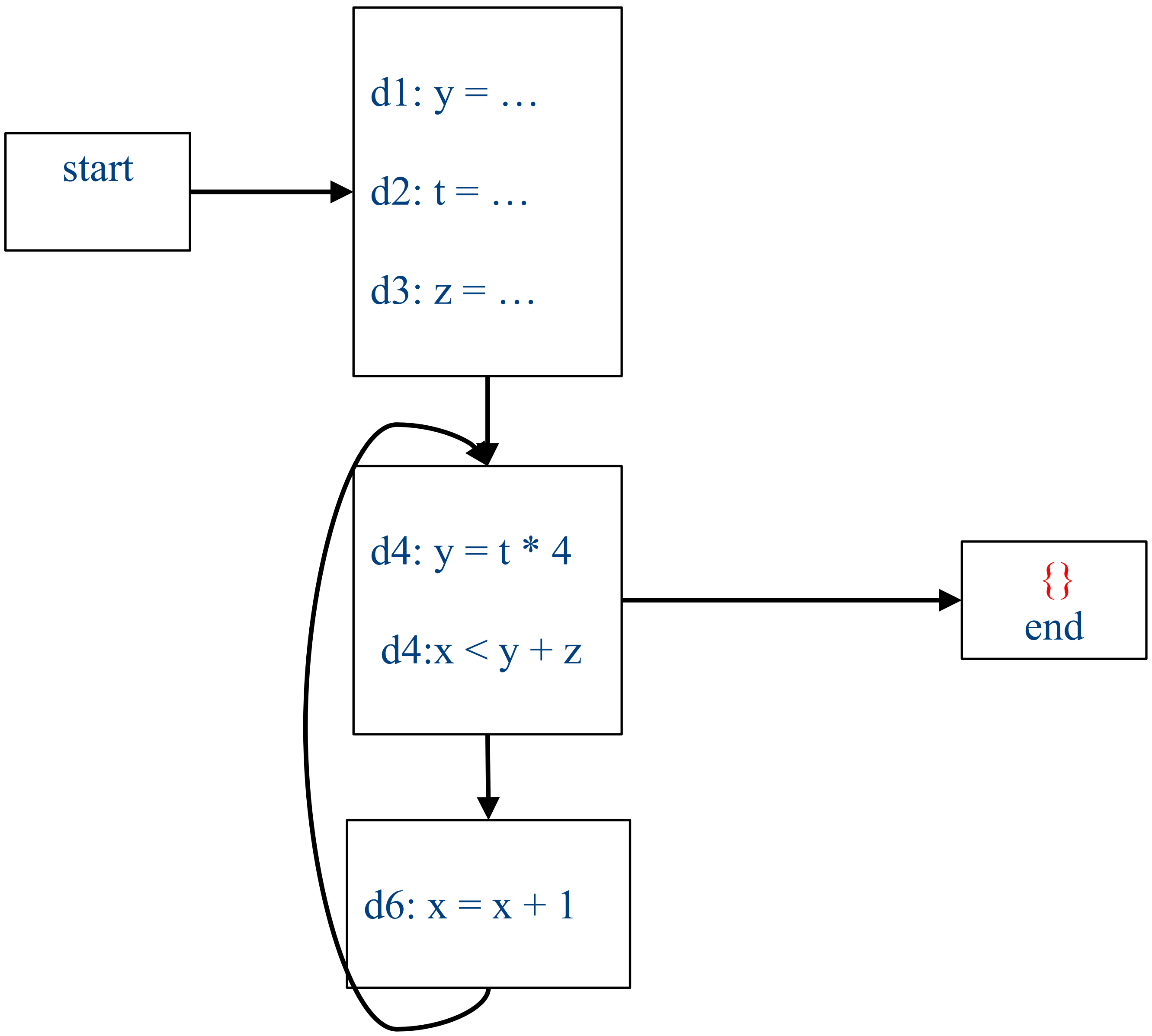
Reaching definitions analysis

- A definition d: t = … reaches a program location if there is a path from the definition to the program location, along which the defined variable is never redefined
- Direction: Forward
- Domain: sets of program locations that are definitions `
- Join operator: union
- Transfer function:

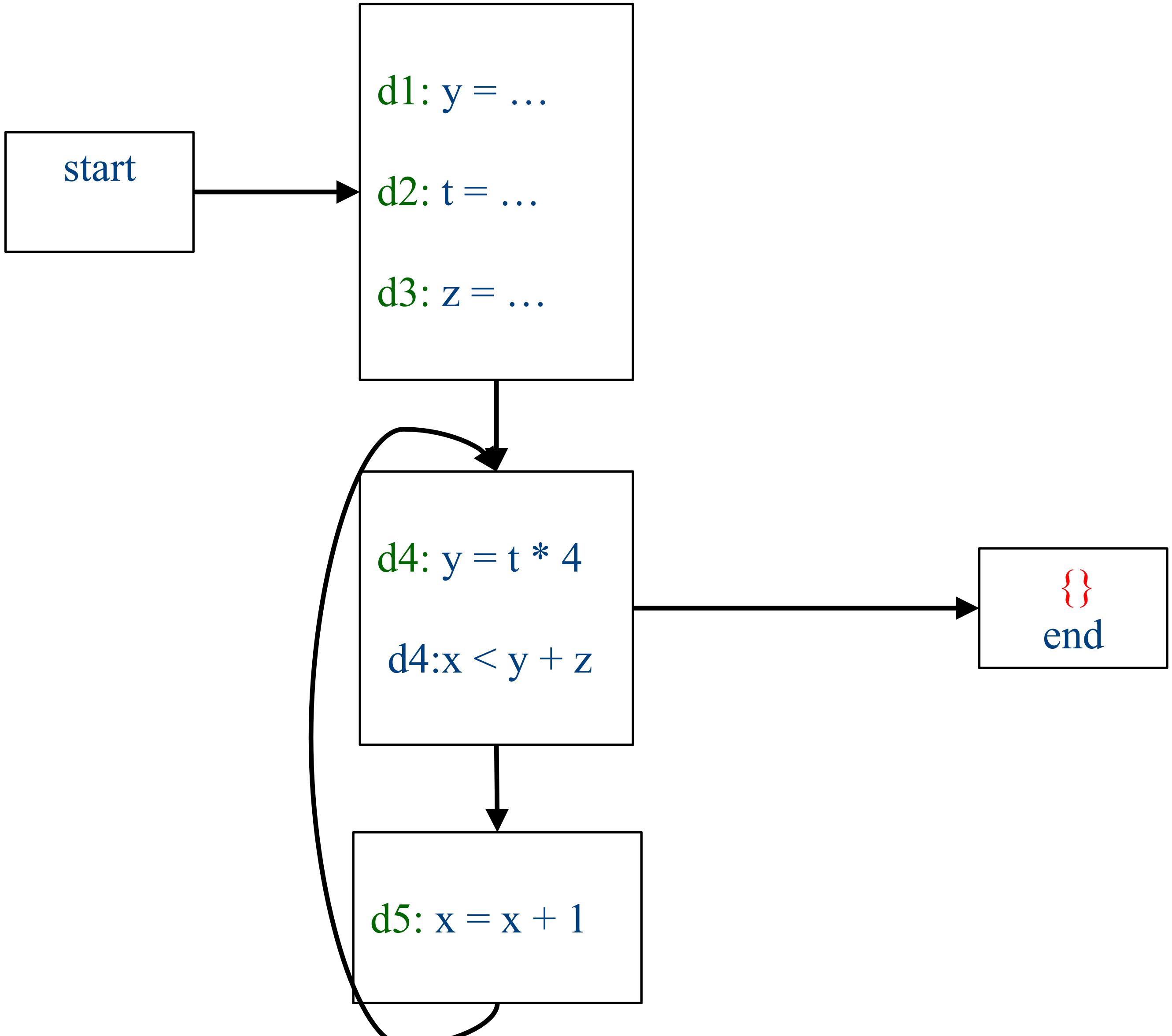
$$f_{d:\ a=b\ op\ c}(RD) = (RD - defs(a)) \cup \{d\}$$
$$f_{d:\ not\text{-}a\text{-}def}(RD) = RD$$

  – Where *defs*(a) is the set of locations defining a (statements of the form a=...)
- Initial value: {}

