Compilation

Lecture 9

Optimizations

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Optimization points

source code → Front end → IR → Code generator → target code

User
profile program
change algorithm

Compiler
intraprocedural IR
Interprocedural IR
IR optimizations

Compiler
register allocation
instruction selection
peephole transformations

now

IR Optimization

- Making code “better”

Overview of IR optimization

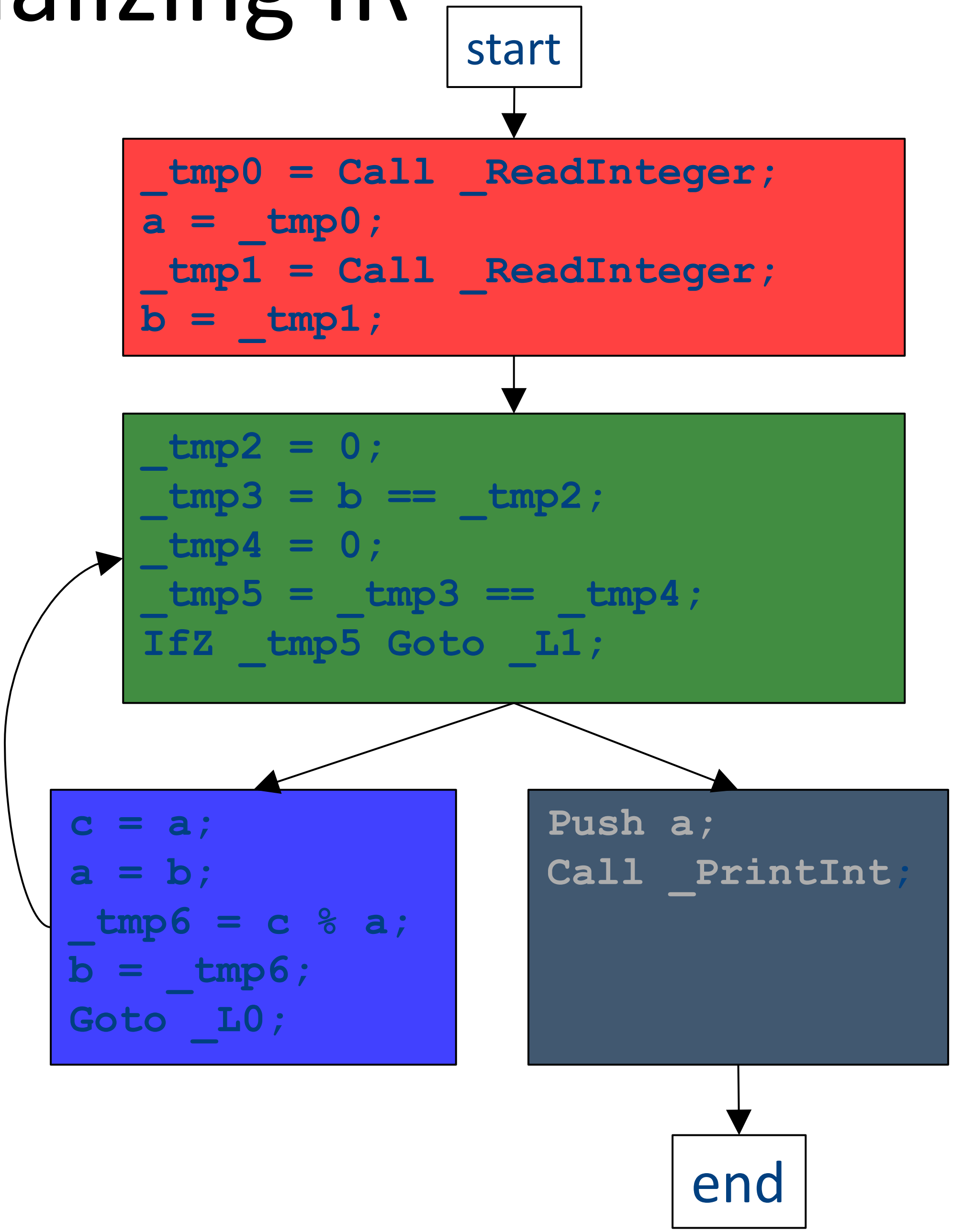
- Formalisms and Terminology
  - Control-flow graphs
  - Basic blocks
- Local optimizations
  - Speeding up small pieces of a procedure
- Global optimizations
  - Speeding up procedure as a whole
- The dataflow framework
  - Defining and implementing a wide class of optimizations

Program Analysis

- In order to optimize a program, the compiler has to be able to reason about the properties of that program
- An analysis is called **sound** if it never asserts an incorrect fact about a program
- All the analyses we will discuss in this class are sound
  - *(Why?)*

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```



Basic blocks

- A basic block is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence
- Informally, a sequence of instructions that always execute as a group

Control-Flow Graphs

- A control-flow graph (CFG) is a graph of the basic blocks in a function
- The term CFG is overloaded – from here on out, we'll mean "control-flow graph" and not "context free grammar"
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block
- There is a dedicated node for the start and end of a function

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b
  …
  v2 = a op b
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b   [or: v1 = a]
  …
  v2 = a op b   [or: v2 = a]
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b   [or: v1 = a]
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Copy Propagation

- If we have a variable assignment
  v1 = v2
  then as long as v1 and v2 are not reassigned, we can rewrite expressions of the form
  a = … v1 …
  as
  a = … v2 …
  provided that such a rewrite is legal

Dead Code Elimination

- An assignment to a variable v is called dead if the value of that assignment is never read anywhere
- Dead code elimination removes dead assignments from IR
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned

Other types of local optimizations

- Arithmetic Simplification
  - Replace "hard" operations with easier ones
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- Constant Folding
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite `x = 4 * 5;` as `x = 20;`

Optimizations and analyses

- Most optimizations are only possible given some analysis of the program's behavior
- In order to implement an optimization, we will talk about the corresponding program analyses

Available expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the available expressions in a program
- An expression is called available if some variable in the program holds the value of that expression
- In common subexpression elimination, we replace an available expression by the variable holding its value
- In copy propagation, we replace the use of a variable by the available expression it holds

Finding available expressions

- Initially, no expressions are available
- Whenever we execute a statement
  a = b *op* c:
  - Any expression holding **a** is invalidated
  - The expression **a = b *op* c** becomes available
- **Idea:** Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable

Available expressions example

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
 { b = x, d = x, e = a + b }
f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = d;
 { b = x, d = a + b, e = a + b }
d = b;
 { b = x, d = x, e = a + b }
f = e;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
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f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```

Live variables

- The analysis corresponding to dead code elimination is called liveness analysis
- A variable is live at a point in a program if later in the program its value will be read before it is written to again
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables

Computing live variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order
- Initially, some small set of values are known to be live (which ones depends on the particular program)
- When we see the statement a = b op c:
  - Just before the statement, a is not alive, since its value is about to be overwritten
  - Just before the statement, both b and c are alive, since we're about to read their values
  - (what if we have a = a + b?)

Liveness analysis

```
 { b }
a = b;
 { a, b }
c = a;
 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }
f = e;
 { b, d } - given
```

Which statements are dead?

Dead Code Elimination

```
 { b }
a = b;
 { a, b }
c = a;
 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }
f = e;
 { b, d }
```

Which statements are dead?

Dead Code Elimination

```
 { b }
a = b;
 { a, b }

 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b, e }
d = a;
 { b, d, e }

 { b, d }
```

Liveness analysis II

```
 { b }
a = b;


 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Liveness analysis II

```
 { b }
a = b;

 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;

 { a, b }
d = a + b;
 { a, b, d }
e = d;
 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;

 { a, b }
d = a + b;
 { a, b, d }

 { a, b }
d = a;
 { b, d }
```

Liveness analysis III

```
 { b }
a = b;


 { a, b }
d = a + b;


 { a, b }
d = a;
 { b, d }
```

Which statements are dead?

Dead code elimination

```
 { b }
a = b;
```

Which statements are dead?

```
 { a, b }
d = a + b;
```

```
 { a, b }
d = a;
 { b, d }
```

Dead code elimination

```
 { b }
a = b;

 { a, b }

 { a, b }
d = a;
 { b, d }
```

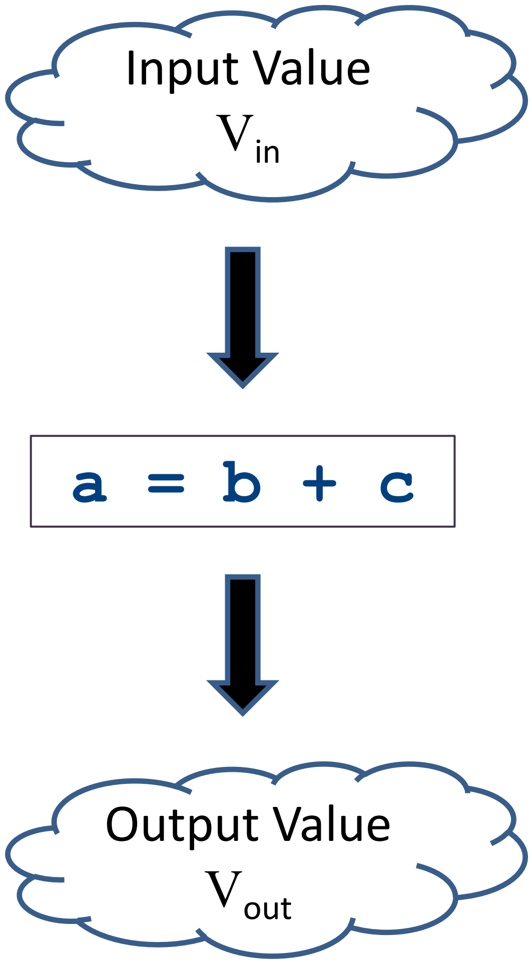
Dead code elimination

```
a = b;
```

If we further apply copy propagation this statement can be eliminated too

```
d = a;
```

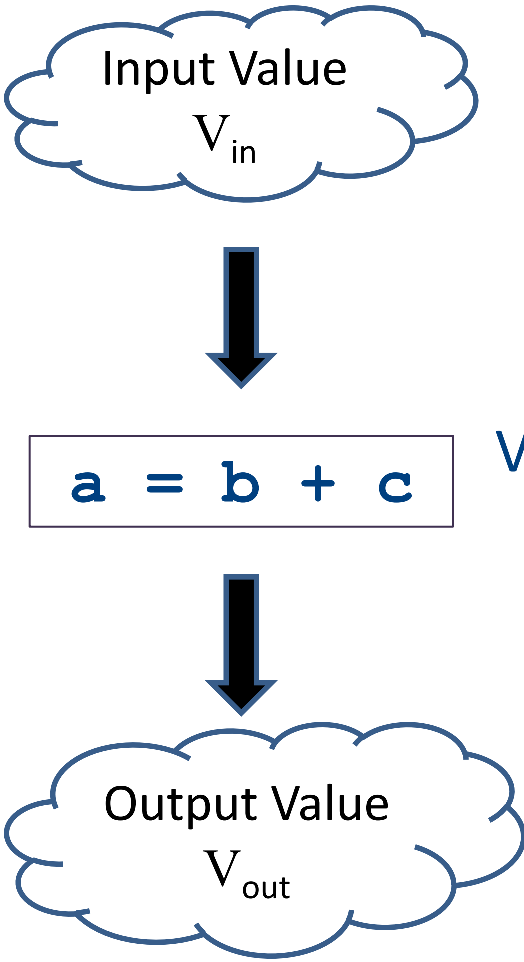
Formalizing local analyses



Transfer Function

$$V_{out} = f_{\texttt{a=b+c}}(V_{in})$$

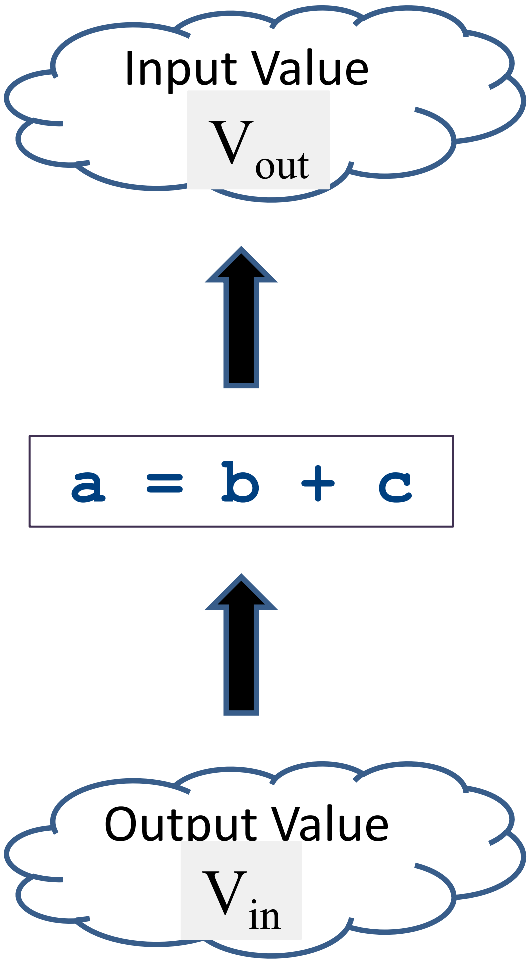
Available Expressions



$V_{out} = (V_{in} \setminus \{e \mid e \text{ contains } \mathbf{a}\}) \cup \{a=b+c\}$

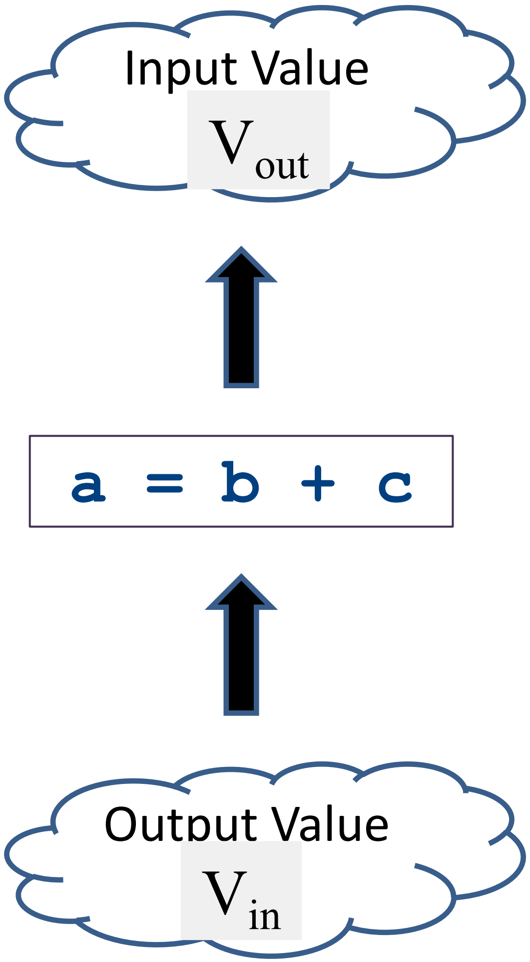
Expressions of the forms
`a`=… and x=…**`a`**…

Live Variables



$$V_{in} = (V_{out} \setminus \{\mathtt{a}\}) \cup \{\mathtt{b},\mathtt{c}\}$$

Live Variables



$$V_{in} = (V_{out} \setminus \{\mathtt{a}\}) \cup \{\mathtt{b,c}\}$$

Information for a local analysis

- What direction are we going?
  - Sometimes forward (available expressions)
  - Sometimes backward (liveness analysis)
- How do we update information after processing a statement?
  - What are the new semantics?
  - What information do we know initially?

Formalizing local analyses

- Define an analysis of a basic block as a quadruple (D, V, F, I) where
  - **D** is a direction (forwards or backwards)
  - **V** is a set of values the program can have at any point
  - **F** is a family of transfer functions defining the meaning of any expression as a function f : **V** → **V**
  - **I** is the initial information at the top (or bottom) of a basic block

Available Expressions

- **Direction:** Forward
- **Values:** Sets of expressions assigned to variables
- **Transfer functions:** Given a set of variable assignments V and statement a = b + c:
  - Remove from V any expression containing a as a subexpression
  - Add to V the expression a = b + c
  - Formally: $V_{out} = (V_{in} \setminus \{e \mid e \text{ contains } \mathbf{a}\}) \cup \{a = b + c\}$
- **Initial value:** Empty set of expressions

Liveness Analysis

- **Direction:** Backward
- **Values:** Sets of variables
- **Transfer functions:** Given a set of variable assignments V and statement a = b + c:
- Remove a from V (any previous value of a is now dead.)
- Add b and c to V (any previous value of b or c is now live.)
- Formally: $V_{in} = (V_{out} \setminus \{\mathbf{a}\}) \cup \{\mathbf{b},\mathbf{c}\}$
- **Initial value:** Depends on semantics of language
  - E.g., function arguments and return values (pushes)
  - Result of local analysis of other blocks as part of a global analysis

Running local analyses

- Given an analysis **(D, V, F, I)** for a basic block
- Assume that **D** is "forward;" analogous for the reverse case
- Initially, set OUT[**entry**] to **I**
- For each statement **s**, in order:
  - Set IN[**s**] to OUT[**prev**], where **prev** is the previous statement
  - Set OUT[**s**] to f_s(IN[**s**]), where f_s is the transfer function for statement **s**

Global Optimizations

High-level goals

- Generalize analysis mechanism
  - Reuse common ingredients for many analyses
  - Reuse proofs of correctness
- Generalize from basic blocks to entire CFGs
  - Go from local optimizations to global optimizations

Global analysis

- A global analysis is an analysis that works on a control-flow graph as a whole
- Substantially more powerful than a local analysis
  - (Why?)
- Substantially more complicated than a local analysis
  - (Why?)

Local vs. global analysis

- Many of the optimizations from local analysis can still be applied globally
  - Common sub-expression elimination
  - Copy propagation
  - Dead code elimination
- Certain optimizations are possible in global analysis that aren't possible locally:
  - e.g. code motion: Moving code from one basic block into another to avoid computing values unnecessarily
- Example global optimizations:
  - Global constant propagation
  - Partial redundancy elimination

Loop invariant code motion example

```
while (t < 120) {
  z = z + x - y;
}
```

value of expression x – y is not changed by loop body

