Compilation

0368-3133

Lecture 3a:

Syntax Analysis:

Top-Down parsing

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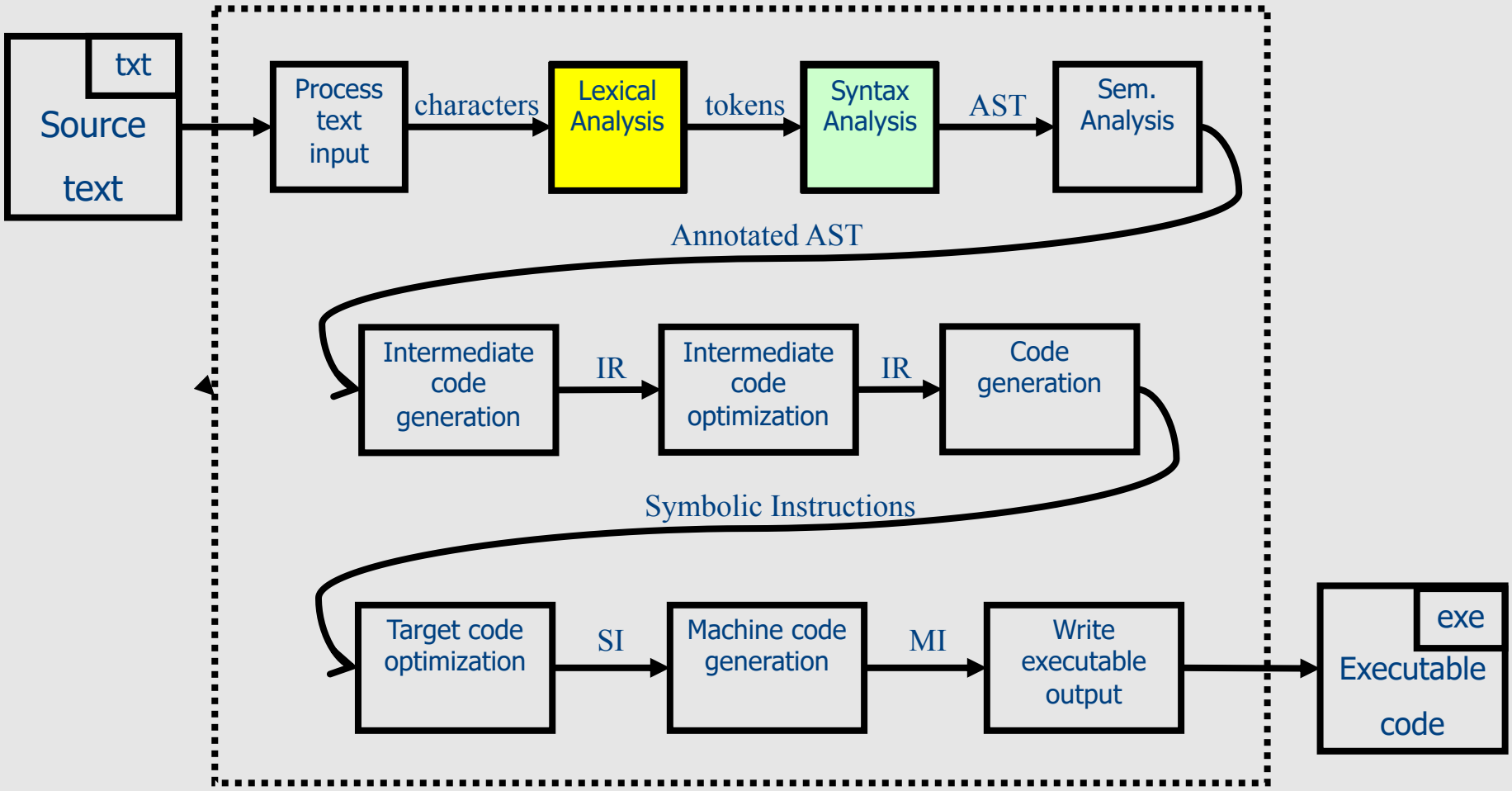
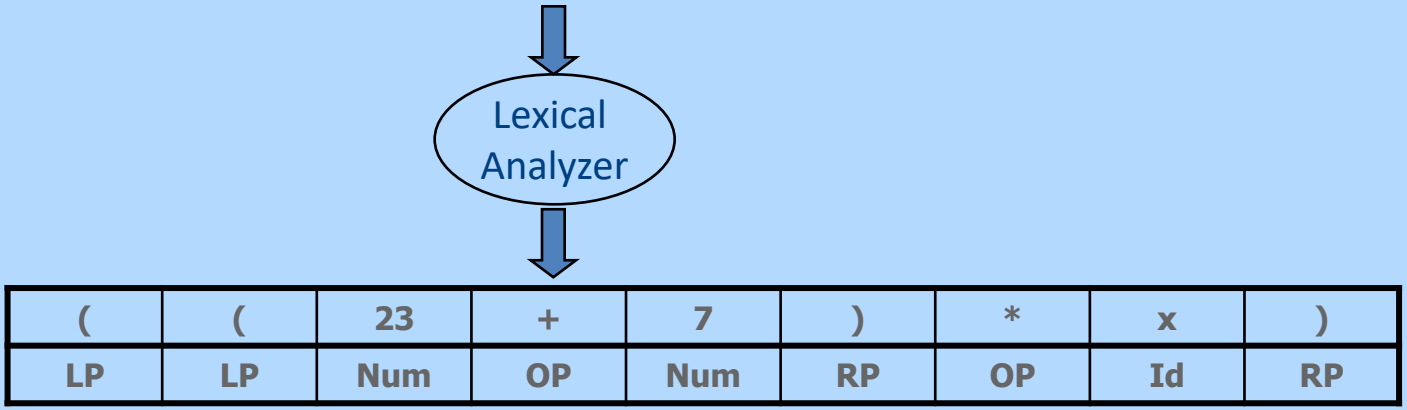
The Real Anatomy of a Compiler



Frontend: Scanning & Parsing

program text ((23 + 7) * x)



Grammar:

E → ... | Id

Id → ‘a’ | ... | ‘z’