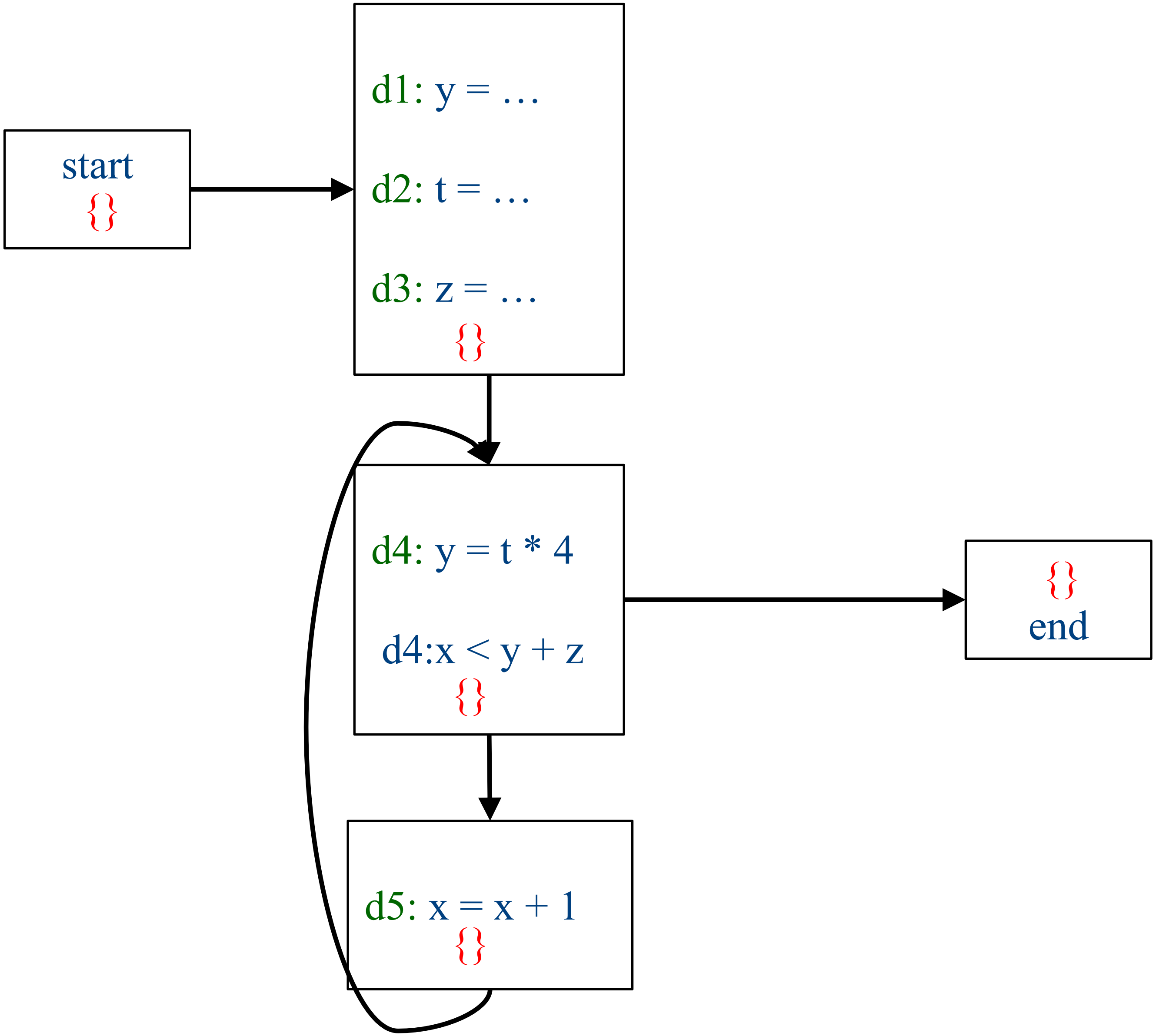
Reaching definitions analysis



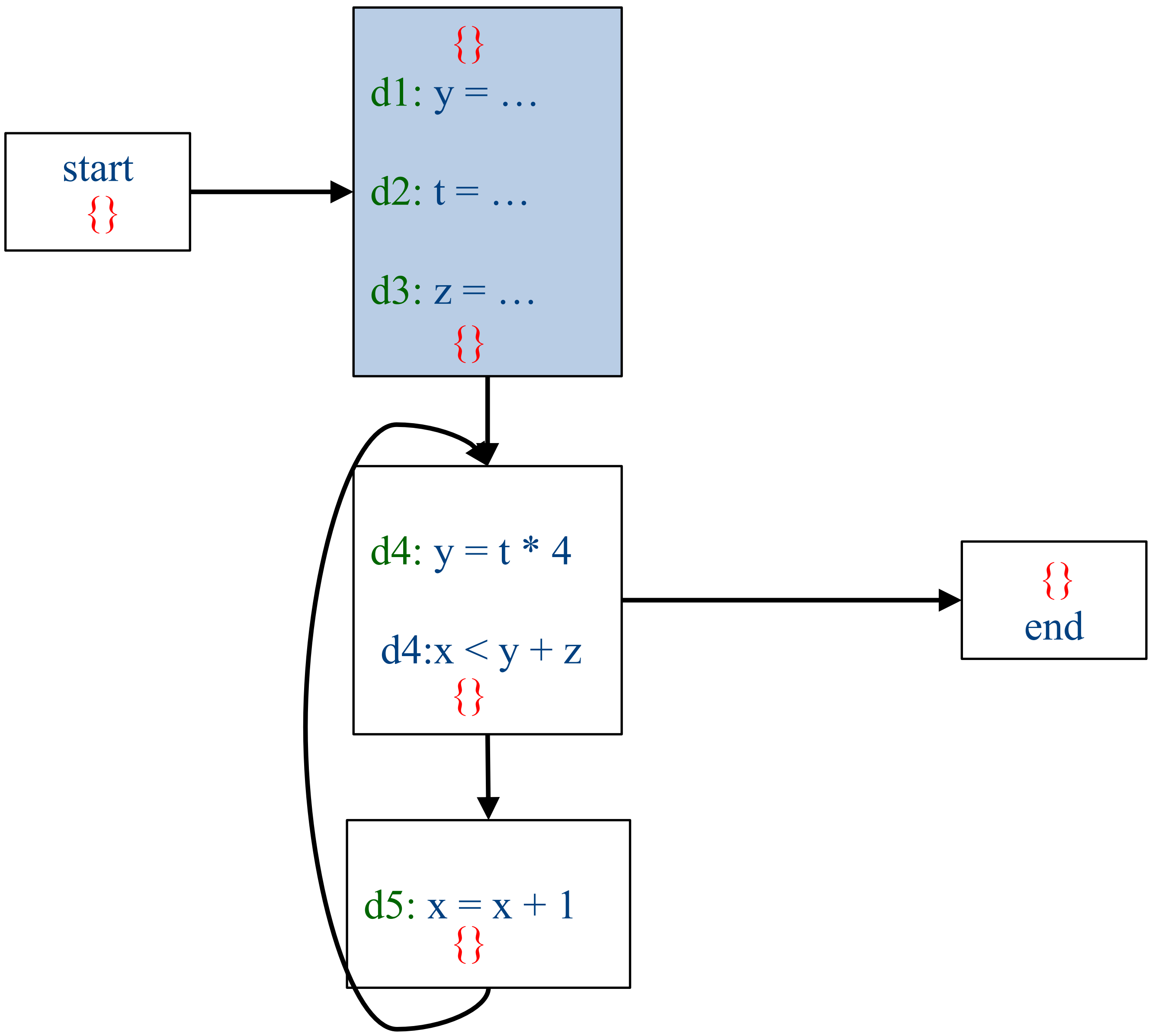
Reaching definitions analysis



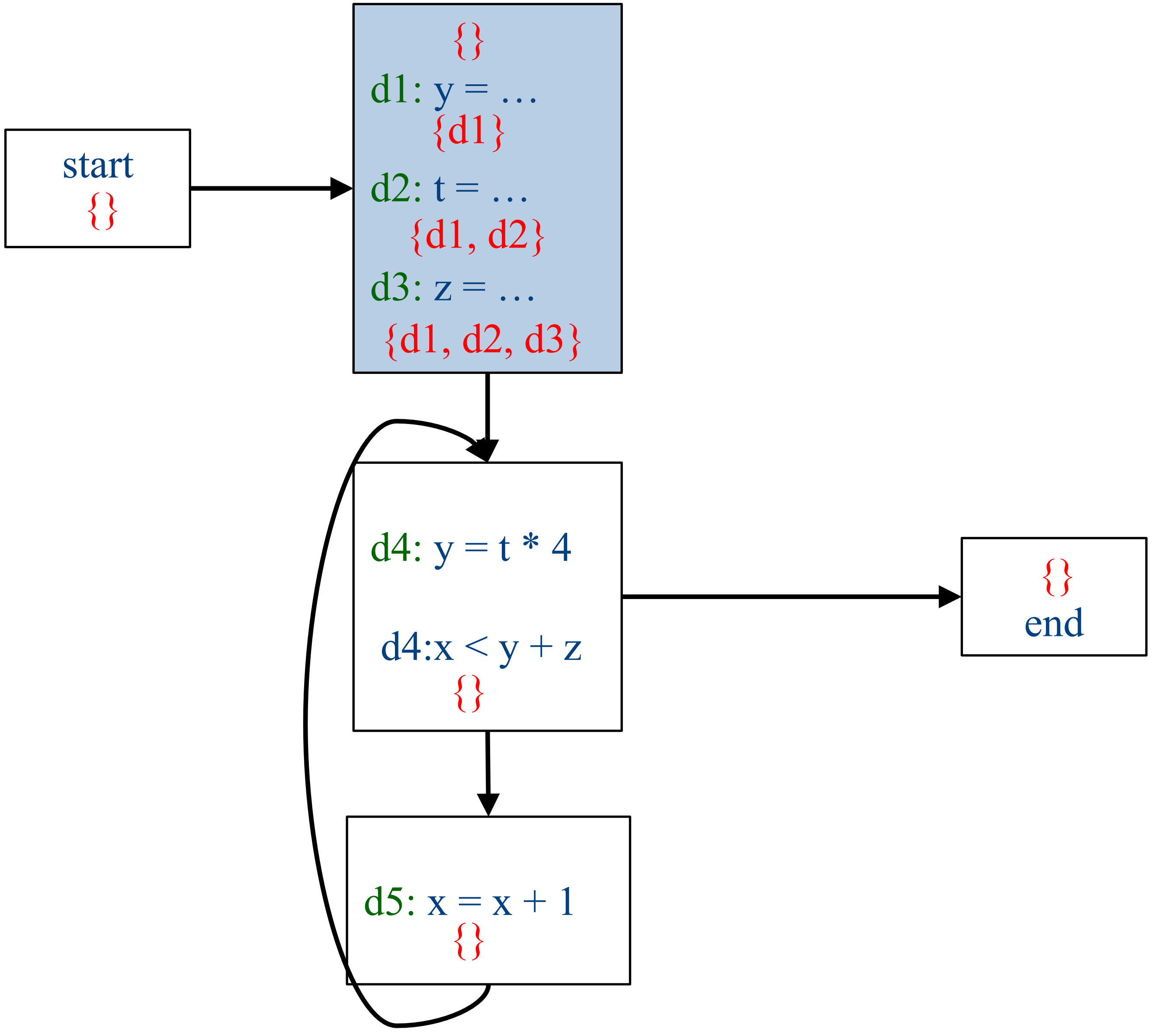
Initialization



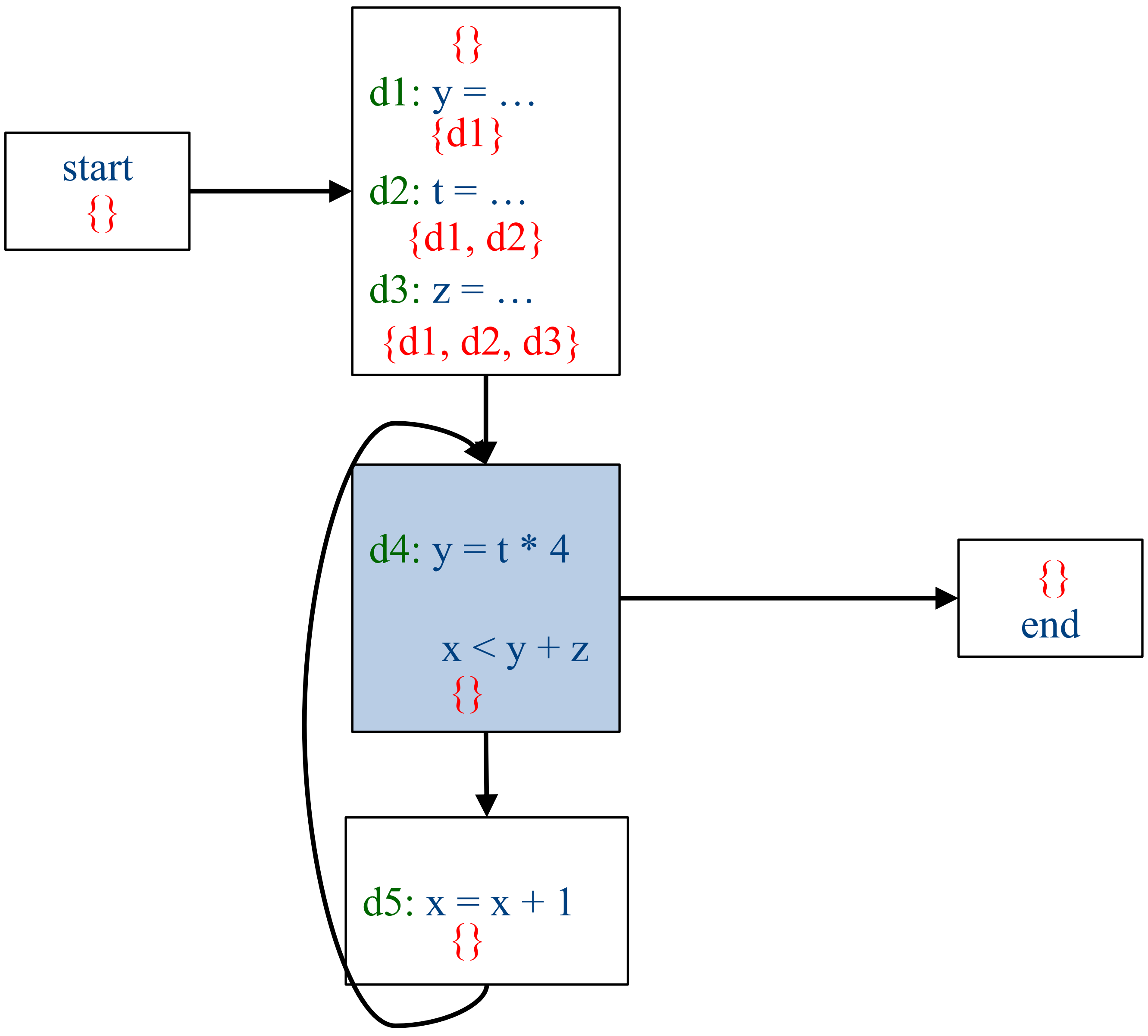
Iteration 1



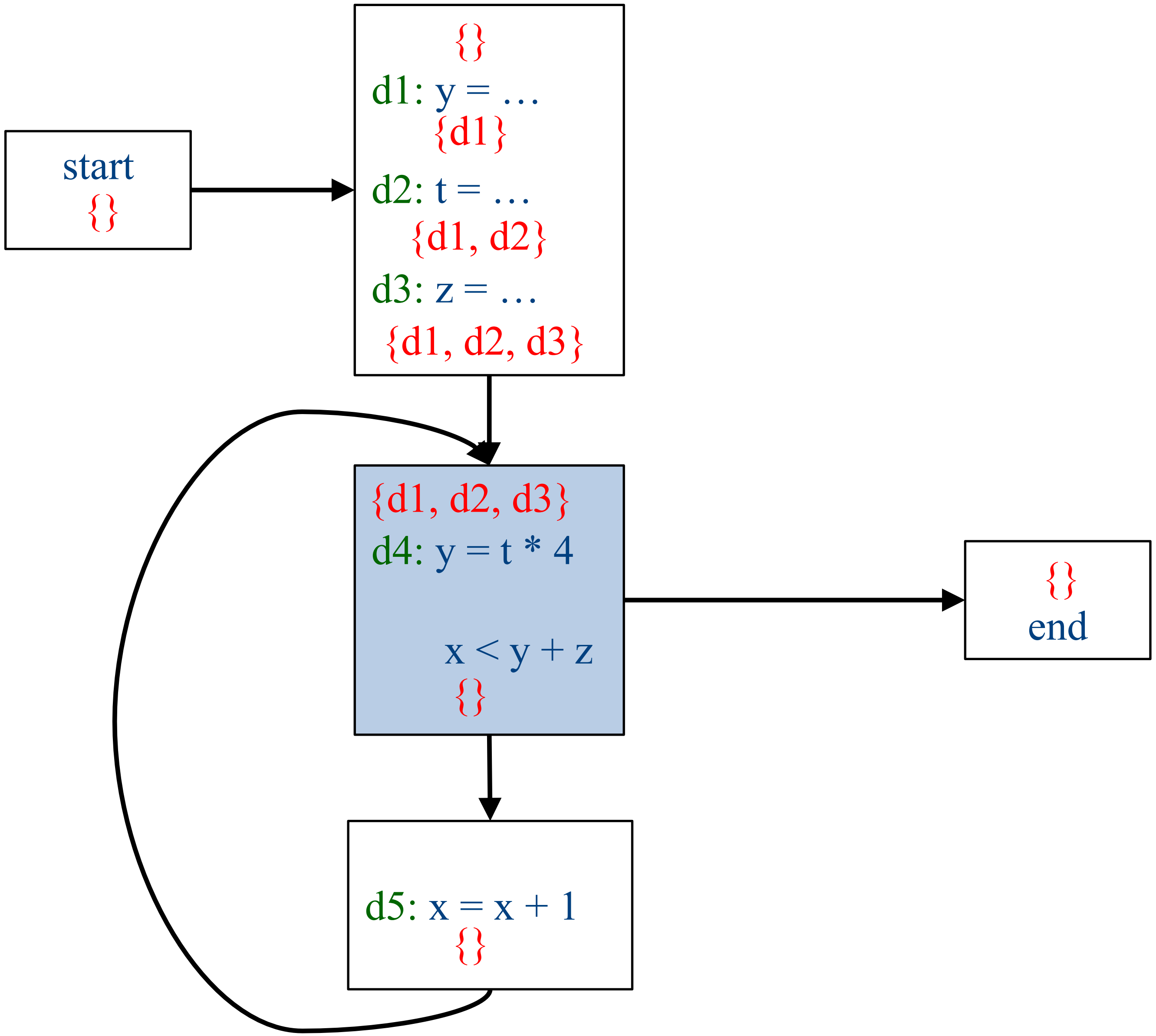
Iteration 1



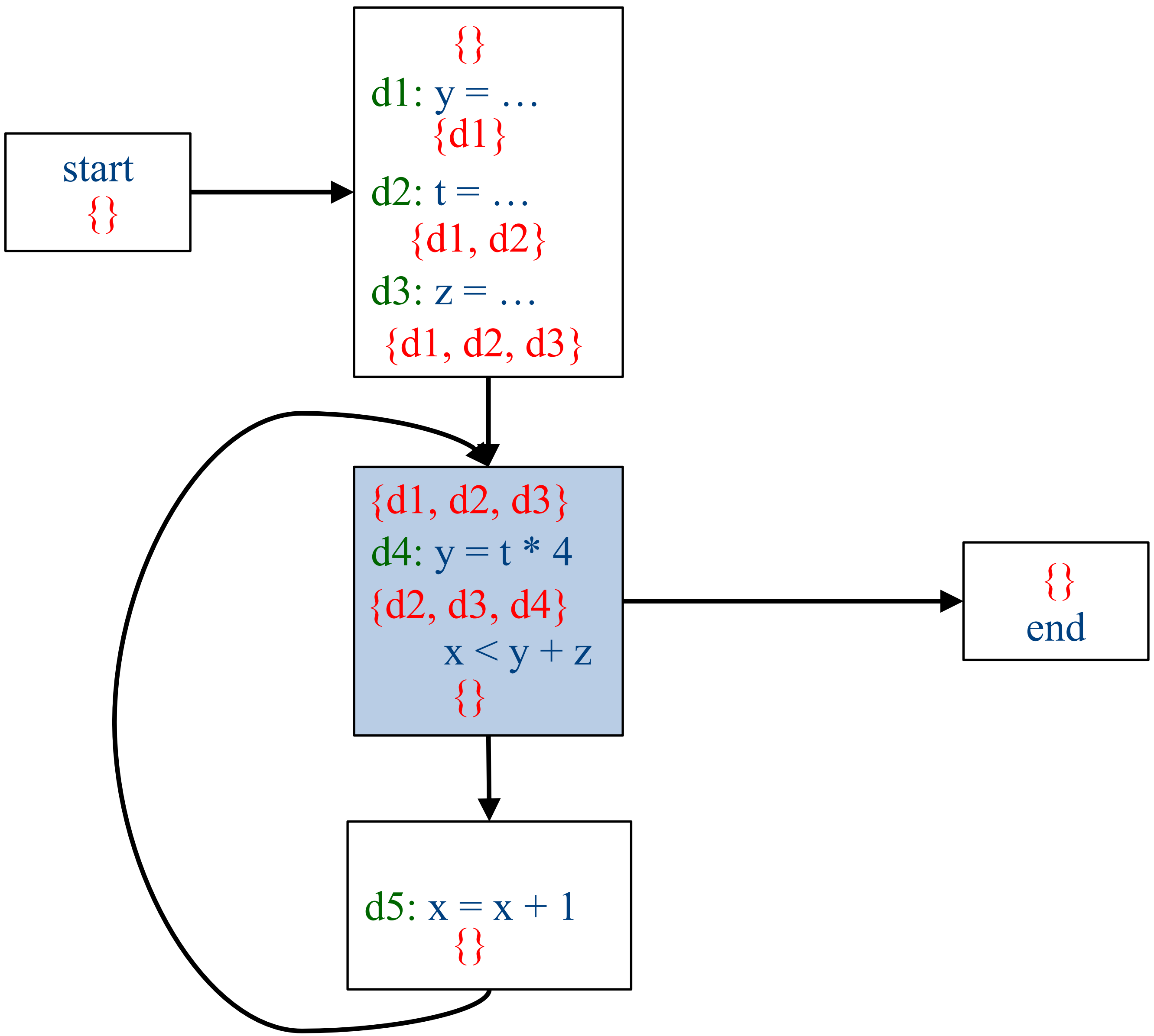
Iteration 2



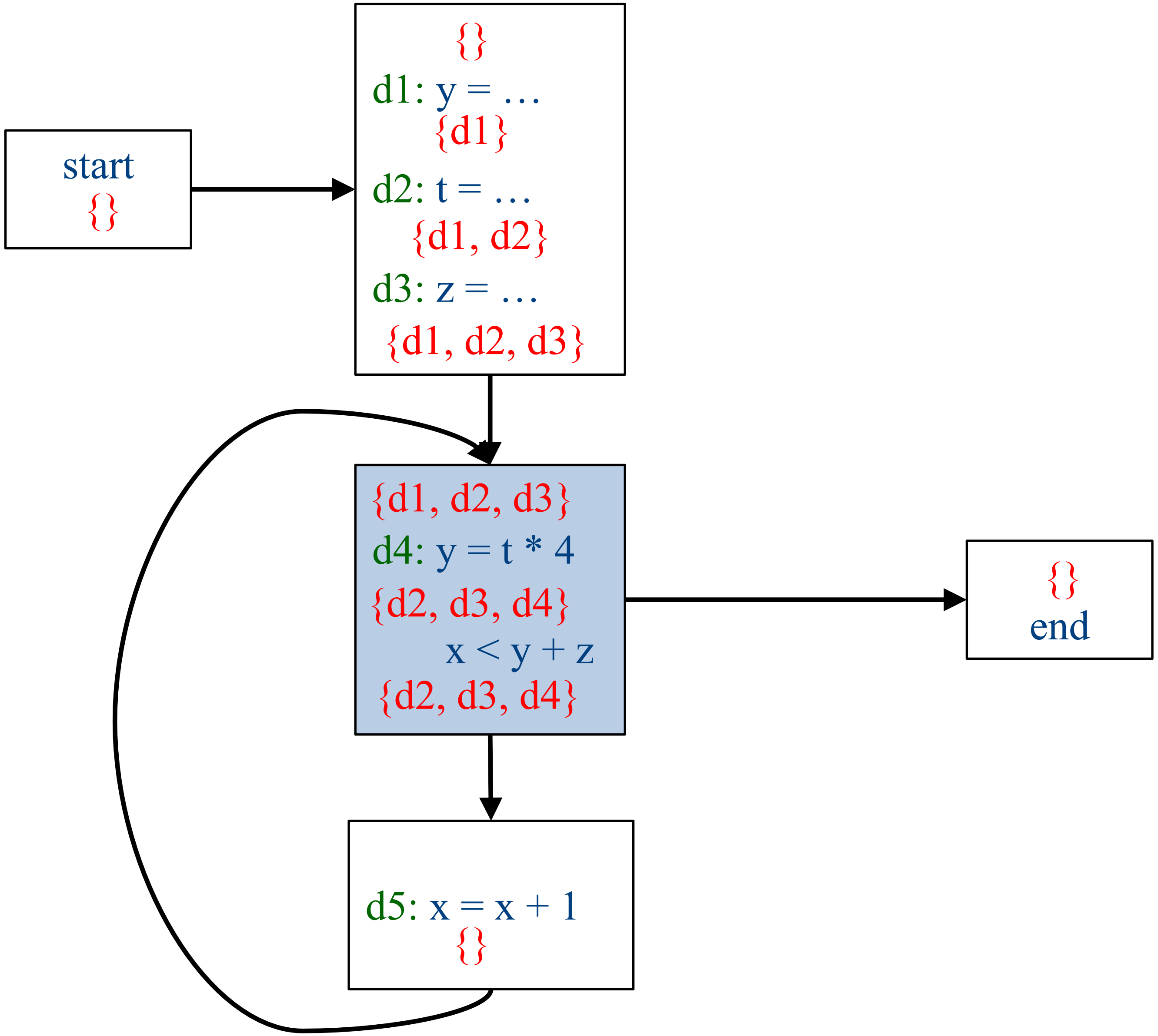
Iteration 2



Iteration 2


start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{}
{}
end
d5: x = x + 1
{}

Iteration 2



start
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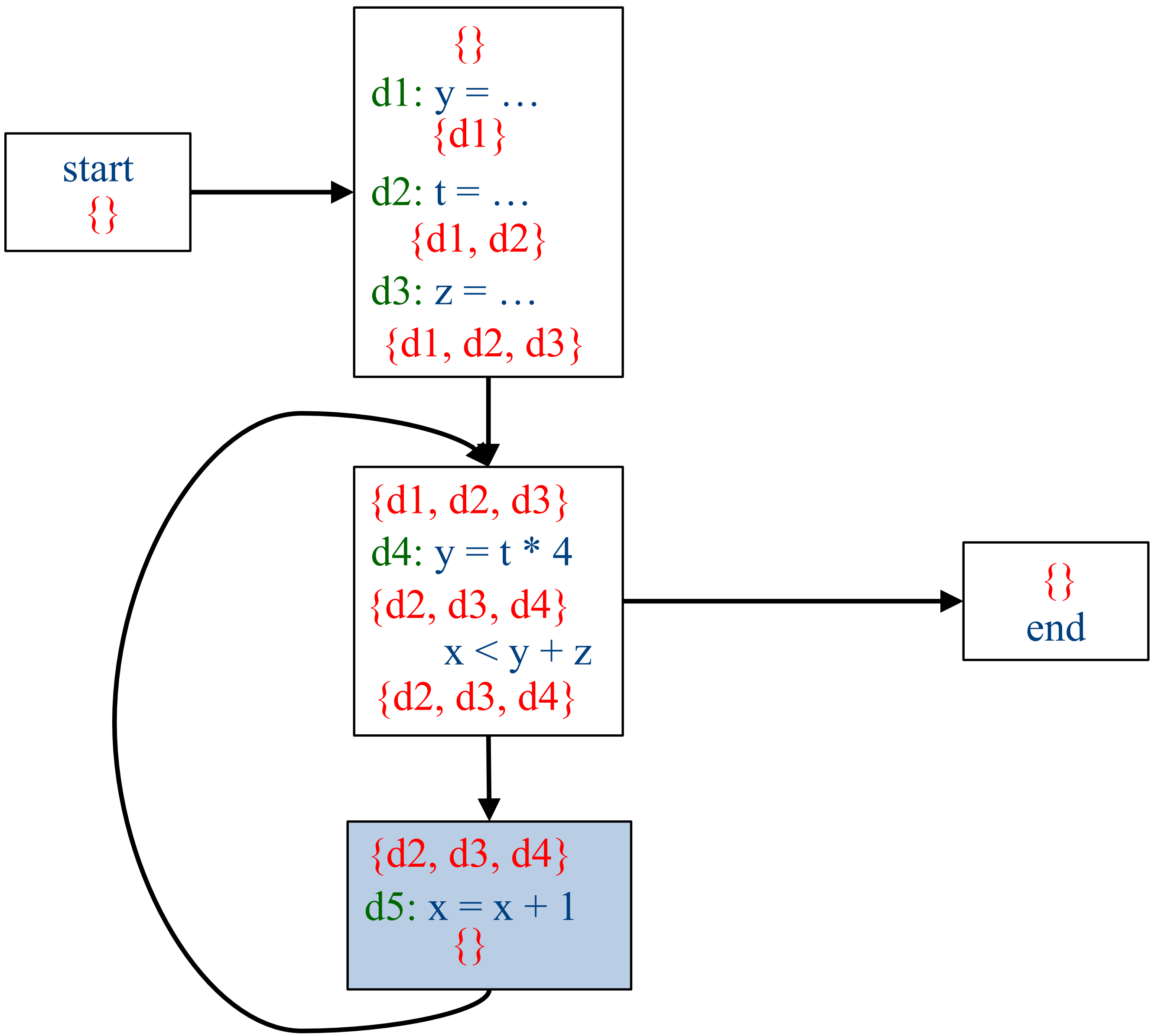
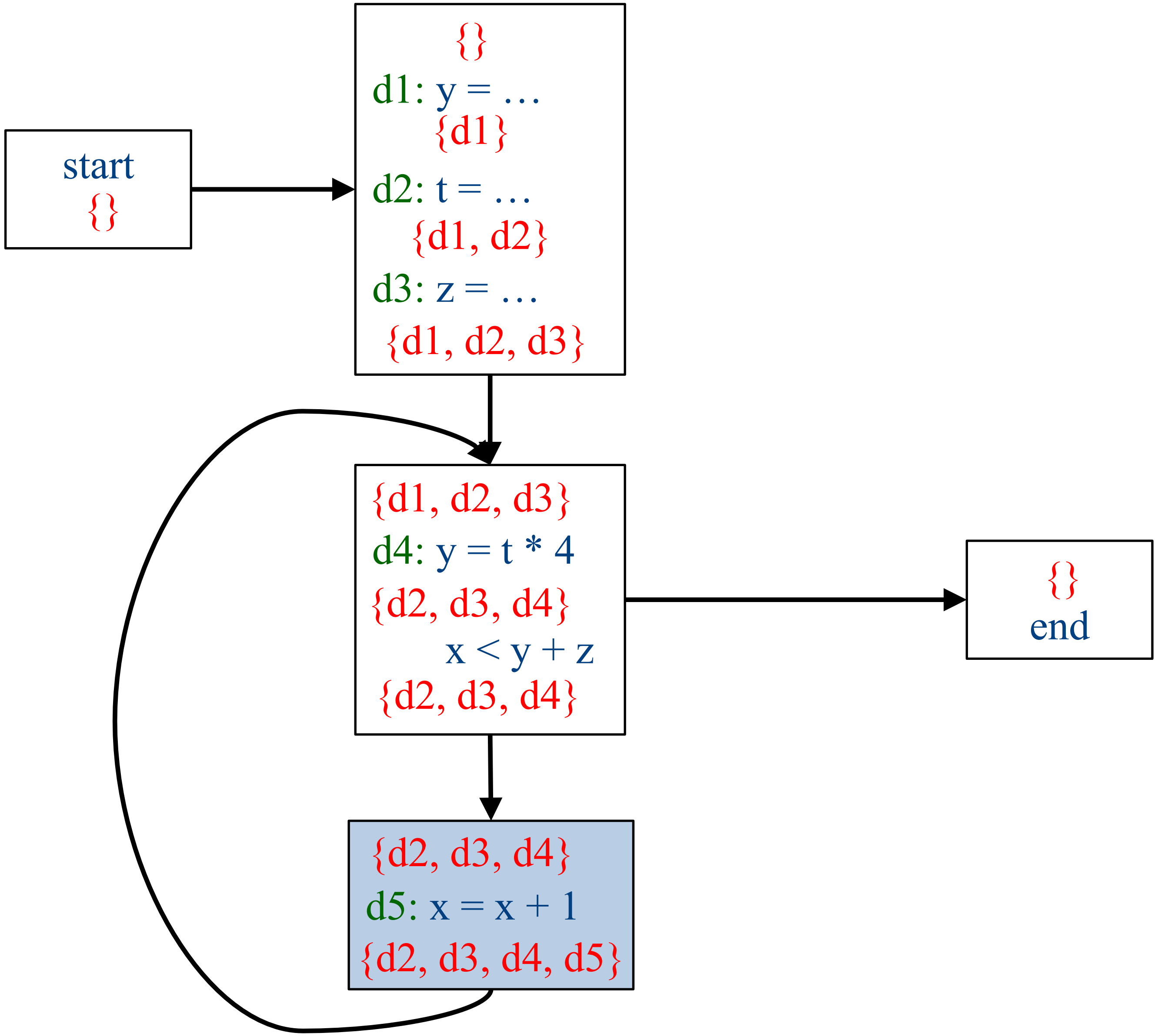
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{d2, d3, d4}
{}
end
d5: x = x + 1
{}