```
w = x - y;
while (t < 120) {
  z = z + w;
}
```

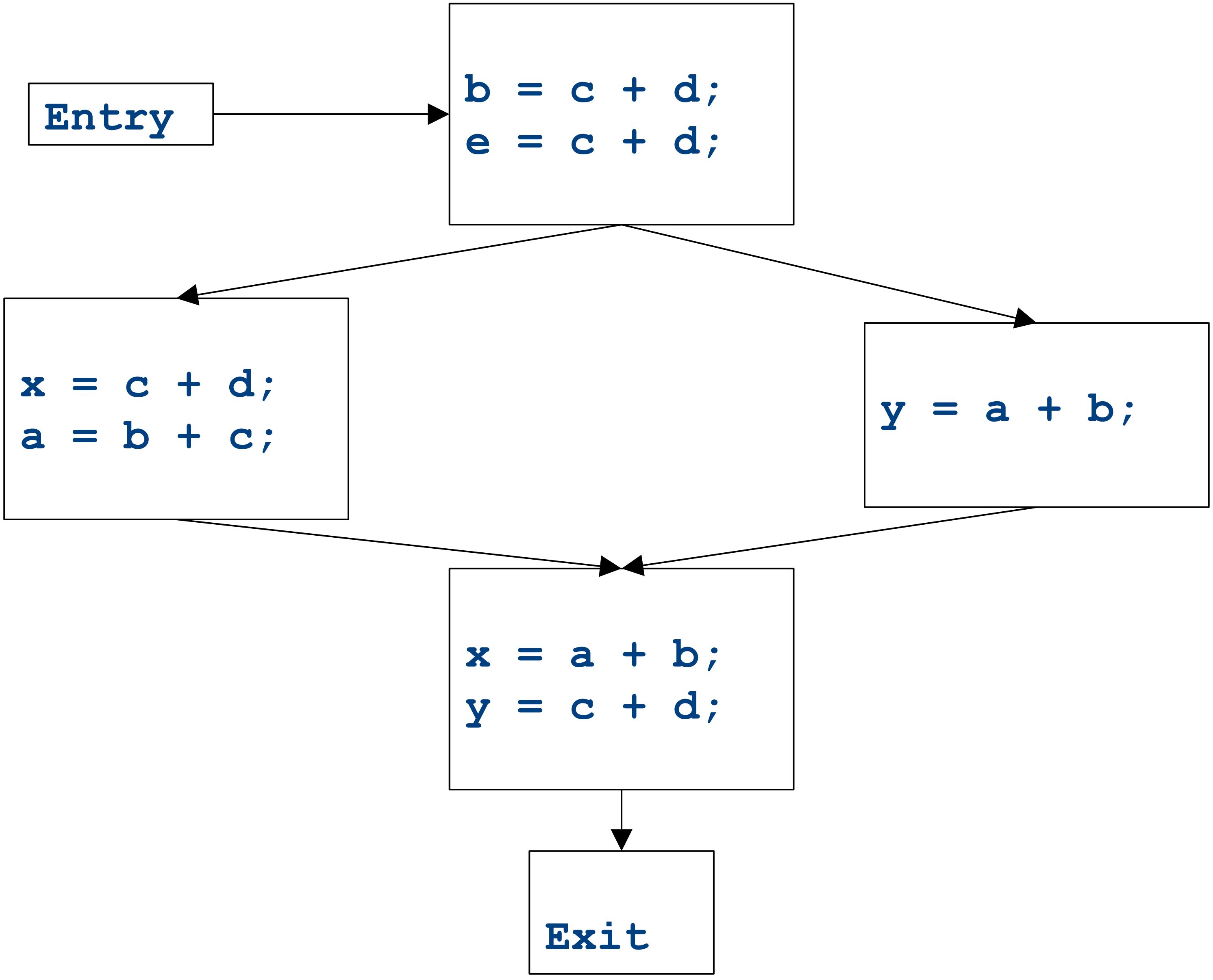
Why global analysis is hard

- Need to be able to handle multiple predecessors/successors for a basic block
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value (but the analysis still needs to terminate!)
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it

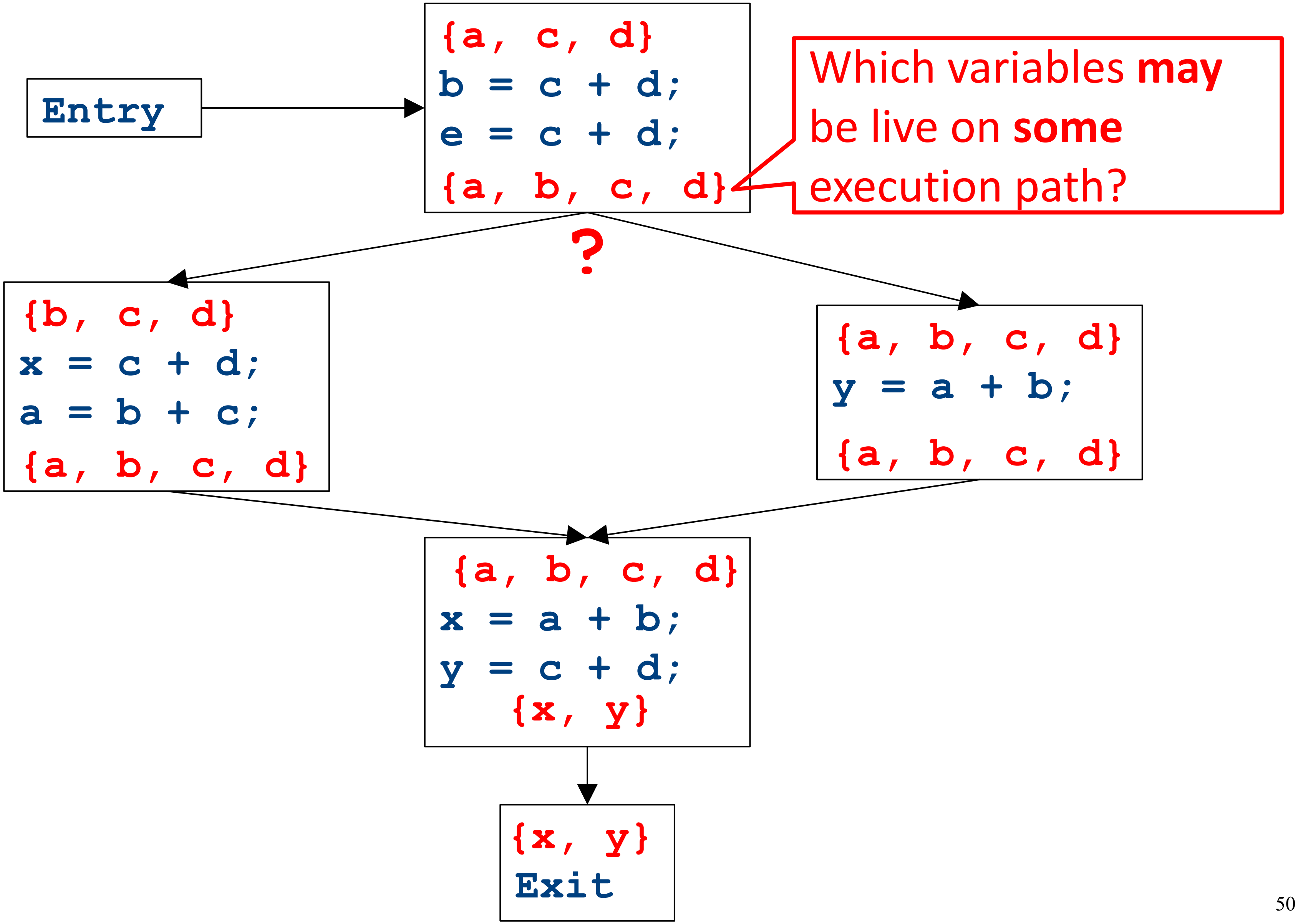
Global dead code elimination

- Local dead code elimination needed to know what variables were live on exit from a basic block
- This information can only be computed as part of a global analysis
- How do we modify our liveness analysis to handle a CFG?

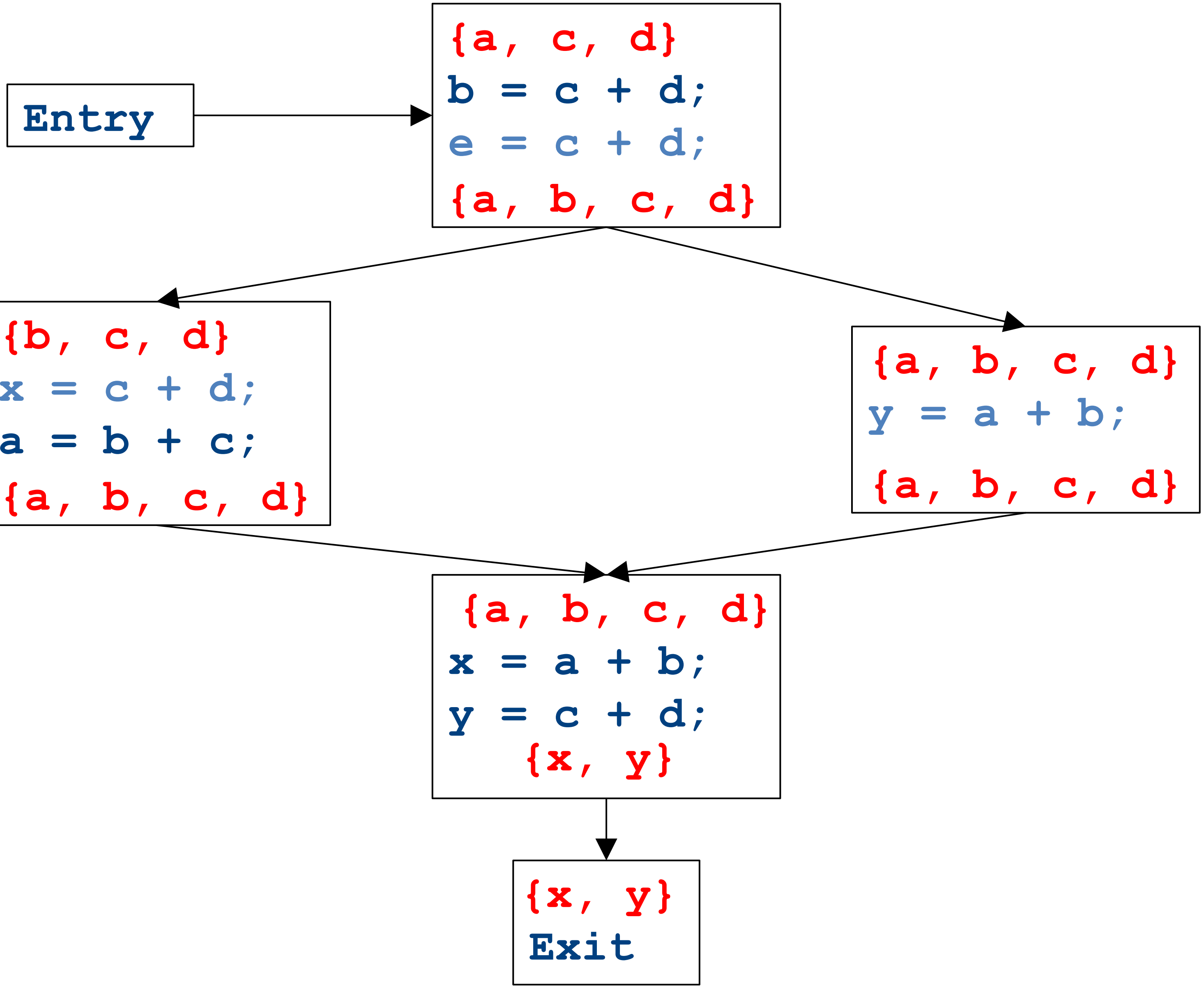
CFGs without loops



CFGs without loops



CFGs without loops



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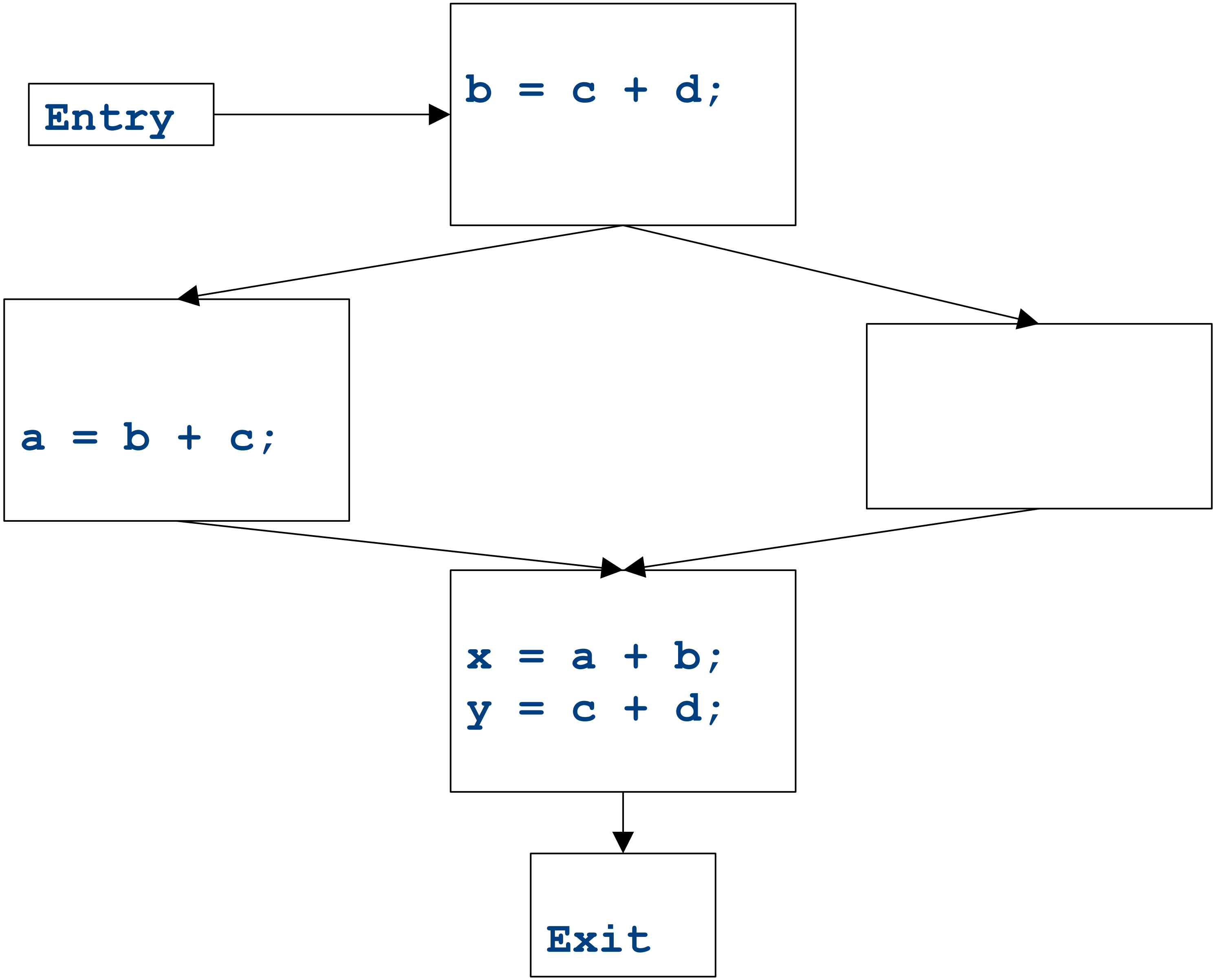

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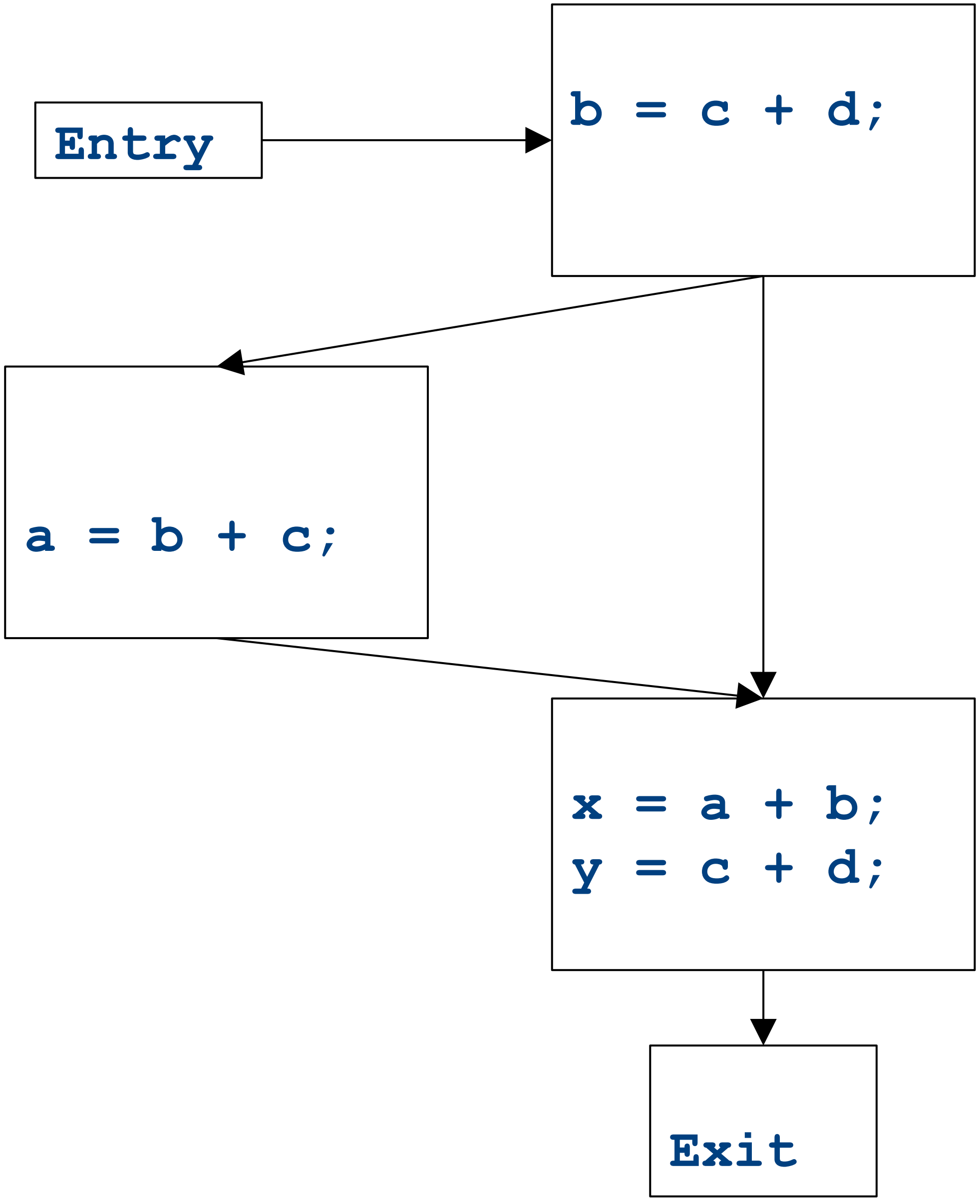
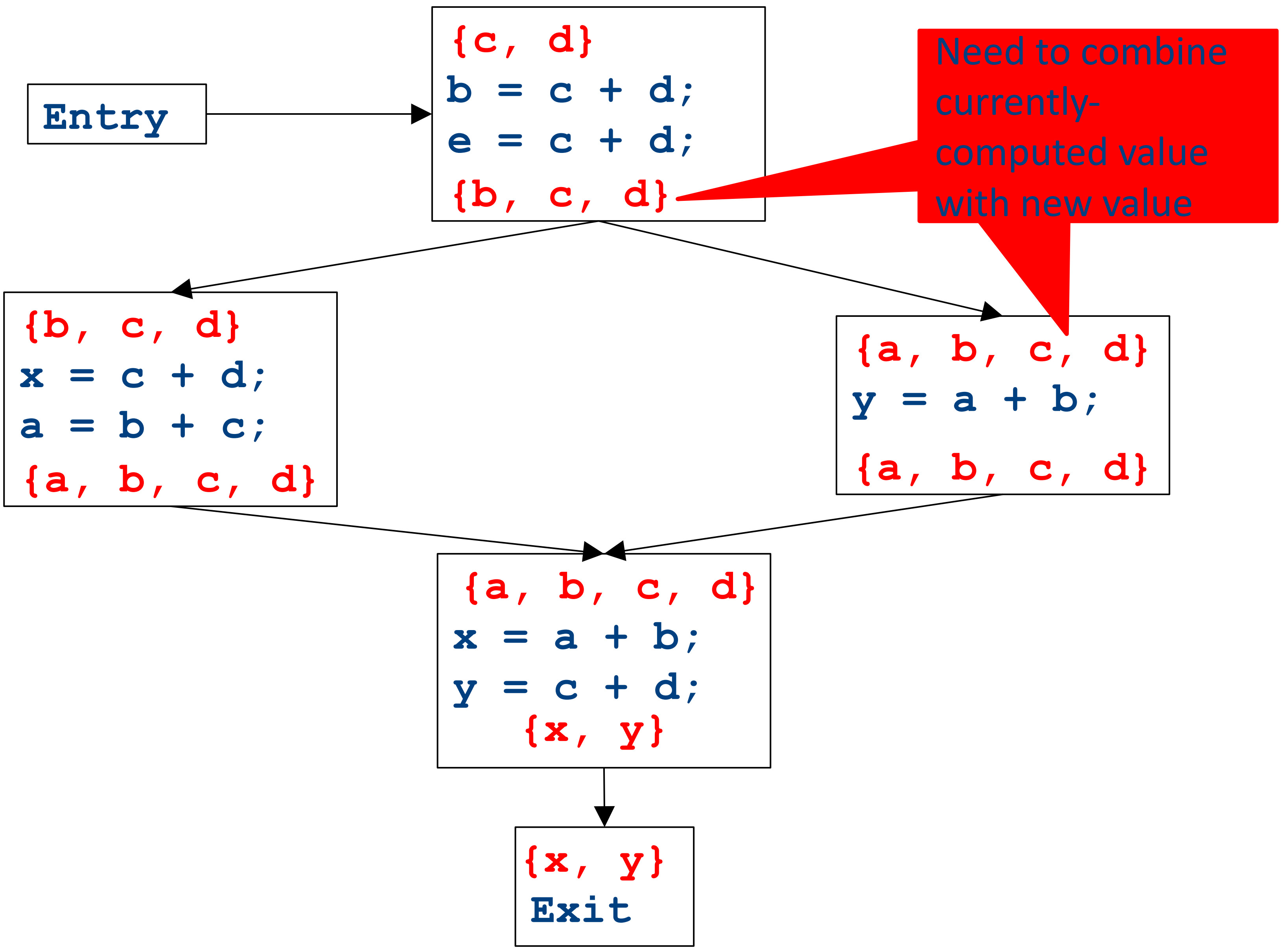


Major changes – part 1

- In a local analysis, each statement has exactly one predecessor
- In a global analysis, each statement may have **multiple** predecessors
- A global analysis must have some means of **combining information** from all predecessors of a basic block

CFGs without loops



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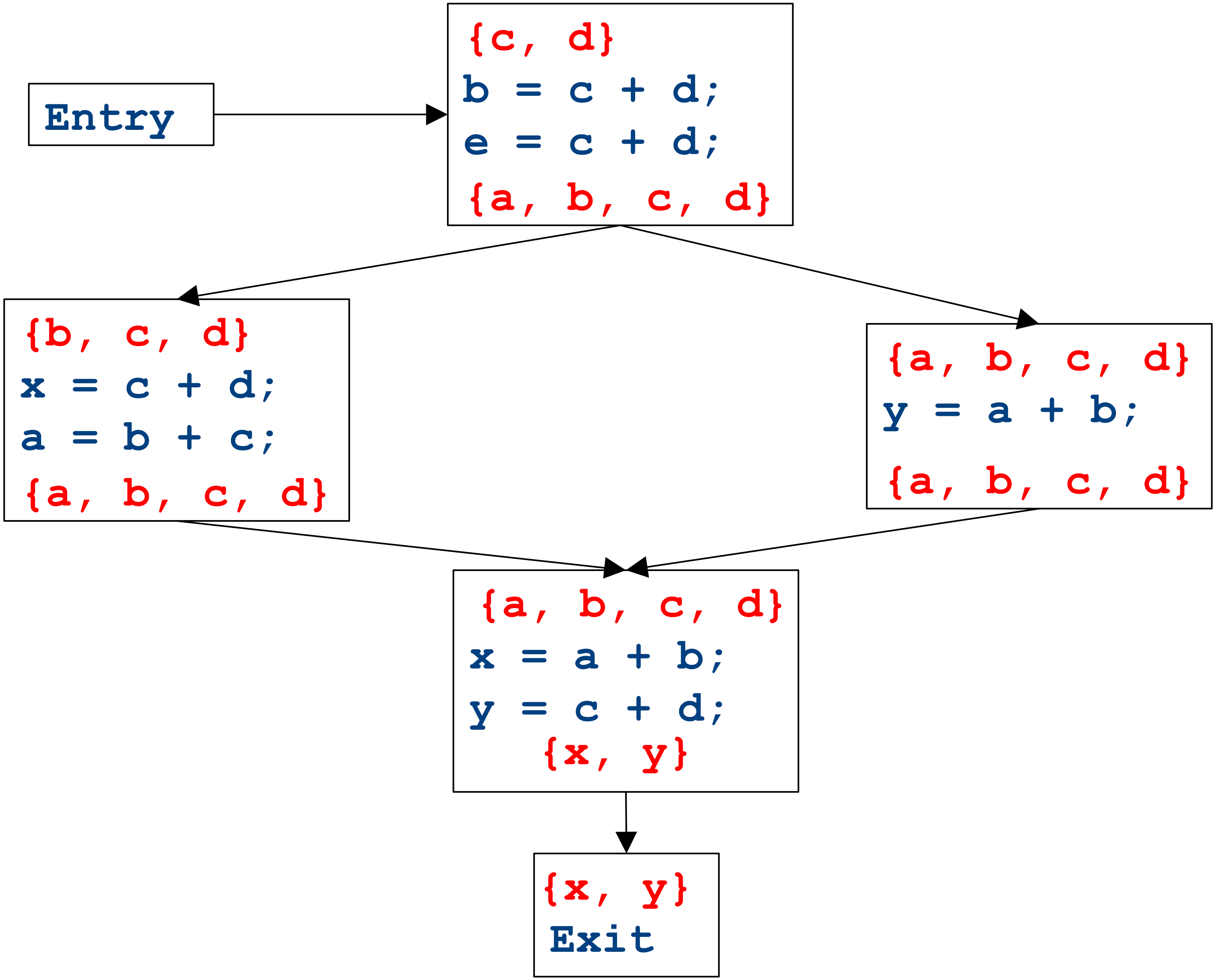

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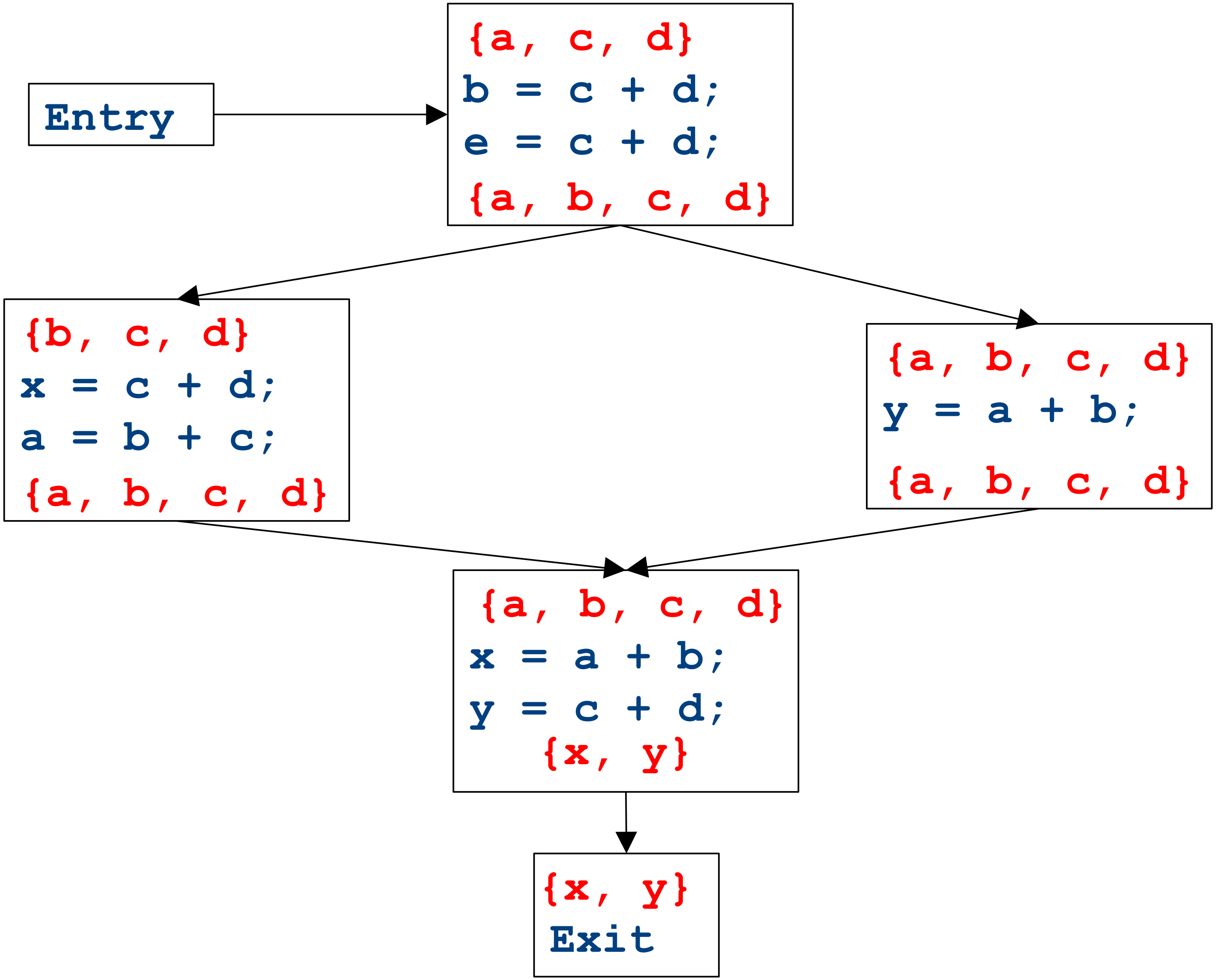
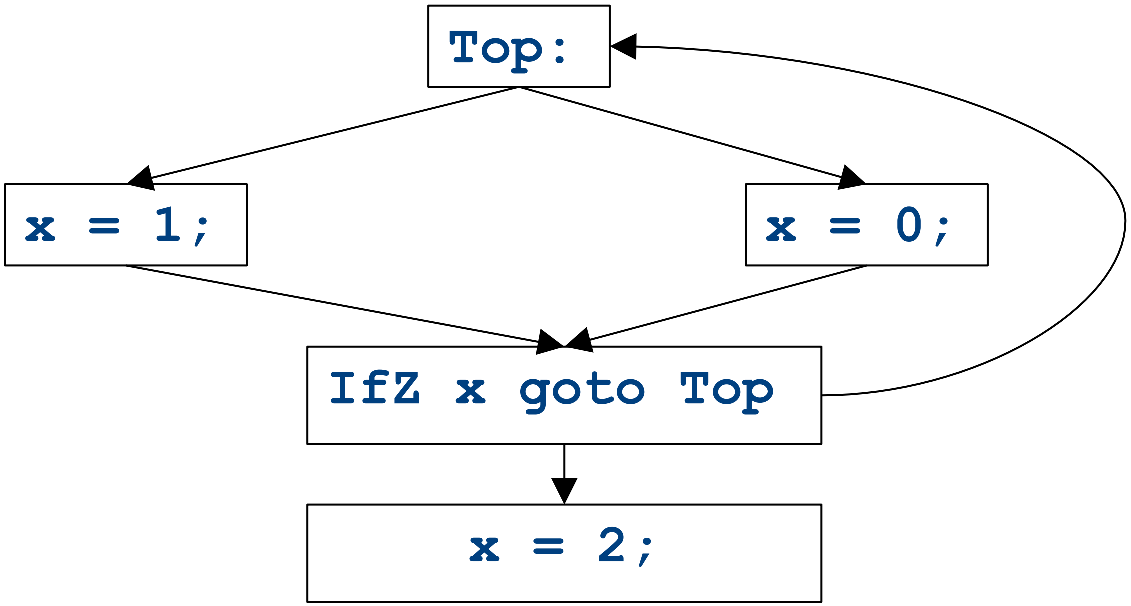


Major changes – part 2

- In a local analysis, there is only one possible path through a basic block
- In a global analysis, there may be **many** paths through a CFG
- May need to recompute values multiple times as more information becomes available
- Need to be careful when doing this not to loop infinitely!
  - (More on that later)
- Can order of computation affect result?

CFGs with loops

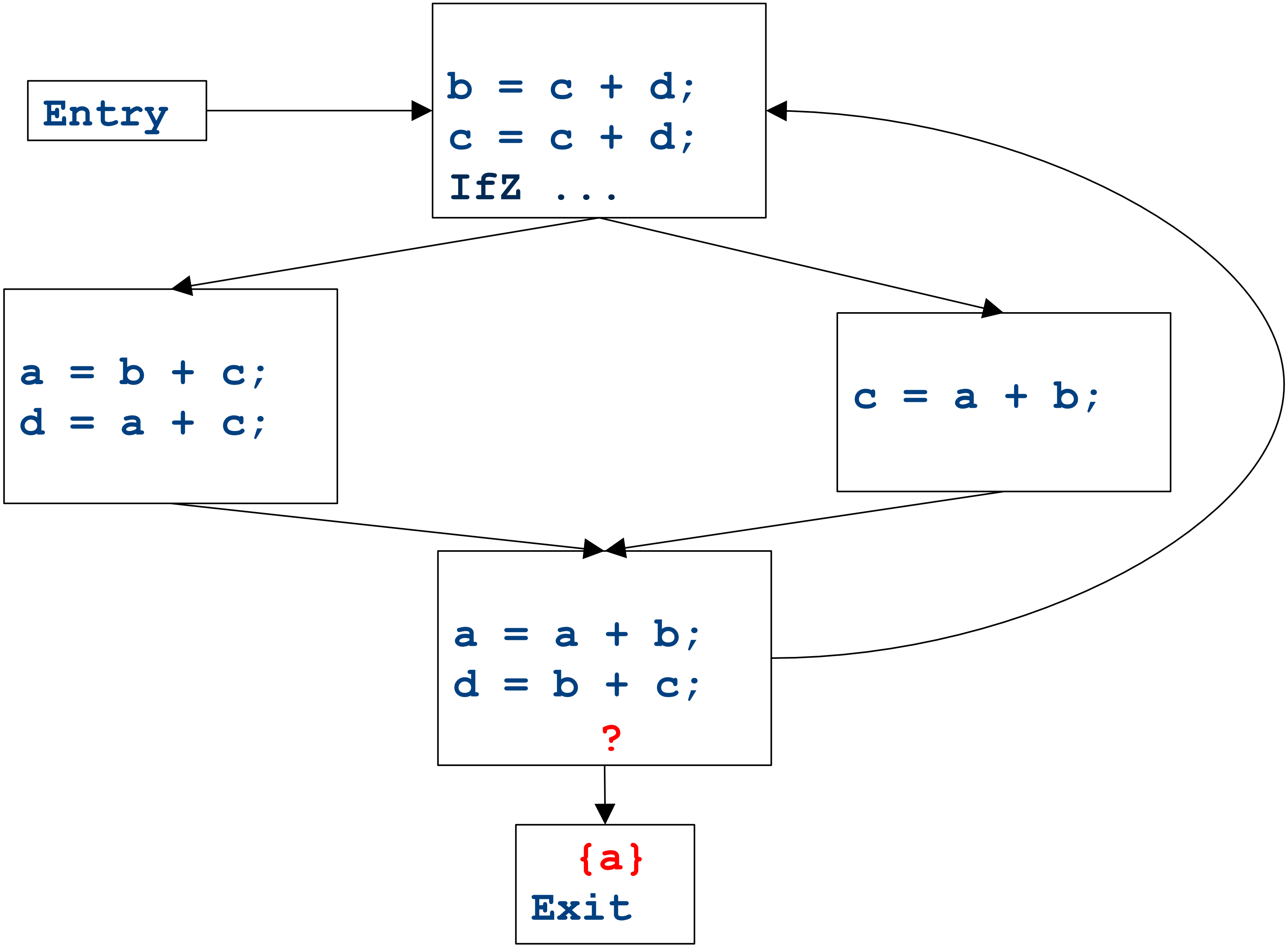
- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths
- When we add loops into the picture, this is no longer true
- Not all possible loops in a CFG can be realized in the actual program



CFGs with loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths
- When we add loops into the picture, this is no longer true