Abstract Syntax Tree

From scanning to parsing

program text

```
((23 + 7) * x)
```

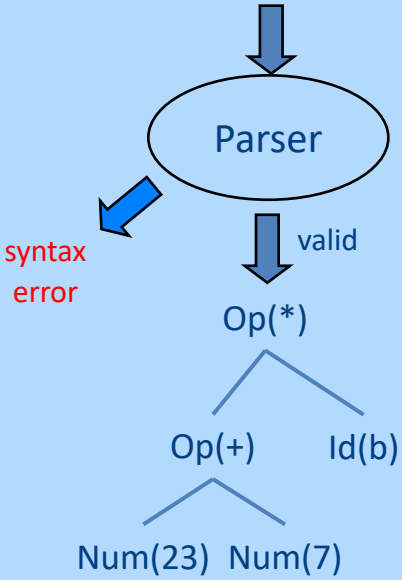
token stream

(	(	23	+	7	)	*	x	)
LP	LP	Num	OP	Num	RP	OP	Id	RP

Grammar:

E → ... | Id

Id → ‘a’ | ... | ‘z’



Abstract Syntax Tree

Reminder

- Context-free languages
  - Grammars
  - Pushdown Automata
- Terminology
  - Derivation
  - Sentential form
  - Parse trees
  - Leftmost/rightmost derivation
  - Ambiguous grammars
- Top-down / Botto-up parsers

Parsing

- Construct a structured representation of the input

- Challenges
  - How do you describe the programming language?
  - How do you check validity of an input?
    - Is a sequence of tokens a valid program in the language?
  - How do you construct the structured representation?
  - Where do you report an error?

Top-down parsing

Predictive parsing

- Given a grammar G and a word w attempt to derive w using G
- Idea
  - Apply production to leftmost nonterminal
  - Pick production rule based on next input token
- General grammar
  - More than one option for choosing the next production based on a token
- Restricted grammars (LL)
  - Know exactly which single rule to apply
  - May require some lookahead to decide

Boolean expressions example

E → LIT | (E OP E) | **not** E
LIT → **true** | **false**
OP → **and** | **or** | **xor**

not (not true or false)

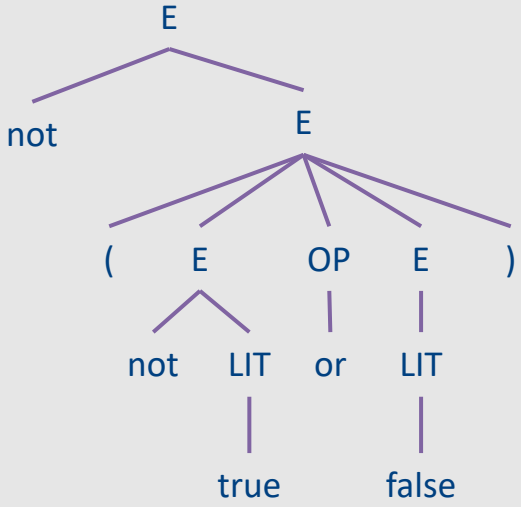
Boolean expressions example

E → LIT | (E OP E) | **not** E
LIT → **true** | **false**
OP → **and** | **or** | **xor**

not (not true or false)

production to apply known from next token

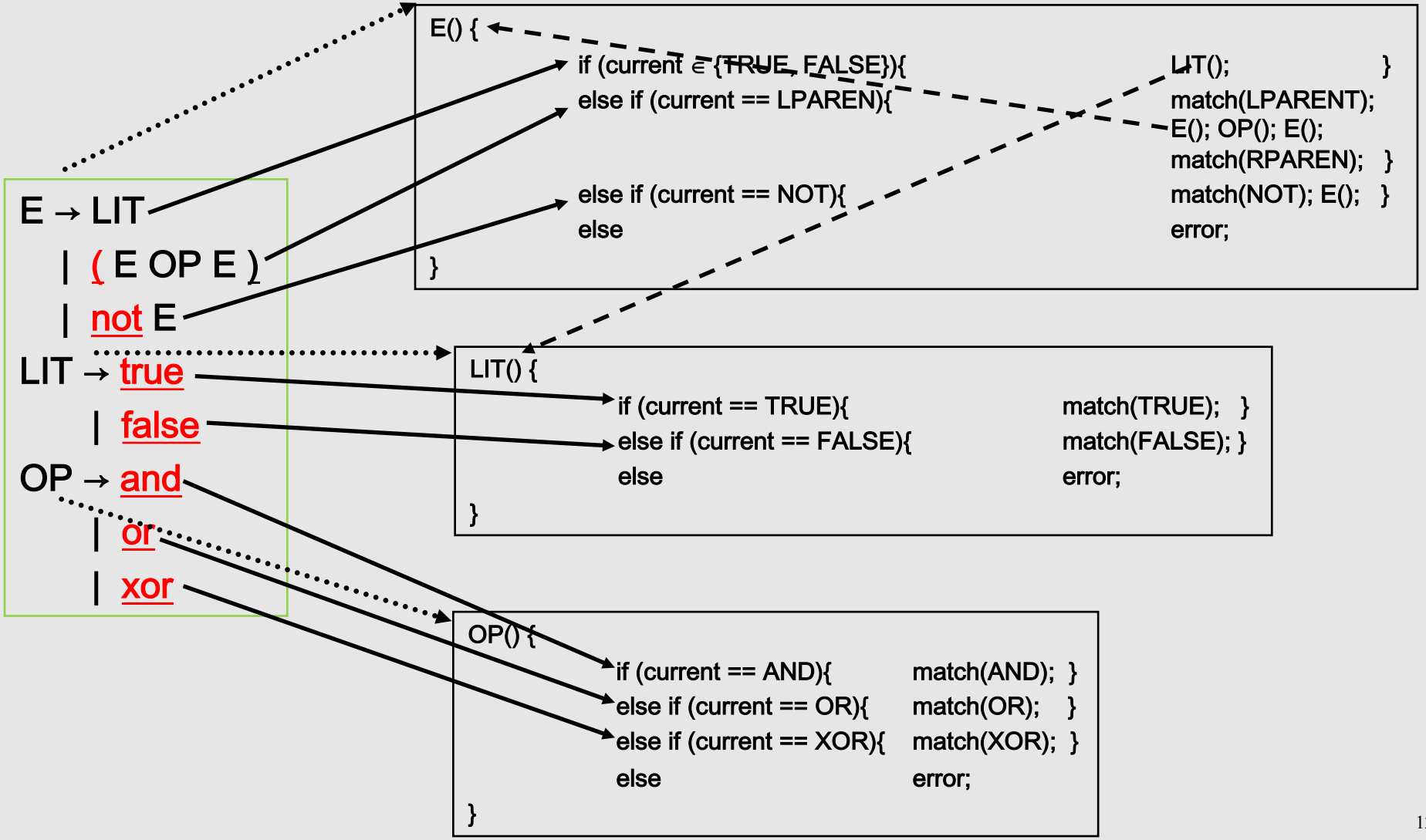
E =>
not E =>
not (E OP E) =>
not (not E OP E) =>
not (not LIT OP E) =>
not (not true OP E) =>
not (not true or E) =>
not (not true or LIT) =>
not (not true or false)