Iteration 3



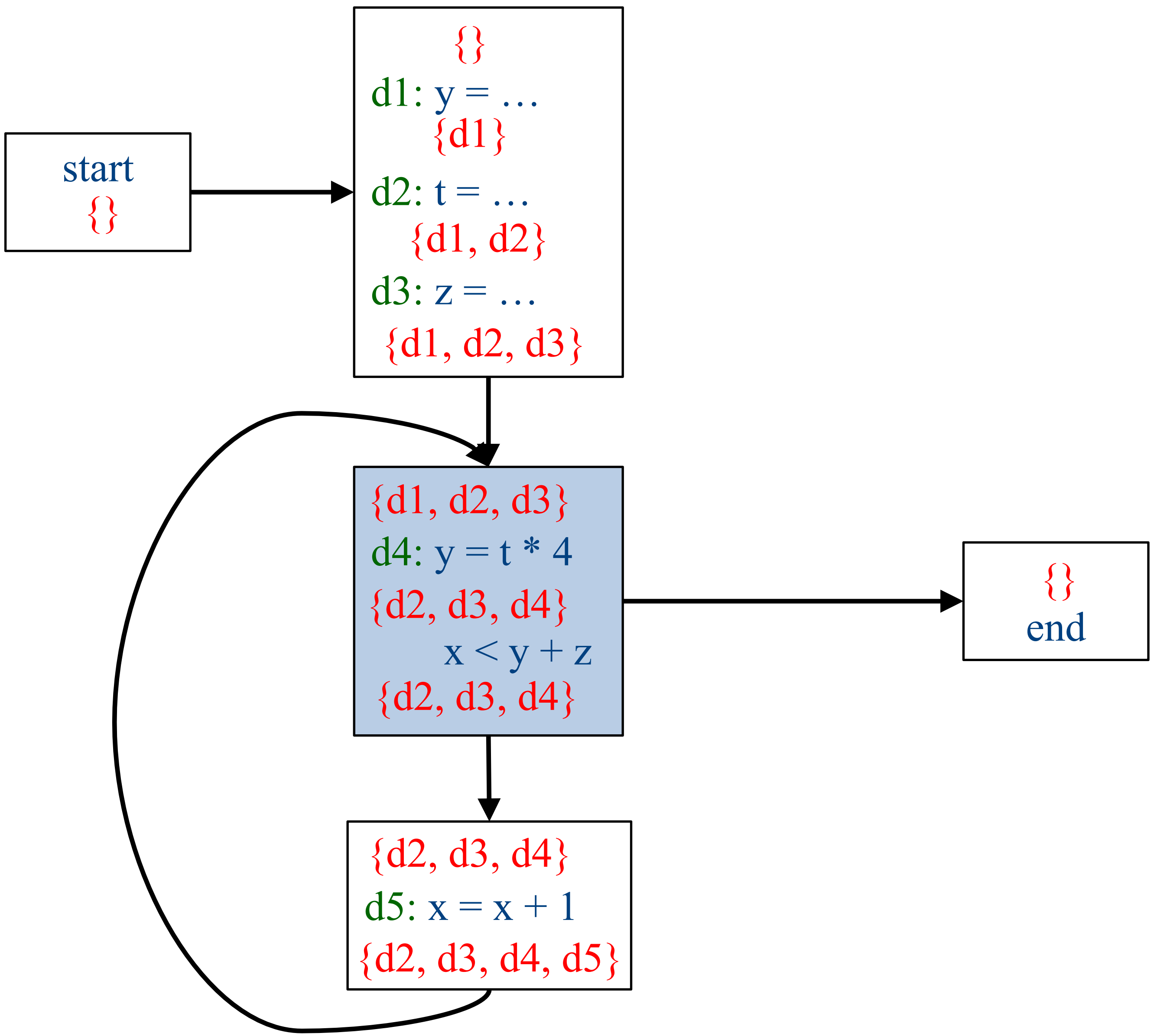
start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{d2, d3, d4}
{}
end
{d2, d3, d4}
d5: x = x + 1
{}

Iteration 3

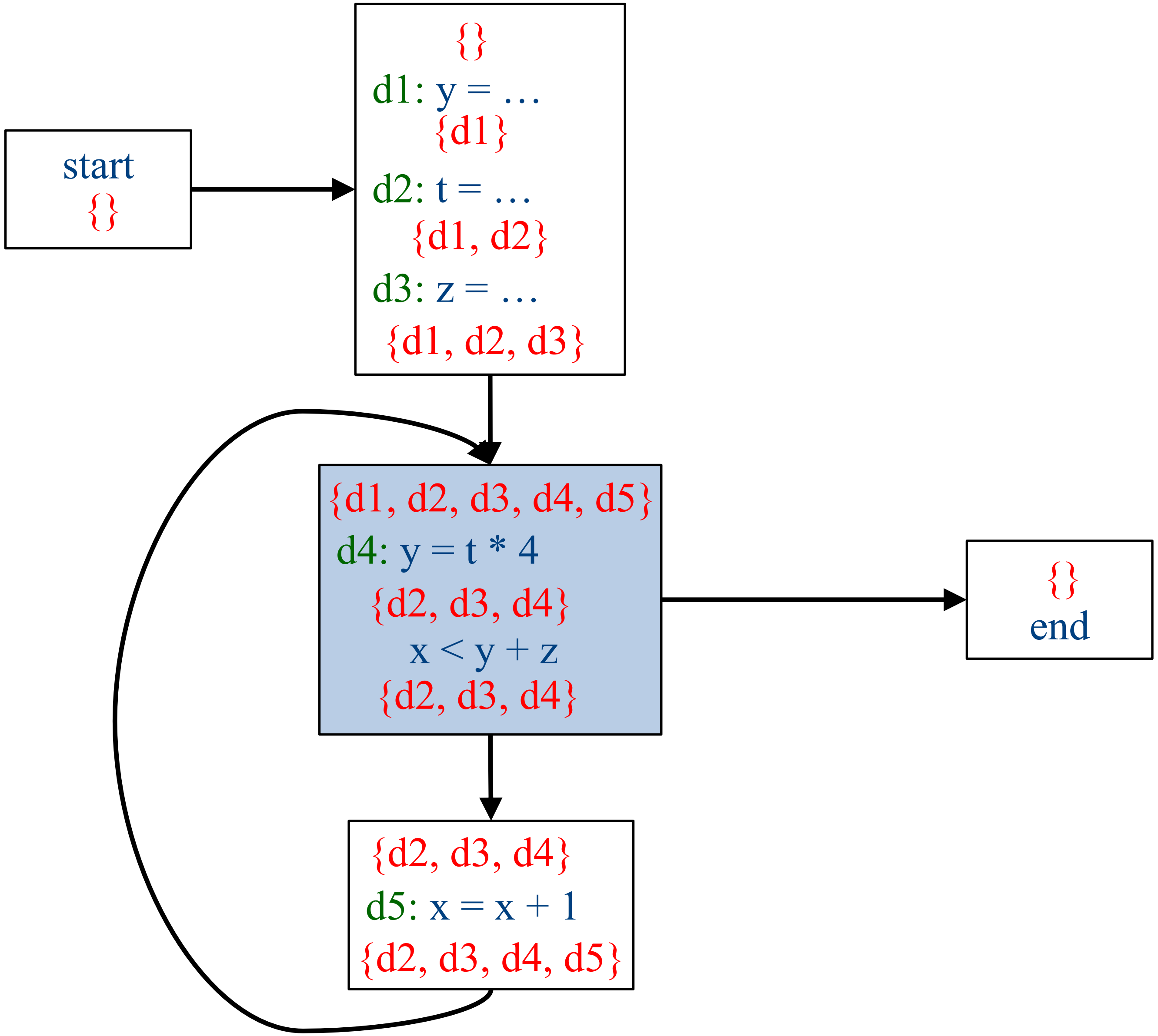


start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{d2, d3, d4}
{}
end
{d2, d3, d4}
d5: x = x + 1
{d2, d3, d4, d5}

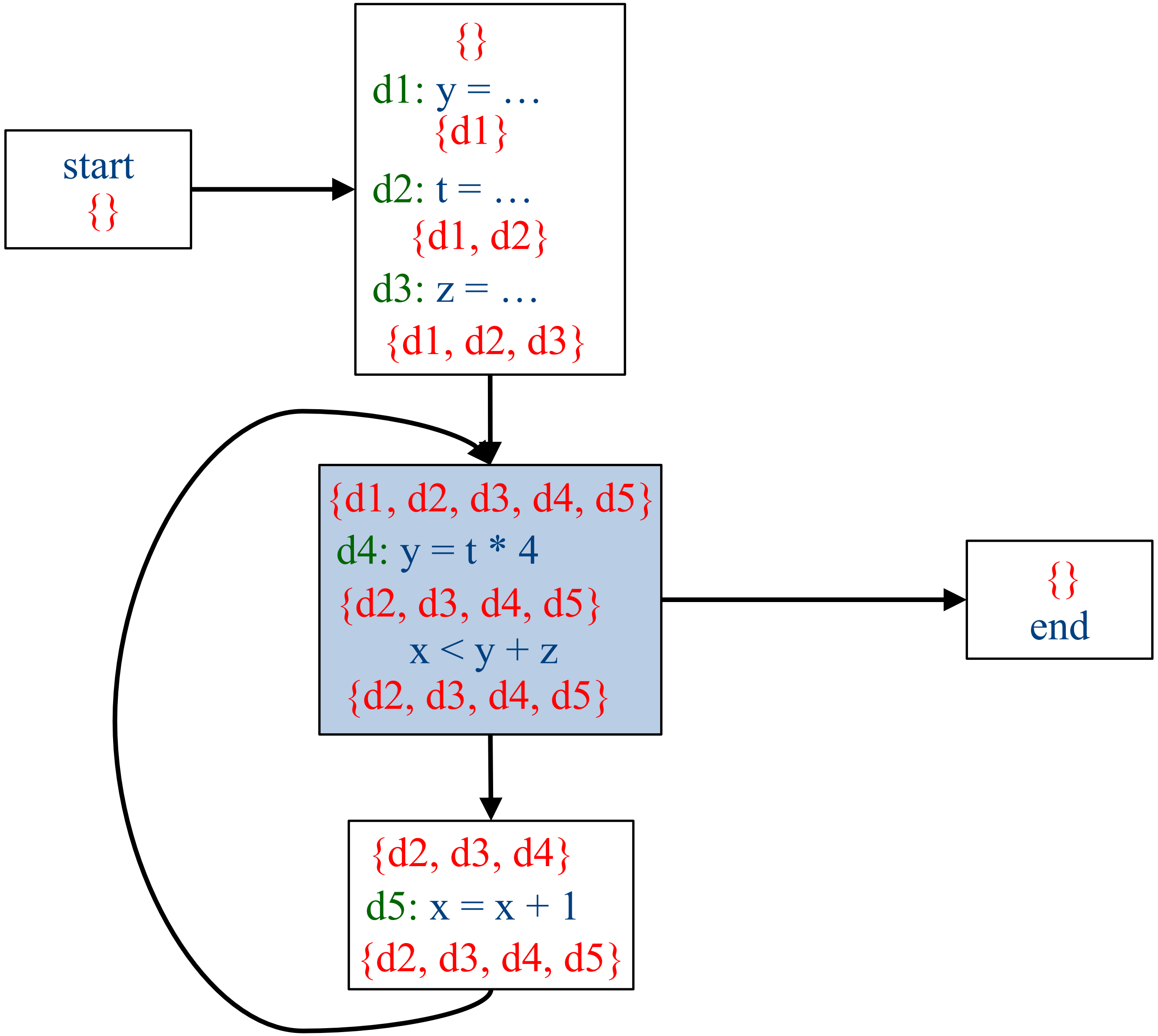
Iteration 4


start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{d2, d3, d4}
{}
end
{d2, d3, d4}
d5: x = x + 1
{d2, d3, d4, d5}

Iteration 4


start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3, d4, d5}
d4: y = t * 4
{d2, d3, d4}
x < y + z
{d2, d3, d4}
{}
end
{d2, d3, d4}
d5: x = x + 1
{d2, d3, d4, d5}

Iteration 4


start
{}
{}
d1: y = …
{d1}
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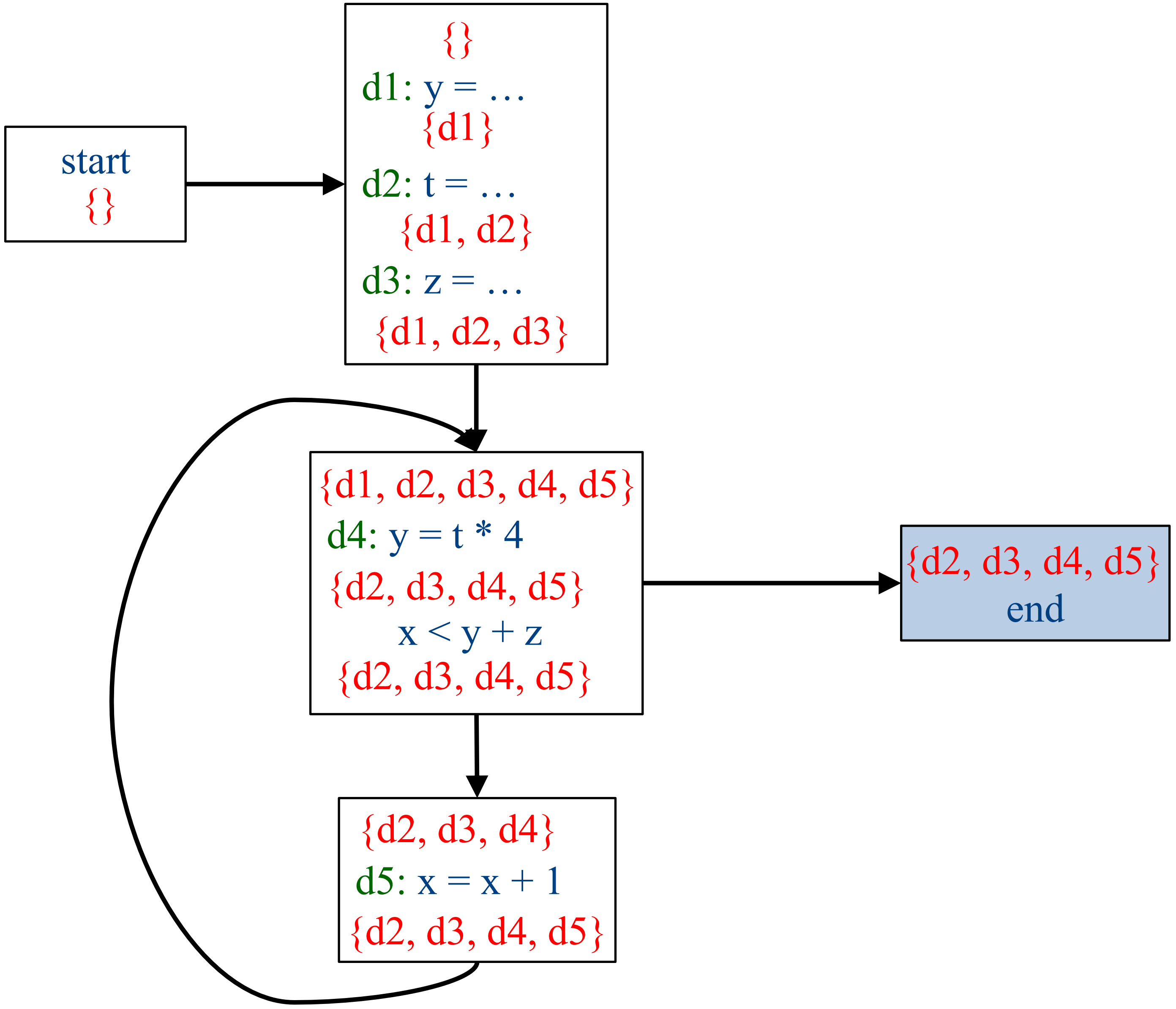
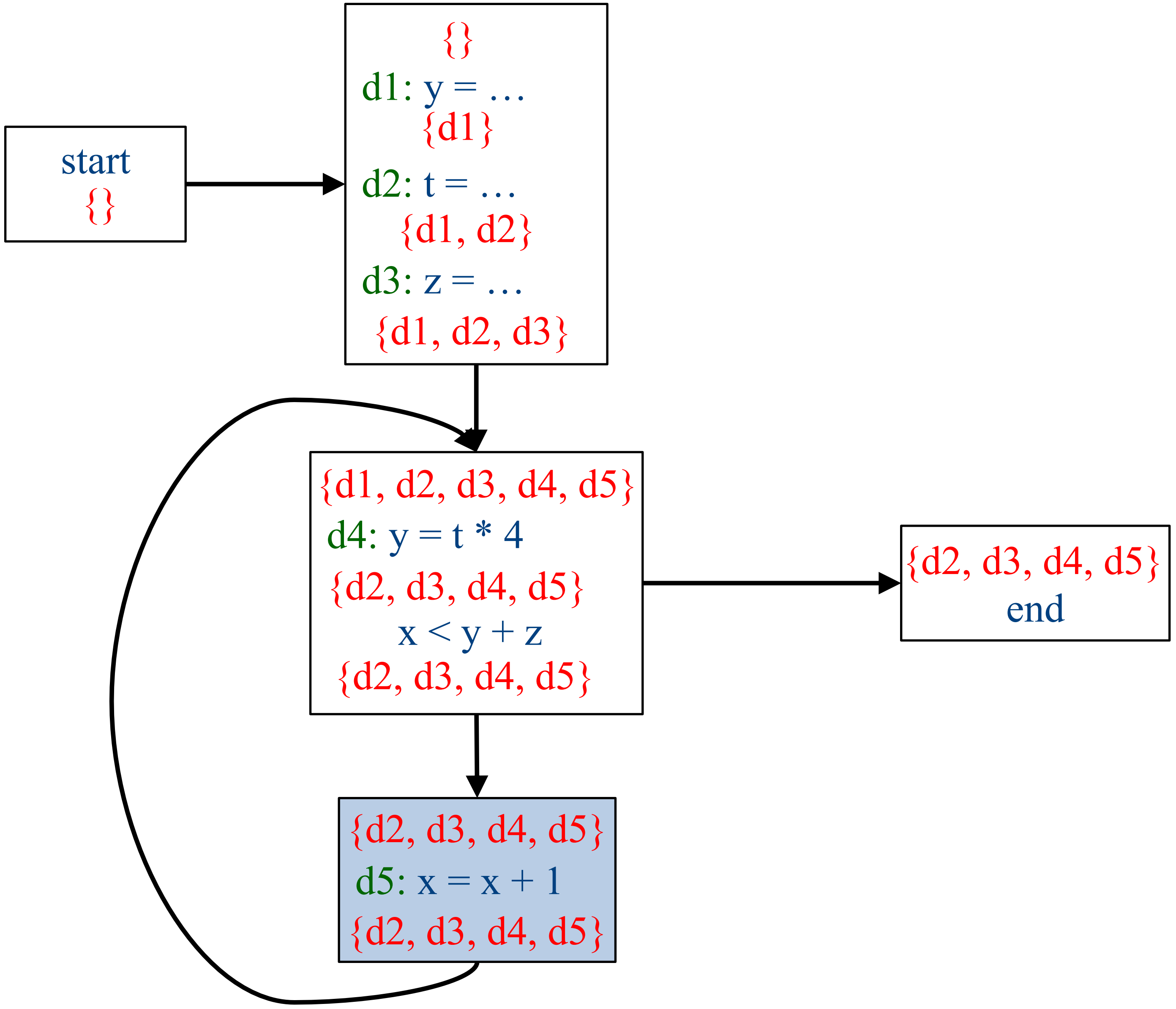
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3, d4, d5}
d4: y = t * 4
{d2, d3, d4, d5}
x < y + z
{d2, d3, d4, d5}
{}
end
{d2, d3, d4}
d5: x = x + 1
{d2, d3, d4, d5}

Iteration 5



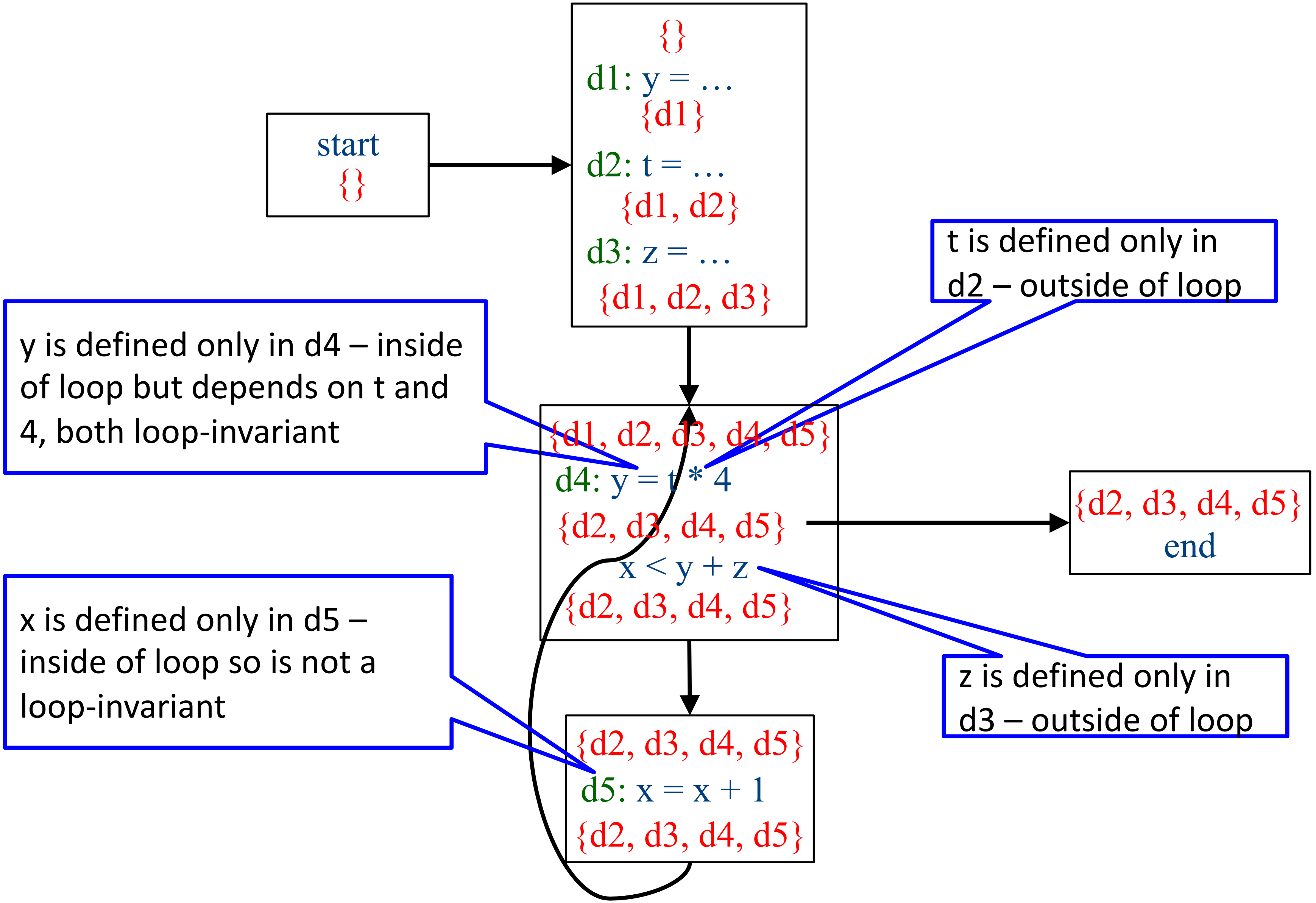
start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3, d4, d5}
d4: y = t * 4
{d2, d3, d4, d5}
x < y + z
{d2, d3, d4, d5}
{d2, d3, d4, d5}
end
{d2, d3, d4}
d5: x = x + 1
{d2, d3, d4, d5}