- Not all possible loops in a CFG can be realized in the actual program
- **Sound approximation:** Assume that every possible path through the CFG corresponds to a valid execution
  - Includes all realizable paths, but some additional paths as well
  - May make our analysis less precise (but still sound)
  - Makes the analysis feasible; we'll see how later

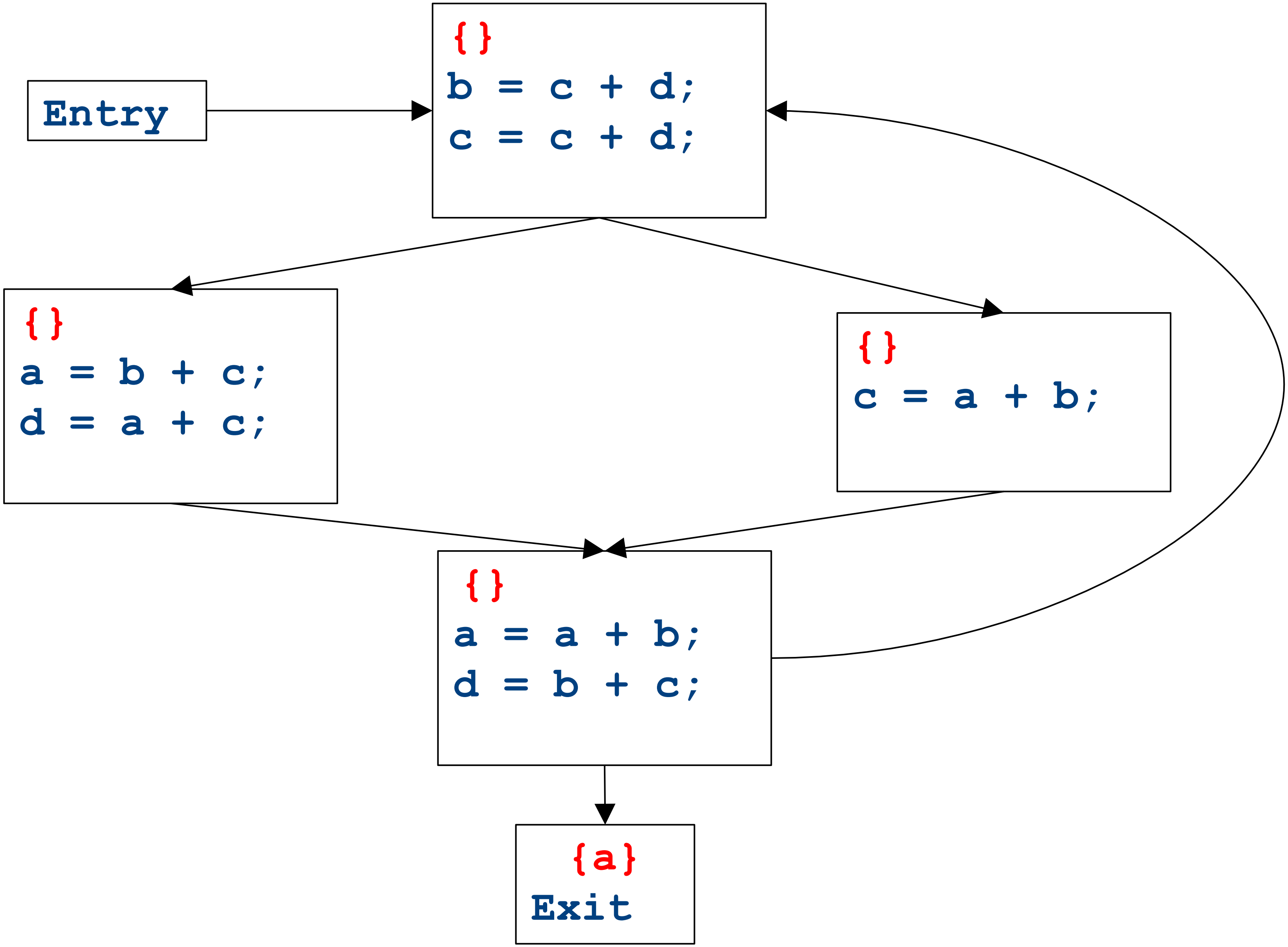
CFGs with loops



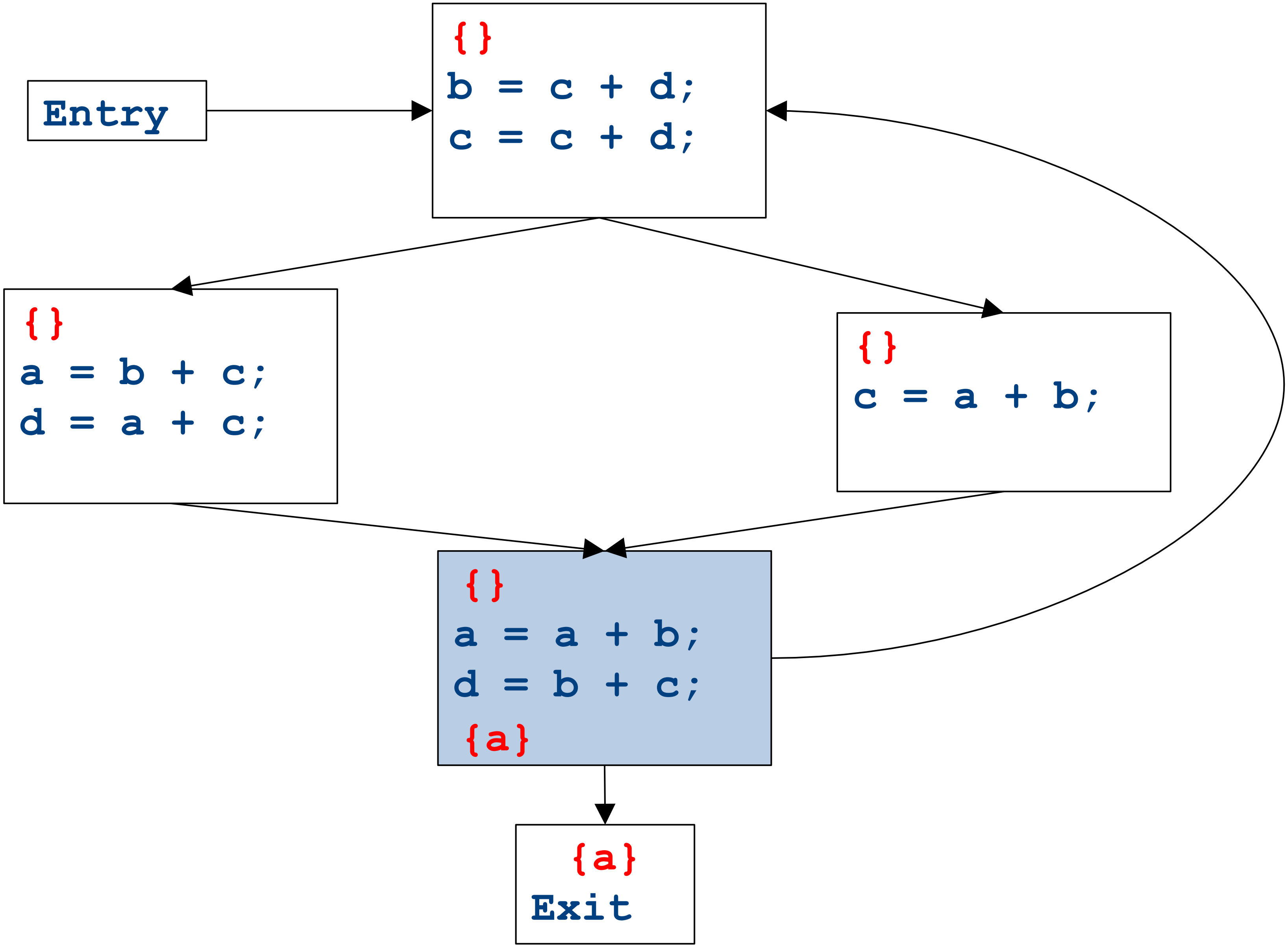
Major changes – part 3

- In a local analysis, there is always a well defined "first" statement to begin processing
- In a global analysis with loops, every basic block might depend on every other basic block
- To fix this, we need to assign initial values to all of the blocks in the CFG

CFGs with loops - initialization



CFGs with loops - iteration



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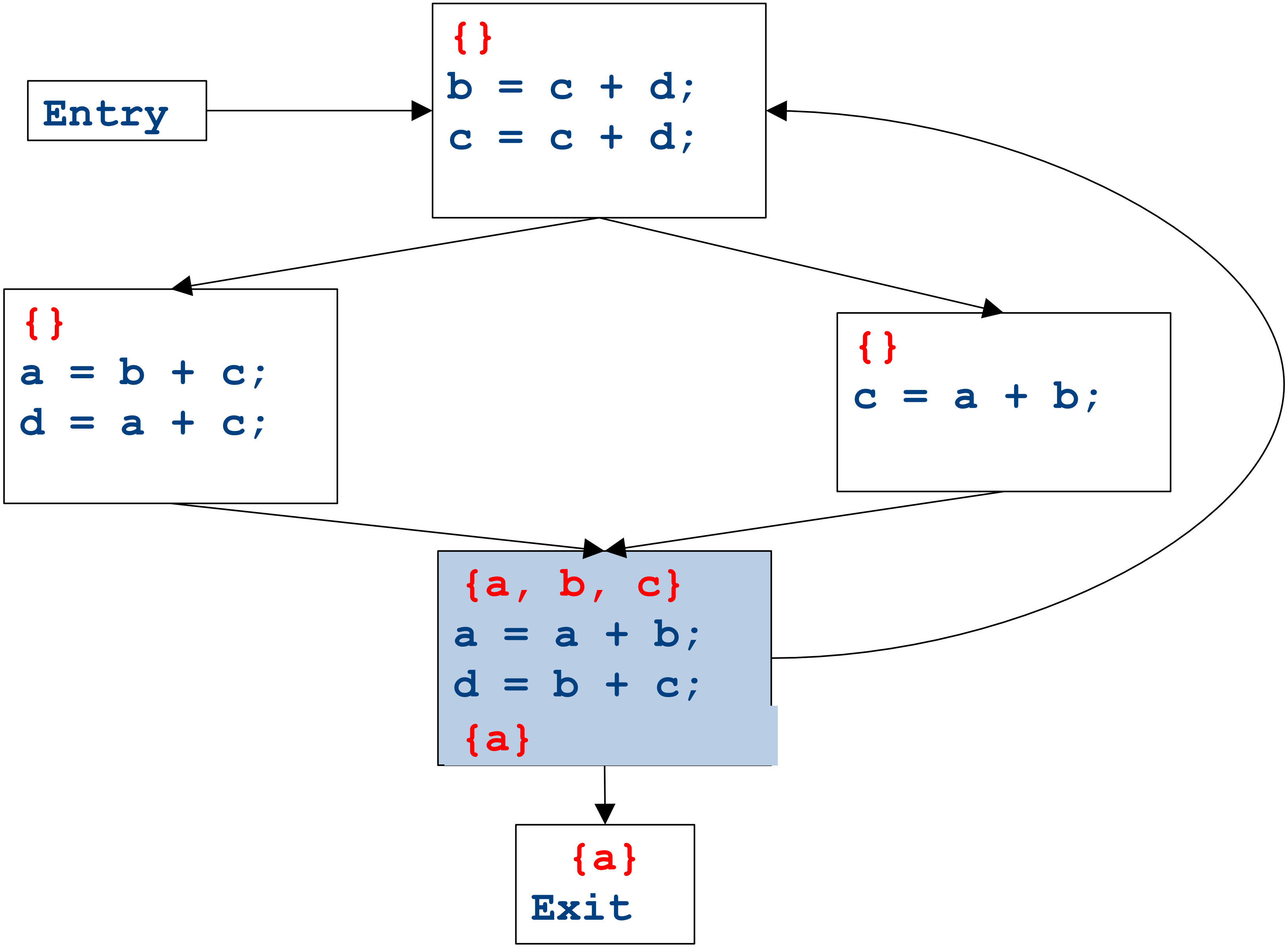

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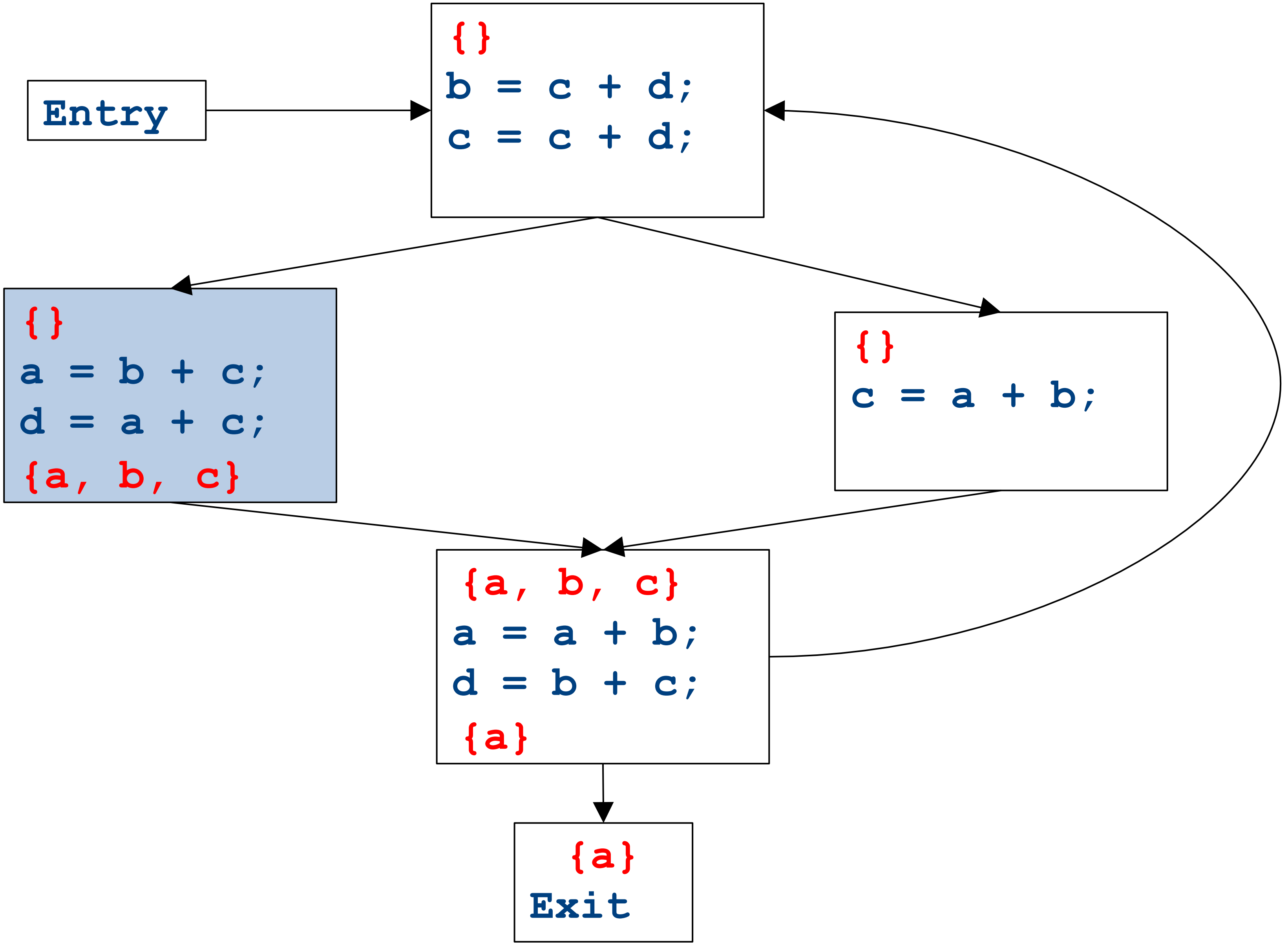

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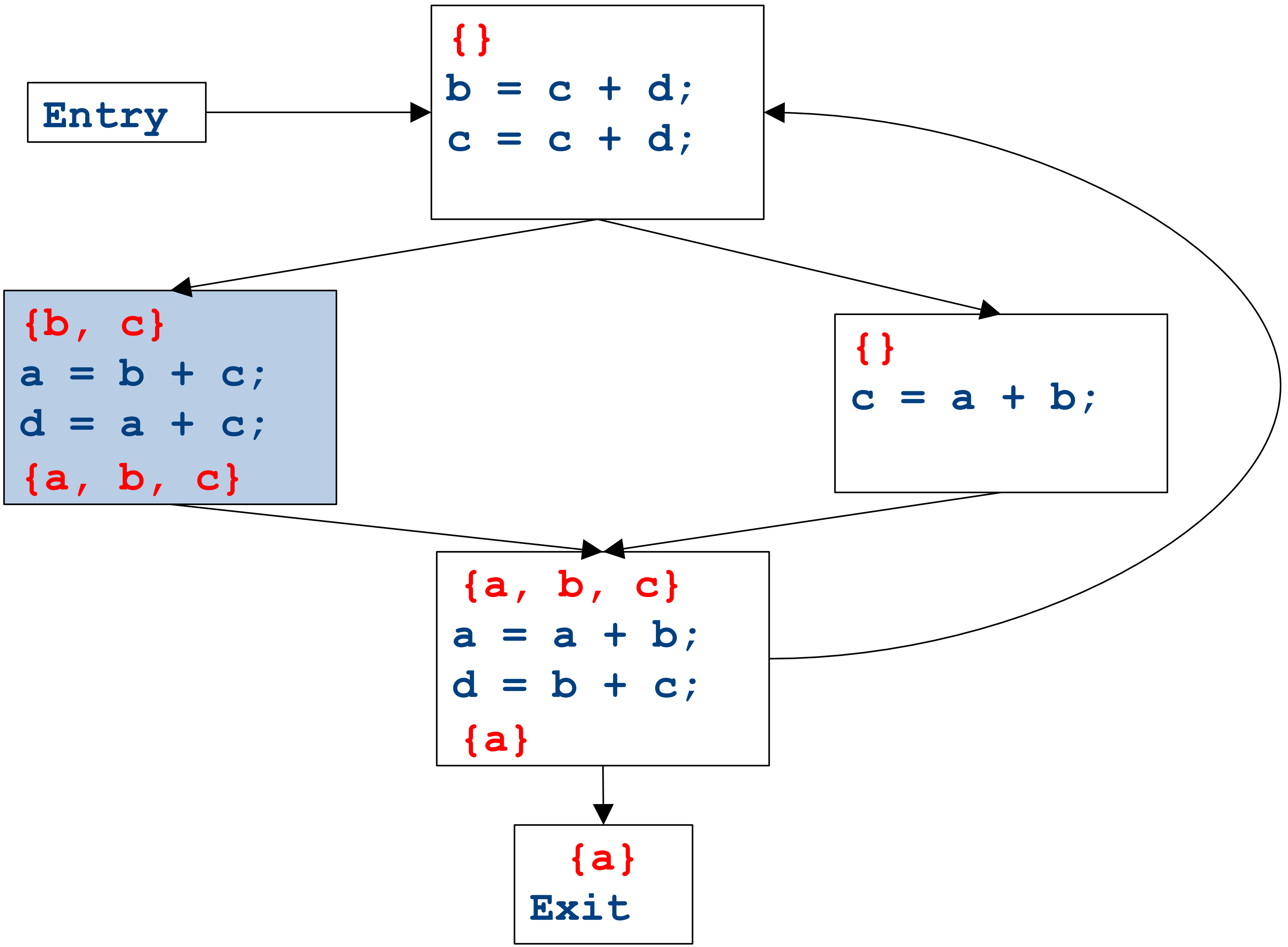

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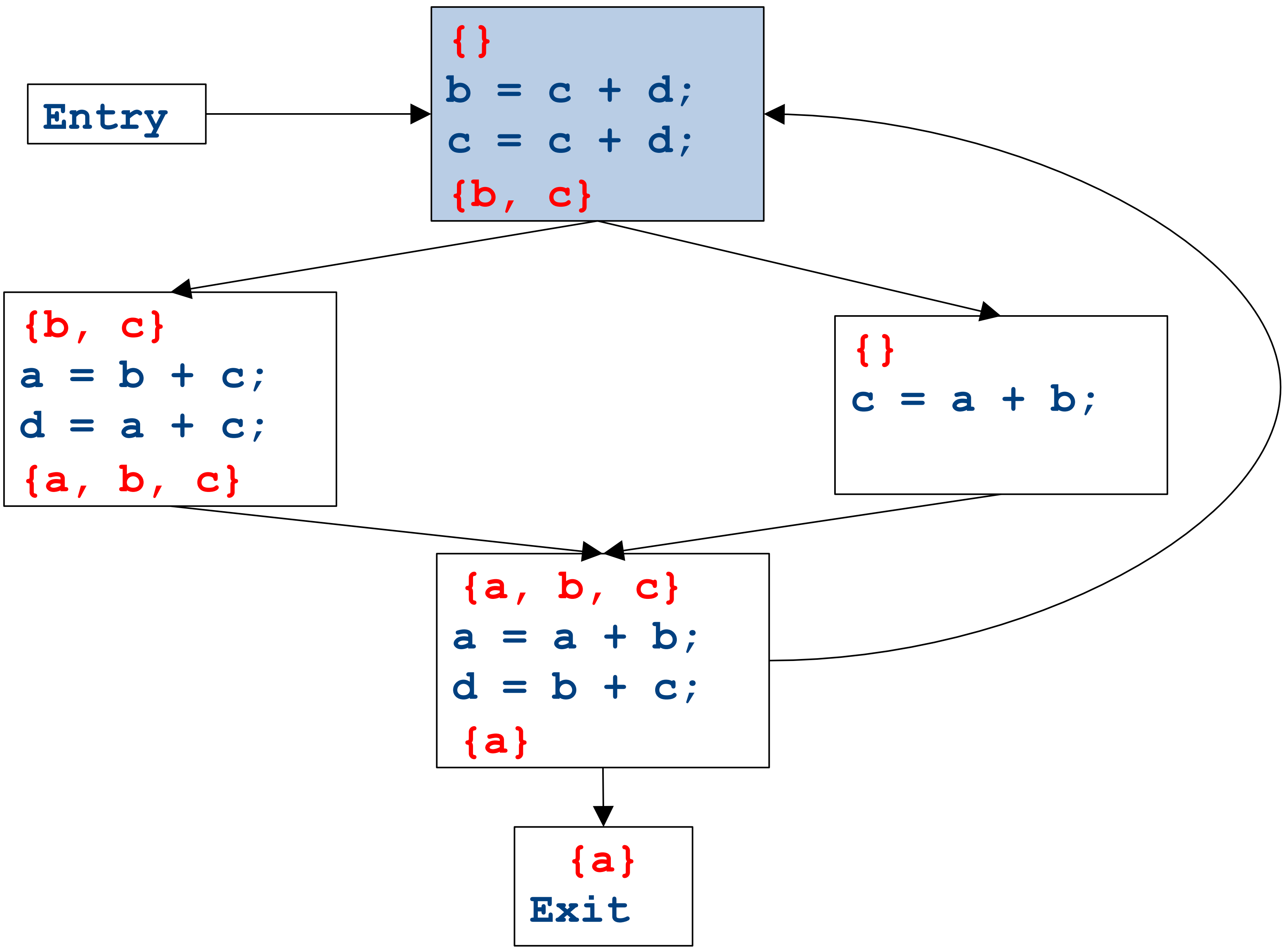

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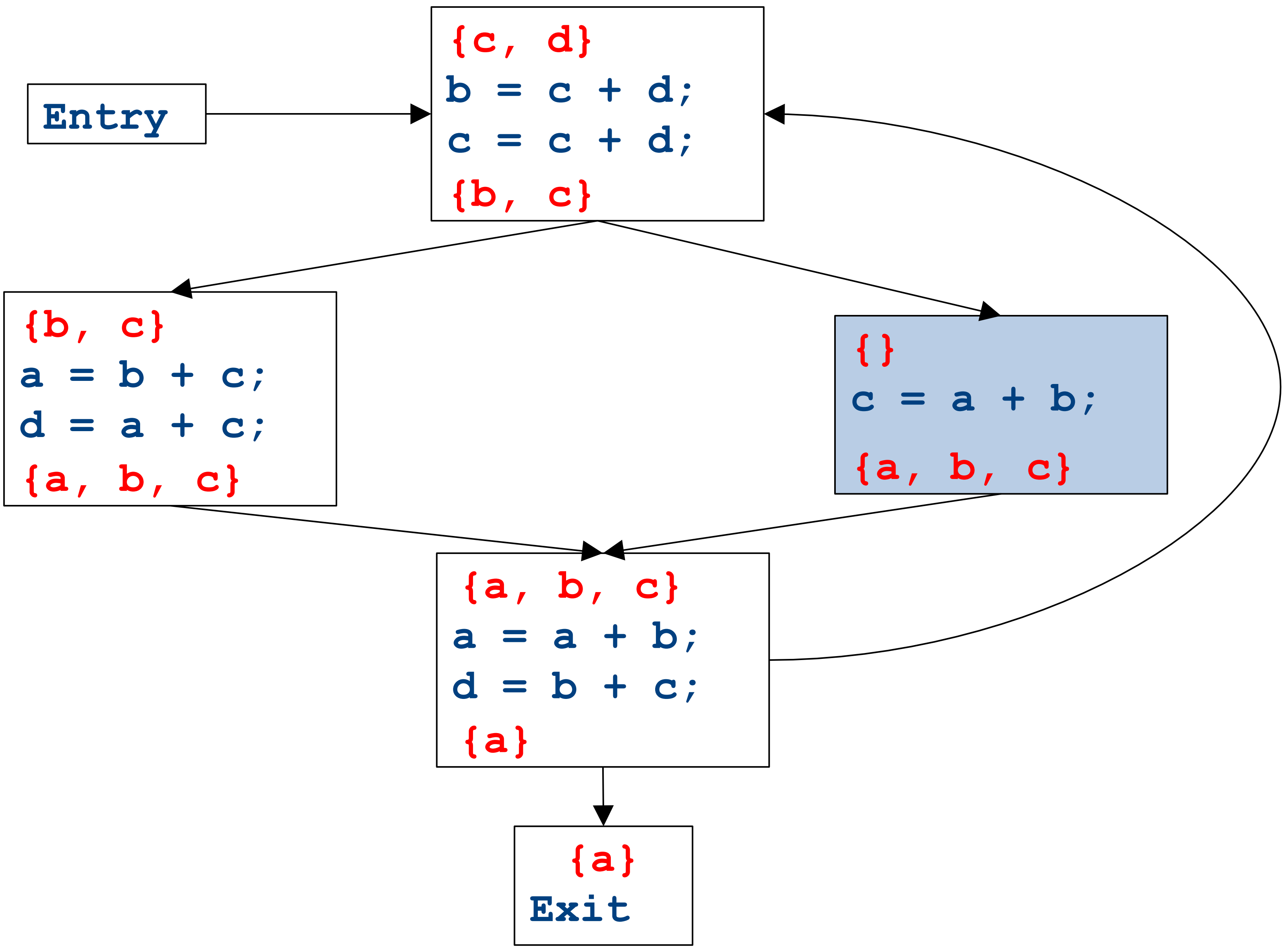

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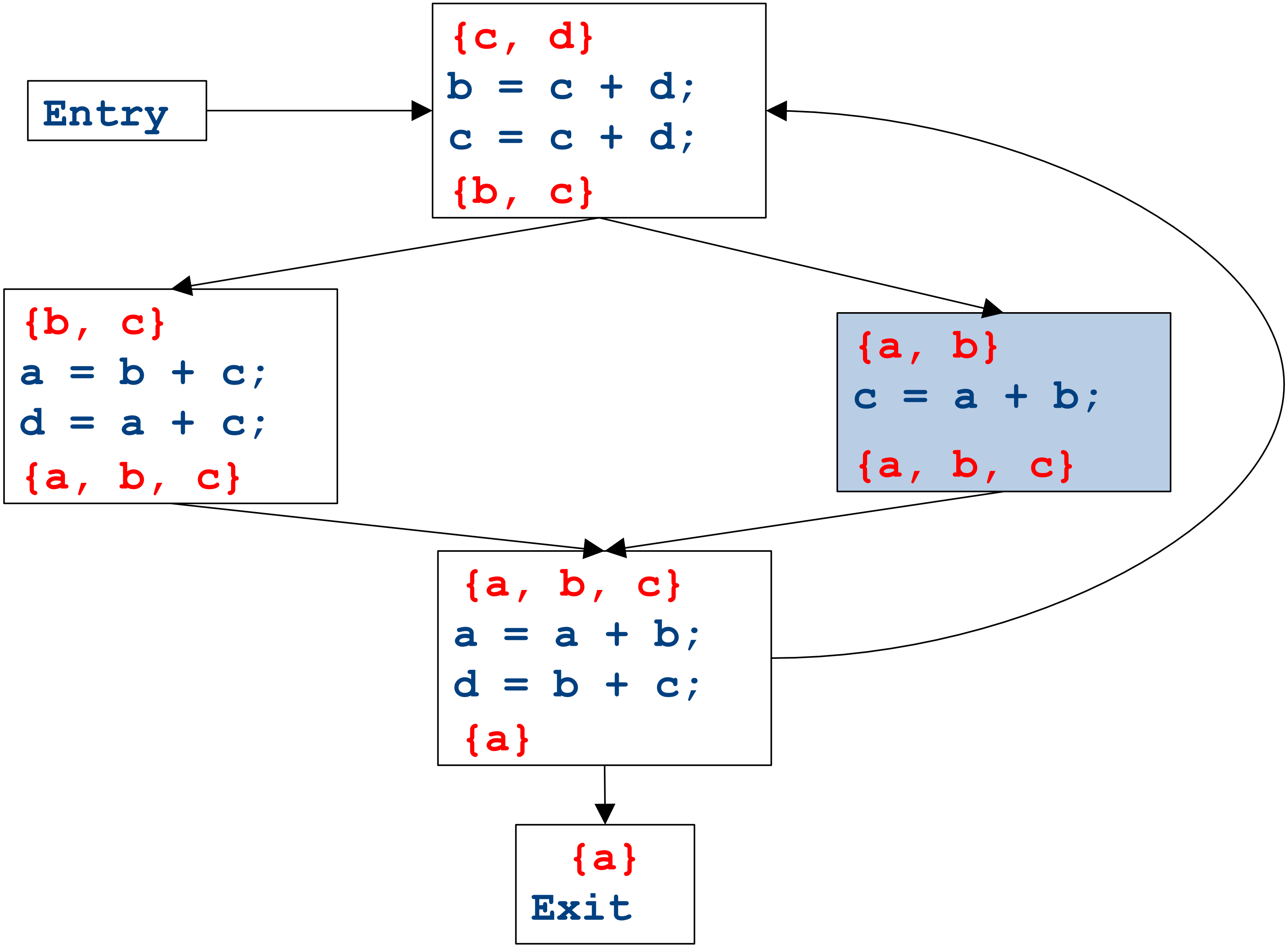
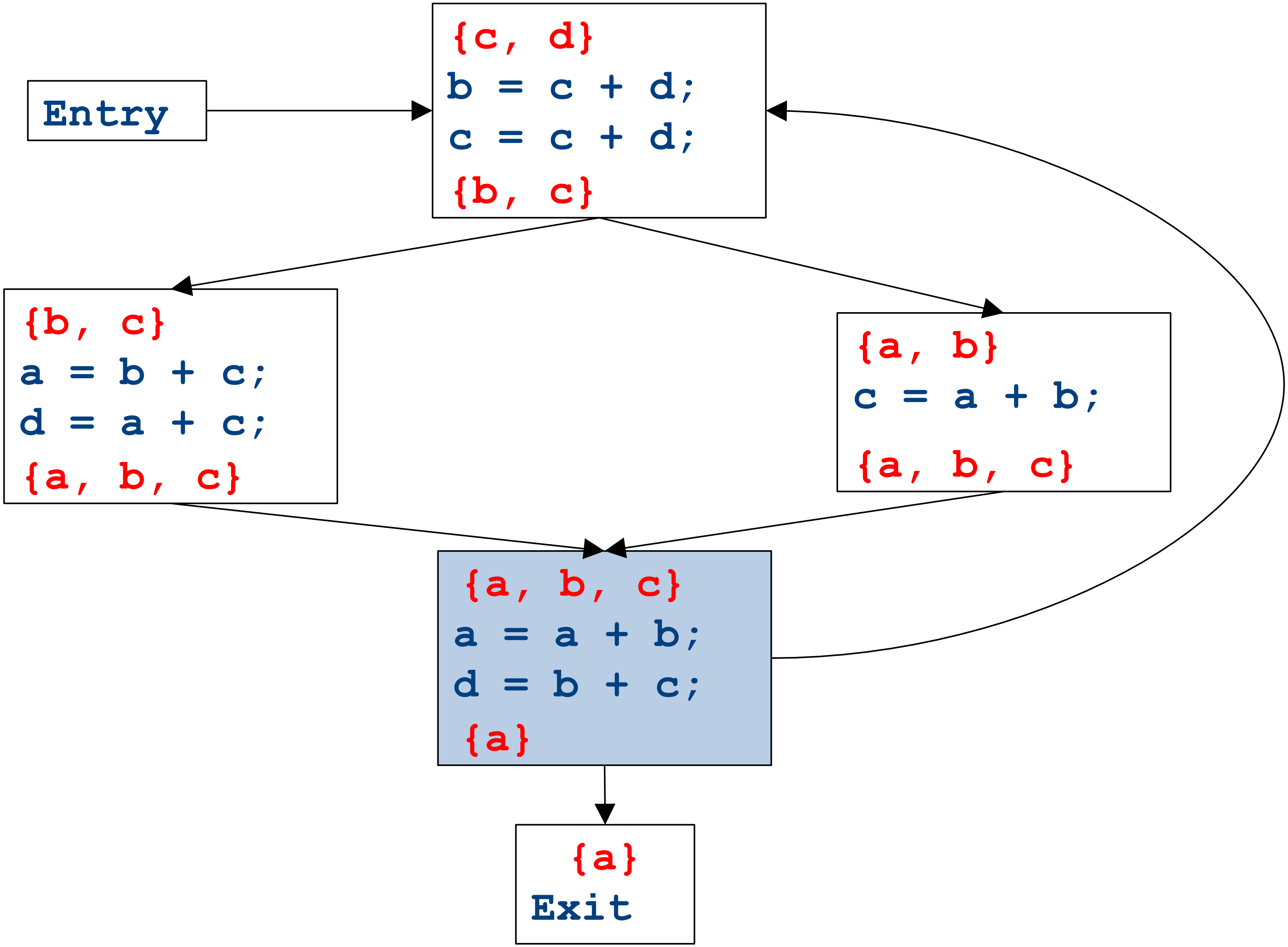
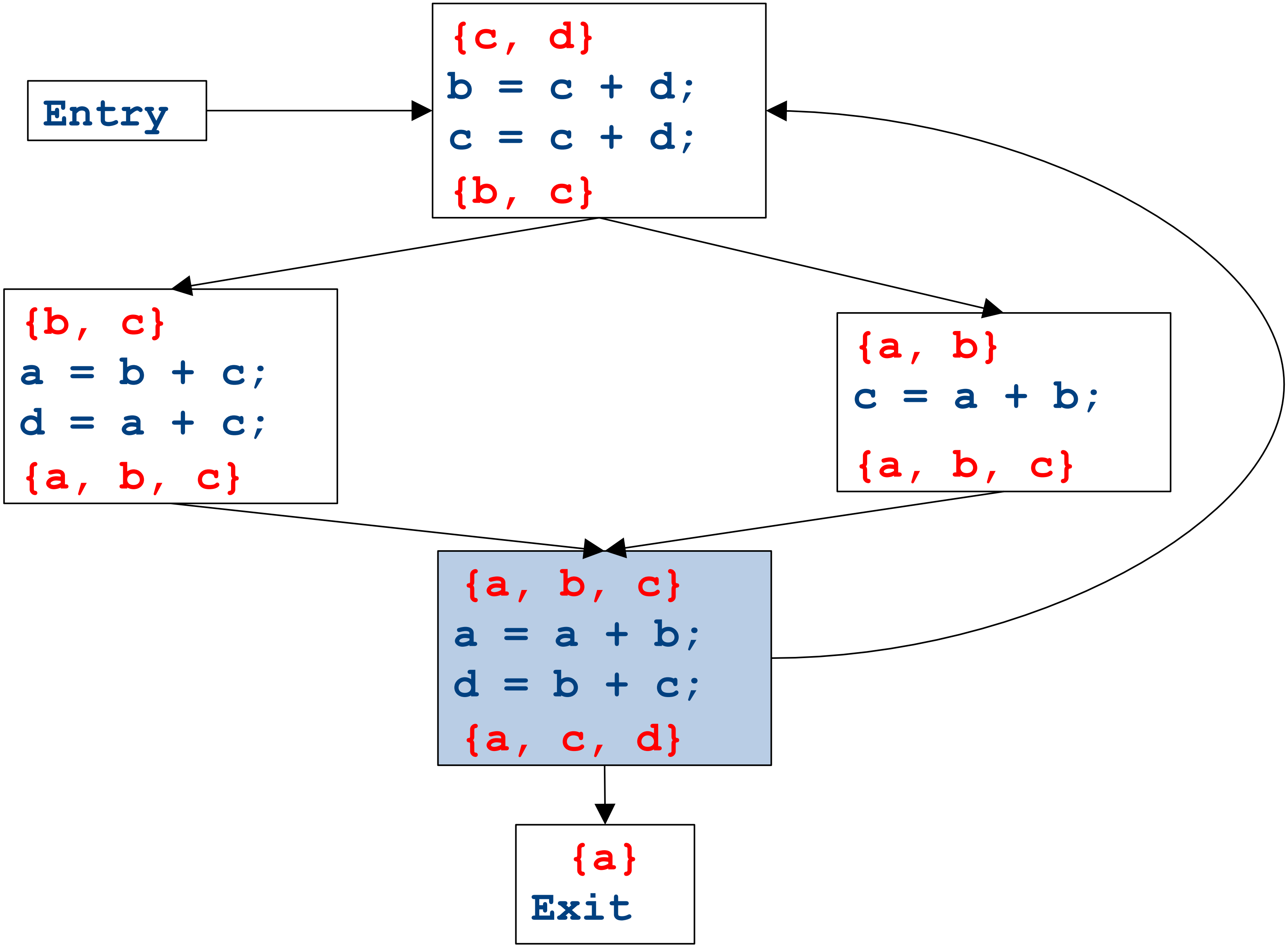


CFGs with loops - iteration



CFGs with loops - iteration



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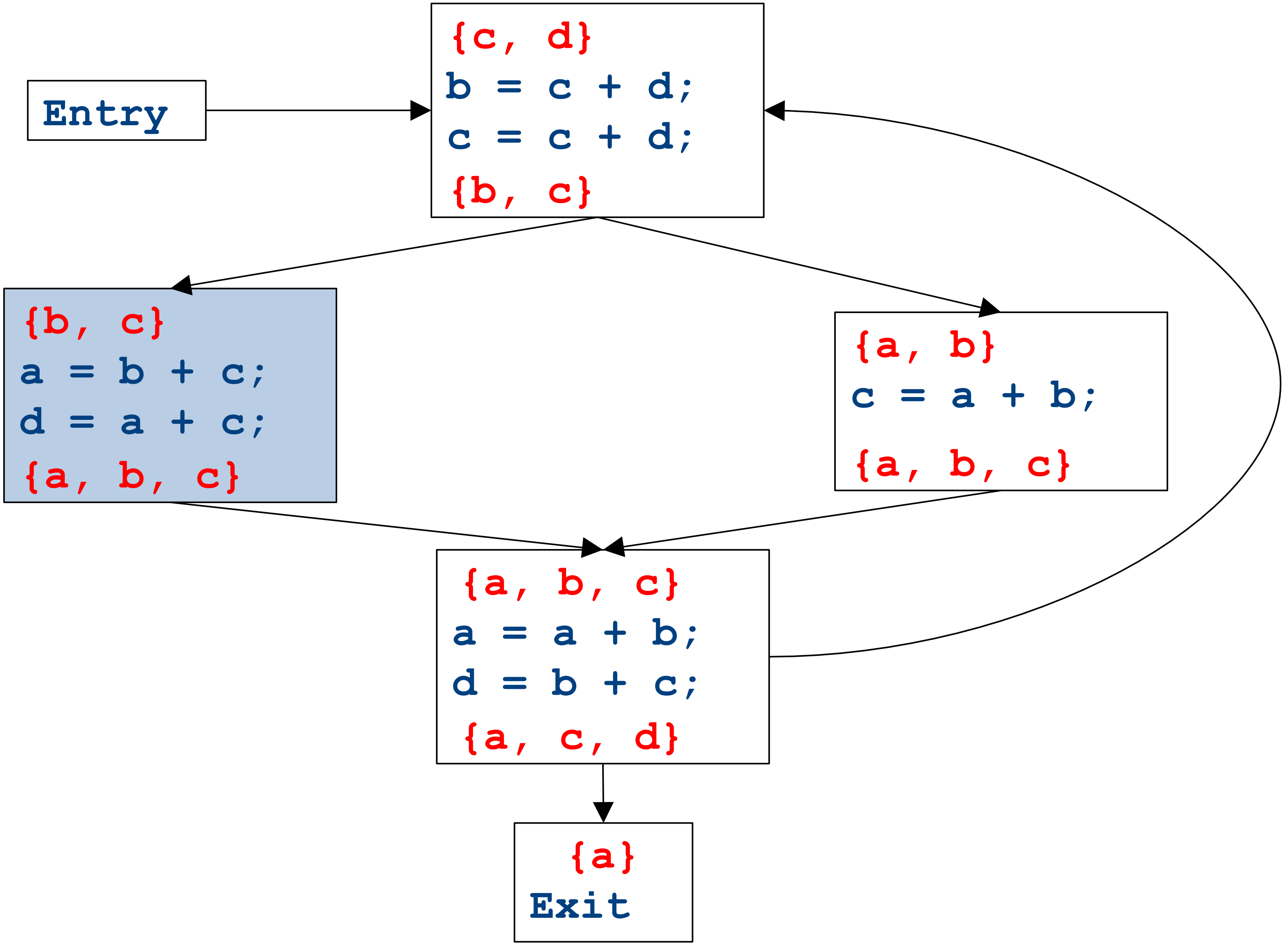

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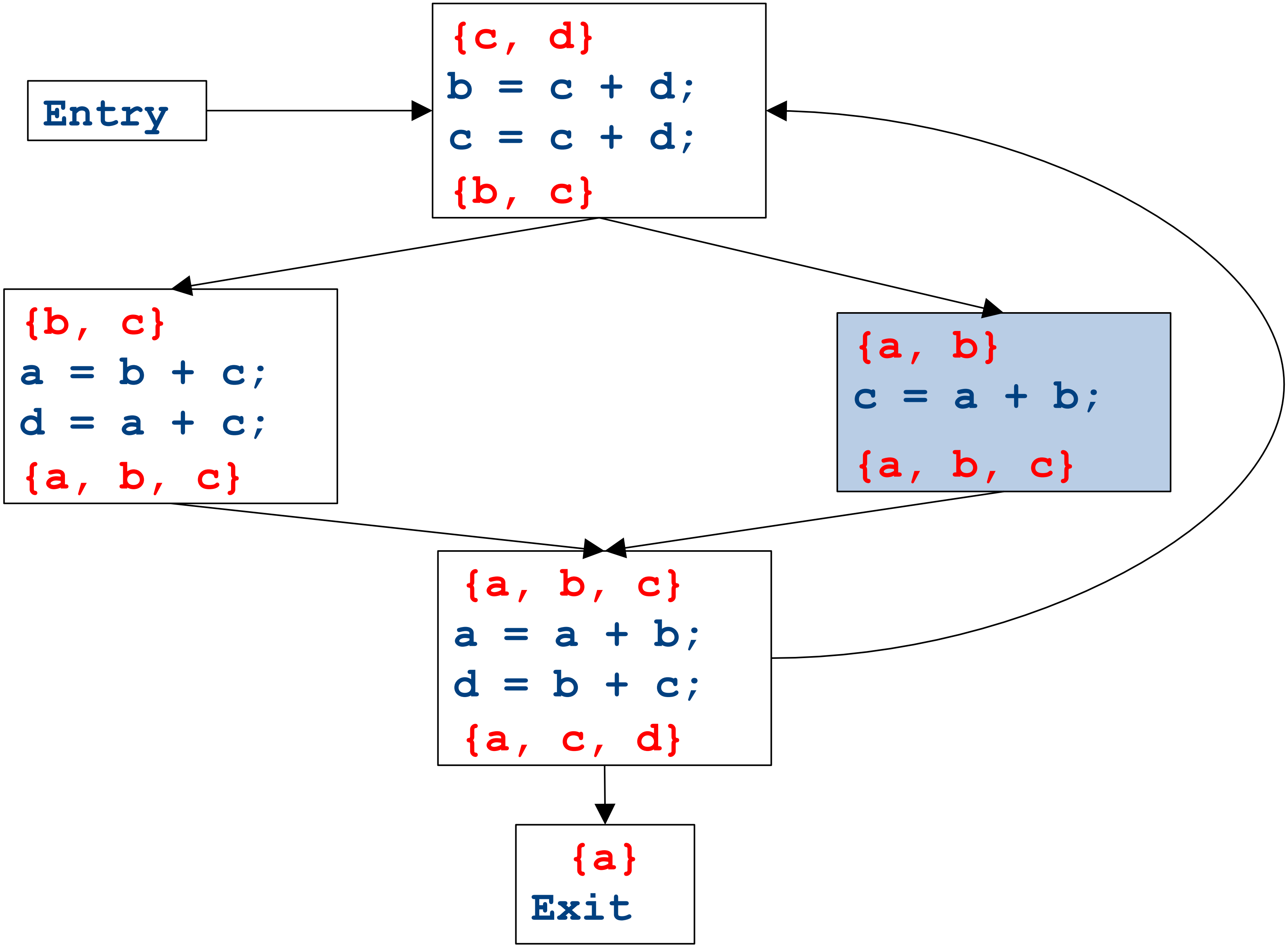
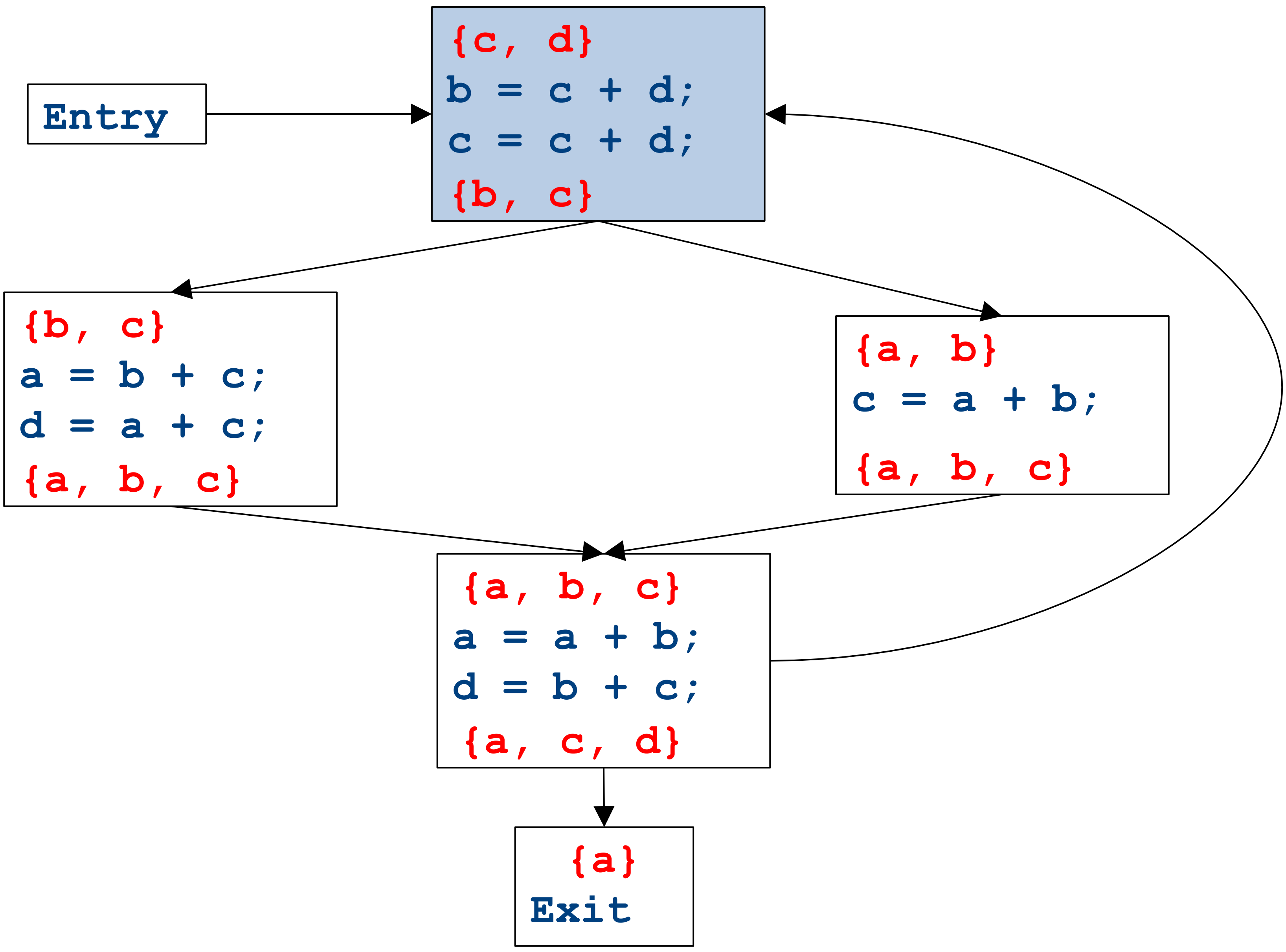
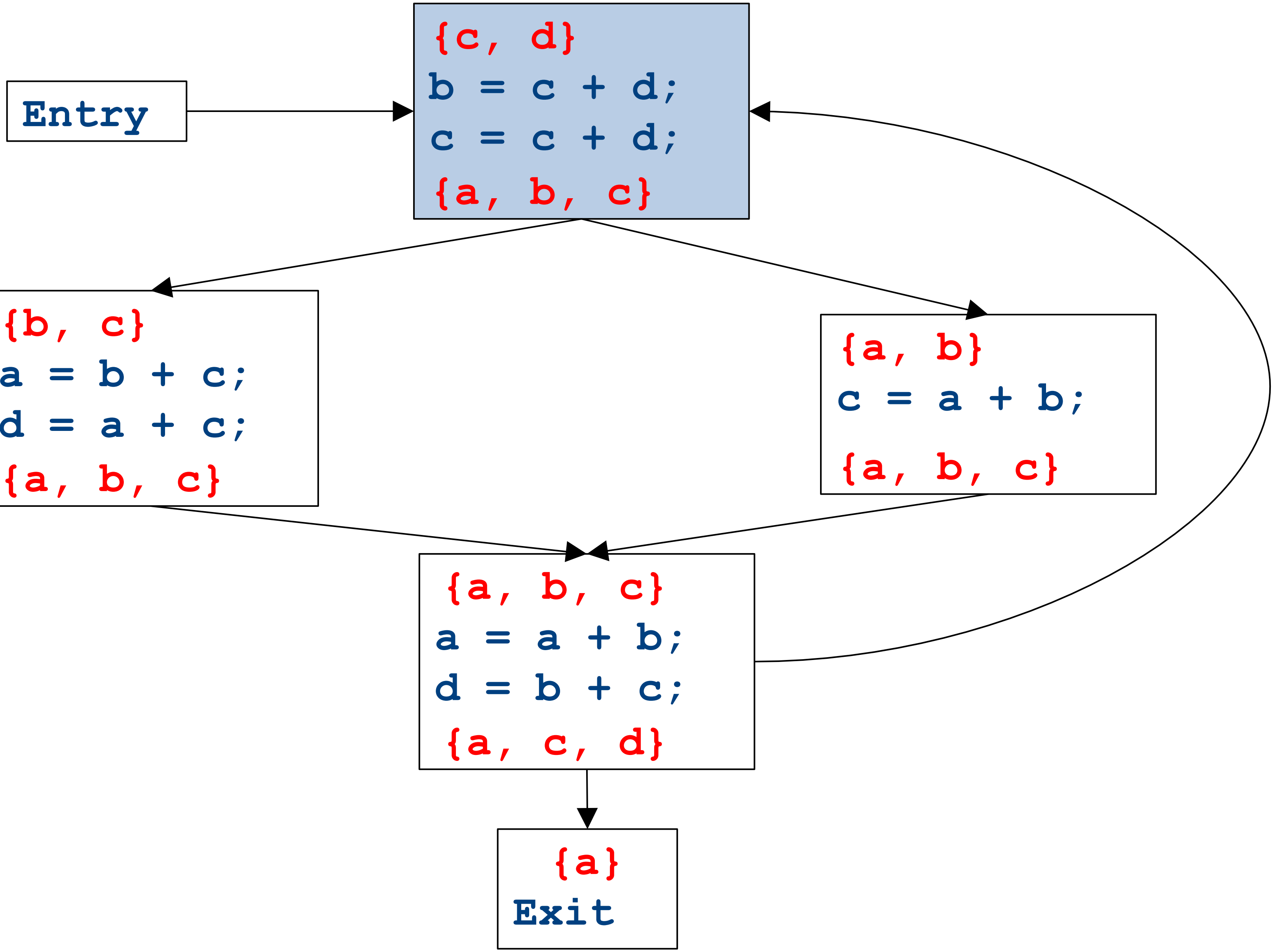


CFGs with loops - iteration



CFGs with loops - iteration



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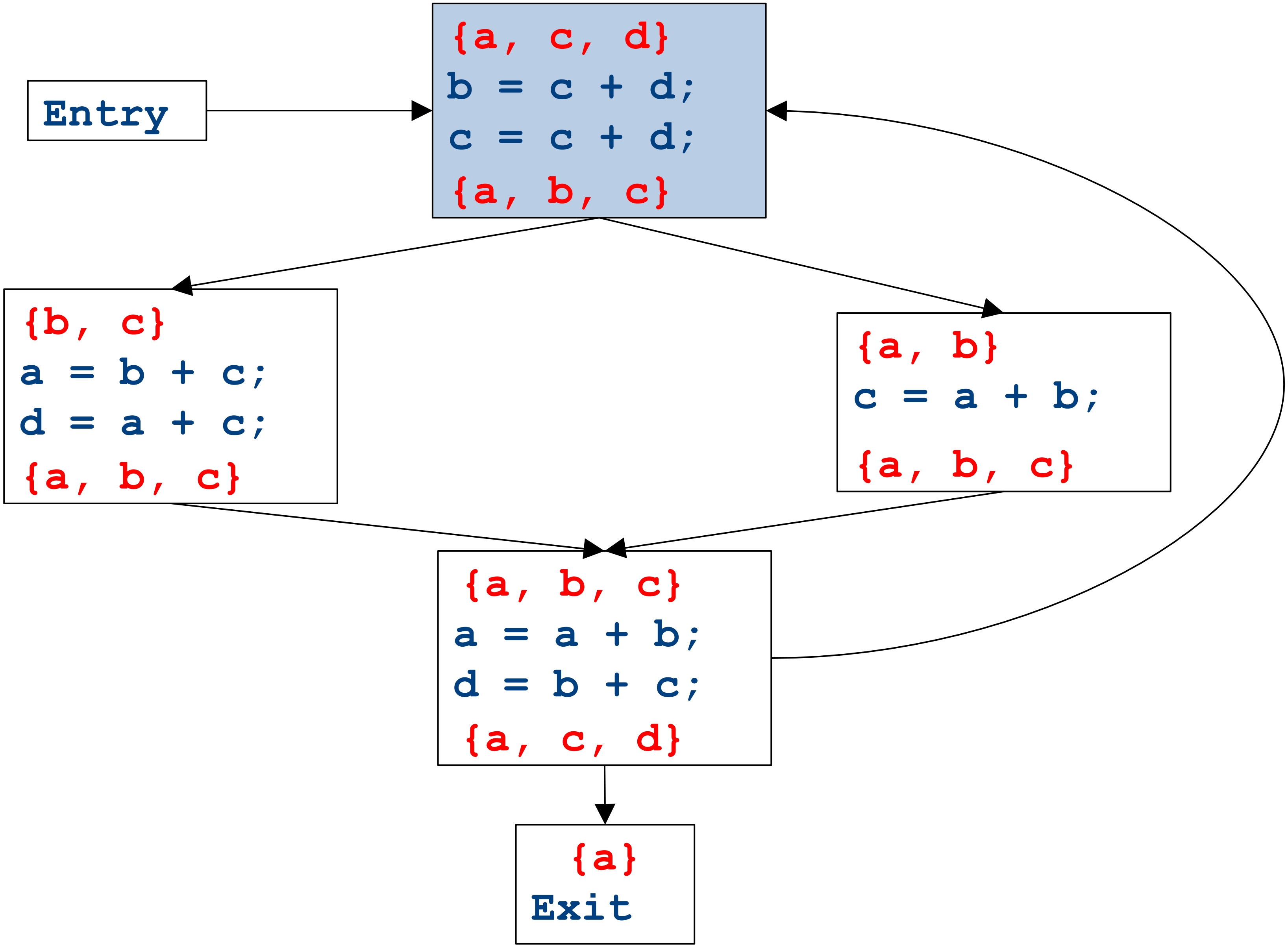

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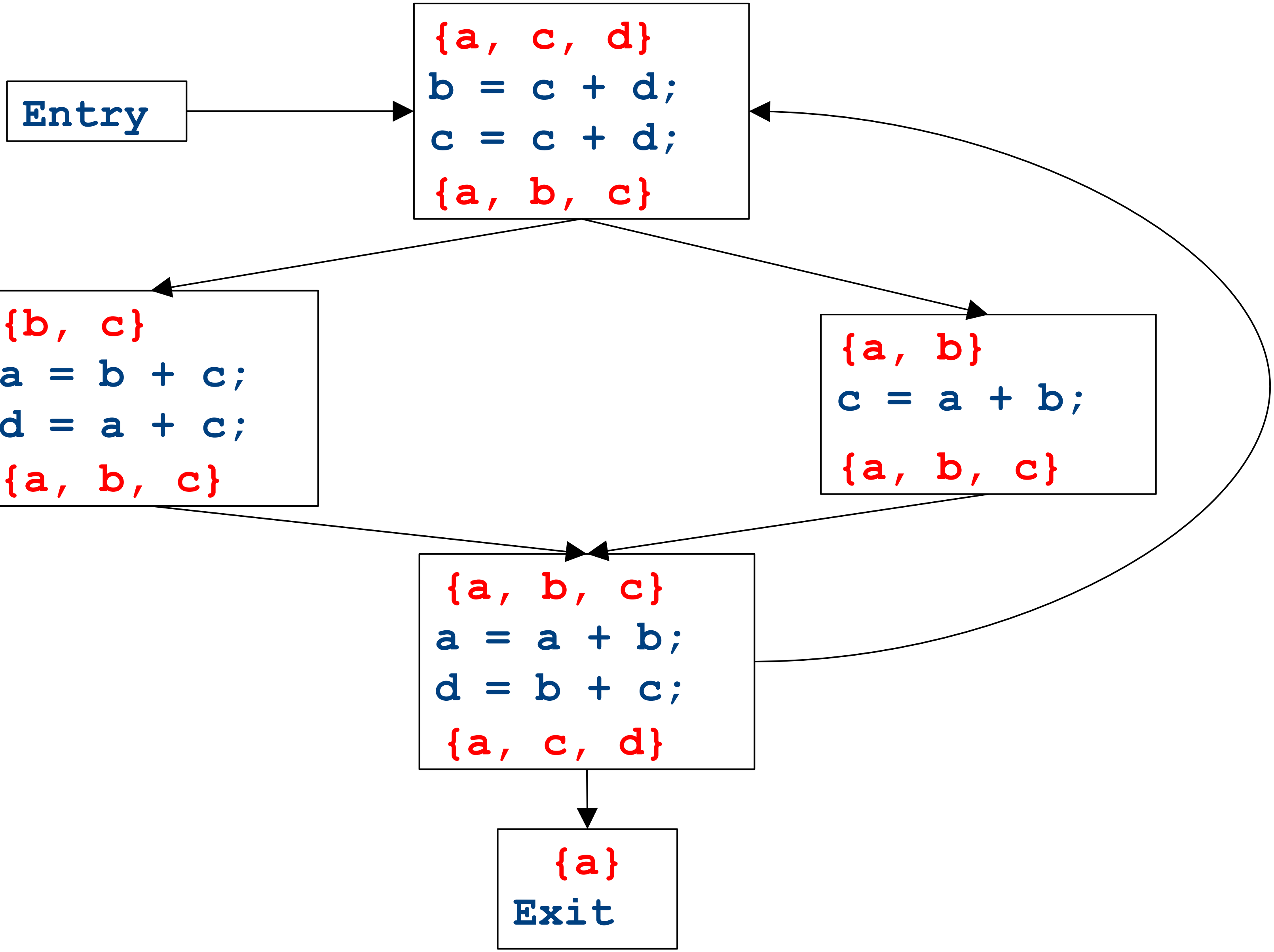


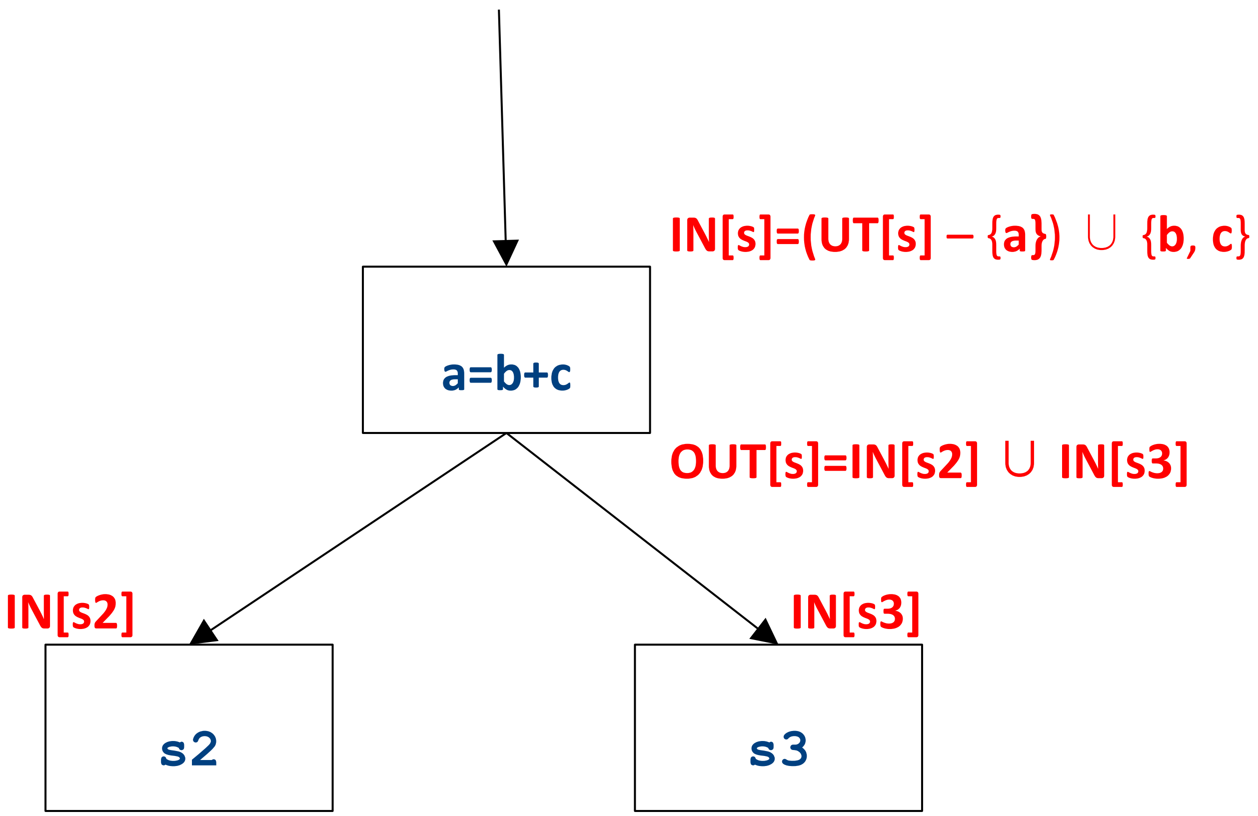
Summary of differences

- Need to be able to handle multiple predecessors/successors for a basic block
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value
  - But the analysis still needs to terminate!