Recursive descent parsing

- Define a function for every nonterminal
- Every function work as follows
  - Find applicable production rule
  - Terminal function checks match with next input token
  - Nonterminal function calls (recursively) other functions
- If there are several applicable productions for a nonterminal, use lookahead

Implementation via recursion



Recursive descent

```
void A() {
  choose an A-production, A → X1X2…Xk;
  for (i=1; i≤ k; i++) {
    if (Xi is a nonterminal)
      call procedure Xi();
    elseif (Xi == current)
      advance input;
    else
      report error;
  }
}
```

- How do you pick the right A-production?
- Generally – try them all and use backtracking
- In our case – use lookahead

Problem 1: productions with common prefix

term → ID | indexed_elem
indexed_elem → ID [expr]

- The function for indexed_elem will never be tried…
  - What happens for input of the form **`ID[expr]`**

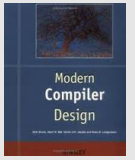
Problem 2: null productions

$S \rightarrow A\ a\ b$

$A \rightarrow a \mid \varepsilon$

```
S() {
  return  A()  ;  match(token('a'))  ;  match(token('b'))
}
A()  {
  match(token('a'))   ||   skip
}
```

- What happens for input "ab"?
- What happens if you flip order of alternatives and try "aab"?



Problem 3: left recursion

$E \rightarrow E$ - term | term

```
E() {
  return E()  ;  match(token('-'))   ;   term()
  ||
  term()
}
```

- What happens with this procedure?
- Recursive descent parsers cannot handle left-recursive grammars

What can we do?

FIRST sets

X → YY | Z Z | Y Z | 1 Y

Y → 4 | ε

Z → 2

L(Z) = {2}

L(Y) = {4, ε}

L(X) = {44, 4, ε, 22, 42, 2, 14, 1}

FIRST sets

X → YY | Z Z | Y Z | 1 Y

Y → 4 | ε

Z → 2

L(Z) = {2}

L(Y) = {4, ε}

L(X) = {44, 4, ε, 22, 42, 2, 14, 1}

FIRST sets

- FIRST(X) = { t | X →* t β} ∪{Ɛ | X →* Ɛ}
  - FIRST(X) = all terminals that α can appear as first in some derivation for X
    - + Ɛ if can be derived from X

- Example:
  - FIRST(**LIT**) = { **true**, **false** }
  - FIRST(**(E OP E)**) = { **(** }
  - FIRST(**not E**) = { **not** }

FIRST sets

- No intersection between FIRST sets => can always pick a single rule
- If the FIRST sets intersect, may need longer lookahead
  - LL(k) = class of grammars in which production rule can be determined using a lookahead of k tokens
  - LL(1) is an important and useful class

Computing FIRST sets

- FIRST (t) = { t } // “t” non terminal

- $\varepsilon \in$ FIRST(X) if
  - X → ε or
  - X → A_1 .. A_k and $\varepsilon \in$ FIRST(A_i) i=1…k

- FIRST(α) $\subseteq$ FIRST(X) if
  - X→ A_1 .. A_k α and $\varepsilon \in$ FIRST(A_i) i=1…k

Computing FIRST sets

- Assume no null productions $A \rightarrow \varepsilon$
  1. Initially, for all nonterminals A, set
     FIRST(A) = { t | $A \rightarrow t\omega$ *for some* ω }
  2. Repeat the following until no changes occur:
     for each nonterminal A
       for each production $A \rightarrow B\omega$
         set FIRST(A) = FIRST(A) ∪ FIRST(B)
- This is known as fixed-point computation

FIRST sets computation example

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM

1. Initialization

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant

2. Iterate 1

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --		

2. Iterate 2

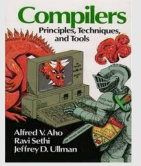
STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --	id constant	

2. Iterate 3 – fixed-point

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --	id constant	
id constant		



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FOLLOW sets

- What do we do with nullable (ε) productions?
  - $A \rightarrow B\ C\ D \quad B \rightarrow \varepsilon\ C \rightarrow \varepsilon$
  - Use what comes afterwards to predict the right production
- For every production rule $A \rightarrow \alpha$
  - FOLLOW(A) = set of tokens that can immediately follow A
- Can predict the alternative A_k for a non-terminal N when the lookahead token is in the set
  - FIRST(A_k) $\rightarrow$ (if A_k is nullable then FOLLOW(N))

FOLLOW sets: Constraints

- $ ∈ FOLLOW(S)
- FIRST(β) – {Ɛ} ⊆ FOLLOW(X)
  - For each A → α X β
- FOLLOW(A) ⊆ FOLLOW(X)
  - For each A → α X β and Ɛ ∈ FIRST(β)

Example: FOLLOW sets

- E→ TX
- T→ (E) | int Y

X→+ E | Ɛ

Y → * T | Ɛ

Terminal	+	(	*	)	int
FOLLOW	int, (	int, (	int, (	_,), $	*,), +, $

Non. Term.	E	T	X	Y
FOLLOW	), $	+,), $	$,)	_,), $

Prediction Table

- $A \rightarrow \alpha$

- $T[A,t] = \alpha$ if $t \in FIRST(\alpha)$
- $T[A,t] = \alpha$ if $\varepsilon \in FIRST(\alpha)$ and $t \in FOLLOW(A)$
  - t can also be $

- T is not well defined → the grammar is not LL(1)

LL(k) grammars

- A grammar is in the class LL(K) when it can be derived via:
  - Top-down derivation
  - Scanning the input from left to right (L)
  - Producing the leftmost derivation (L)
  - With lookahead of k tokens (k)
- A language is said to be LL(k) when it has an LL(k) grammar

LL(1) grammars

- A grammar is in the class LL(1) iff
  - For every two productions $A \rightarrow \alpha$ and $A \rightarrow \beta$ we have
    - $FIRST(\alpha) \cap FIRST(\beta) = \{\}$ // including ε
    - If $\varepsilon \in FIRST(\alpha)$ then $FIRST(\beta) \cap FOLLOW(A) = \{\}$
    - If $\varepsilon \in FIRST(\beta)$ then $FIRST(\alpha) \cap FOLLOW(A) = \{\}$

Problem: Non LL Grammars

Problem: Non LL Grammars

$S \rightarrow A\ a\ b$

$A \rightarrow a \mid \varepsilon$

```
bool S() {
  return  A() && match(token('a')) && match(token('b'));
}

bool A()  {
 return  match(token('a')) || true;
}
```

- What happens for input "ab"?
- What happens if you flip order of alternatives and try "aab"?