Iteration 6



start
{}
{}
d1: y = …
{d1}
d2: t = …
{d1, d2}
d3: z = …
{d1, d2, d3}
{d1, d2, d3, d4, d5}
d4: y = t * 4
{d2, d3, d4, d5}
x < y + z
{d2, d3, d4, d5}
{d2, d3, d4, d5}
end
{d2, d3, d4, d5}
d5: x = x + 1
{d2, d3, d4, d5}

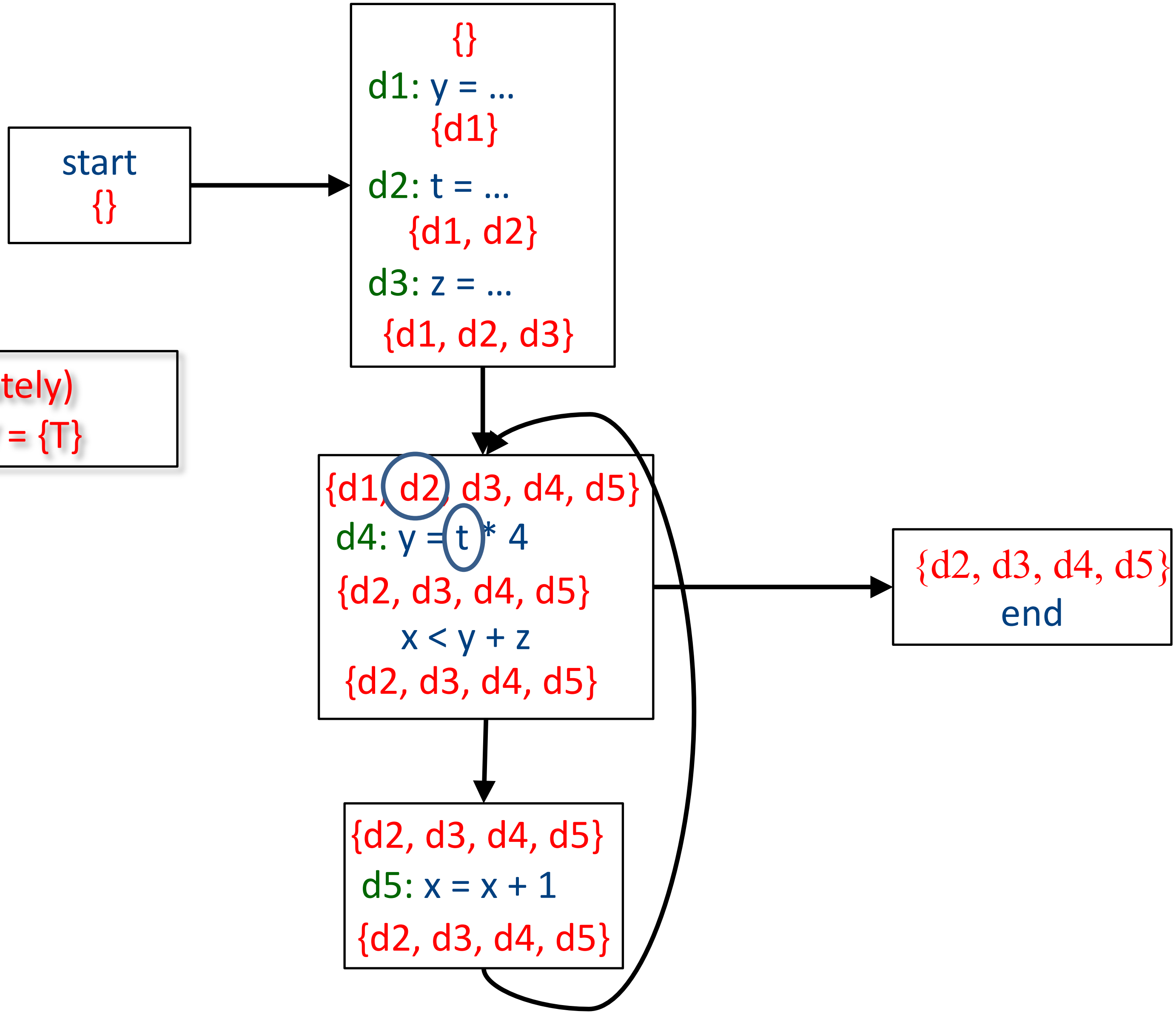
Which expressions are loop invariant?



Inferring loop-invariant expressions

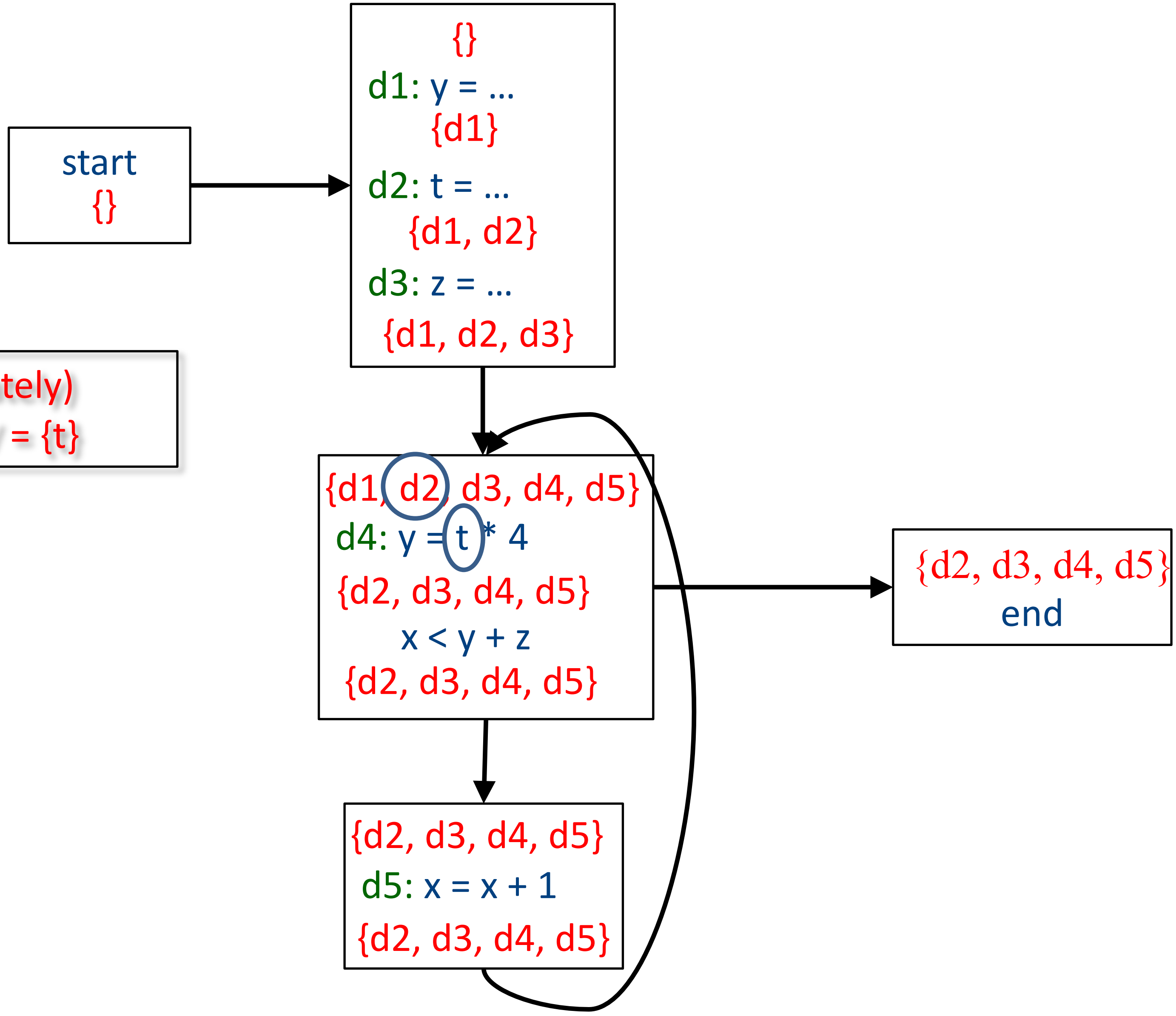
- For a statement s of the form t = a_1 op a_2
- A variable a_i is immediately loop-invariant if all reaching definitions IN[s]={$d_1,\ldots,d_k$} for a_i are outside of the loop
- LOOP-INV = immediately loop-invariant variables and constants
  LOOP-INV = LOOP-INV $\cup$ {x | d: x = a_1 op a_2, d is in the loop, and both a_1 and a_2 are in LOOP-INV}
  - Iterate until fixed-point
- An expression is loop-invariant if all operands are loop-invariants