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it

Global liveness analysis

- Initially, set IN[**s**] = { } for each statement **s**
- Set IN[**exit**] to the set of variables known to be live on exit (language-specific knowledge)
- Repeat until no changes occur:
  - For each statement **s** of the form **a = b + c**, in any order you'd like:
    - Set OUT[**s**] to set union of IN[**p**] for each successor **p** of **s**
    - Set IN[**s**] to (OUT[**s**] – **a**) ∪ {**b**, **c**}.
- Yet another fixed-point iteration!

Global liveness analysis



Why does this work?

- To show correctness, we need to show that
  - The algorithm eventually terminates, and
  - When it terminates, it has a sound answer
- Termination argument:
  - Once a variable is discovered to be live during some point of the analysis, it always stays live
  - Only finitely many variables and finitely many places where a variable can become live
- Soundness argument (sketch):
  - Each individual rule, applied to some set, correctly updates liveness in that set
  - When computing the union of the set of live variables, a variable is only live if it was live on some path leaving the statement

Abstract Interpretation

- Theoretical foundations of program analysis
- Cousot and Cousot 1977
- Abstract meaning of programs
  - Executed at compile time

Another view of local optimization

- In local optimization, we want to reason about some property of the runtime behavior of the program
- Could we run the program and just watch what happens?
- **Idea:** Redefine the semantics of our programming language to give us information about our analysis

Properties of local analysis

- The only way to find out what a program will actually do is to run it
- Problems:
  - The program might not terminate
  - The program might have some behavior we didn't see when we ran it on a particular input
- However, this is not a problem inside a basic block
  - Basic blocks contain no loops
  - There is only one path through the basic block

Assigning new semantics

- Example: Available Expressions
- Redefine the statement **a = b + c to** mean "**a now holds the value of b + c,** and any variable holding the value **a is** now invalid"
- Run the program assuming these new semantics