Problem: Non LL Grammars

$S \rightarrow A\ a\ b$
$A \rightarrow a \mid \varepsilon$

- FIRST(S) = { a } FOLLOW(S) = { $ }
- FIRST(A) = { a, ε } FOLLOW(A) = { a }

- FIRST/FOLLOW conflict

Back to problem 1

term → ID | indexed_elem
indexed_elem → ID [expr]

- FIRST(term) = { ID }
- FIRST(indexed_elem) = { ID }

- FIRST/FIRST conflict

Solution: left factoring

- Rewrite the grammar to be in LL(1)

term → ID | indexed_elem
indexed_elem → ID [expr]

term → ID after_ID
After_ID → [expr] | ε

Intuition: just like factoring x*y + x*z into x*(y+z)

Left factoring – another example

$S \rightarrow$ if E then S else S
| if E then S
| T

$S \rightarrow$ if E then S S'
| T
$S' \rightarrow$ else S | ε

Back to problem 2

$S \rightarrow A\ a\ b$
$A \rightarrow a \mid \varepsilon$

- FIRST(S) = { a } FOLLOW(S) = { }
- FIRST(A) = { a , ε } FOLLOW(A) = { a }

- FIRST/FOLLOW conflict

Solution: substitution

$S \rightarrow A\ a\ b$
$A \rightarrow a \mid \varepsilon$

Substitute A in S

$S \rightarrow a\ a\ b \mid a\ b$

Left factoring

$S \rightarrow a\ after\_A$
$after\_A \rightarrow a\ b \mid b$

Back to problem 3

$E \rightarrow E$ - term | term

- Left recursion cannot be handled with a bounded lookahead

- What can we do?

Left recursion removal

$N \rightarrow N\alpha \mid \beta$

G_1

$N \rightarrow \beta N'$
$N' \rightarrow \alpha N' \mid \varepsilon$

G_2

Can be done algorithmically.
Problem: grammar becomes mangled beyond recognition

- $L(G_1) = \beta, \beta\alpha, \beta\alpha\alpha, \beta\alpha\alpha\alpha, \ldots$
- $L(G_2)$ = same

- For our 3rd example:

$E \rightarrow E$ - term | term

$E \rightarrow$ term TE | term
$TE \rightarrow$ - term TE | ε

LL(k) Parsers

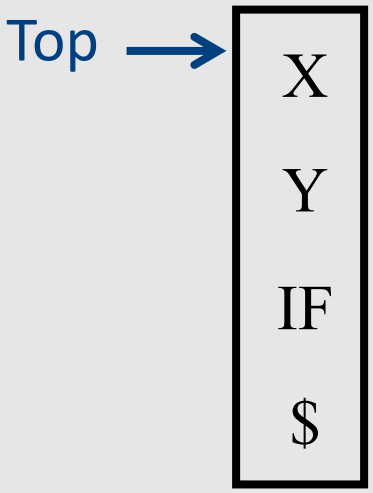
- Recursive Descent
  - Manual construction
  - Uses recursion

- Wanted
  - A parser that can be generated automatically
  - Does not use recursion

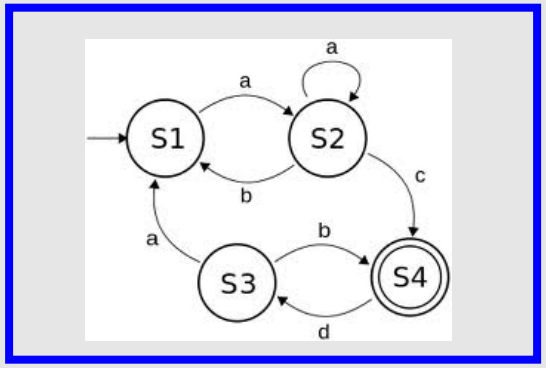
Pushdown Automata (PDA)

Intuition: PDA

- An ε-NFA with the additional power to manipulate **one** stack



stack



control (ε-NFA)

Intuition: PDA

- Think of an ε-NFA with the additional power that it can manipulate a stack
- PDA moves are determined by:
  - The current state (of its "ε-NFA")
  - The current input symbol (or ε)
  - The current symbol on top of its stack

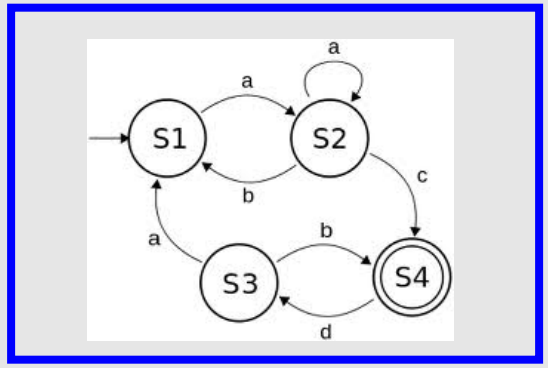
Intuition: PDA

Current

input `if (oops) then stat:= blah else abort`

Top → X Y IF $

stack



control (ε-NFA)

Intuition: PDA

- Moves:
  - Change state
  - Replace the top symbol by 0…n symbols
    - 0 symbols = "pop" ("reduce")
    - 0 < symbols = sequence of "pushes" ("shift")
- Nondeterministic choice of next move

PDA Formalism

- PDA = (Q, Σ, Γ, δ, q_0, $, F):
  - Q: finite set of states
  - Σ: Input symbols alphabet
  - Γ: stack symbols alphabet
  - δ: transition function
  - q_0: start state
  - $: start symbol
  - F: set of final states

Tokens

Non terminals

The Transition Function

- $\delta(q, a, X) = \{ (p_1, \sigma_1), \ldots, (p_n, \sigma_n)\}$
  - Input: triplet
    - A state $q \in Q$
    - An input symbol $a \in \Sigma$ or ε
    - A stack symbol $X \in \Gamma$
  - Output: set of 0 … k actions of the form (p, σ)
    - A state $p \in Q$
    - σ a sequence $X_1 \cdots X_n \in \Gamma^*$ of stack symbols

Actions of the PDA

- Say $(p, \sigma) \in \delta(q, a, X)$
  - If the PDA is in state q and X is the top symbol and a is at the front of the input
  - Then it can
    - Change the state to p.
    - Remove a from the front of the input
      - (but a may be ε).
    - Replace X on the top of the stack by σ.