Computing LOOP-INV



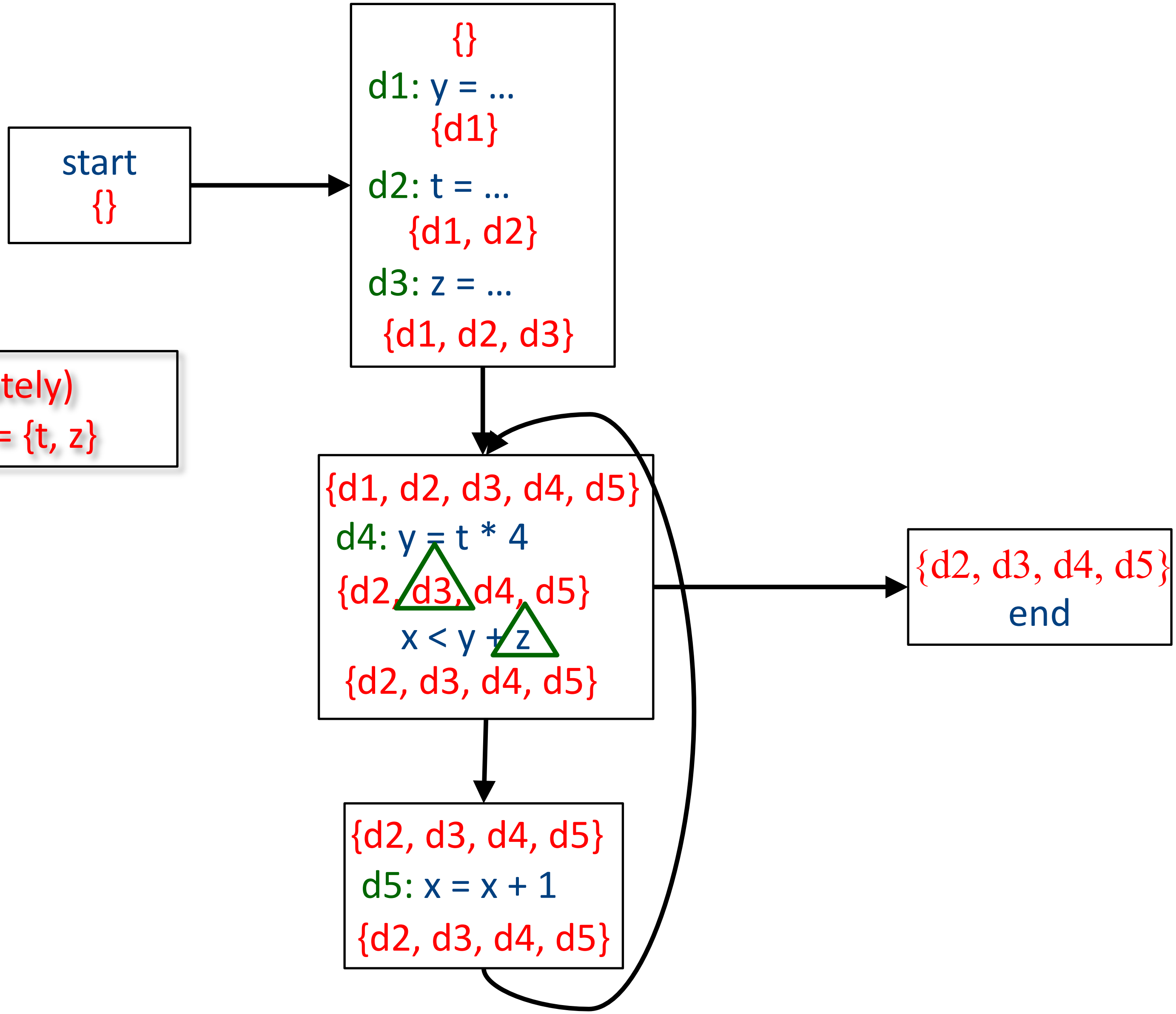
(immediately)
LOOP-INV = {T}

Computing LOOP-INV



(immediately)
LOOP-INV = {t}

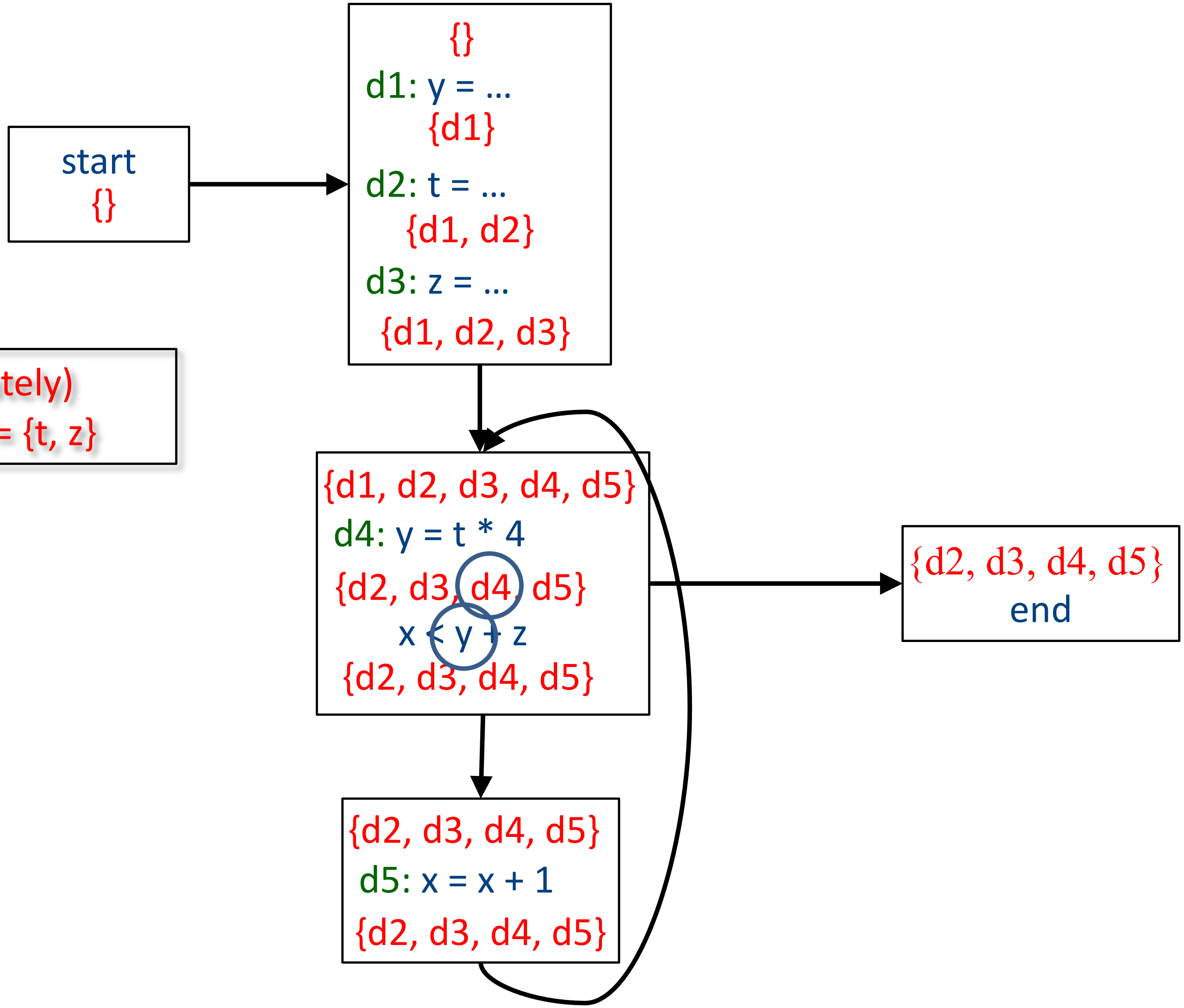
Computing LOOP-INV



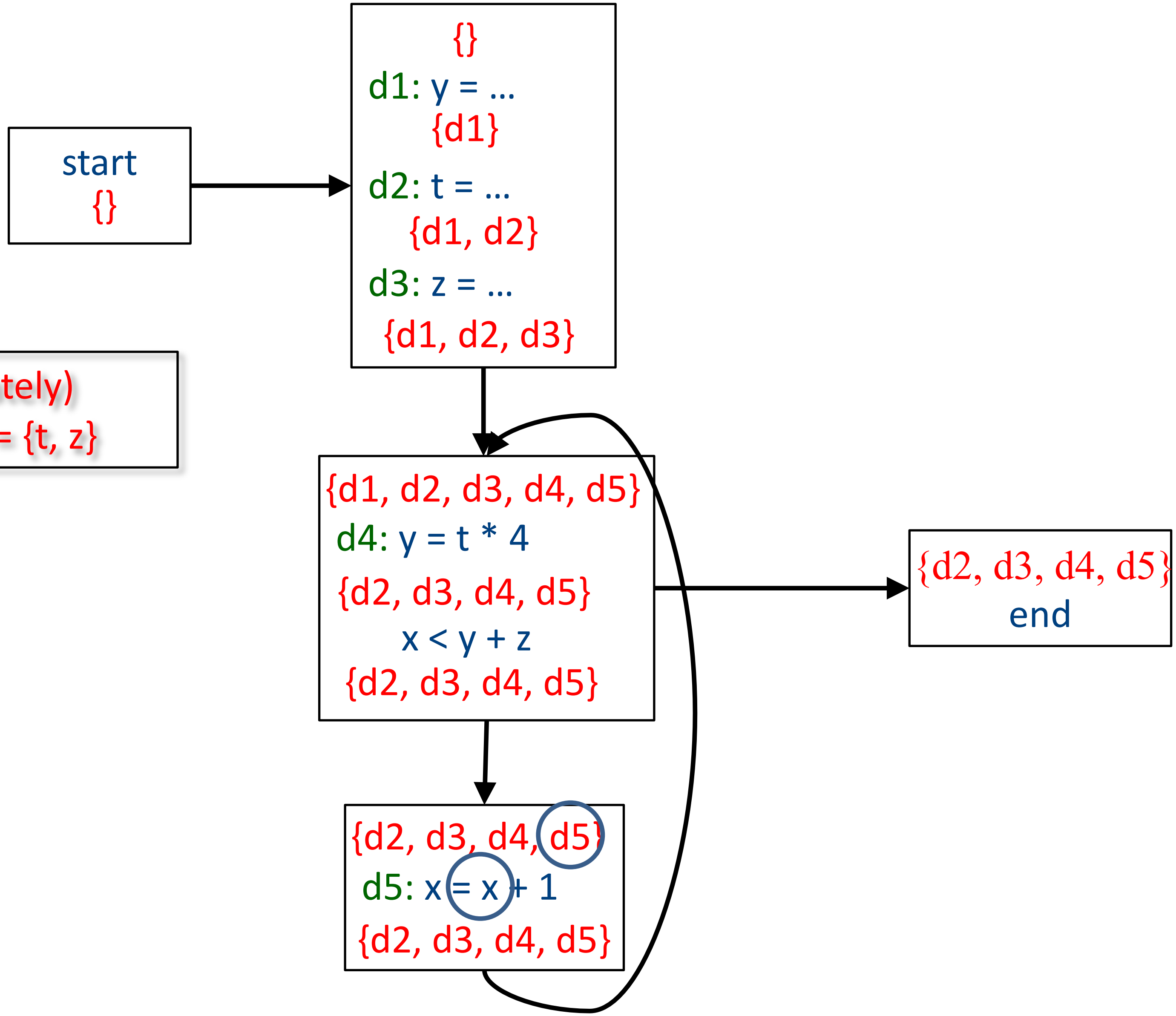
(immediately)
LOOP-INV = {t, z}

Computing LOOP-INV

(immediately)
LOOP-INV = {t, z}

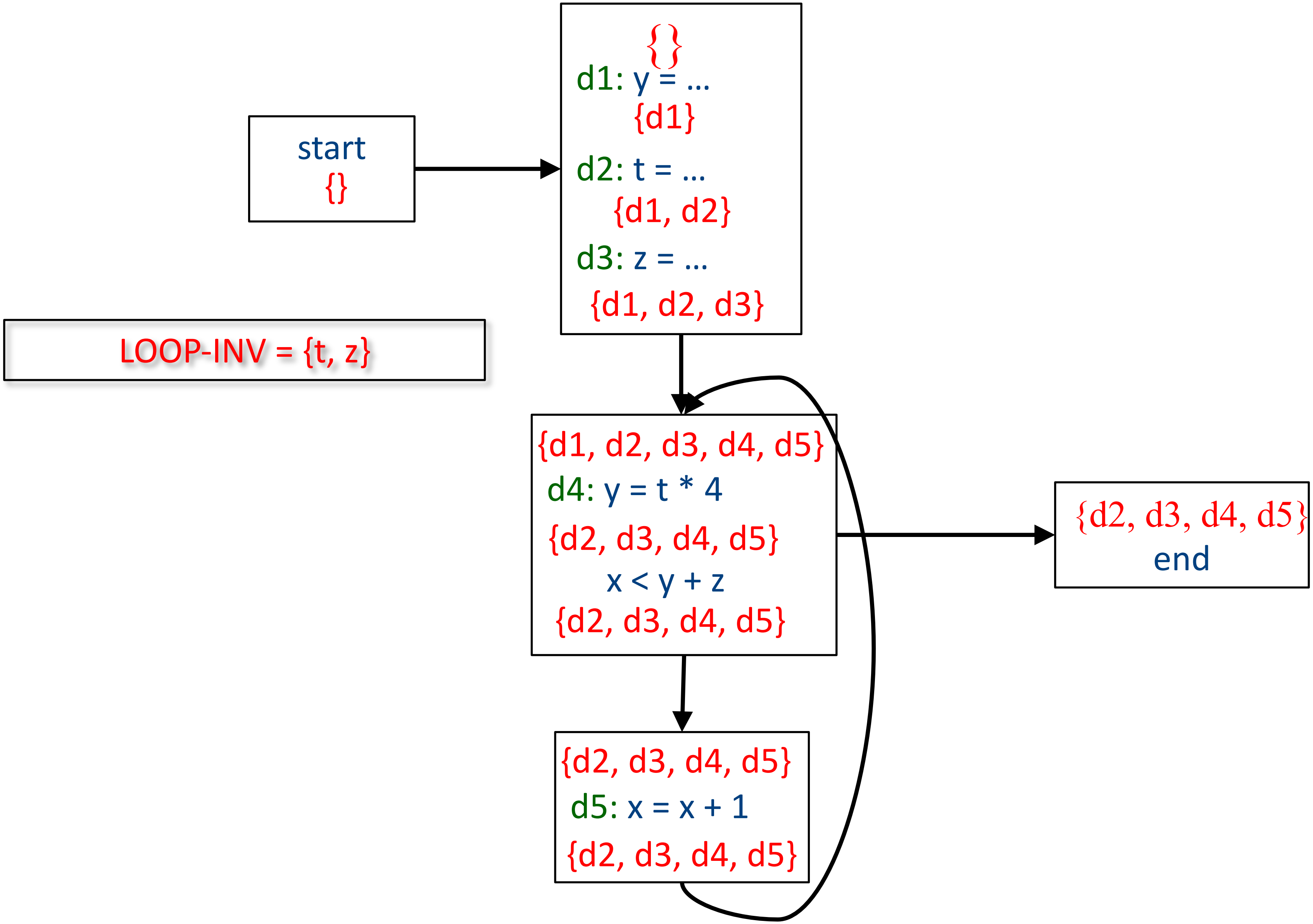


Computing LOOP-INV

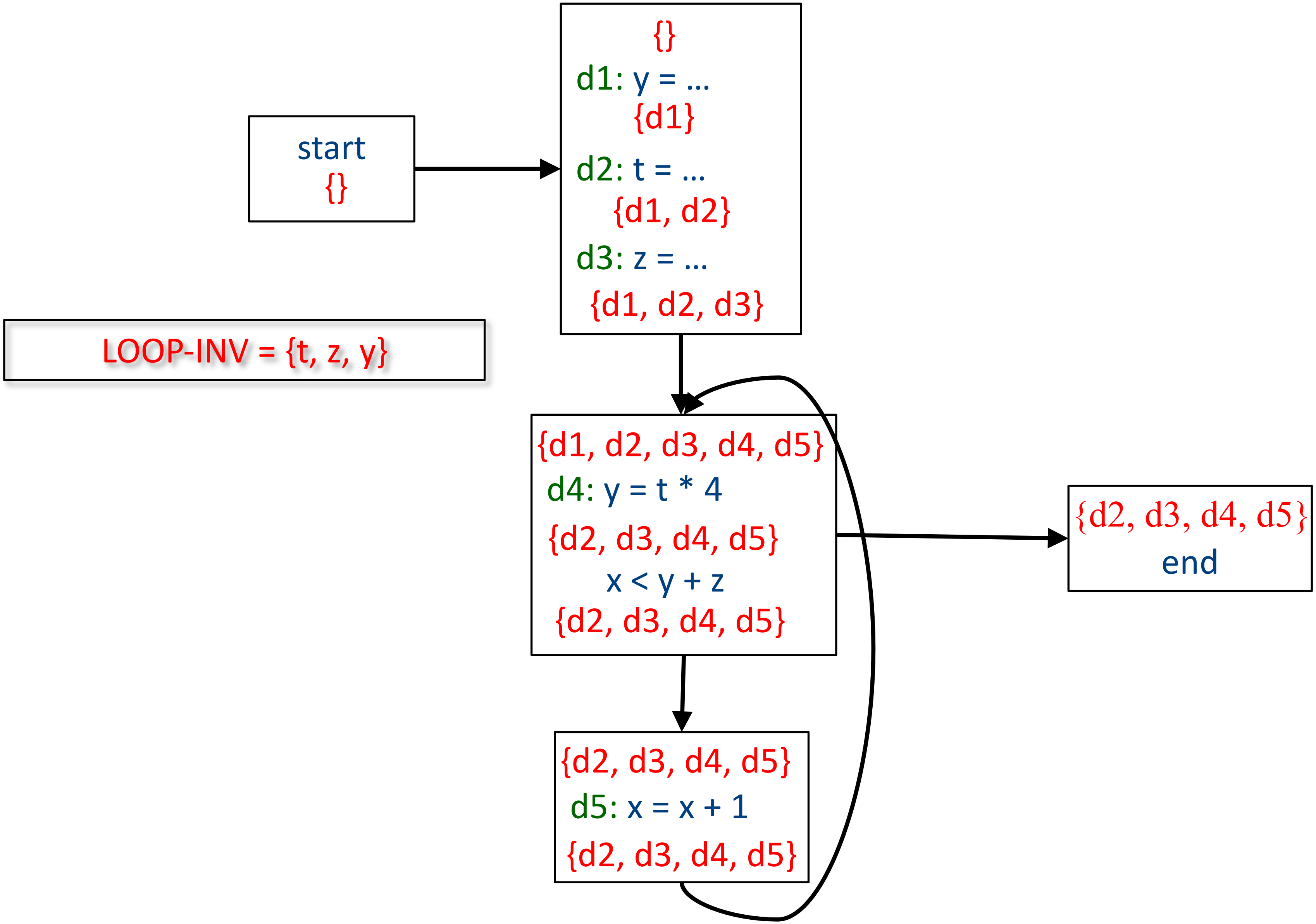


(immediately)
LOOP-INV = {t, z}

Computing LOOP-INV



Computing LOOP-INV



Induction variables

j is a linear function of the induction variable with multiplier 4

```
while (i < x) {
    j = a + 4 * i
    a[j] = j
    i = i + 1
}
```

i is incremented by a loop-invariant expression on each iteration – this is called an induction variable

Strength-reduction

Prepare initial value

```
j = a + 4 * i
while (i < x) {
    j = j + 4
    a[j] = j
    i = i + 1
}
```

Increment by multiplier

Compilation

0368-3133

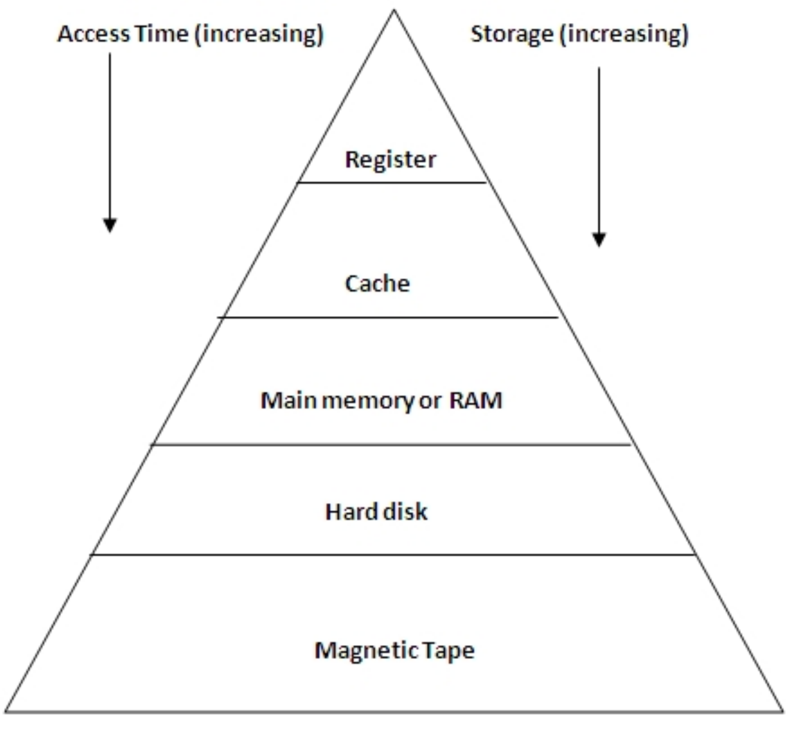
Lecture 10b

Register Allocation

Noam Rinetzky

Registers

- **Dedicated memory** locations that
  - can be accessed quickly,
  - can have computations performed on them, and



Registers

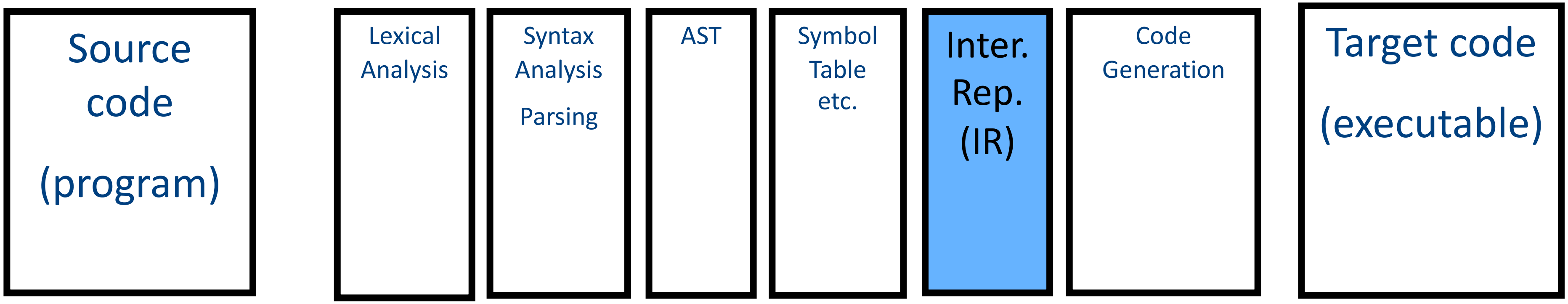
- **Dedicated memory** locations that
  - can be accessed quickly,
  - can have computations performed on them, and

- Usages
  - Operands of instructions
  - Store temporary results
  - Can (should) be used as loop indexes due to frequent arithmetic operation
  - Used to manage administrative info
    - e.g., runtime stack

Register allocation

- Number of registers is **limited**
- Need to **allocate** them in a clever way
  - Using registers intelligently is a critical step in any compiler
    - A good register allocator can generate code orders of magnitude better than a bad register allocator

Register Allocation: IR



Simple approach

- Straightforward solution:
  - Allocate each variable in activation record
  - At each instruction, bring values needed into registers, perform operation, then store result to memory

```
x = y + z
```

```
mov 16(%ebp), %eax
mov 20(%ebp), %ebx
add %ebx, %eax
mov %eax, 24(%ebp)
```

- Problem: program execution very inefficient– moving data back and forth between memory and registers

Register allocation

- In **TAC**, there is an unlimited number of variables (temporaries)
- On a physical machine there is a small number of registers:
  - **x86** has **4** general-purpose registers and a number of specialized registers
  - **MIPS** has **24** general-purpose registers and **8** special-purpose registers

- Register allocation is the process of assigning variables to registers and managing data transfer in and out of registers

simple code generation

- assume machine instructions of the form
- `LD reg, mem`
- `ST mem, reg`
- `OP reg,reg,reg` (*)

Fixed number of Registers!

- We will assume that we have all registers available for any usage
  - Ignore registers allocated for stack management
  - Treat all registers as general-purpose

Plan

- Goal: Reduce number of temporaries (registers)
  - Machine-agnostic optimizations
    - Assume unbounded number of registers
  - Machine-dependent optimization
    - Use at most K registers
    - K is machine dependent

Generating Compound Expressions

- Use registers to store temporaries
  - Why can we do it?

- Maintain a counter for temporaries in c
- Initially: c = 0
- **cgen**(e_1 *op* e_2) = {

  Let A = **cgen**(e_1)
  c = c + 1
  Let B = **cgen**(e_2)
  c = c + 1
  Emit(_tc = A *op* B;) // _tc is a register
  Return _tc

  }

Improving **cgen** for expressions

- Observation – naïve translation needlessly generates temporaries for leaf expressions
- Observation – temporaries used exactly once
  – Once a temporary has been read it can be reused for another sub-expression
- **cgen**(e_1 *op* e_2) = {
  Let _t1 = **cgen**(e_1)
  Let _t2 = **cgen**(e_2)
  Emit(_t1 =_t1 *op* _t2;)
  Return _t1
  }
- Temporaries **cgen**(e_1) can be reused in **cgen**(e_2)

Register Allocation

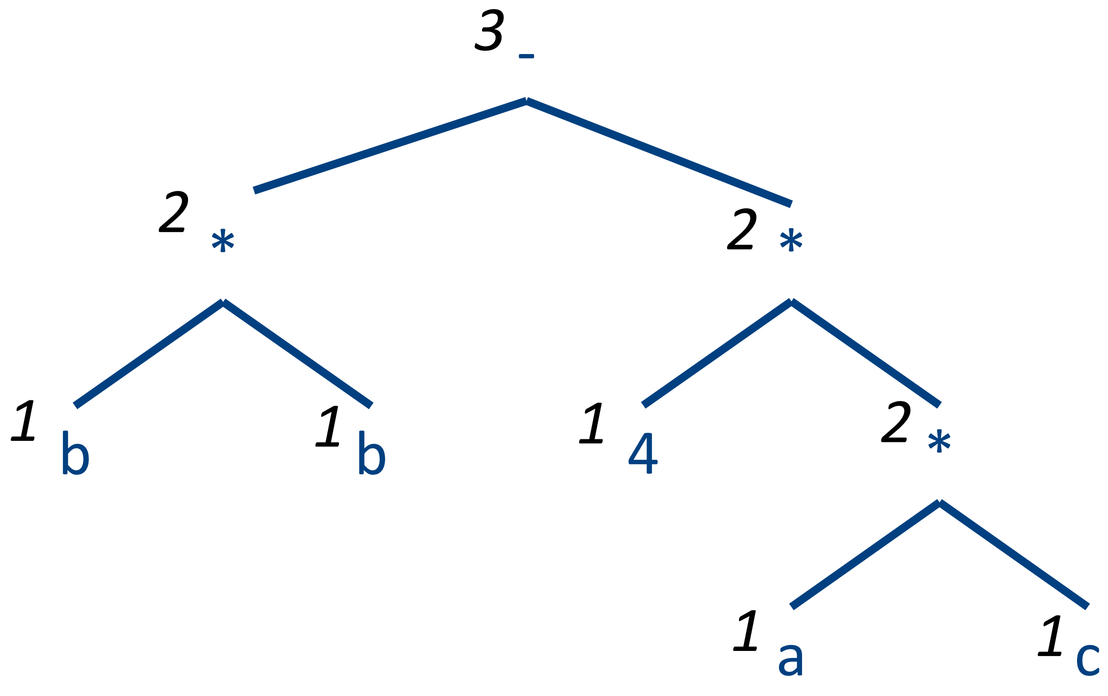
- Machine-agnostic optimizations
    - Assume unbounded number of registers
  - Expression trees
  - Basic blocks

- Machine-dependent optimization
    - K registers
    - Some have special purposes
  - Control flow graphs (whole program)

Sethi-Ullman translation

- Algorithm by Ravi Sethi and Jeffrey D. Ullman to emit optimal TAC
  - Minimizes number of temporaries for a **single expression**

Example (optimized): b*b-4*a*c



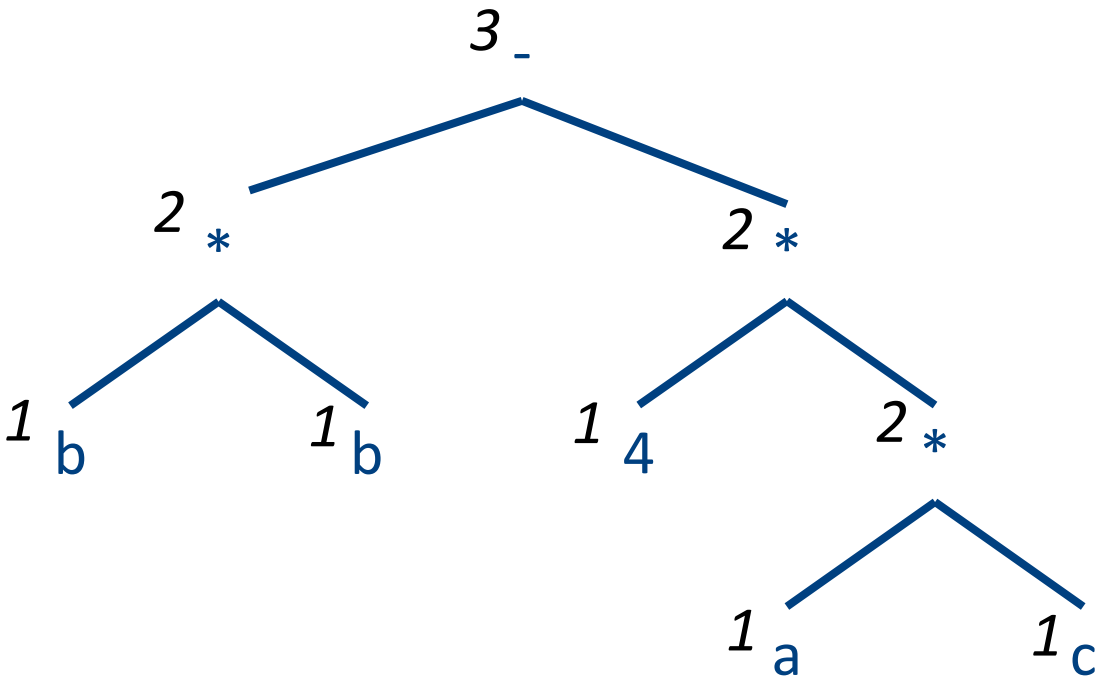
Generalizations

- More than two arguments for operators
  - Function calls
- Multiple effected registers
  - Multiplication
- Spilling
  - Need more registers than available
- Register/memory operations

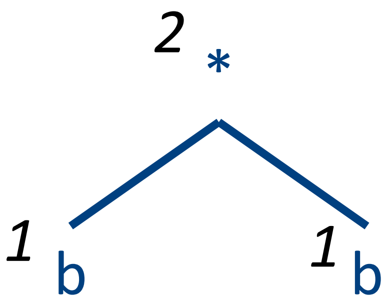
Simple **Spilling** Method

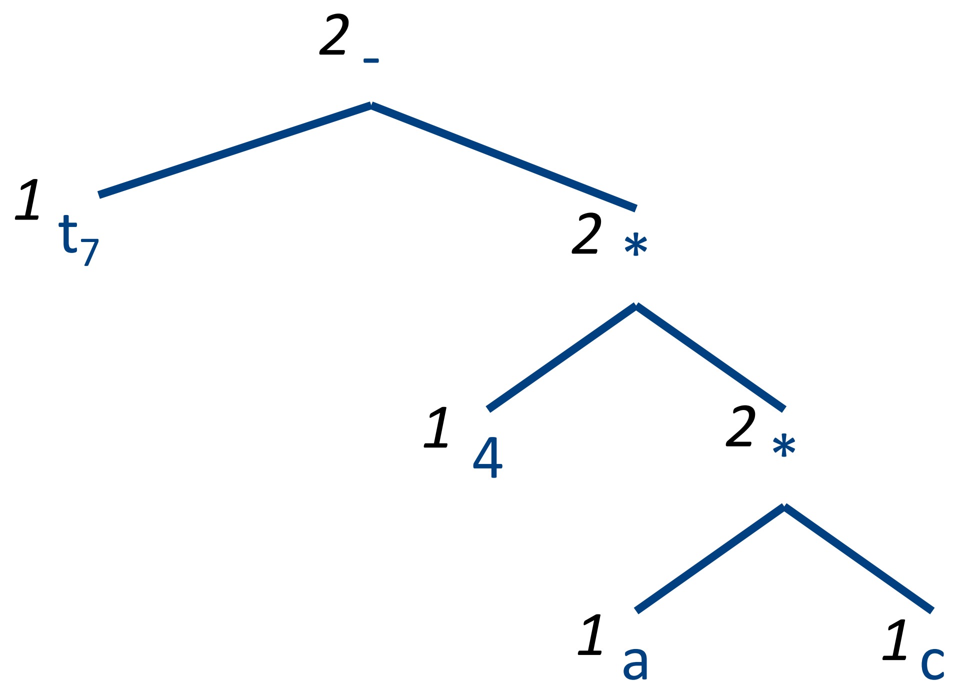
- Heavy tree – Needs more registers than available
- A “heavy” tree contains a “heavy” subtree whose dependents are “light”
- Simple spilling
  - Generate code for the light tree
  - Spill the content into memory and replace subtree by temporary
  - Generate code for the resultant tree