- Treat the optimizer as an interpreter for these new semantics

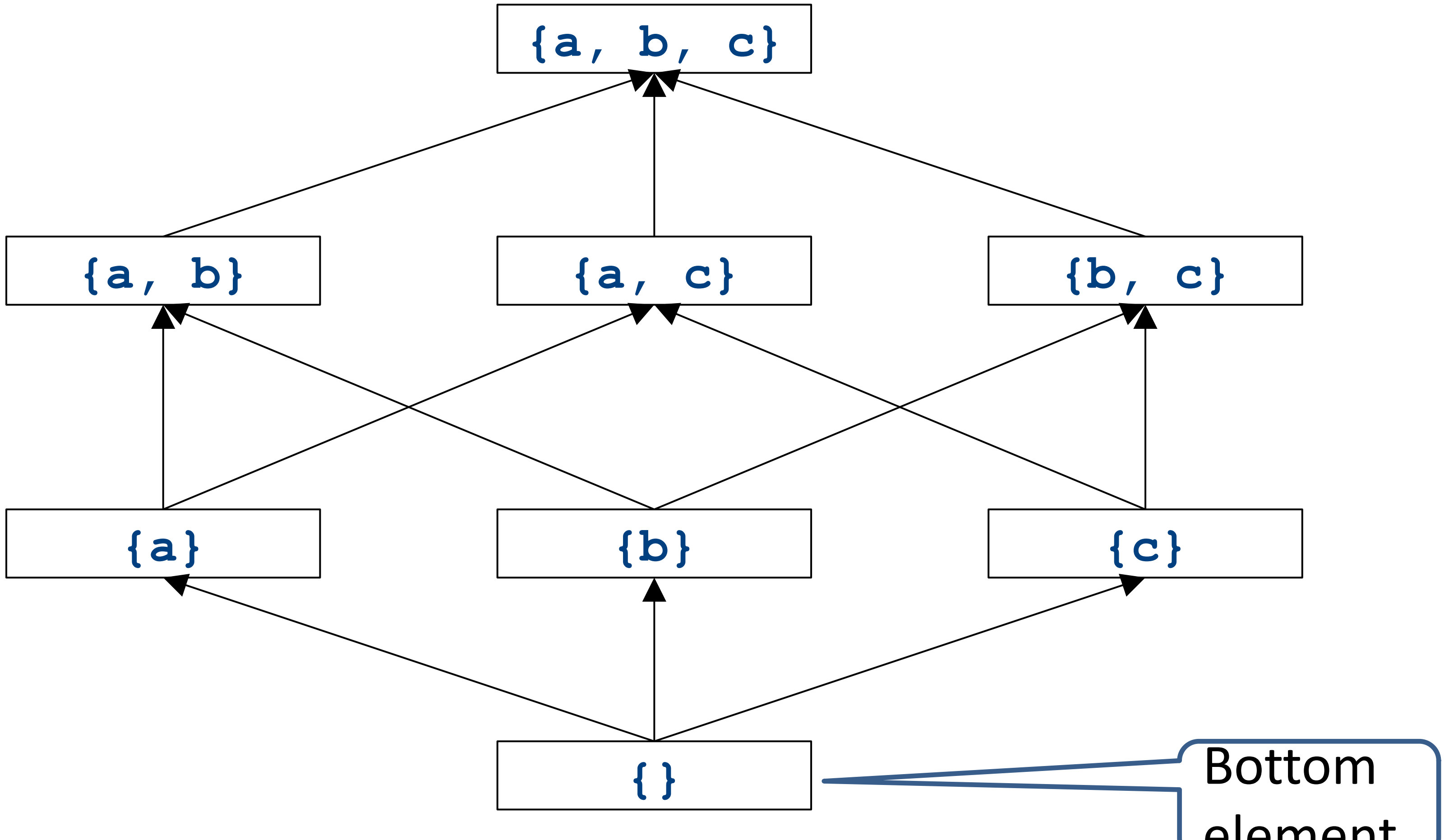
Theory to the rescue

- Building up all of the machinery to design this analysis was tricky
- The key ideas, however, are mostly independent of the analysis:
  - We need to be able to compute functions describing the behavior of each statement
  - We need to be able to merge several subcomputations together
  - We need an initial value for all of the basic blocks
- There is a beautiful formalism that captures many of these properties

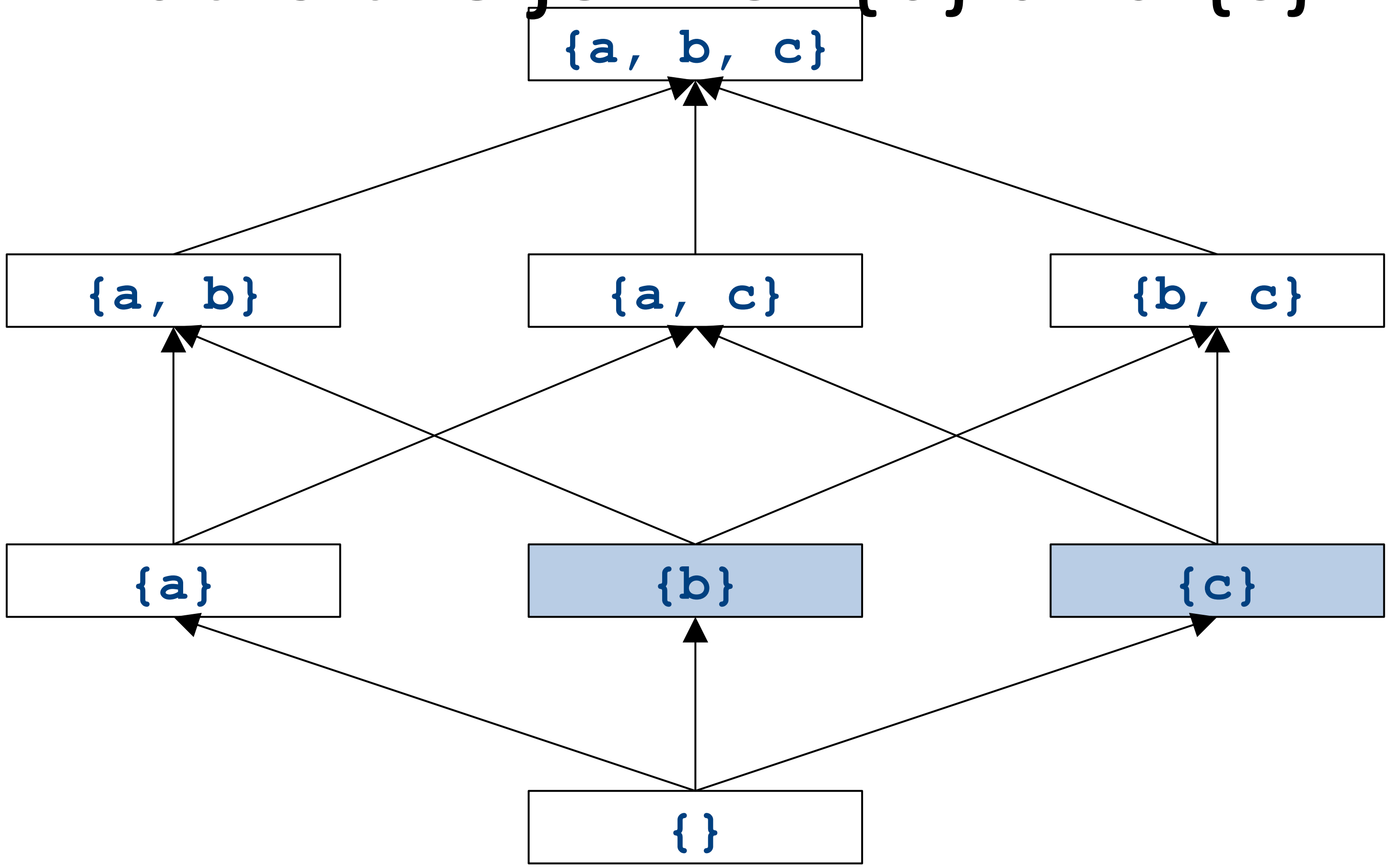
Join semilattices

- A join semilattice is a ordering defined on a set of elements
- Any two elements have some join that is the smallest element larger than both elements
- There is a unique bottom element, which is smaller than all other elements
- Intuitively:
  - The join of two elements represents combining information from two elements by an overapproximation
- The bottom element represents “no information yet” or “the least conservative possible answer”

Join semilattice for liveness



What is the join of {b} and {c}?



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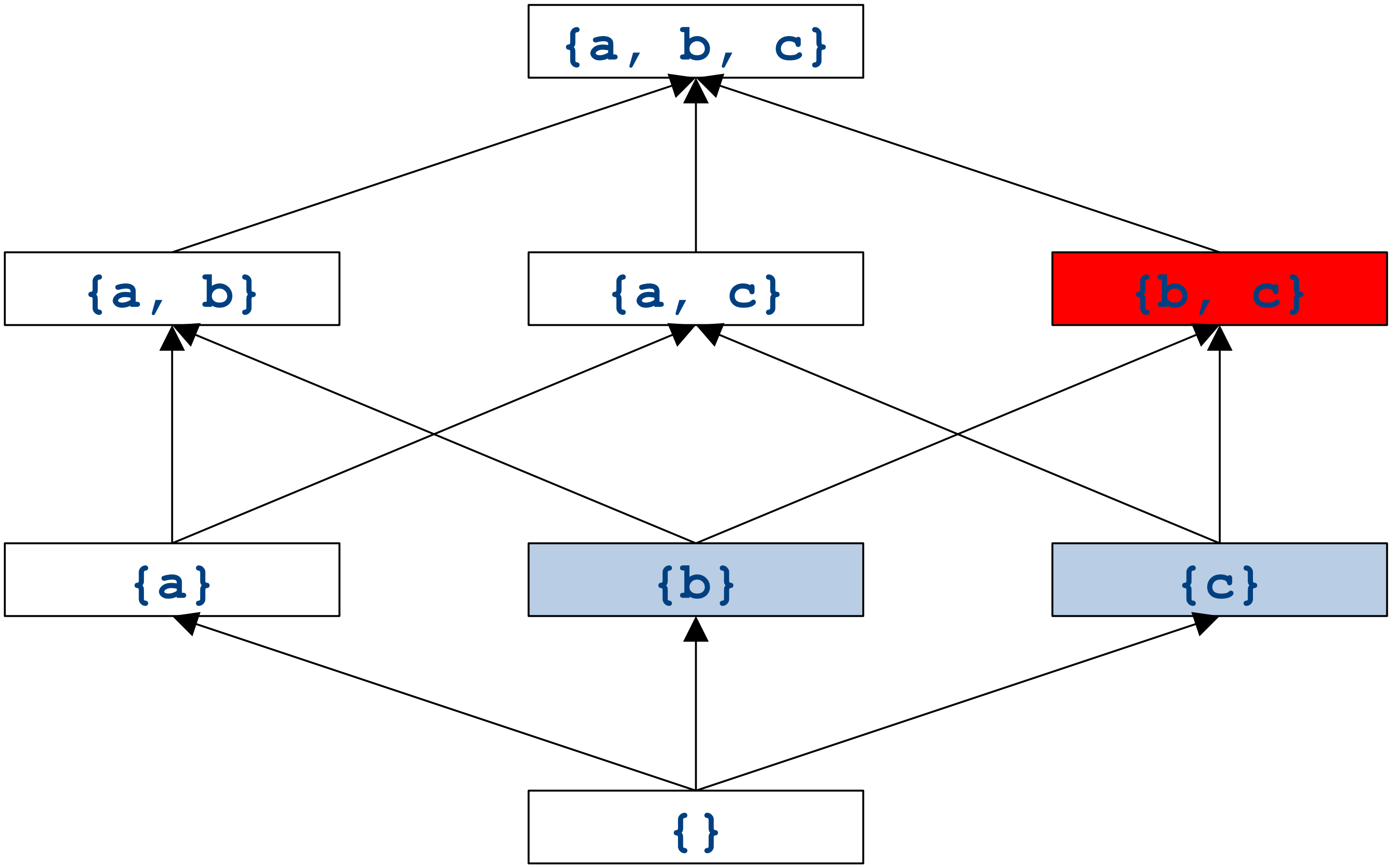
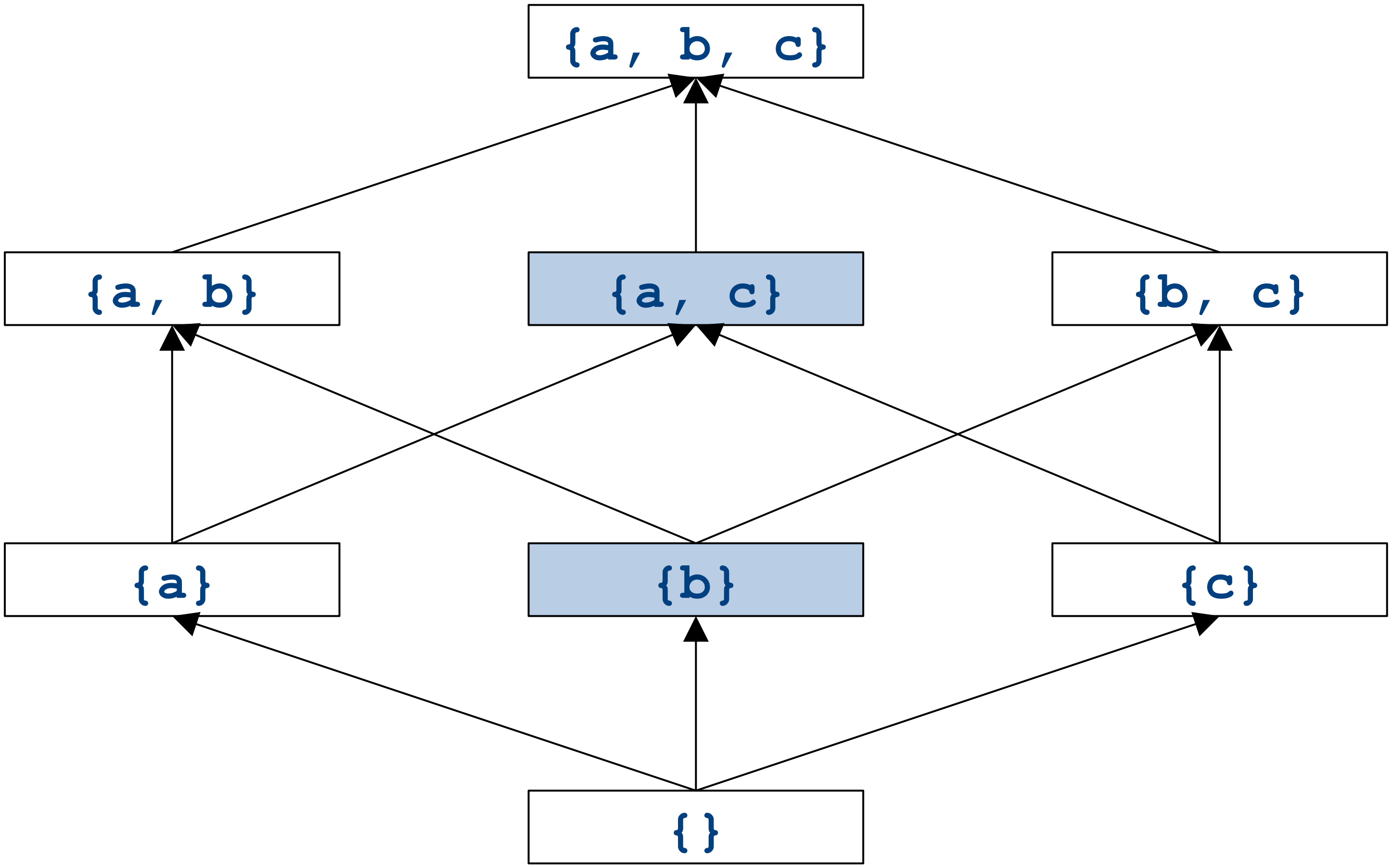


What is the join of {b} and {a,c}?



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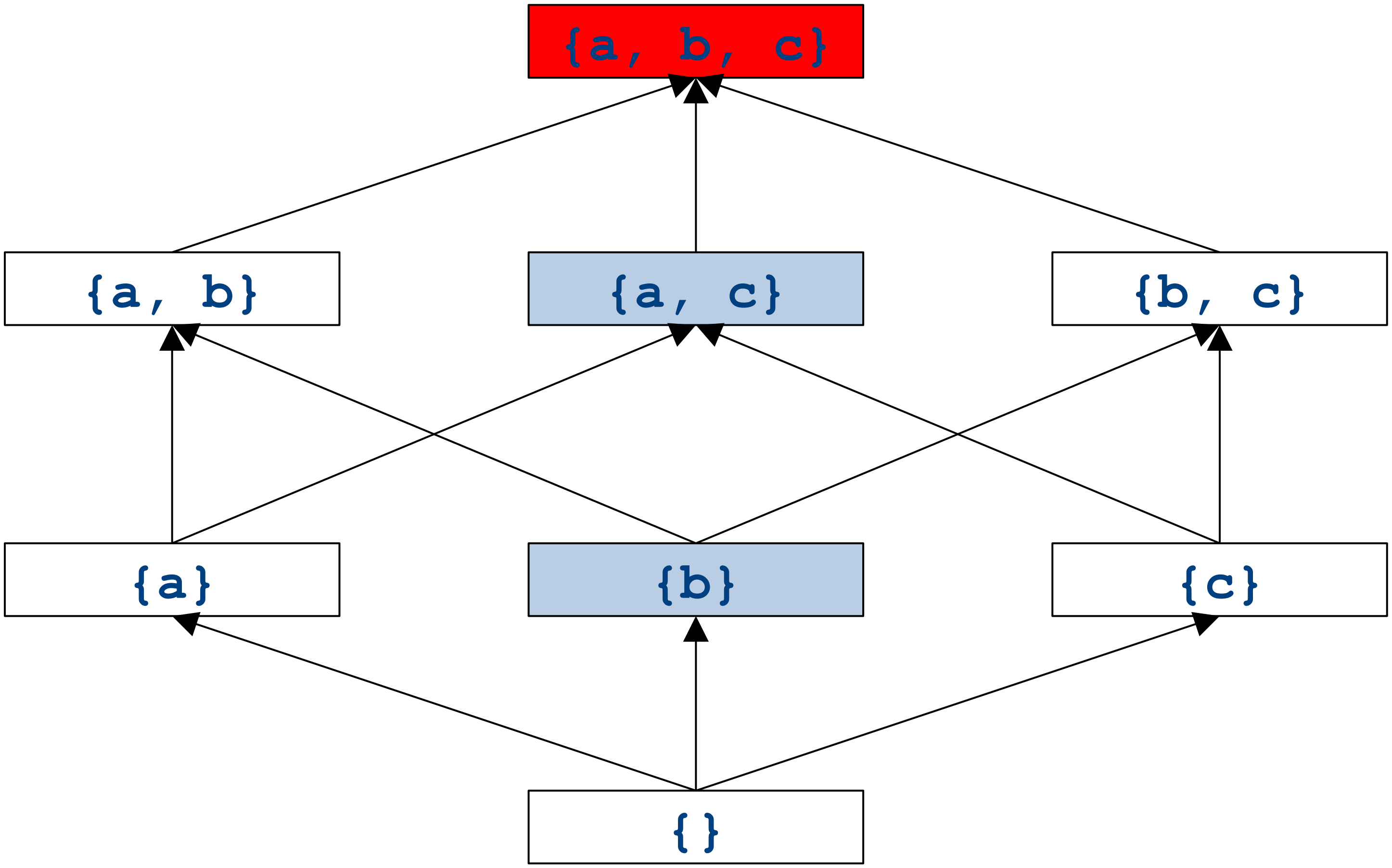

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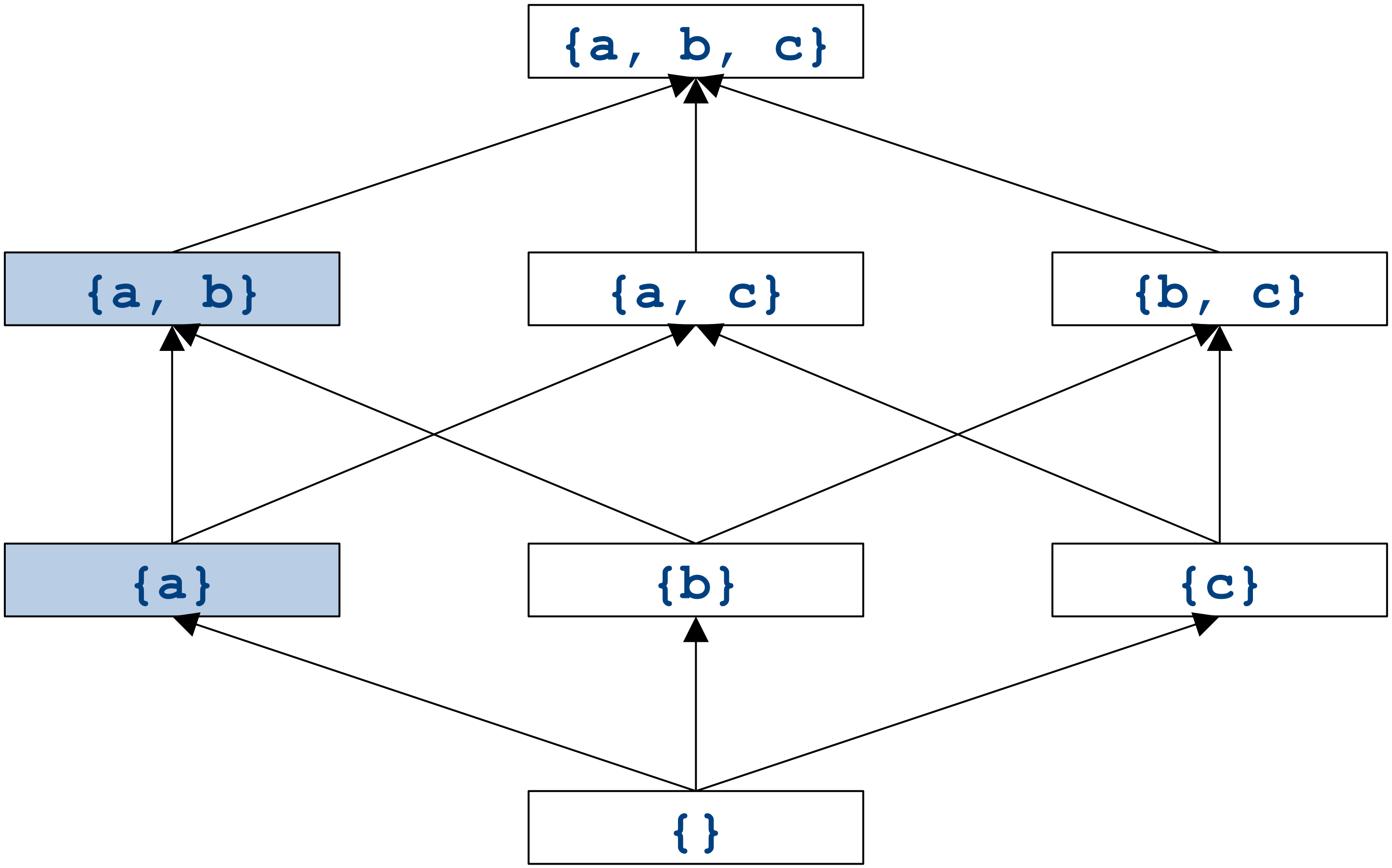

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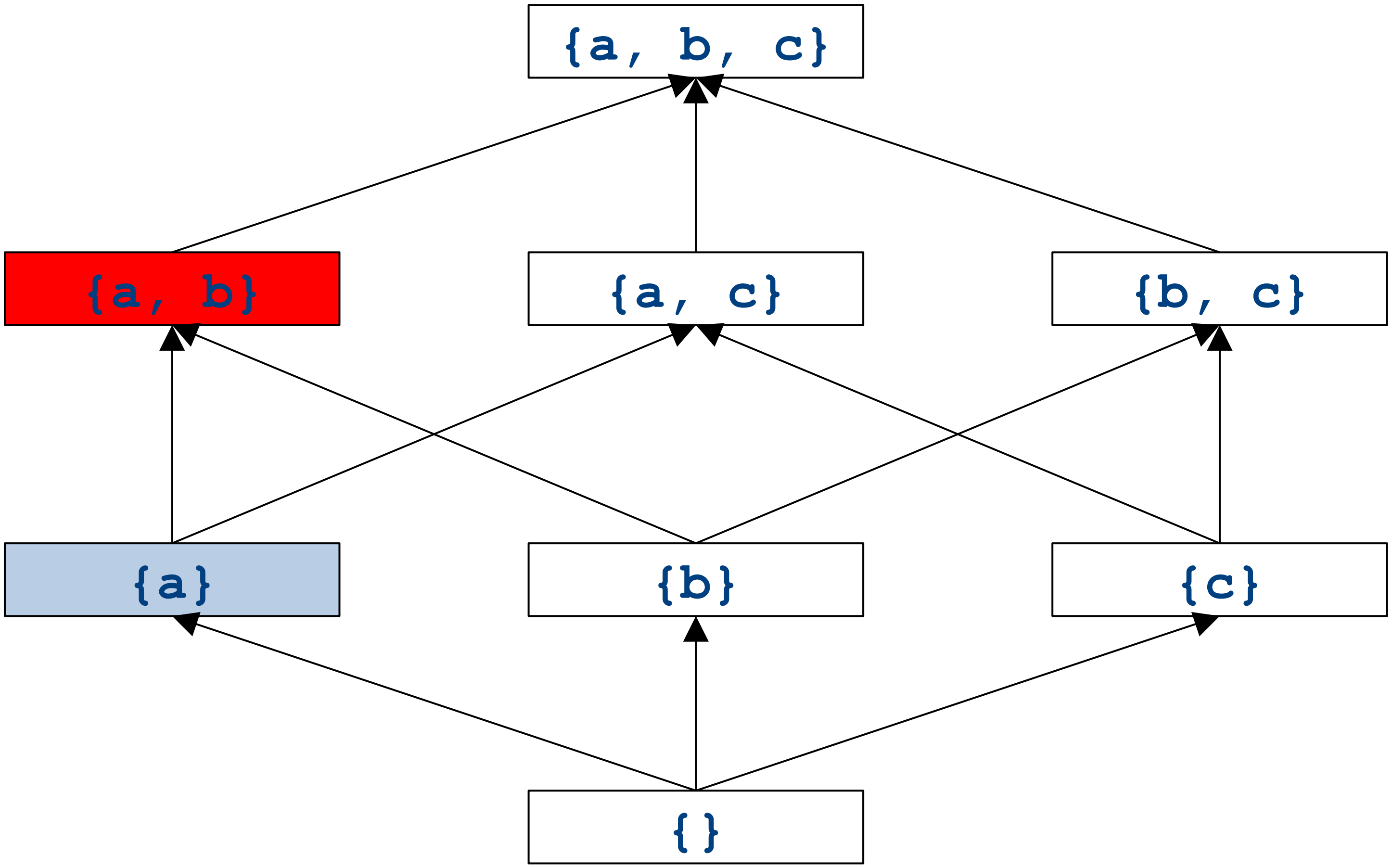
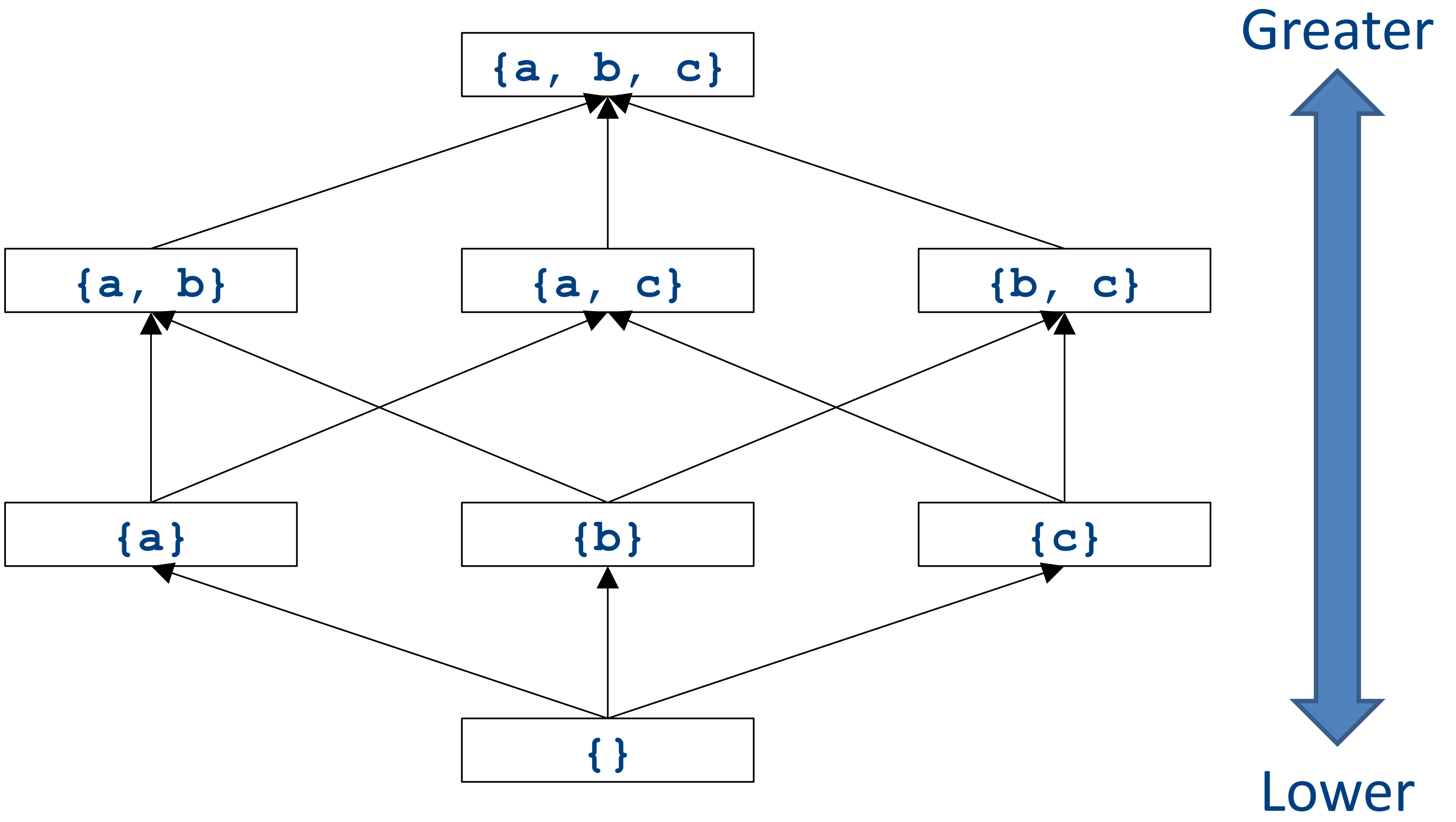


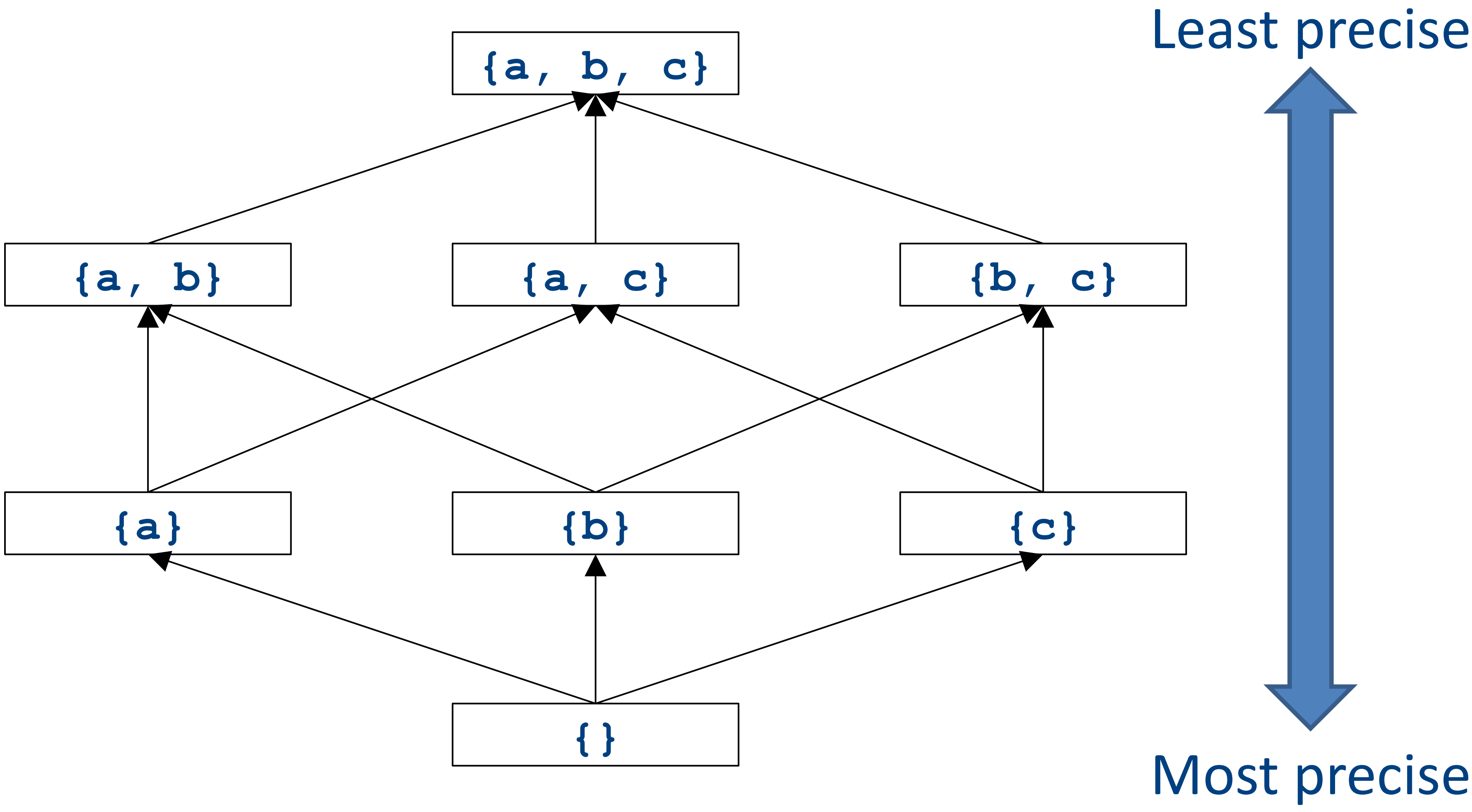
Formal definitions

- A join semilattice is a pair $(V, \sqcup)$, where
- V is a domain of elements
- $\sqcup$ is a join operator that is
  - commutative: $x \sqcup y = y \sqcup x$
  - associative: $(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)$
  - idempotent: $x \sqcup x = x$
- If $x \sqcup y = z$, we say that z is the join or (**l**east **u**pper **b**ound) of x and y
- Every join semilattice has a bottom element denoted $\perp$ such that $\perp \sqcup x = x$ for all x

Join semilattices and ordering



Join semilattices and ordering



Join semilattices and orderings

- Every join semilattice $(V, \sqcup)$ induces an ordering relationship $\sqsubseteq$ over its elements