Example: Deterministic PDA

- Design a PDA to accept $\{0^n1^n \mid n > 1\}$.
- The states:
  - q = We have not seen 1 so far
    - start state
  - p = we have seen at least one 1 and no 0s since
  - f = final state; accept.

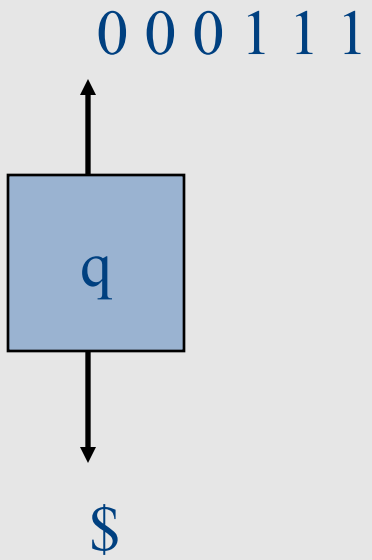
Example: Stack Symbols

- $ = start symbol.
  - Also marks the bottom of the stack,
  - Indicates when we have counted the same number of 1's as 0's.
- X = "counter"
  - used to count the number of 0s we saw

Example: Transitions

- $\delta(q, 0, \$) = \{(q, X\$)\}$.
- $\delta(q, 0, X) = \{(q, XX)\}$.
  - These two rules cause one X to be pushed onto the stack for each 0 read from the input.
- $\delta(q, 1, X) = \{(p, \varepsilon)\}$.
  - When we see a 1, go to state p and pop one X.
- $\delta(p, 1, X) = \{(p, \varepsilon)\}$.
  - Pop one X per 1.
- $\delta(p, \varepsilon, \$) = \{(f, \$)\}$.
  - Accept at bottom.

Actions of the Example PDA



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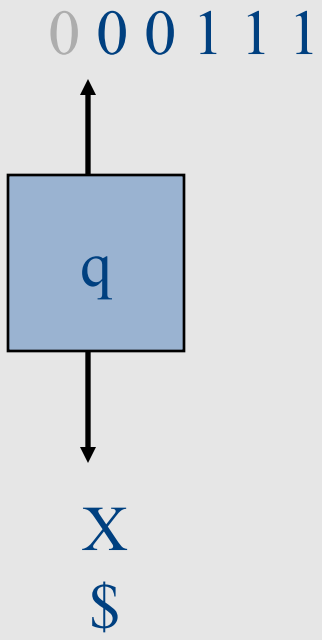

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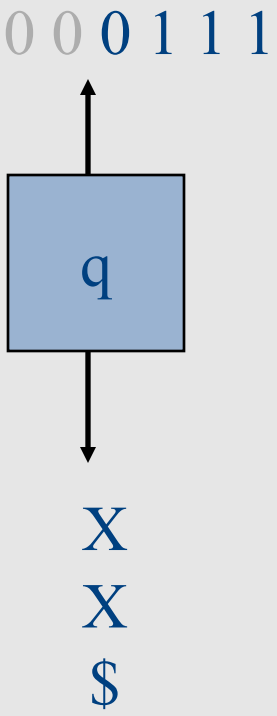

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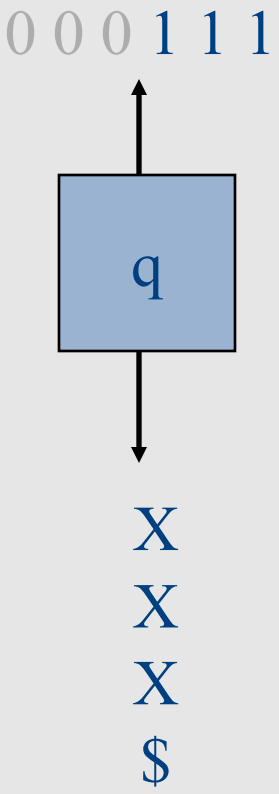

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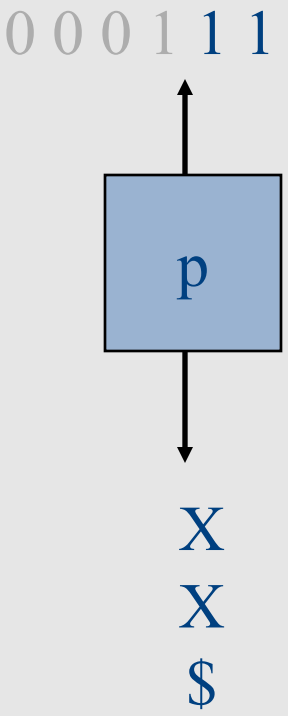

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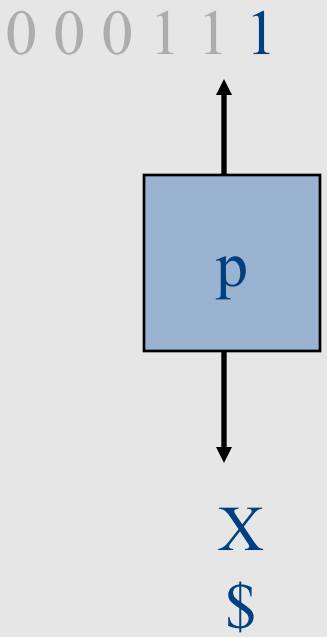

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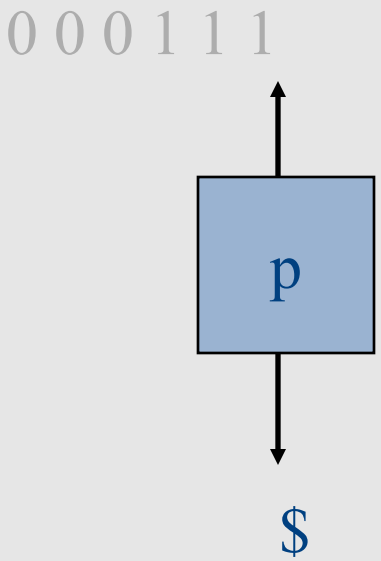
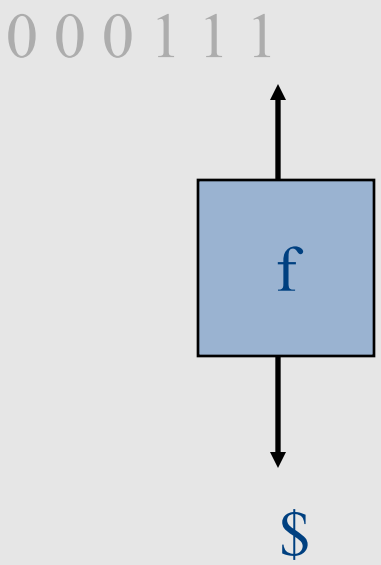