Example (optimized): x:=b*b-4*a*c



Example (spilled): x := b*b-4*a*c






```
t7 := b * b
```

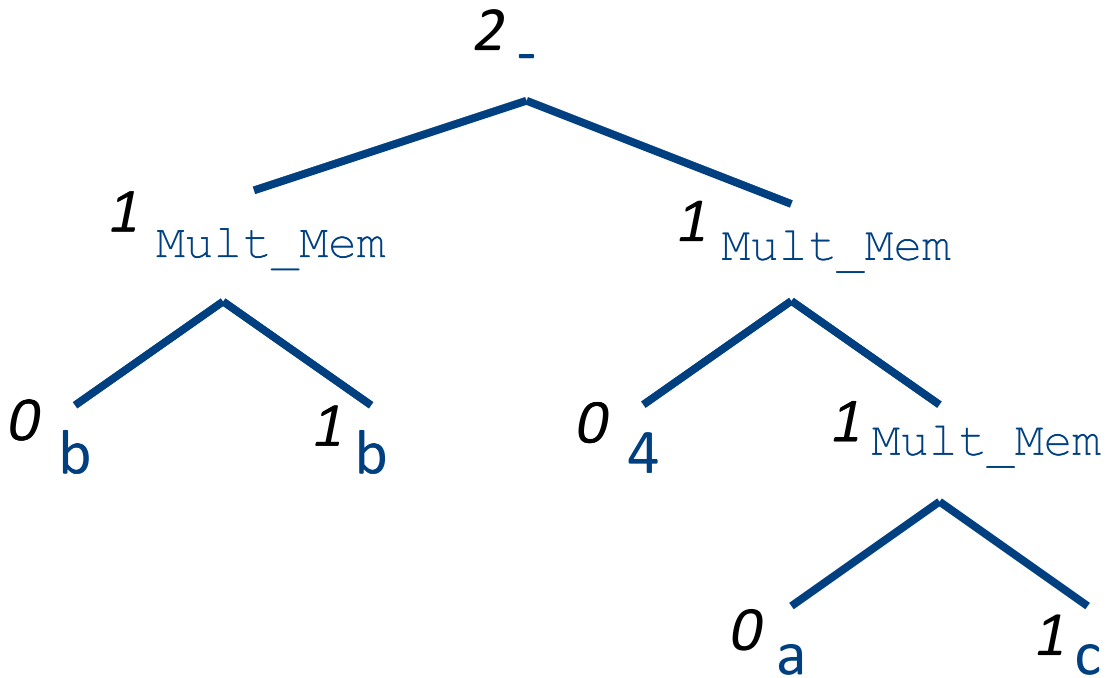
```
x := t7 - 4 * a * c
```

Register Memory Operations

- Add_Mem X, R1
- Mult_Mem X, R1
- No need for registers to store right operands

Hidden Registers

Example: b*b-4*a*c



Can We do Better?

- Yes: Increase view of code
  - Simultaneously allocate registers for multiple expressions
- But: Lose per expression optimality
  - Works well in practice

Register Allocation

- Machine-agnostic optimizations
    - Assume unbounded number of registers
  - Expression trees
  - Basic blocks
- Machine-dependent optimization
    - K registers
    - Some have special purposes
  - Control flow graphs (whole program)

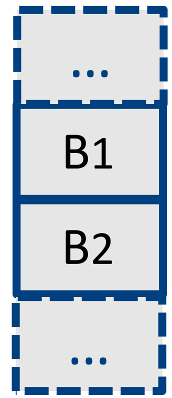
Basic Blocks

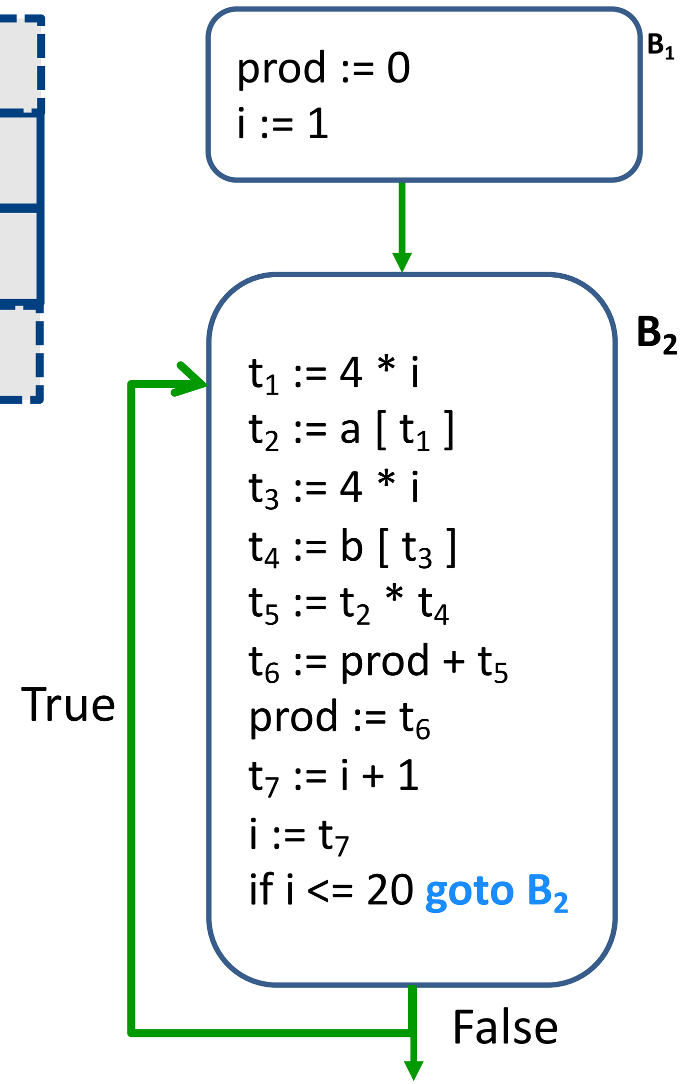
- **basic block** is a sequence of instructions with
  - **single entry** (to first instruction), no jumps to the middle of the block
  - **single exit** (last instruction)
  - code execute as a sequence from first instruction to last instruction without any jumps
- edge from one basic block B1 to another block B2 when the last statement of B1 may jump to B2

control flow graph

- A directed graph G=(V,E)
- nodes V = basic blocks
- edges E = control flow
  - (B1,B2) ∈ E when control from B1 flows to B2

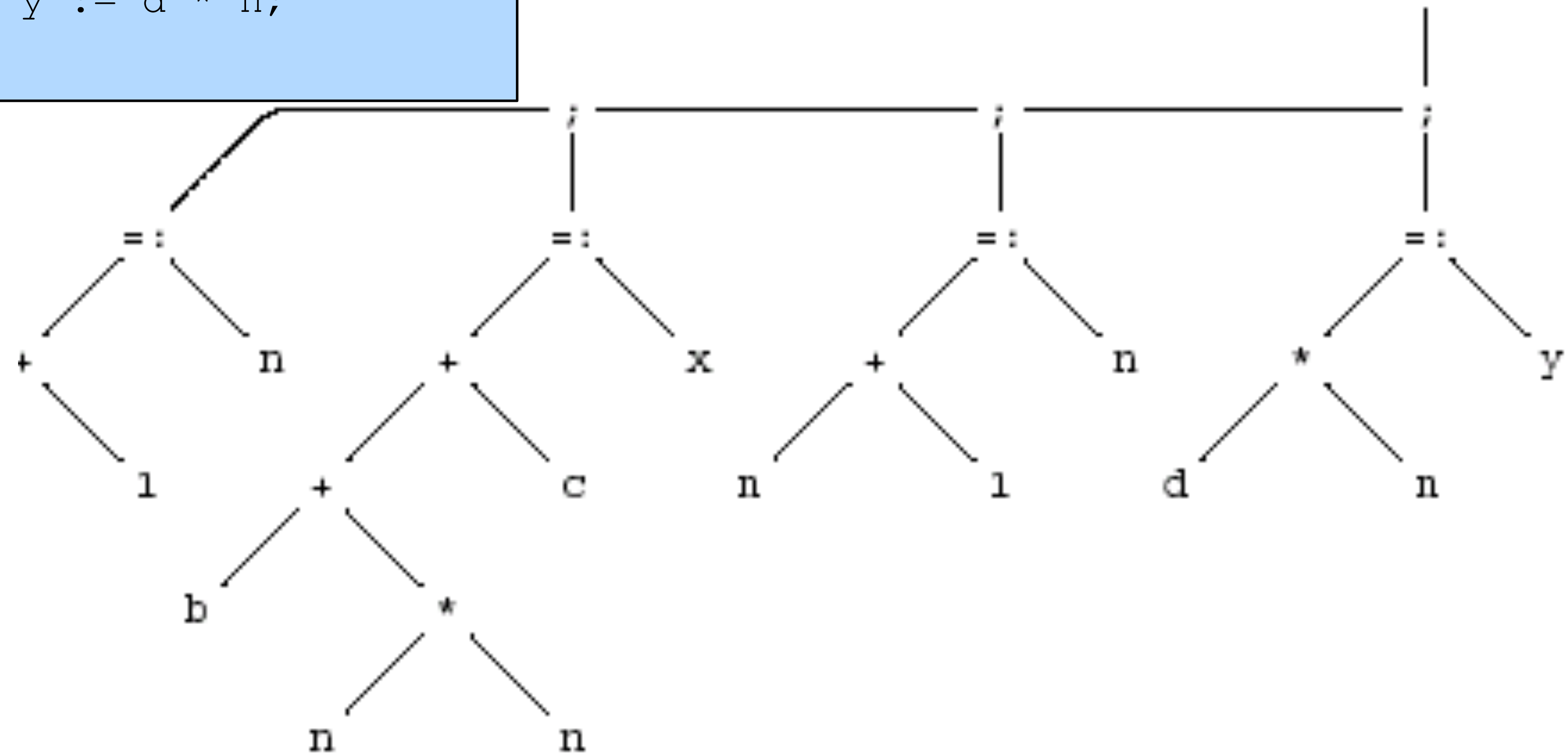
- Leaders-based construction
  - Target of jump instructions
  - Instructions following jumps





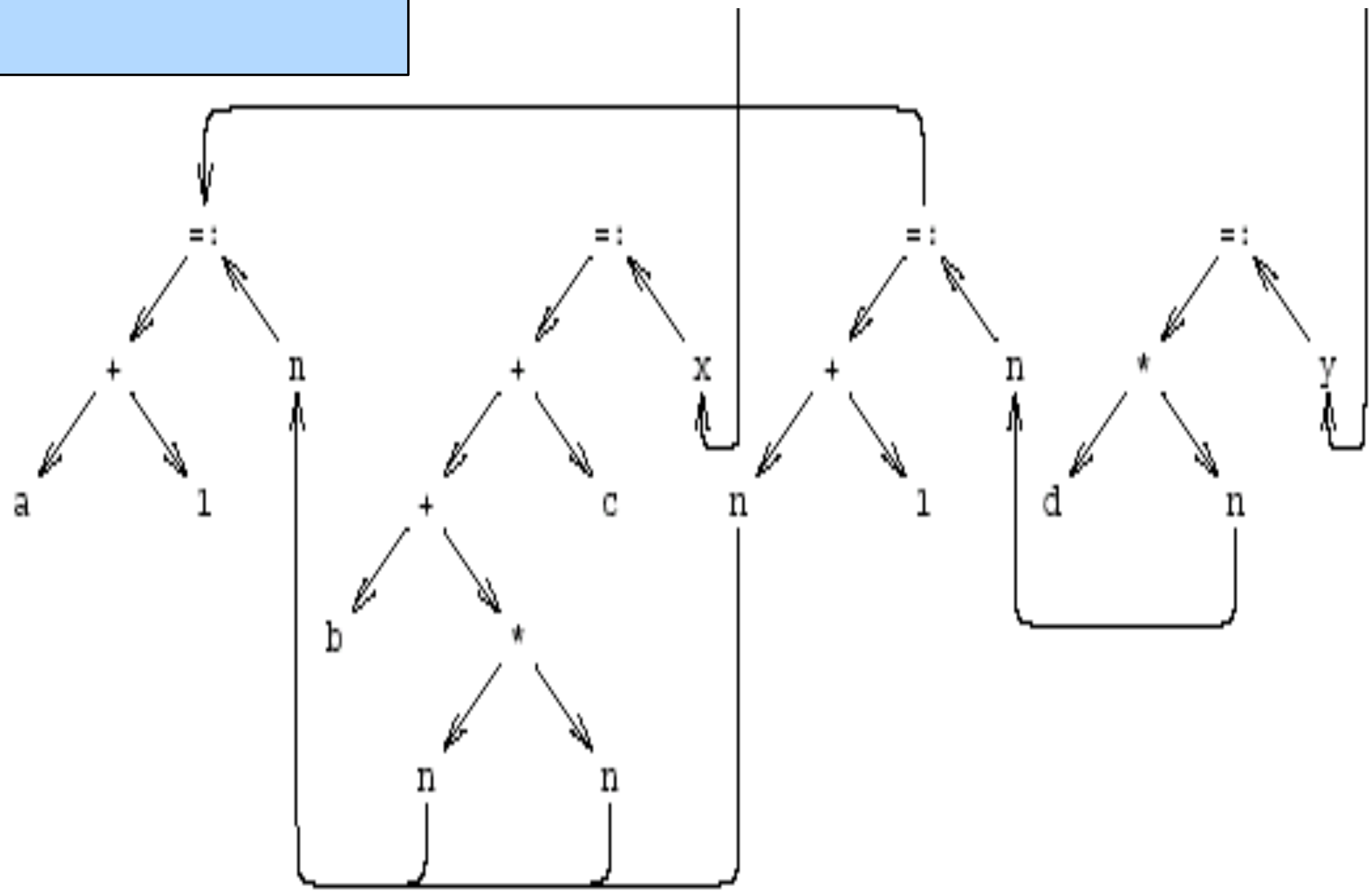
AST for a Basic Block

```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```



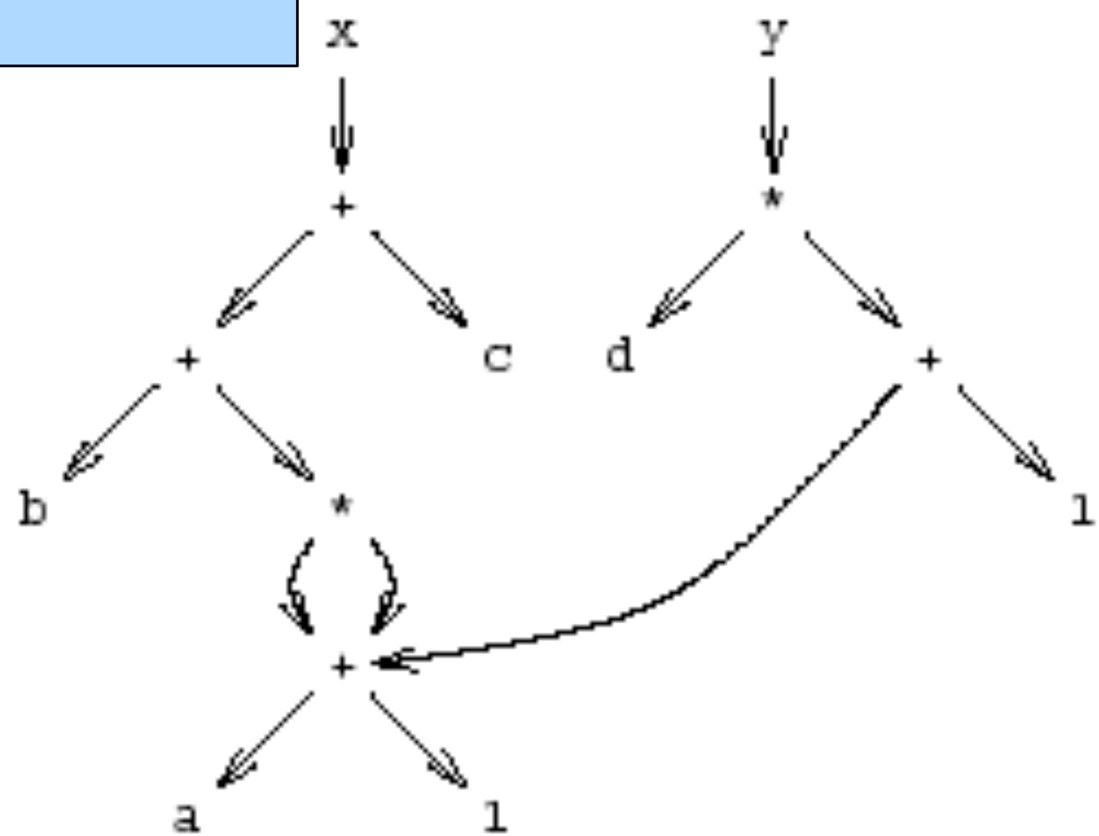
Dependency graph