- Define $x \sqsubseteq y$ iff $x \sqcup y = y$
- Need to prove
  - Reflexivity: $x \sqsubseteq x$
  - Antisymmetry: If $x \sqsubseteq y$ and $y \sqsubseteq x$, then $x = y$
  - Transitivity: If $x \sqsubseteq y$ and $y \sqsubseteq z$, then $x \sqsubseteq z$

An example join semilattice

- The set of natural numbers and the **max** function
- Idempotent
  - **max**{a, a} = a
- Commutative
  - **max**{a, b} = **max**{b, a}
- Associative
  - **max**{a, **max**{b, c}} = **max**{**max**{a, b}, c}
- Bottom element is 0:
  - **max**{0, a} = a
- What is the ordering over these elements?

A join semilattice for liveness

- Sets of live variables and the set union operation
- Idempotent:
  - $x \cup x = x$
- Commutative:
  - $x \cup y = y \cup x$
- Associative:
  - $(x \cup y) \cup z = x \cup (y \cup z)$
- Bottom element:
  - The empty set: $\emptyset \cup x = x$
- What is the ordering over these elements?

Semilattices and program analysis

- Semilattices naturally solve many of the problems we encounter in global analysis
- How do we combine information from multiple basic blocks?
- What value do we give to basic blocks we haven't seen yet?
- How do we know that the algorithm always terminates?

Semilattices and program analysis

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  - Actually, we still don't! More on that later

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A general framework

- A global analysis is a tuple $(\mathbb{D}, \mathbb{V}, \sqsubseteq, \mathbb{F}, \mathbb{I})$, where
  - $\mathbb{D}$ is a direction (forward or backward)
    - The order to visit statements within a basic block, not the order in which to visit the basic blocks
  - $\mathbb{V}$ is a set of values
  - $\sqcup$ is a join operator over those values
  - $\mathbb{F}$ is a set of transfer functions f : **V → V**
  - $\mathbb{I}$ is an initial value
- The only difference from local analysis is the introduction of the join operator

Running global analyses

- Assume that $(\mathbb{D}, \mathbb{V}, \sqcup, \mathbb{F}, \mathbb{I})$ is a forward analysis
- Set OUT[**s**] = $\perp$ for all statements **s**
- Set OUT[**entry**] = $\mathbb{I}$
- Repeat until no values change:
  - For each statement **s** with predecessors $\mathbf{p_1}$, $\mathbf{p_2}$, … , $\mathbf{p_n}$:
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] $\sqcup$ OUT[$\mathbf{p_2}$] $\sqcup$ … $\sqcup$ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s (IN[**s**])
- The order of this iteration does not matter
  - This is sometimes called chaotic iteration

For comparison

- Set OUT[**s**] = $\perp$ for all statements **s**
- Set OUT[**entry**] = **I**
- Repeat until no values change:
  - For each statement **s** with predecessors $\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_n}$:
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] $\sqcup$ OUT[$\mathbf{p_2}$] $\sqcup$ … $\sqcup$ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s (IN[**s**])

- Set IN[**s**] = {} for all statements **s**
- Set OUT[**exit**] = the set of variables known to be live on exit
- Repeat until no values change:
  - For each statement **s** of the form **a=b+c**:
    - Set OUT[**s**] = set union of IN[**x**] for each successor **x** of **s**
    - Set IN[**s**] = (OUT[**s**]-{a}) $\cup$ {b,c}

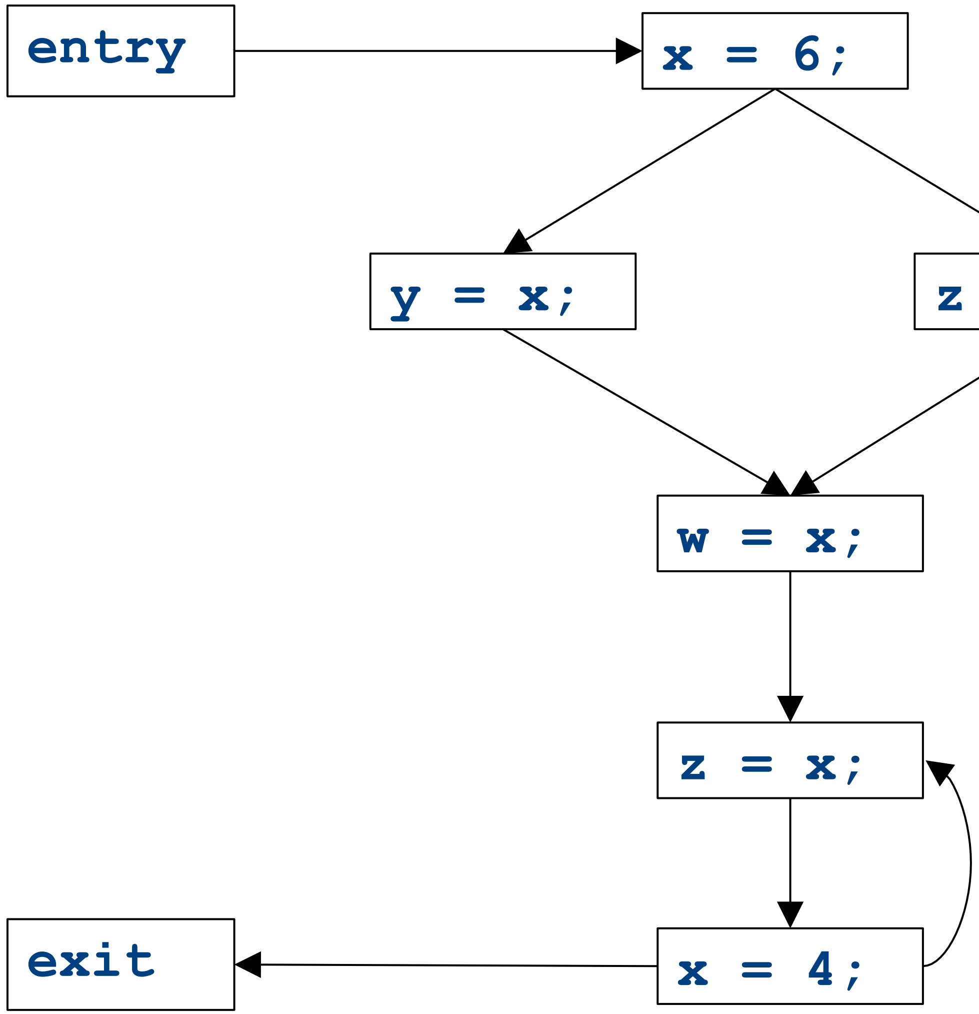
The dataflow framework

- This form of analysis is called the dataflow framework
- Can be used to easily prove an analysis is sound
- With certain restrictions, can be used to prove that an analysis eventually terminates
  - Again, more on that later

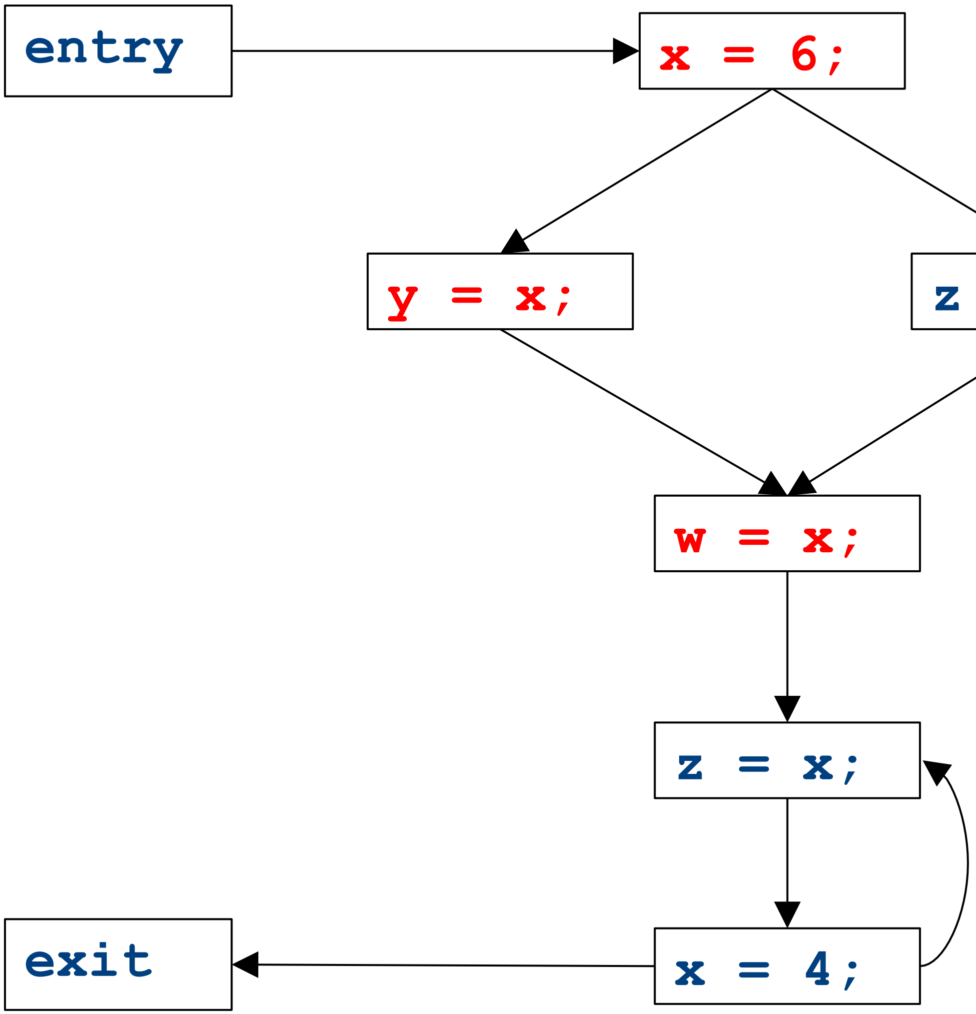
Global constant propagation

- Constant propagation is an optimization that replaces each variable that is known to be a constant value with that constant
- An elegant example of the dataflow framework

Global constant propagation



Global constant propagation



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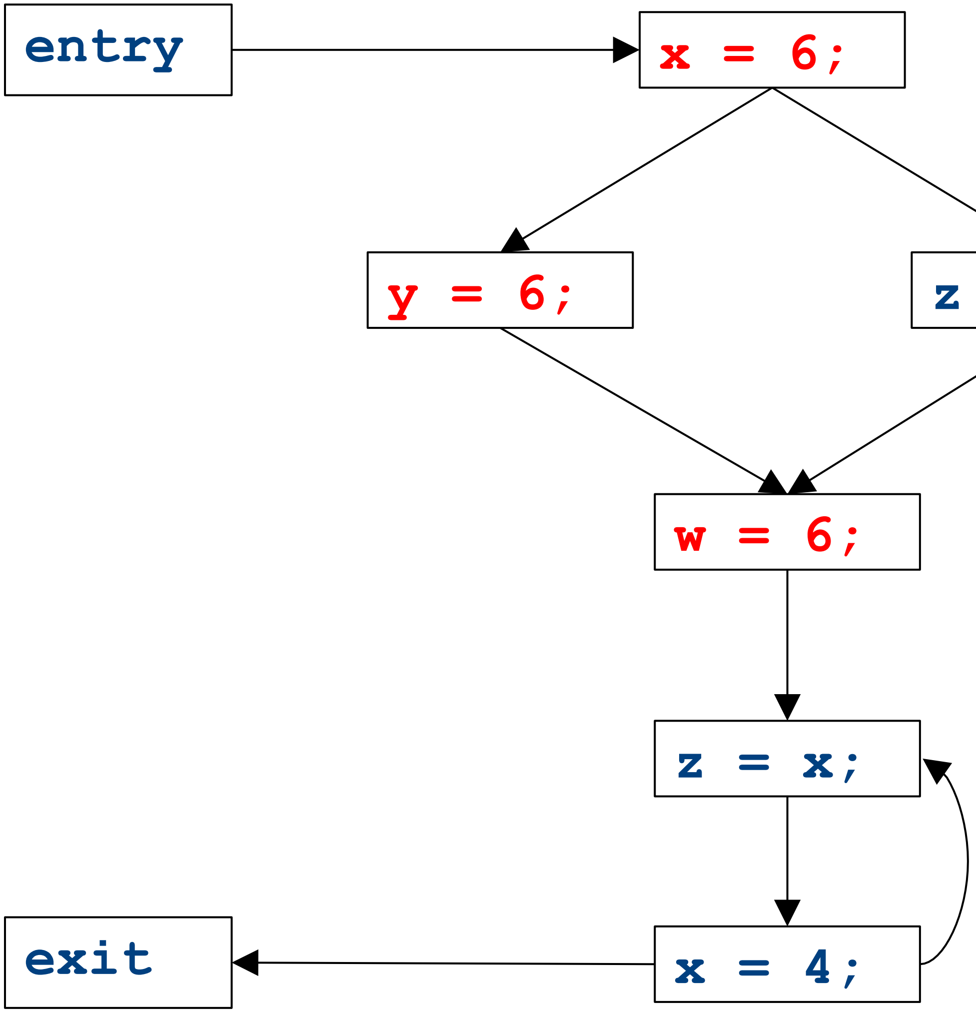


Constant propagation analysis

- In order to do a constant propagation, we need to track what values might be assigned to a variable at each program point
- Every variable will either
  - Never have a value assigned to it,
  - Have a single constant value assigned to it,
  - Have two or more constant values assigned to it, or
  - Have a known non-constant value.
  - Our analysis will propagate this information throughout a CFG to identify locations where a value is constant

Properties of constant propagation

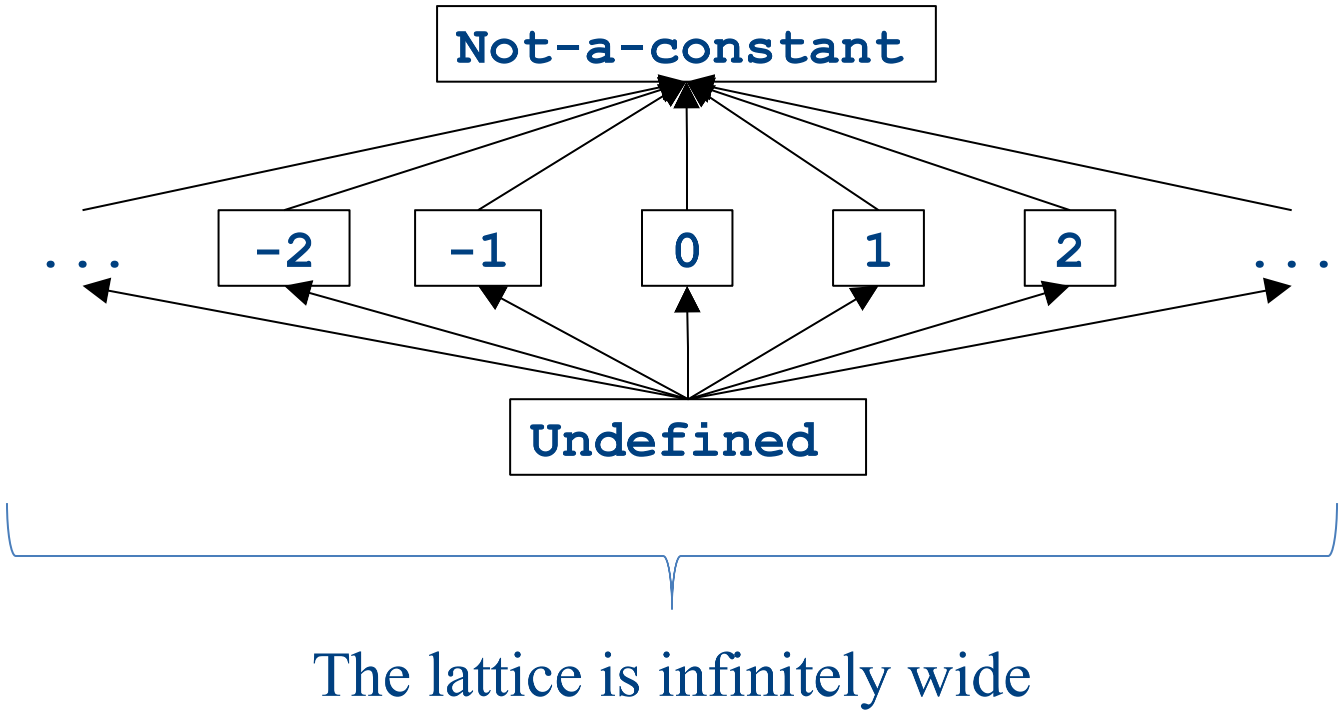