Actions of the Example PDA



Example: Non Deterministic PDA

- A PDA that accepts palindromes
  - L {pp’ ∈ Σ* | p’=reverse(p)}

LL(k) parsing via pushdown automata

- Pushdown automaton uses
  - Prediction stack
  - Input stream
  - Transition table
    - nonterminals x tokens -> production alternative
    - Entry indexed by nonterminal N and token t contains the alternative of N that must be predicated when current input starts with t

LL(k) parsing via pushdown automata

- Two possible moves
  - Prediction
    - When top of stack is nonterminal N, pop N, lookup table[N,t]. If table[N,t] is not empty, push table[N,t] on prediction stack, otherwise – syntax error
  - Match
    - When top of prediction stack is a terminal T, must be equal to next input token t. If (t == T), pop T and consume t. If (t ≠ T) syntax error
- Parsing terminates when prediction stack is empty
  - If input is empty at that point, success. Otherwise, syntax error

Example transition table

(1) E → LIT

(2) E → (E OP E)

(3) E → not E

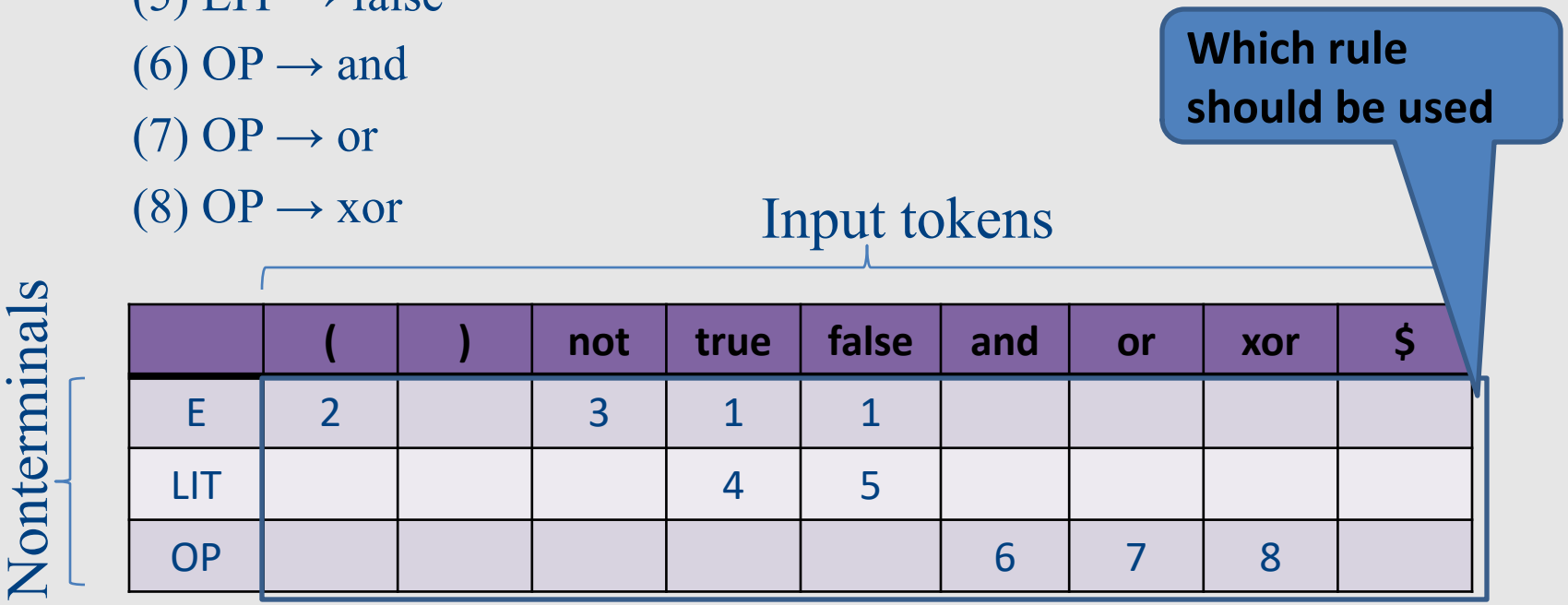
(4) LIT → true

(5) LIT → false

(6) OP → and

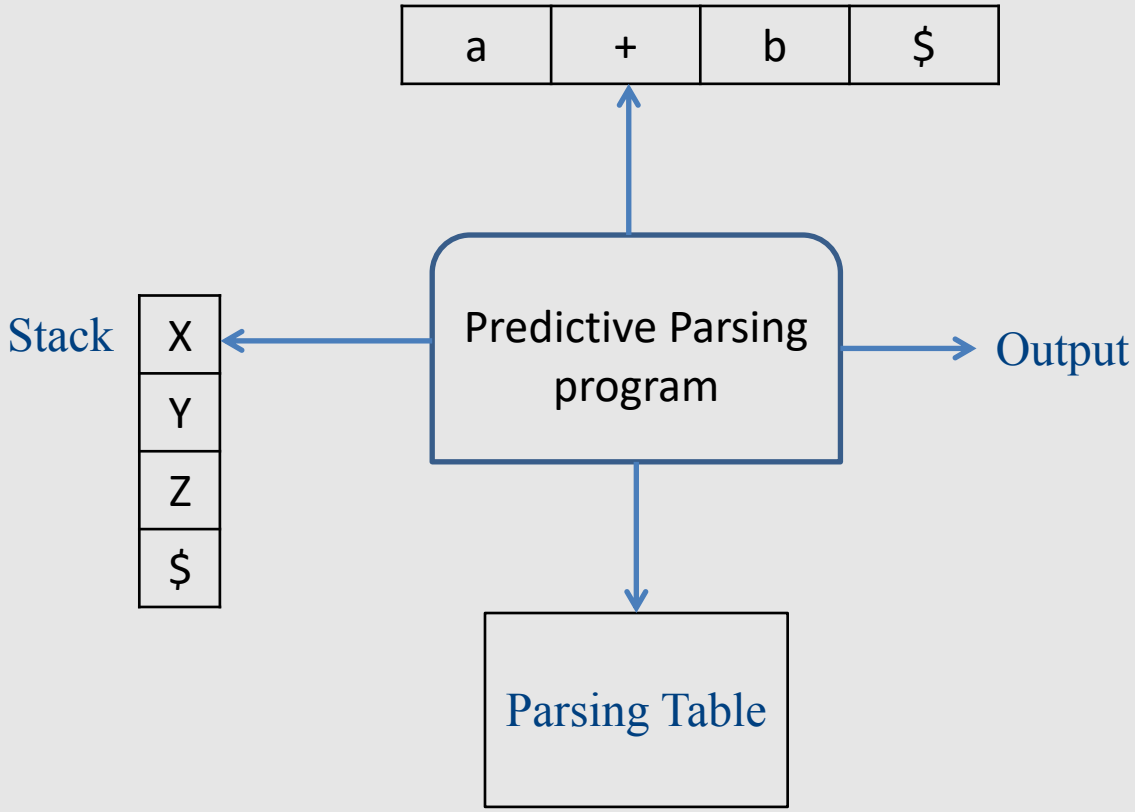
(7) OP → or

(8) OP → xor



	(	)	not	true	false	and	or	xor	$
E	2		3	1	1				
LIT				4	5				
OP						6	7	8	

Model of non-recursive predictive parser



Running parser example

aacbb$

$A \rightarrow aAb \mid c$

Input suffix	Stack content	Move
aacbb$	A$	predict(A,a) = A → aAb
aacbb$	aAb$	match(a,a)
acbb$	Ab$	predict(A,a) = A → aAb
acbb$	aAbb$	match(a,a)
cbb$	Abb$	predict(A,c) = A → c
cbb$	cbb$	match(c,c)
bb$	bb$	match(b,b)
b$	b$	match(b,b)
$	$	match($,$) – success

	a	b	c
A	A → aAb		A → c

Erorrs

Handling Syntax Errors

- Report and locate the error
- Diagnose the error
- Correct the error
- Recover from the error in order to discover more errors
  - without reporting too many “strange” errors

Error Diagnosis

- Line number
  - may be far from the actual error
- The current token
- The expected tokens
- Parser configuration

Error Recovery

- Becomes less important in interactive environments
- Example heuristics:
  - Search for a semi-column and ignore the statement
  - Try to “replace” tokens for common errors