```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```

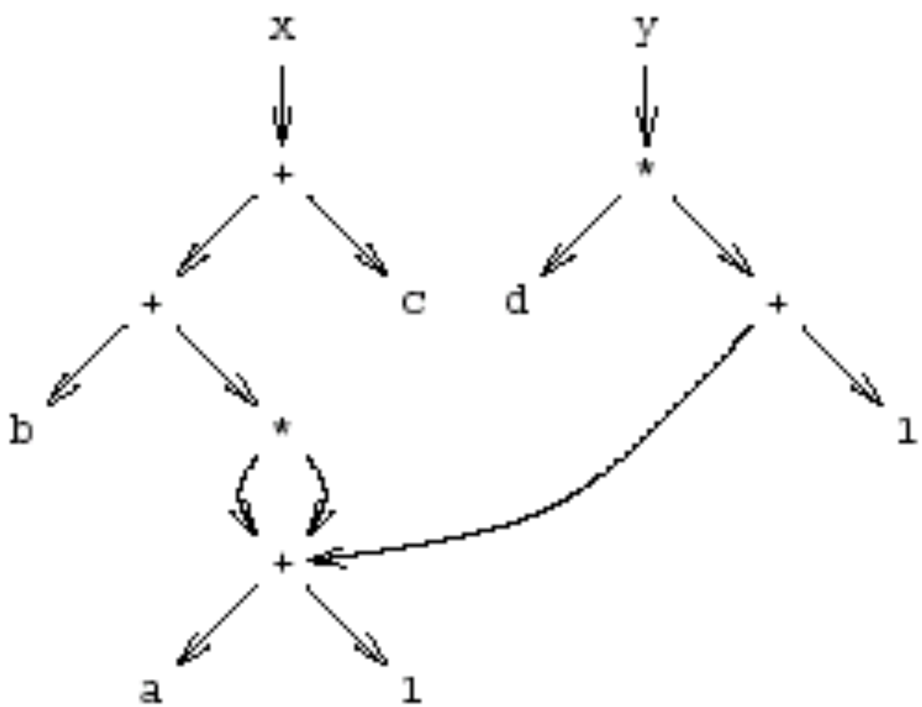


Simplified Data Dependency Graph

```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```



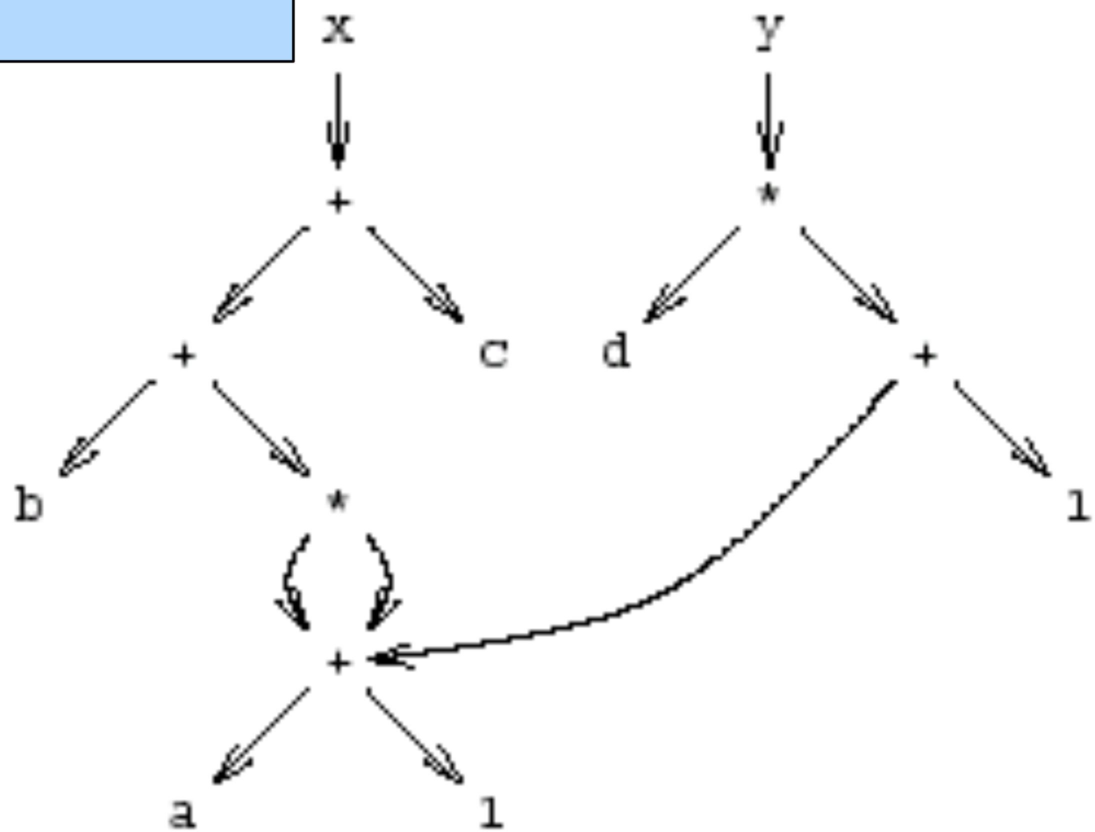
Pseudo Register Target Code



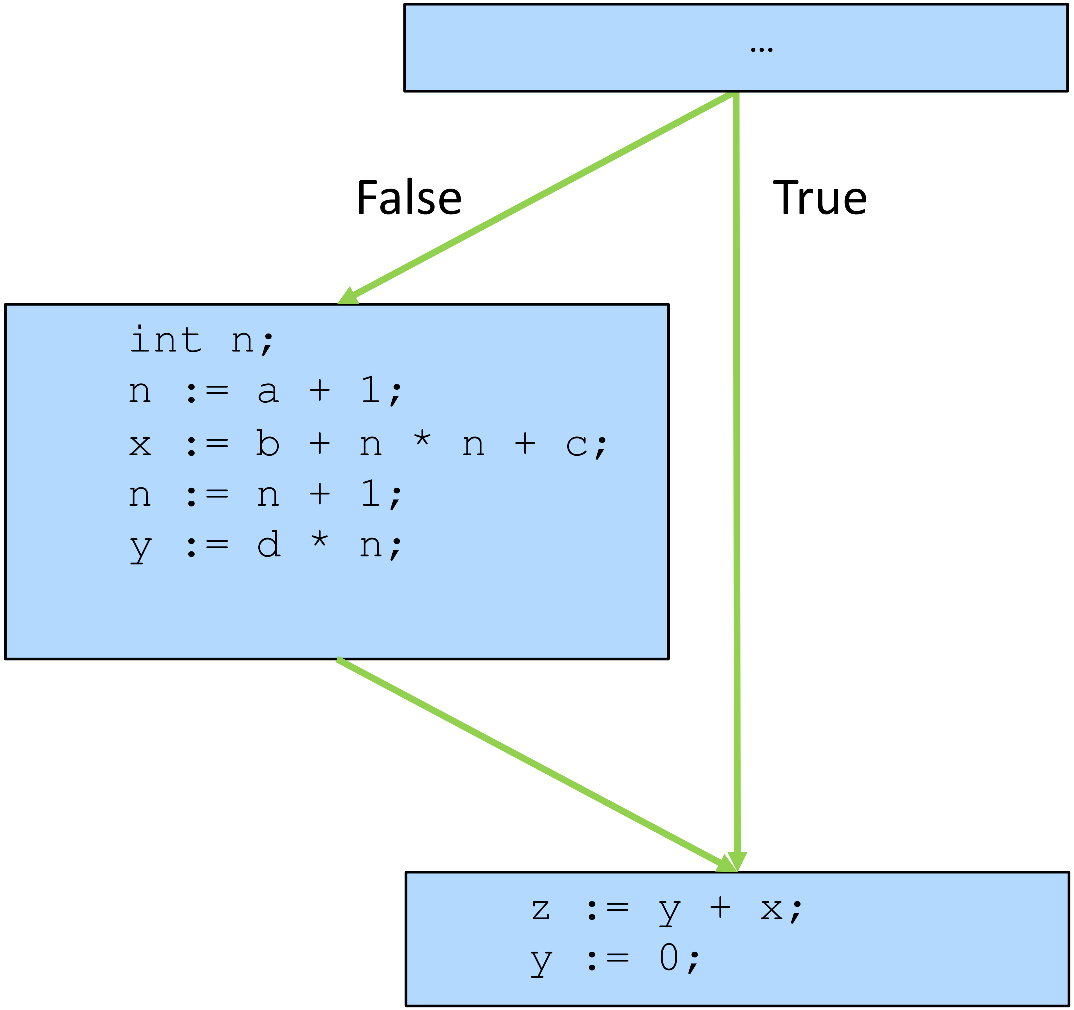

```
Load_Mem     a,R1
Add_Const    1,R1
Load_Reg     R1,X1

Load_Reg     X1,R1
Mult_Reg     X1,R1
Add_Mem      b,R1
Add_Mem      c,R1
Store_Reg    R1,x

Load_Reg     X1,R1
Add_Const    1,R1
Mult_Mem     d,R1
Store_Reg    R1,y
```

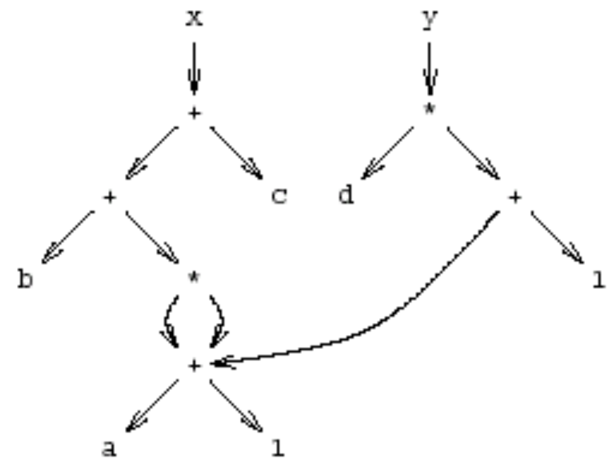
```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```

Question: Why "y"?

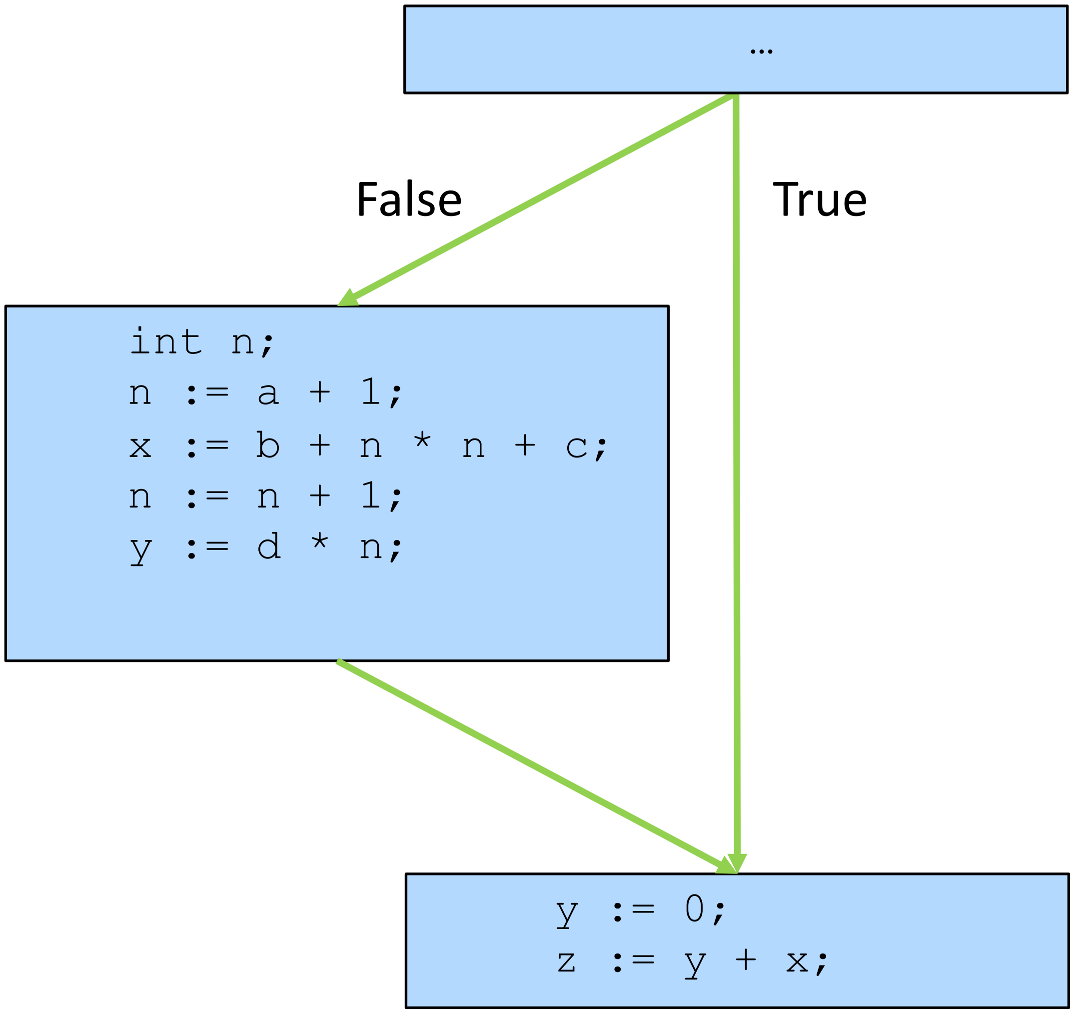


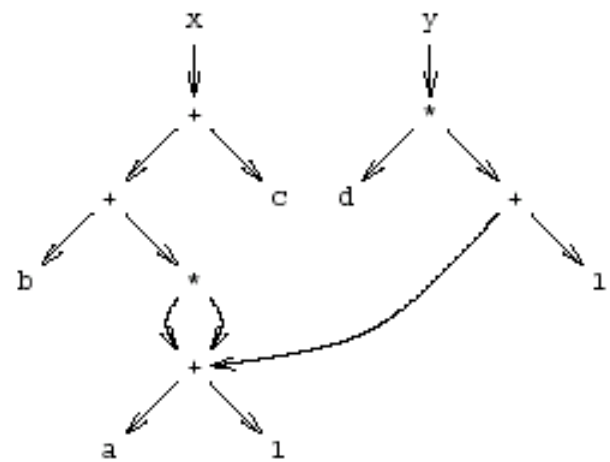
Question: Why "y"?



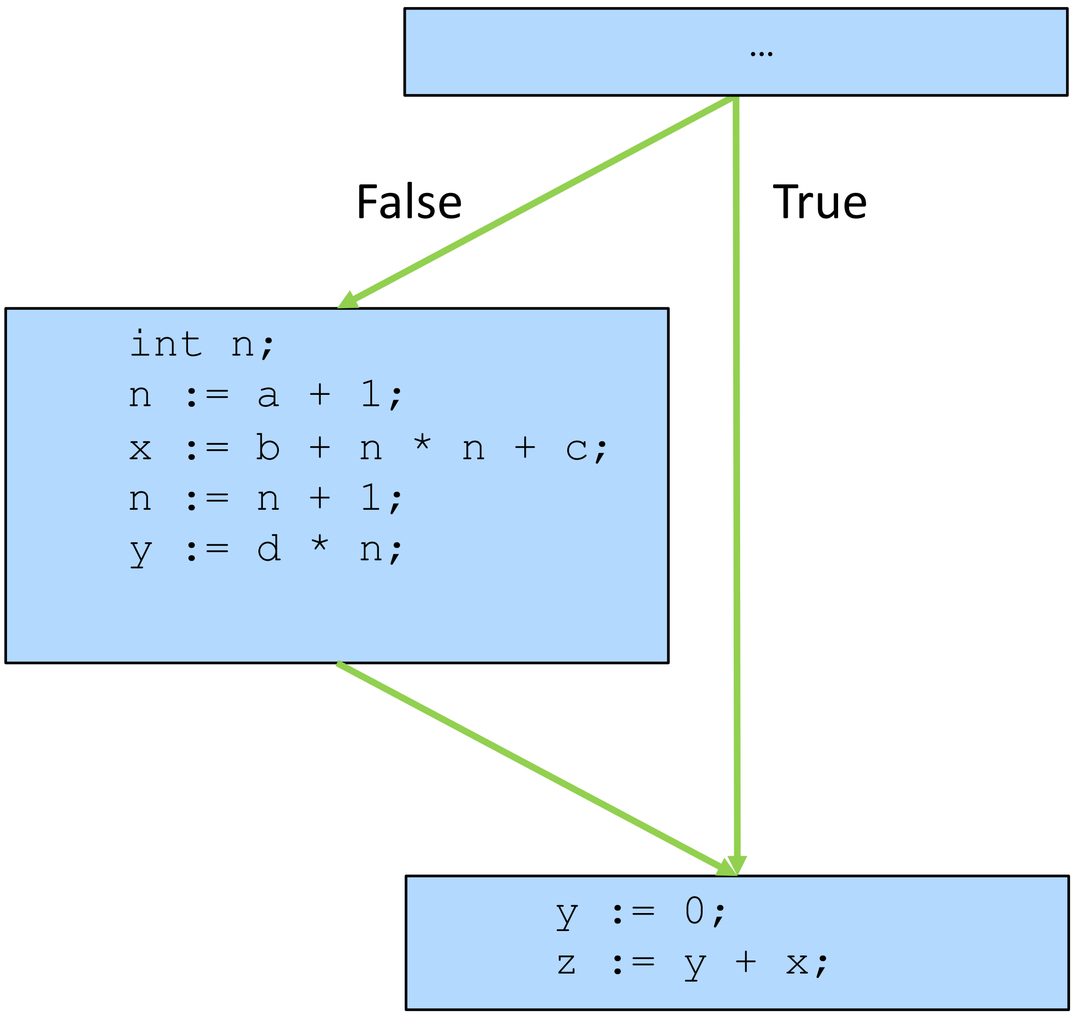


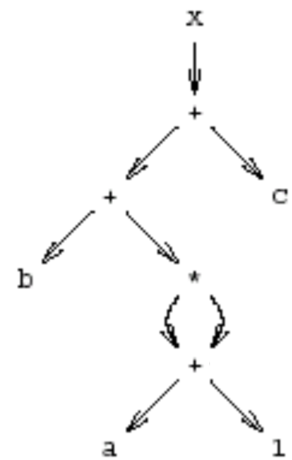
Question: Why "y"?



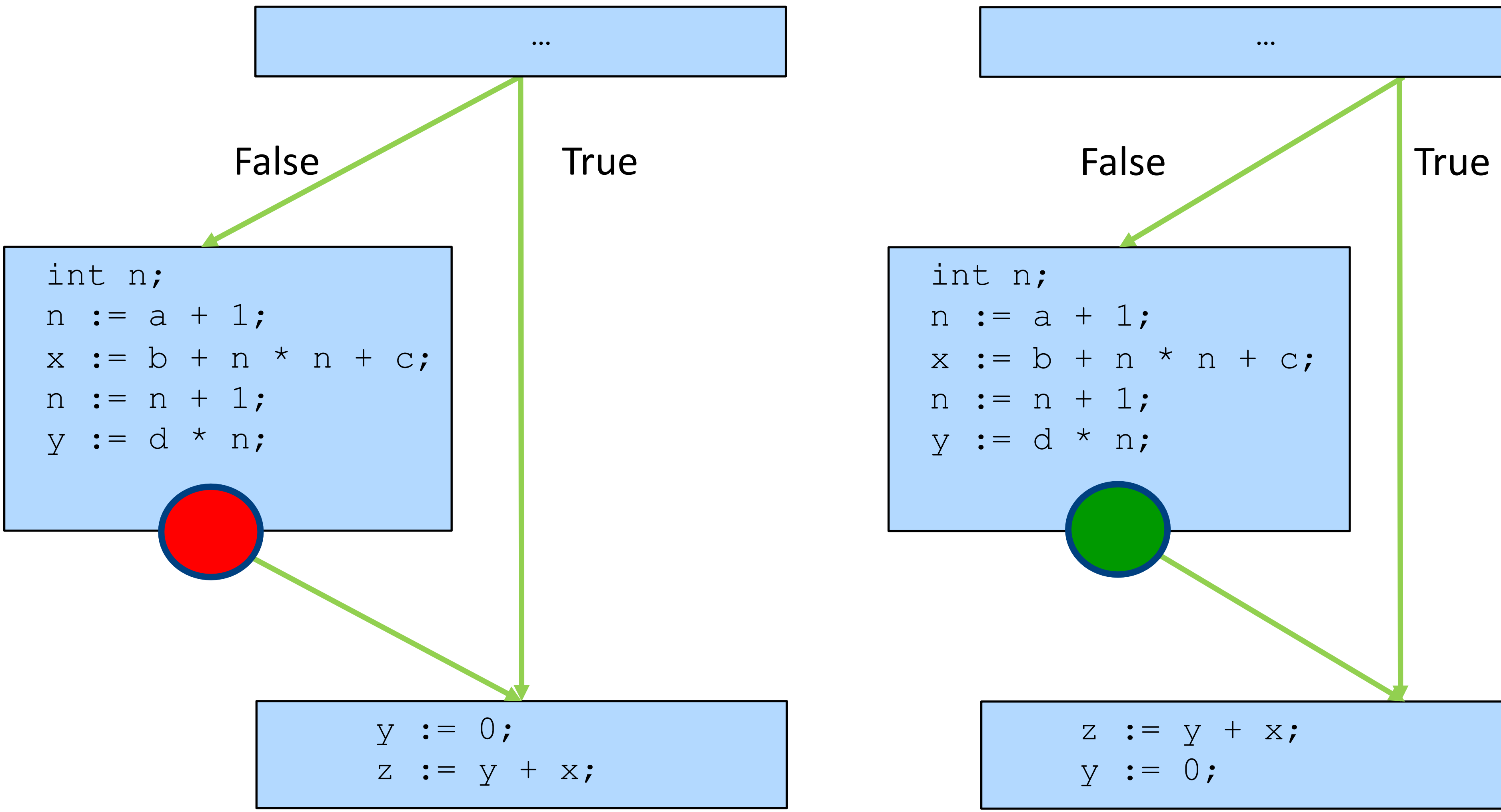


Question: Why “y”?

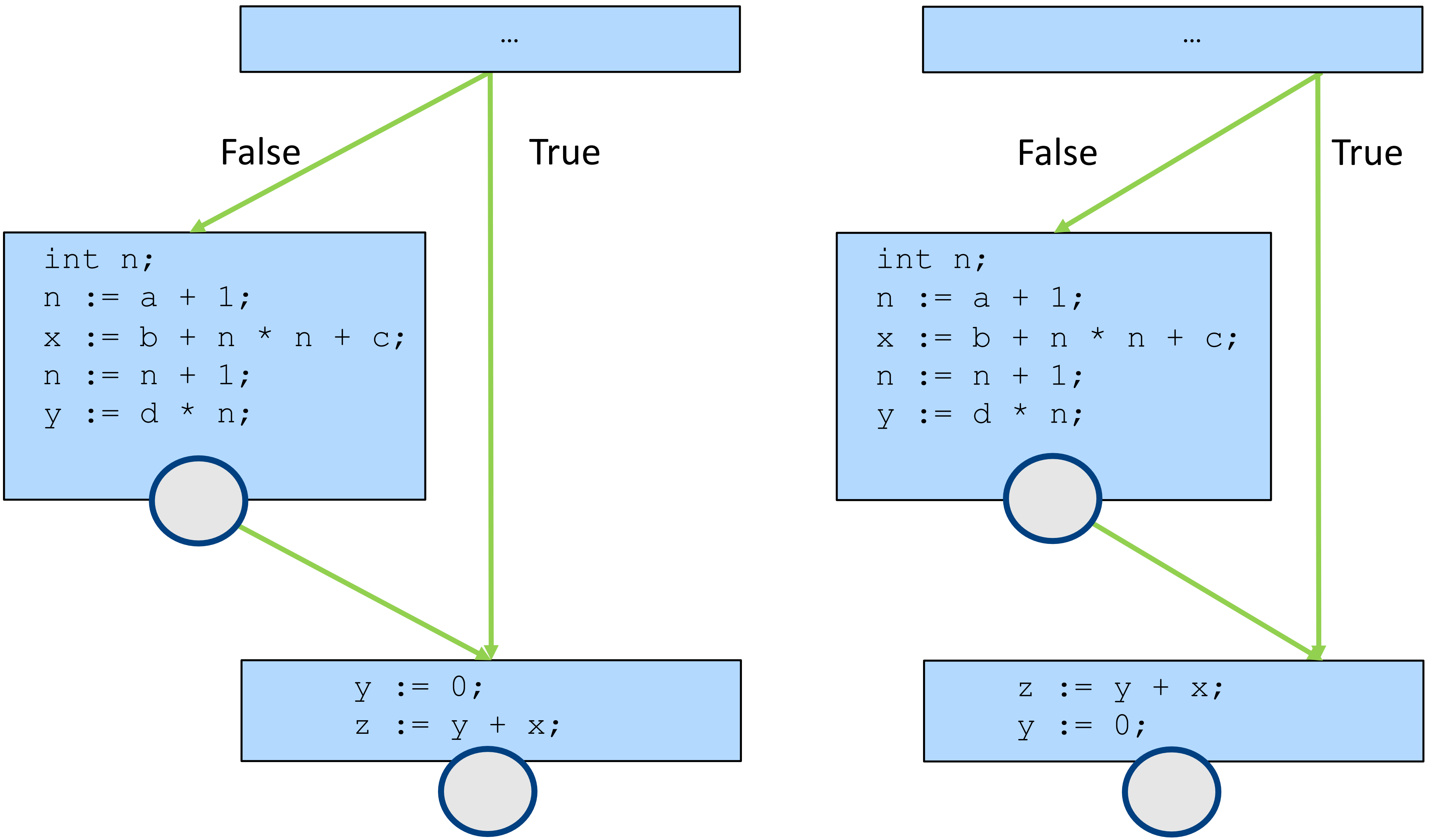




`y`, dead or alive?



x, dead or alive?



Register Allocation for B.B.

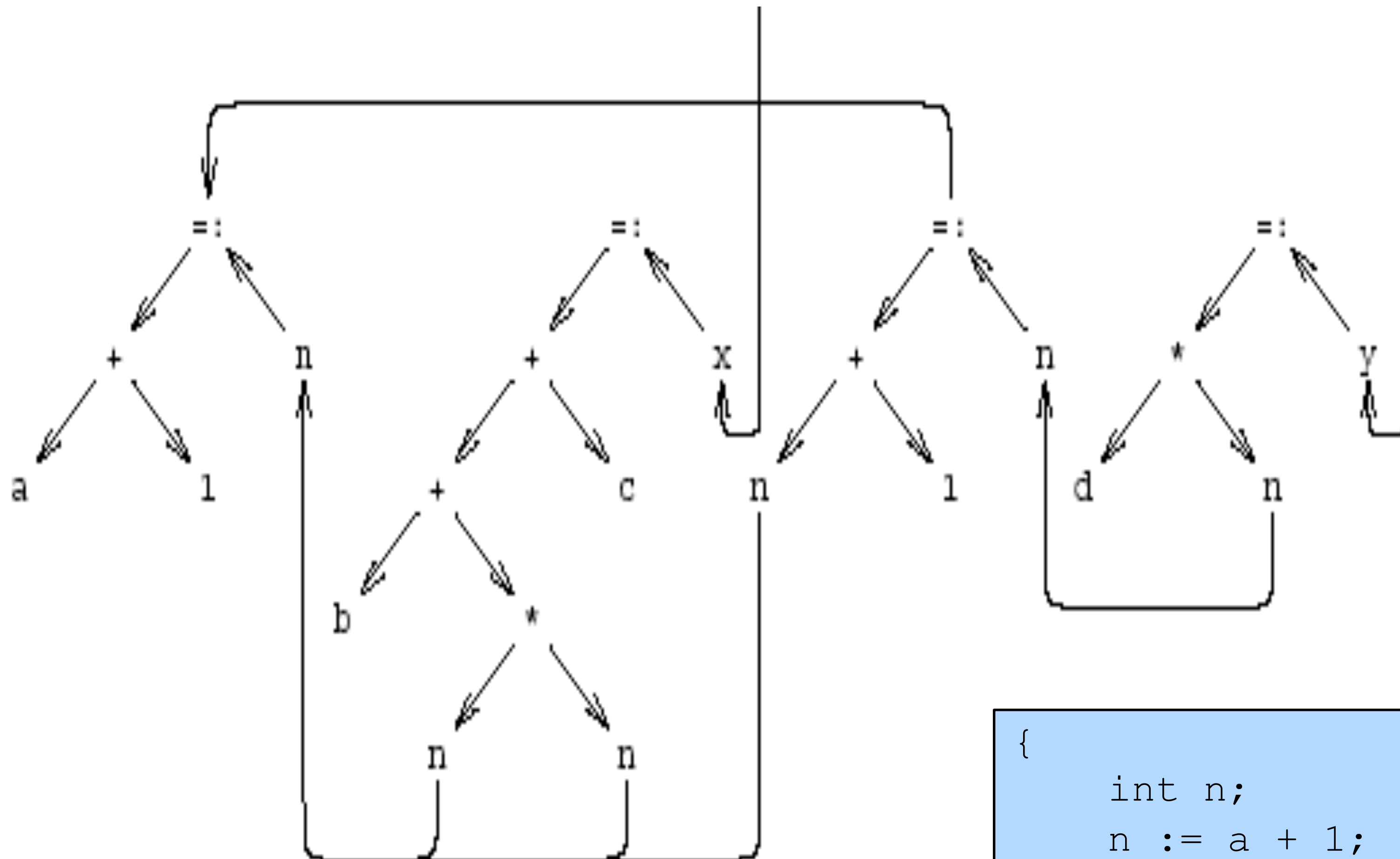
- Dependency graphs for basic blocks
- Transformations on dependency graphs
- From dependency graphs into code
  - Instruction selection
    - linearizations of dependency graphs
  - Register allocation
    - At the basic block level

Dependency graphs

- TAC imposes an order of execution
  - But the compiler can reorder assignments as long as the program results are not changed

- Define a partial order on assignments
  - $a < b \Leftrightarrow a$ must be executed before b
  - Represented as a directed graph
    - Nodes are assignments
    - Edges represent dependency
  - Acyclic for basic blocks

Running Example




```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```

Sources of dependency

- Data flow inside expressions
  - Operator depends on operands
  - Assignment depends on assigned expressions
- Data flow between statements
  - From assignments to their use

  - Pointers complicate dependencies

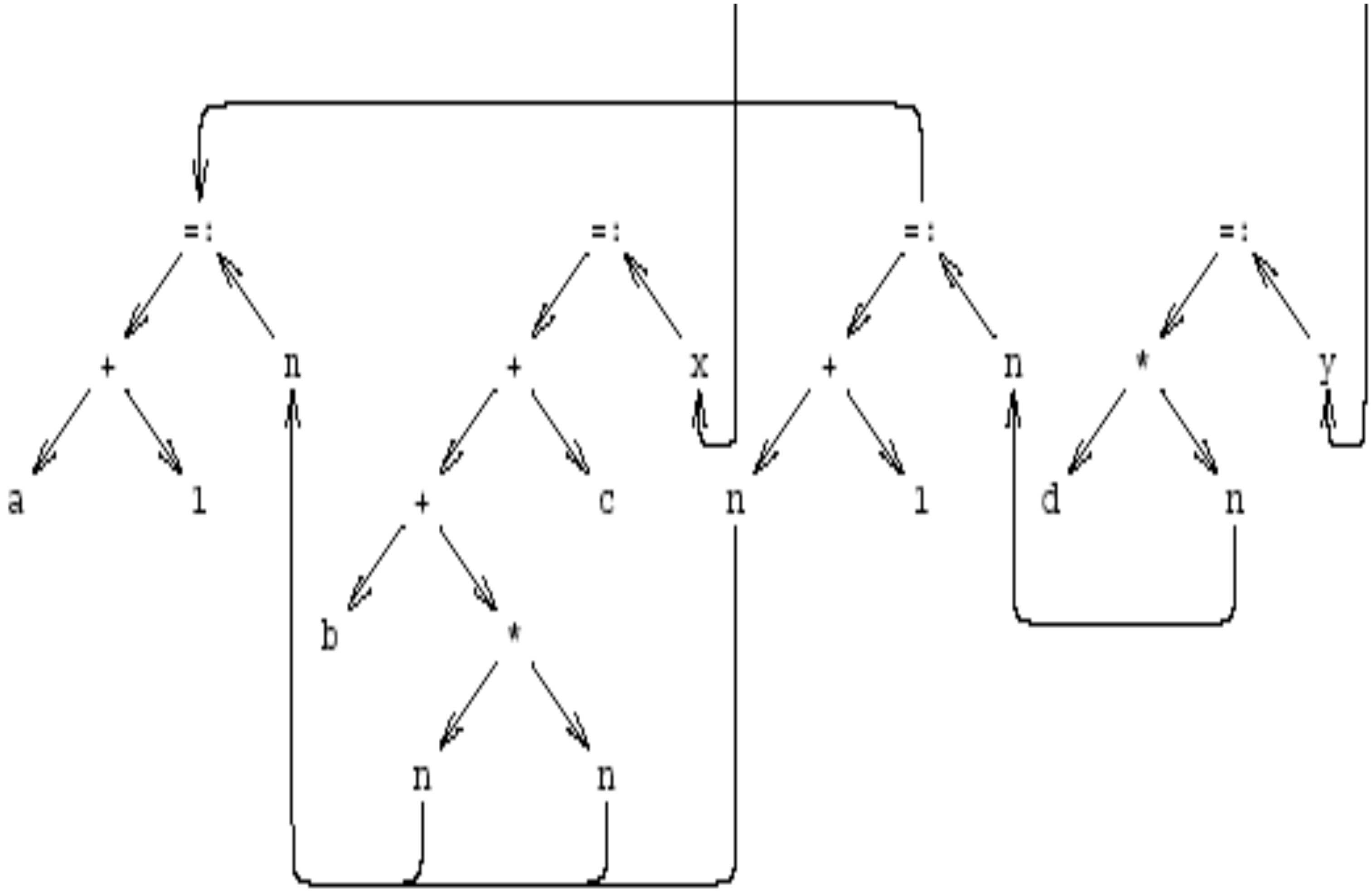
Sources of dependency

- Order of subexpresion evaluation is immaterial
  - As long as inside dependencies are respected
- The order of uses of a variable X are immaterial as long as:
  - X is used between dependent assignments
  - Before next assignment to X

Creating Dependency Graph from AST

- Nodes AST becomes nodes of the graph
- Replaces arcs of AST by dependency arrows
  - Operator $\rightarrow$ Operand
  - Create arcs from assignments to uses
  - Create arcs between assignments of the same variable
- Select output variables (roots)
- Remove ; nodes and their arrows

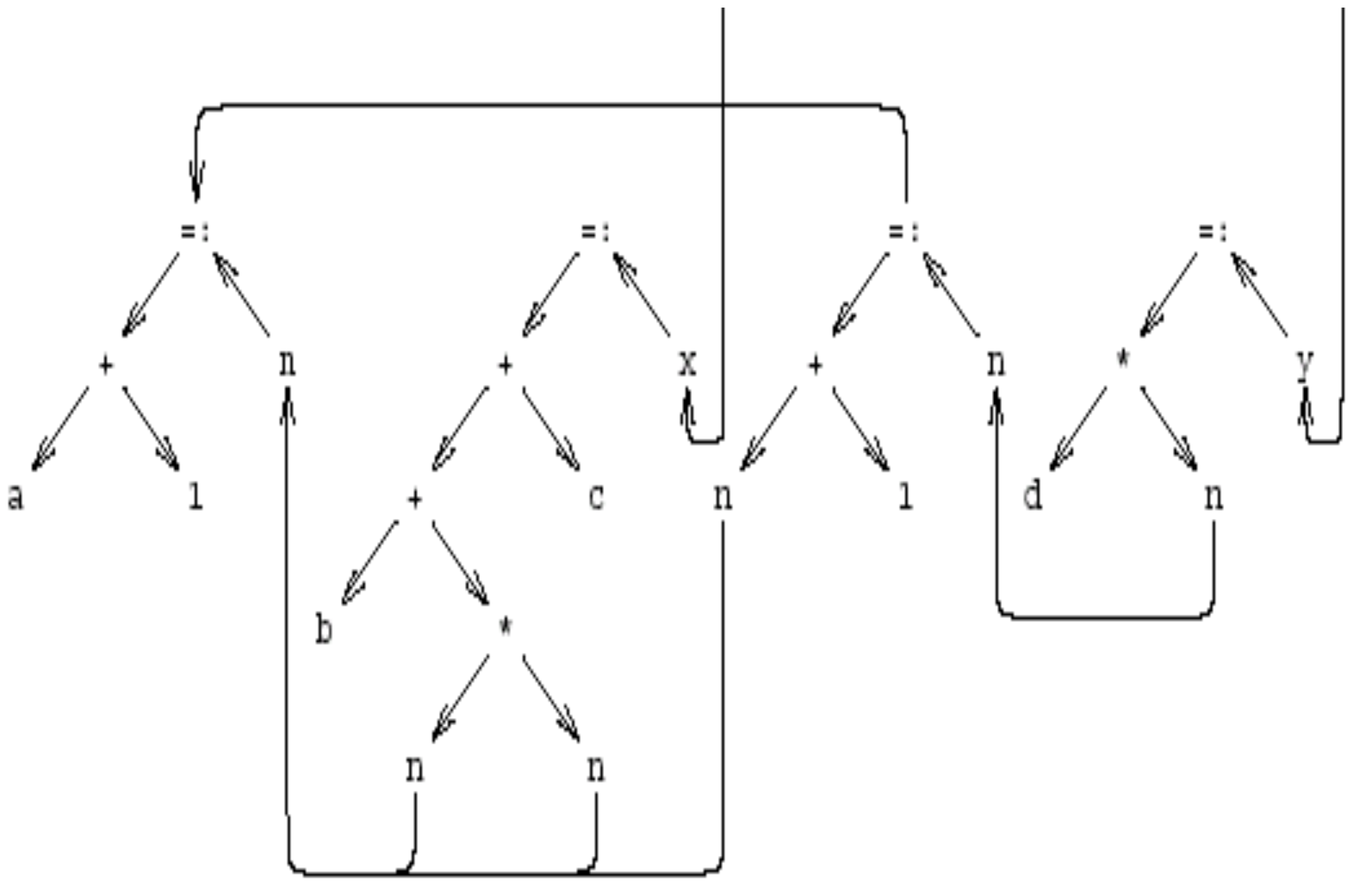
Running Example



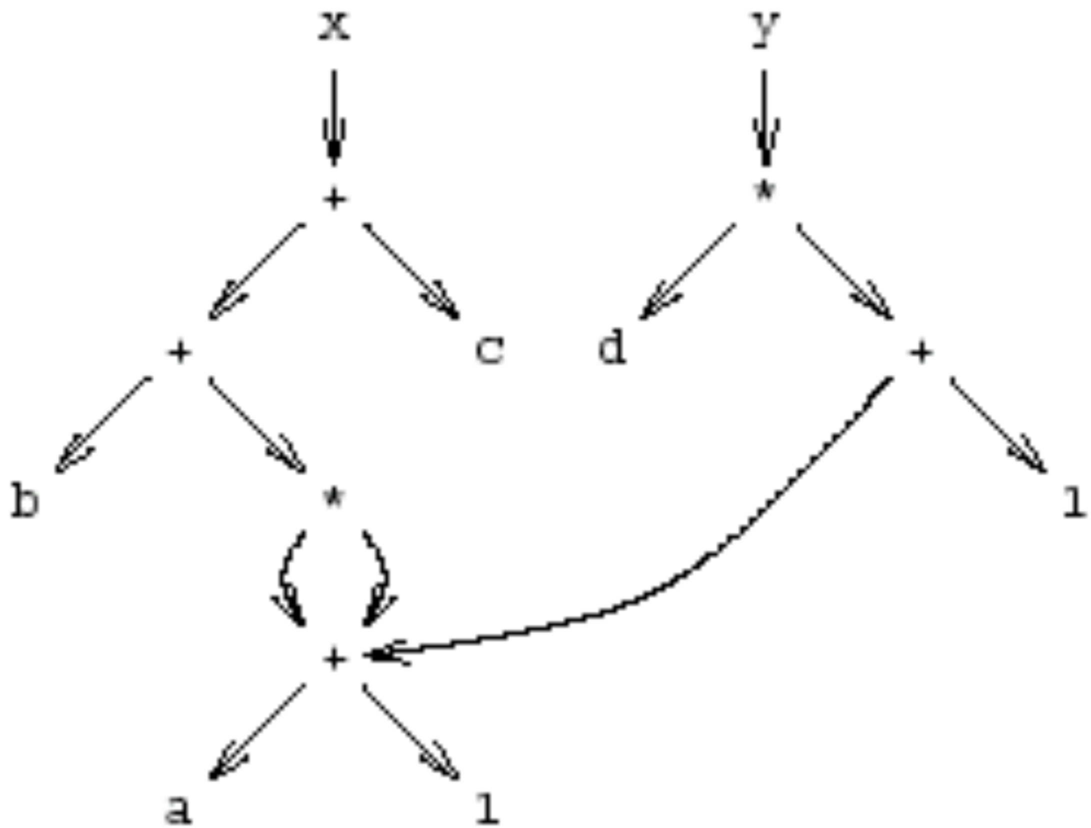
Dependency Graph Simplifications

- Short-circuit assignments
  - Connect variables to assigned expressions
  - Connect expression to uses
- Eliminate nodes not reachable from roots

Running Example



Cleaned-Up Data Dependency Graph



Common Subexpressions

- Repeated subexpressions
- Examples

  x = a * a +   2 * a * b + b * b;

  y = a * a –   2 * a * b + b * b;

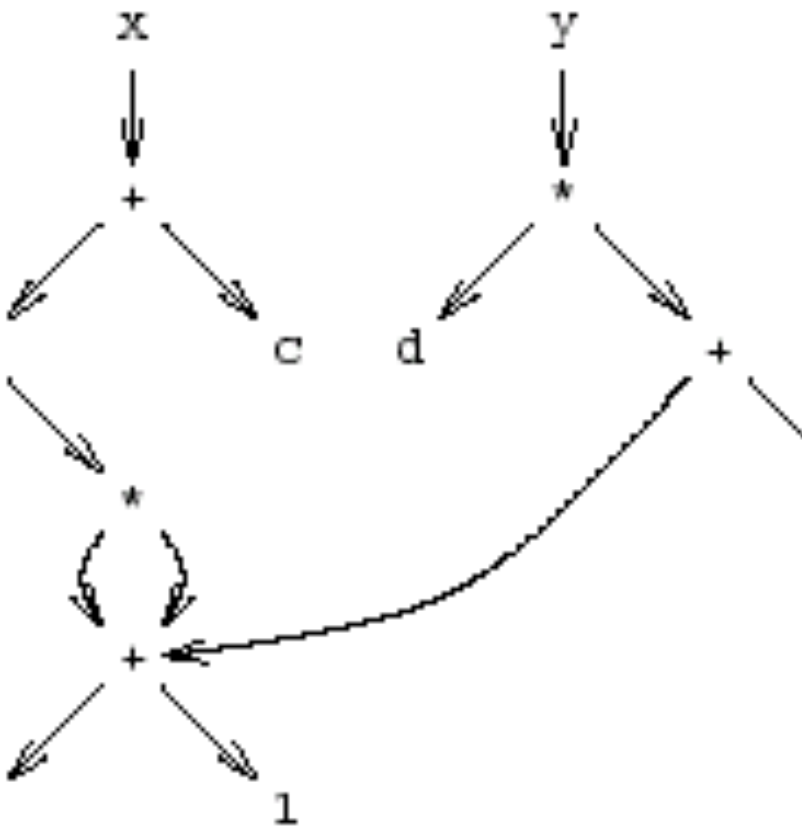
  n[i] := n[i] +m[i]

- Can be eliminated by the compiler
  - In the case of basic blocks rewrite the DAG

From Dependency Graph into Code

- Linearize the dependency graph
  - Instructions must follow dependency
- Many solutions exist
- Select the one with small runtime cost
- Assume infinite number of registers
  - Symbolic registers
  - Assign registers later
    - May need additional spill
  - Possible Heuristics
    - Late evaluation
    - Ladders

Pseudo Register Target Code




```
Load_Mem    a,R1
Add_Const   1,R1
Load_Reg    R1,X1

Load_Reg    X1,R1
Mult_Reg    X1,R1
Add_Mem     b,R1
Add_Mem     c,R1
Store_Reg   R1,x

Load_Reg    X1,R1
Add_Const   1,R1
Mult_Mem    d,R1
Store_Reg   R1,y
```

Non optimized vs Optimized Code

```
Load_Mem     a,R1
Add_Const    1,R1
Load_Reg     R1,X1

Load_Reg     X1,R1
Mult_Reg     X1,R1
Add_Mem      b,R1
Add_Mem      c,R1
Store_Reg    R1,x

Load_Reg     X1,R1
Add_Const    1,R1
Mult_Mem     d,R1
Store_Reg    R1,y
```

```
{
  int n;
  n := a + 1;
  x := b + n * n + c;
  n := n + 1;
  y := d * n;
}
```

```
Load_Mem     a,R1
Add_Const    1,R1
Load_Reg     R1,R2

Load_Reg     R2,R1
Mult_Reg     R2,R1
Add_Mem      b,R1
Add_Mem      c,R1
Store_Reg    R1,x

Load_Reg     R2,R1
Add_Const    1,R1
Mult_Mem     d,R1
Store_Reg    R1,y
```

```
ıd_Mem      a,R1
l_Const     1,R1
ıd_Reg      R1,R2

.t_Reg      R1,R2
l_Mem       b,R2
l_Mem       c,R2
ıre_Reg     R2,x

l_Const     1,R1
.t_Mem      d,R1
ıre_Reg     R1,y
```

Register Allocation

- Maps symbolic registers into physical registers
  - Reuse registers as much as possible
  - Graph coloring (next)
    - Undirected graph
    - Nodes = Registers (Symbolic and real)
    - Edges = Interference
    - May require spilling

Register Allocation for Basic Blocks

- Heuristics for code generation of basic blocks
- Works well in practice
- Fits modern machine architecture
- Can be extended to perform other tasks
  - Common subexpression elimination
- But basic blocks are small
- Can be generalized to a procedure

Problem	Technique	Quality
Expression trees, using register-register or memory-register instructions	Weighted trees; Figure 4.30	
with sufficient registers:		Optimal
with insufficient registers:		Optimal
Dependency graphs, using register-register or memory-register instructions	Ladder sequences; Section 4.2.5.2	Heuristic
Expression trees, using any instructions with cost function	Bottom-up tree rewriting; Section 4.2.6	
with sufficient registers:		Optimal
with insufficient registers:		Heuristic
Register allocation when all interferences are known	Graph coloring; Section 4.2.7	Heuristic

The End