  - Refrain from reporting 3 subsequent errors
- Globally optimal solutions
  - For every input w, find a valid program w’ with a “minimal-distance” from w

Illegal input example

abcbb$

A → aAb | c

Input suffix	Stack content	Move
abcbb$	A$	predict(A,a) = A → aAb
abcbb$	aAb$	match(a,a)
bcbb$	Ab$	predict(A,b) = ERROR

	a	b	c
A	A → aAb		A → c

Error handling in LL parsers

c$

S → a c | b S

Input suffix	Stack content	Move
c$	S$	predict(S,c) = ERROR

- Now what?
  - Predict b S anyway "missing token b inserted in line XXX"

	a	b	c
S	S → a c	S → b S	

Error handling in LL parsers

c$

S → a c | b S

Input suffix	Stack content	Move
bc$	S$	predict(b,c) = S → bS
bc$	bS$	match(b,b)
c$	S$	Looks familiar?

- Result: infinite loop

	a	b	c
S	S → a c	S → b S	

Error handling and recovery

- x = a * (p+q * (-b * (r-s);
  - Where should we report the error?
  - The valid prefix property

The Valid Prefix Property

- For every prefix tokens
  - $t_1, t_2, \ldots, t_i$ that the parser identifies as legal:
    - there exists tokens $t_{i+1}, t_{i+2}, \ldots, t_n$ such that $t_1, t_2, \ldots, t_n$ is a syntactically valid program

- If every token is considered as single character:
  - For every prefix word u that the parser identifies as legal there exists w such that u.w is a valid program

Recovery is tricky

- Heuristics for dropping tokens, skipping to semicolon, etc.

Building the Parse Tree

Adding semantic actions

- Can add an action to perform on each production rule
- Can build the parse tree
  - Every function returns an object of type Node
  - Every Node maintains a list of children
  - Function calls can add new children

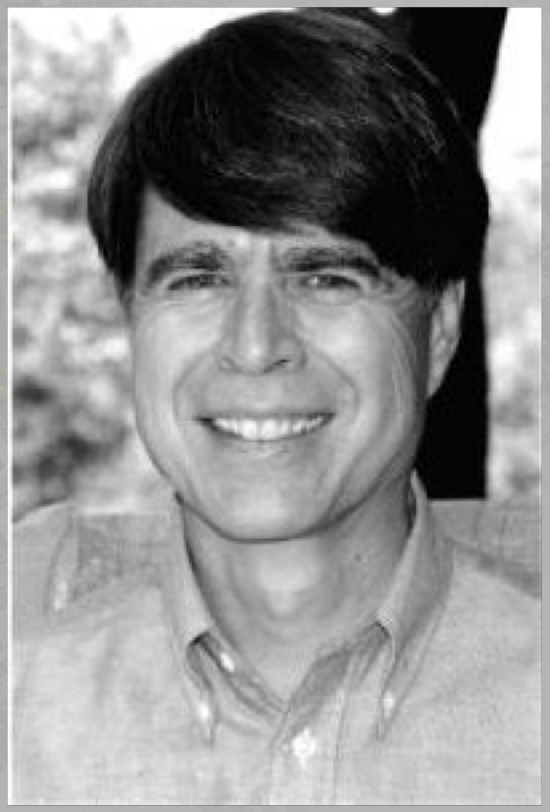
Building the parse tree

```
Node E() {
  result = new Node();
  result.name = "E";
  if (current ∈ {TRUE, FALSE}) // E → LIT
    result.addChild(LIT());
  else if (current == LPAREN)  // E → ( E OP E )
    result.addChild(match(LPAREN));
    result.addChild(E());
    result.addChild(OP());
    result.addChild(E());
    result.addChild(match(RPAREN));
  else if (current == NOT)          // E → not E
    result.addChild(match(NOT));
    result.addChild(E());
  else error;
    return result;
}
```

Parser for Fully Parenthesized Expers

```
static int Parse_Expression(Expression **expr_p) {
  Expression *expr = *expr_p = new_expression() ;
  /* try to parse a digit */
  if (Token.class == DIGIT) {
        expr->type='D';  expr->value=Token.repr -'0';
        get_next_token();
        return 1;       }
/* try parse parenthesized expression */
if (Token.class == '(') {
      expr->type='P';   get_next_token();
      if (!Parse_Expression(&expr->left))  Error("missing expression");
      if (!Parse_Operator(&expr->oper))  Error("missing operator");
      if (Token.class != ')') Error("missing )");
      get_next_token();
      return 1; }
return 0;
}
```

Earley Parsing



Jay Earley, PhD

Earley Parsing

- Invented by Jay Earley [PhD. 1968]
- Handles arbitrary context free grammars
  - Can handle ambiguous grammars
- Complexity $O(N^3)$ when N = |input|
- Uses dynamic programming
  - Compactly encodes ambiguity

Dynamic programming

- Break a problem P into subproblems $P_1 \ldots P_k$
  - Solve P by combining solutions for $P_1 \ldots P_k$
  - Memo-ize (store) solutions to subproblems instead of re-computation
- Bellman-Ford shortest path algorithm
  - Sol(x,y,i) = minimum of
    - Sol(x,y,i-1)
    - Sol(t,y,i-1) + weight(x,t) for edges (x,t)

Earley Parsing

- Dynamic programming implementation of a recursive descent parser
  - S[N+1] Sequence of sets of "Earley states"
    - N = |INPUT|
    - Earley state (item) s is a sentential form + aux info
  - S[i] All parse tree that can be produced (by a RDP) after reading the first i tokens
    - S[i+1] built using S[0] … S[i]

Earley Parsing

- Parse arbitrary grammars in $O(|input|^3)$
  - $O(|input|^2)$ for unambigous grammer
  - Linear for most LR(k) langaues (next lesson)
- Dynamic programming implementation of a recursive descent parser
  - S[N+1] Sequence of sets of “Earley states”
    - N = |INPUT|
    - Earley states is a sentential form + aux info
  - S[i] All parse tree that can be produced (by an RDP) after reading the first i tokens
    - S[i+1] built using S[0] … S[i]

Earley States

- s = < constituent, back >
  - constituent (dotted rule) for $A \rightarrow \alpha\beta$
    - $A \rightarrow \bullet\alpha\beta$ predicated constituents
    - $A \rightarrow \alpha\bullet\beta$ in-progress constituents
    - $A \rightarrow \alpha\beta\bullet$ completed constituents
  - back previous Early state in derivation

Earley States

- s = < constituent, back >
  - constituent (dotted rule) for $A \rightarrow \alpha\beta$
    - $A \rightarrow \bullet\alpha\beta$ predicated constituents
    - $A \rightarrow \alpha\bullet\beta$ in-progress constituents
    - $A \rightarrow \alpha\beta\bullet$ completed constituents
  - back previous Early state in derivation

Earley Parser

Input = x[1…N]

S[0] = <E'→ •E, 0>; S[1] = … S[N] = {}

for i = 0 ... N do

  until S[i] does not change do

    foreach s ∈ S[i]

---

if s = <A→…•a…, b> and a=x[i+1] then // scan

  S[i+1] = S[i+1] ∪ {<A→…a•…, b> }

---

if s = <A→ … •X …, b> and X→α then // predict

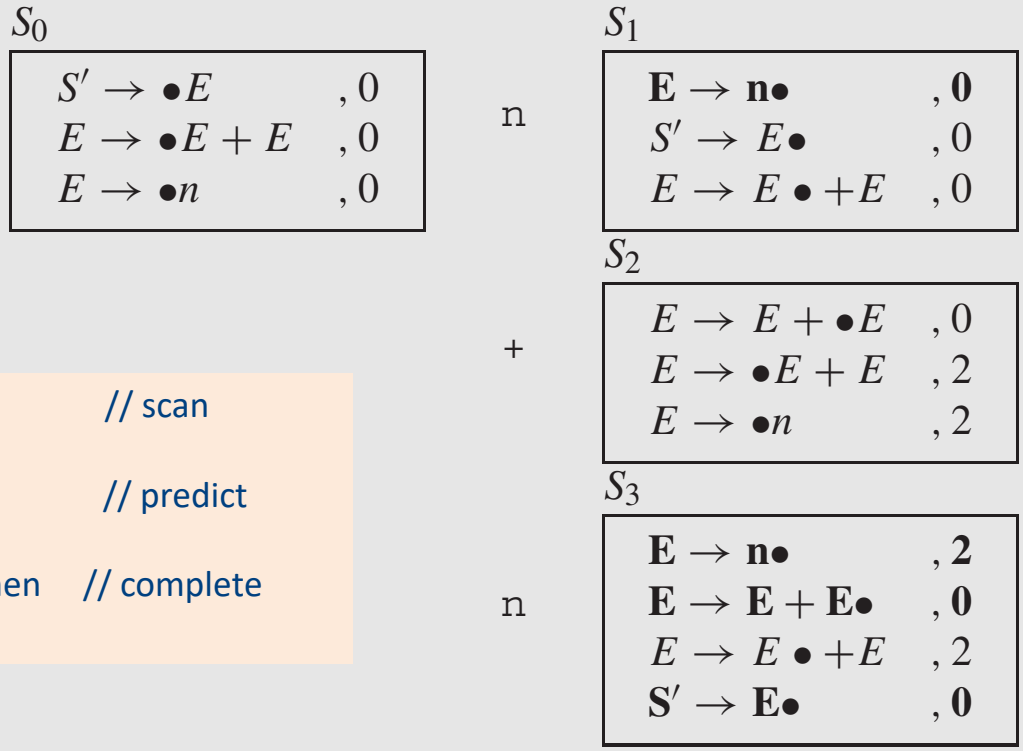
  S[i] = S[i] ∪ {<X→•α, i > }

---

if s = < A→ … • , b> and <X→…•A…, k> ∈ S[b] then // complete

  S[i] = S[i] ∪{<X→…A•…, k> }

Example



if s = <A→…•a…, b> and a=x[i+1] then // scan
S[i+1] = S[i+1] ∪ {<A→…a•…, b> }
if s = <A→ … •X …, b> and X→α then // predict
S[i] = S[i] ∪ {<X→•α, i > }
if s = < A→ … • , b> and <X→…•A…, k> ∈ S[b] then // complete
S[i] = S[i] ∪{<X→…A•…, k> }

FIGURE 1. Earley sets for the grammar $E \rightarrow E + E \mid n$ and the input `n + n`. Items in bold are ones which correspond to the input's derivation.