- For now, consider just some single variable **x**
- At each point in the program, we know one of three things about the value of **x**:
  - **x** is definitely not a constant, since it's been assigned two values or assigned a value that we know isn't a constant
  - **x** is definitely a constant and has value **k**
  - We have never seen a value for **x**
- Note that the first and last of these are **not** the same!
  - The first one means that there may be a way for **x** to have multiple values
  - The last one means that **x** never had a value at all

Defining a join operator

- The join of any two different constants is **Not-a-Constant**
  - (If the variable might have two different values on entry to a statement, it cannot be a constant)
- The join of **Not a Constant** and any other value is **Not-a-Constant**
  - (If on some path the value is known not to be a constant, then on entry to a statement its value can't possibly be a constant)
- The join of **Undefined** and any other value is that other value
  - (If **x** has no value on some path and does have a value on some other path, we can just pretend it always had the assigned value)

A semilattice for constant propagation

- One possible semilattice for this analysis is shown here (for each variable):



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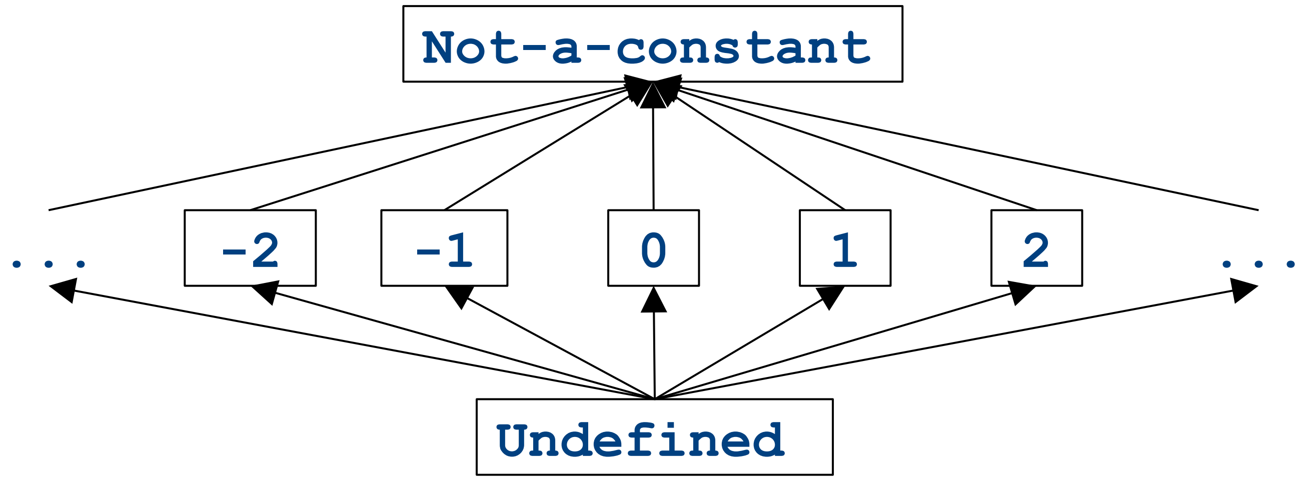
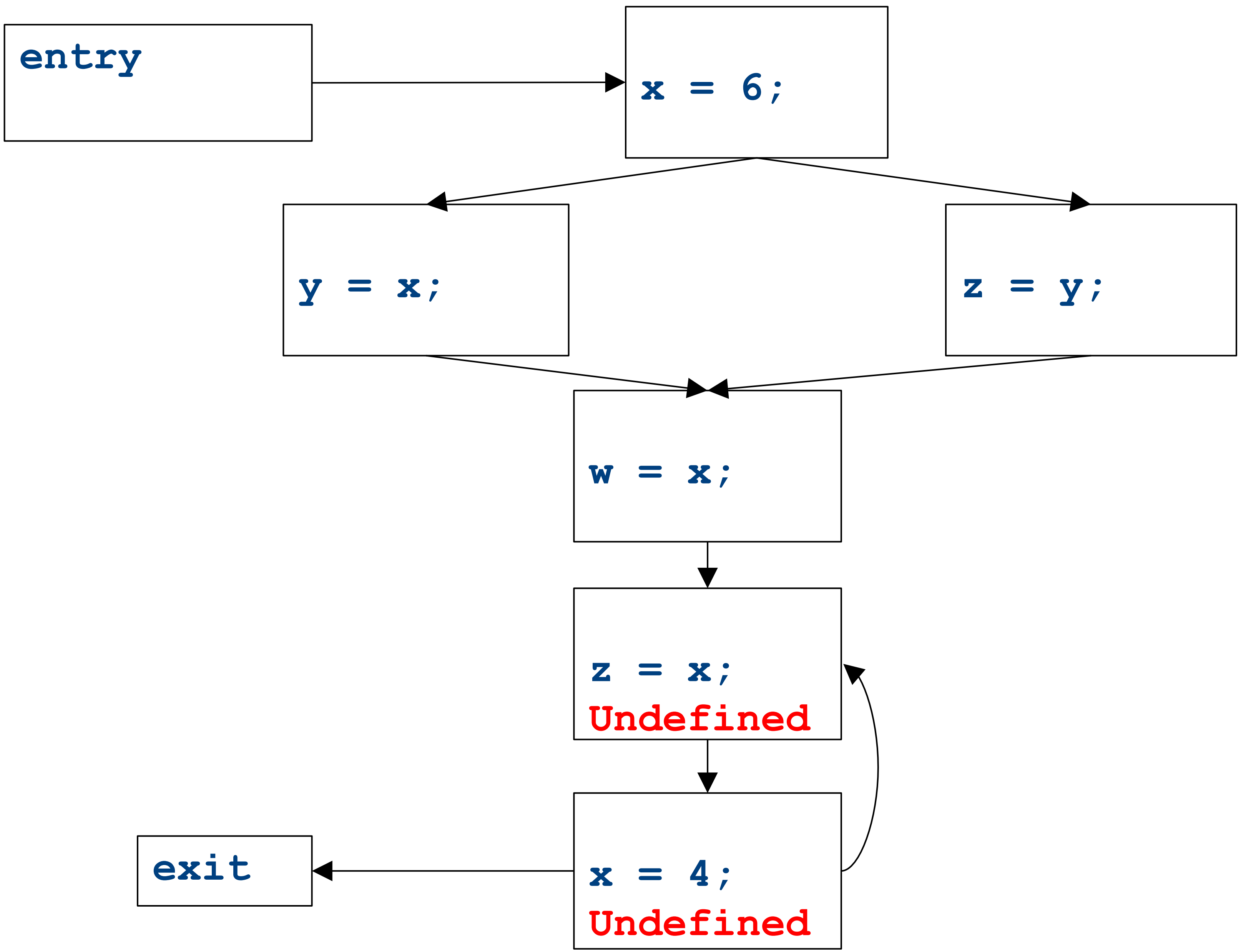


- Note:
  - The join of any two different constants is **Not-a-Constant**
  - The join of **Not a Constant** and any other value is **Not-a-Constant**
  - The join of **Undefined** and any other value is that other value

Global constant propagation



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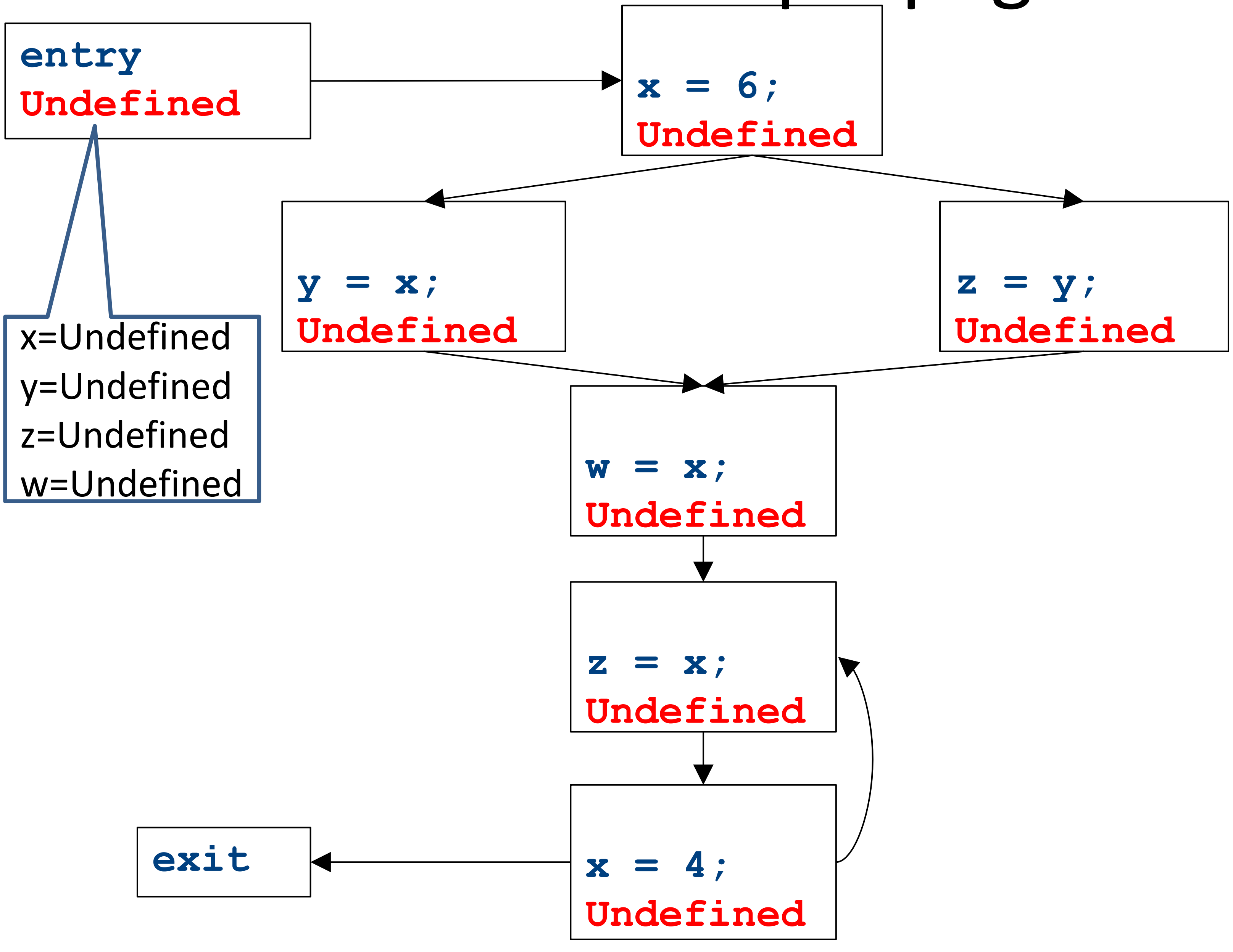

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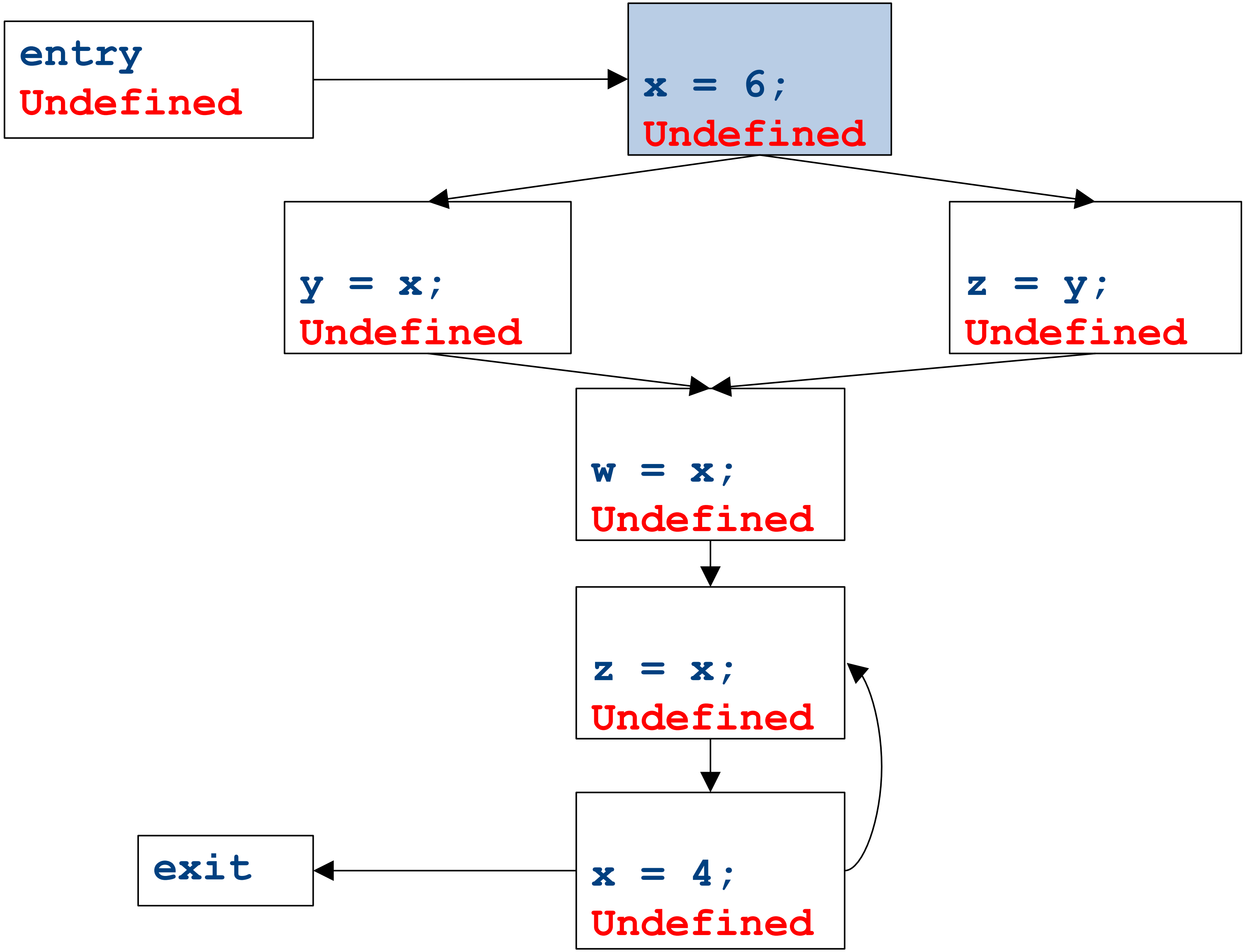

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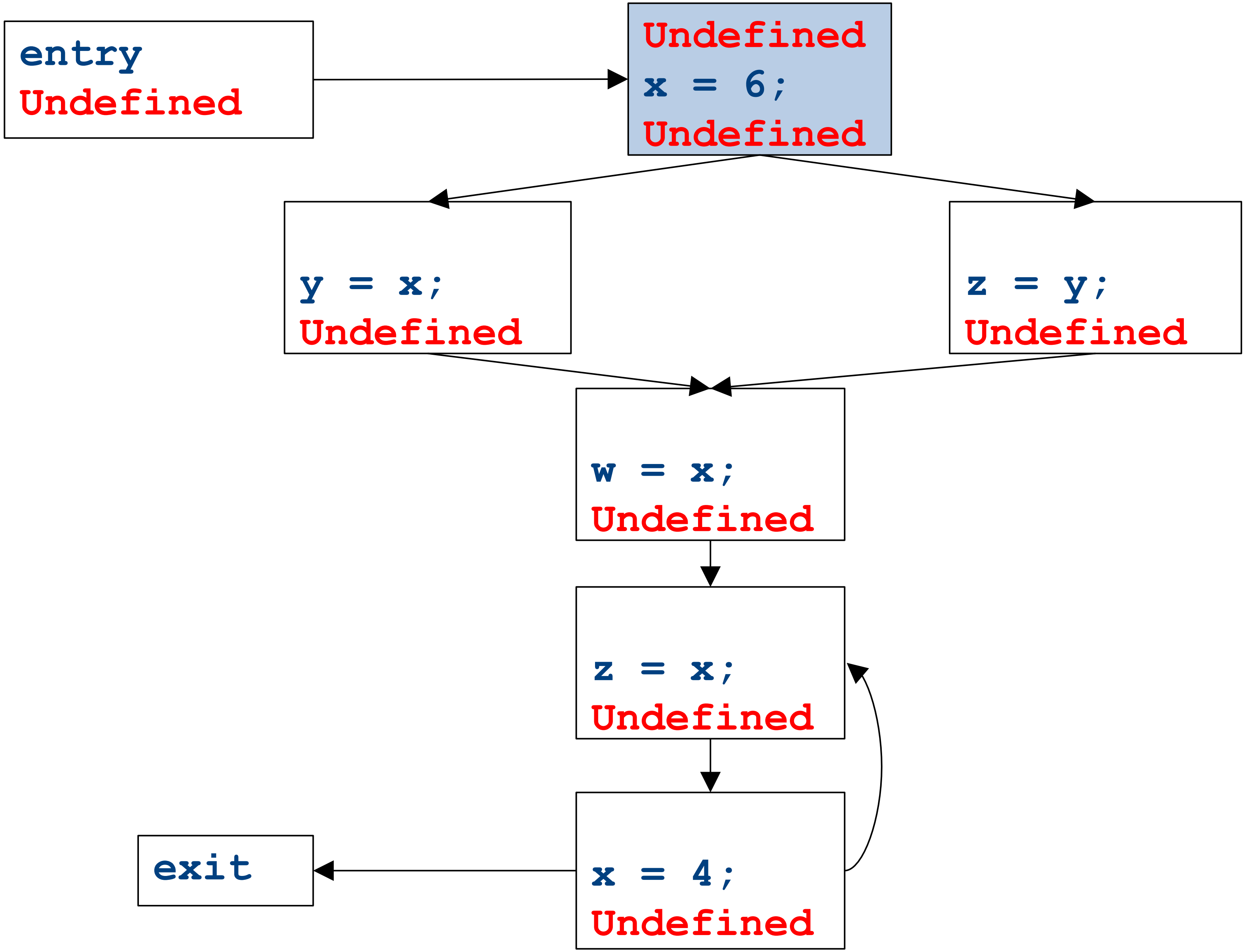

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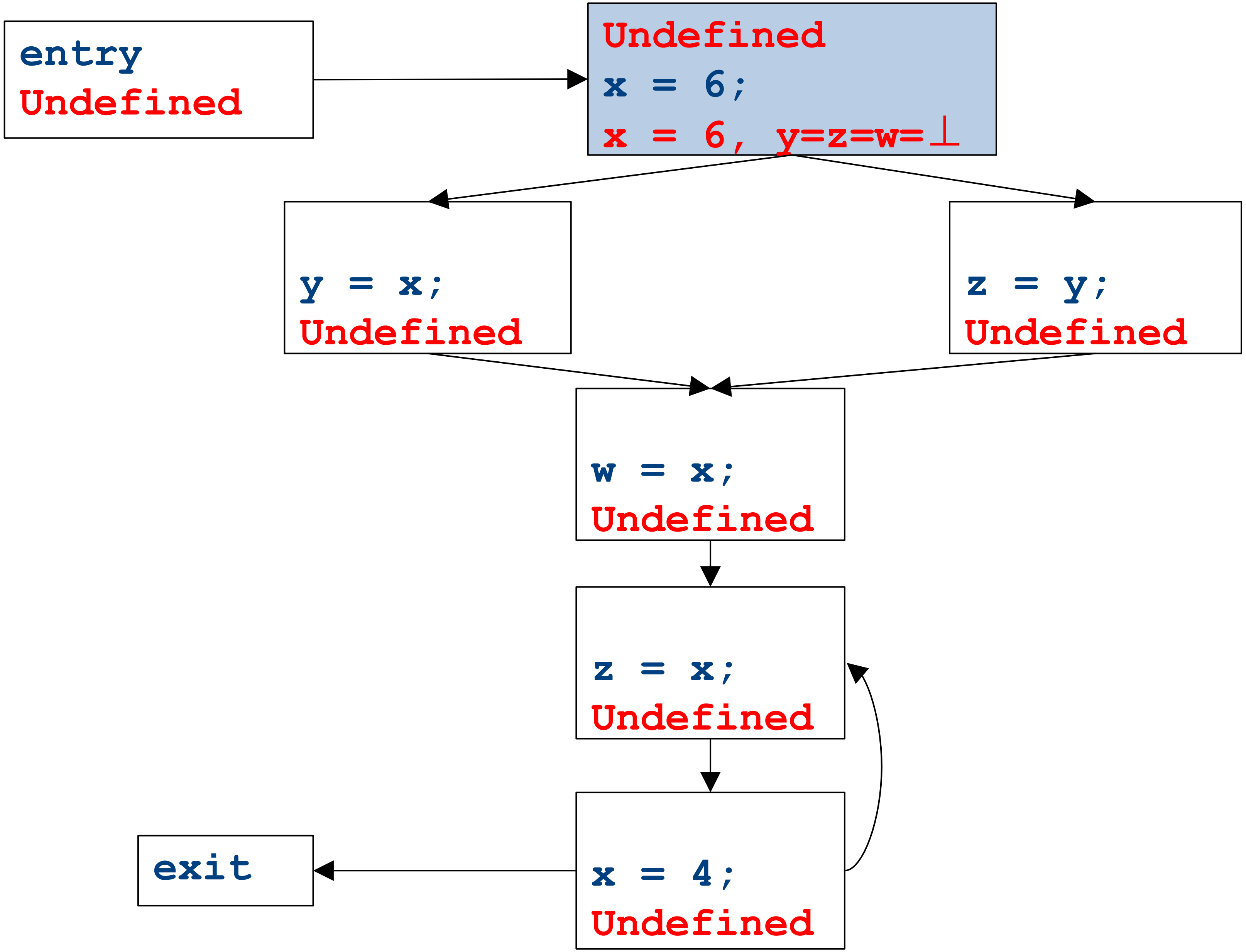

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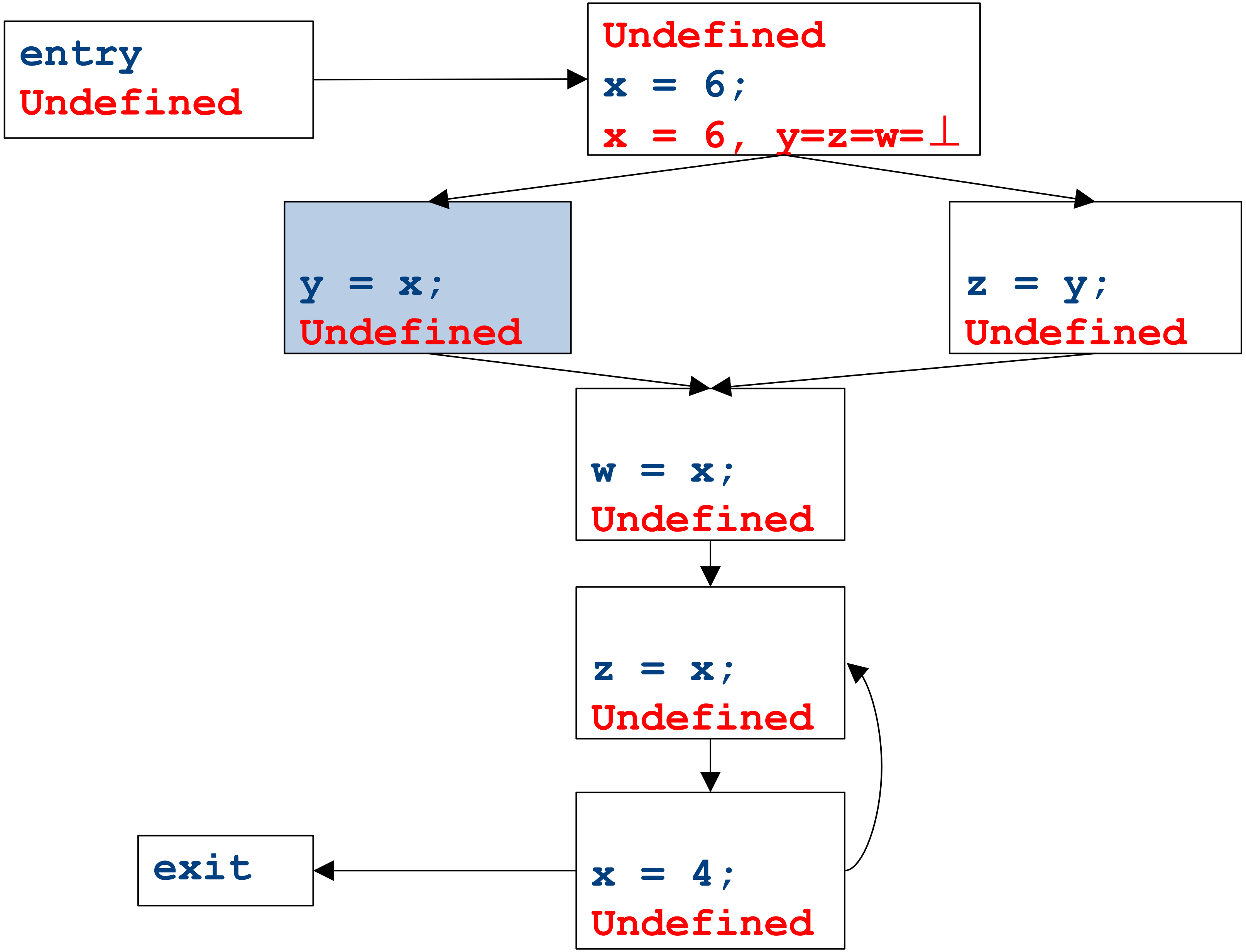

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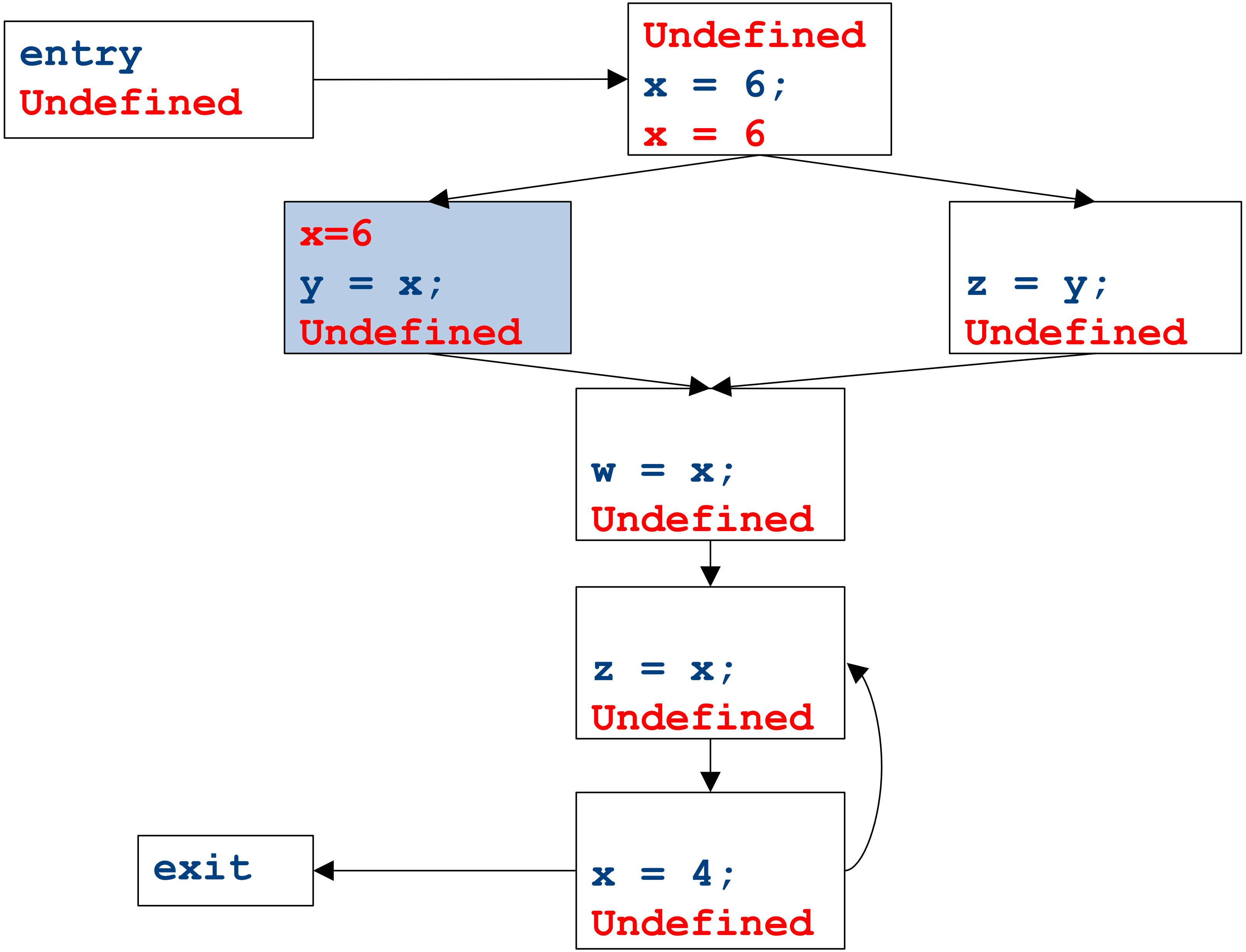

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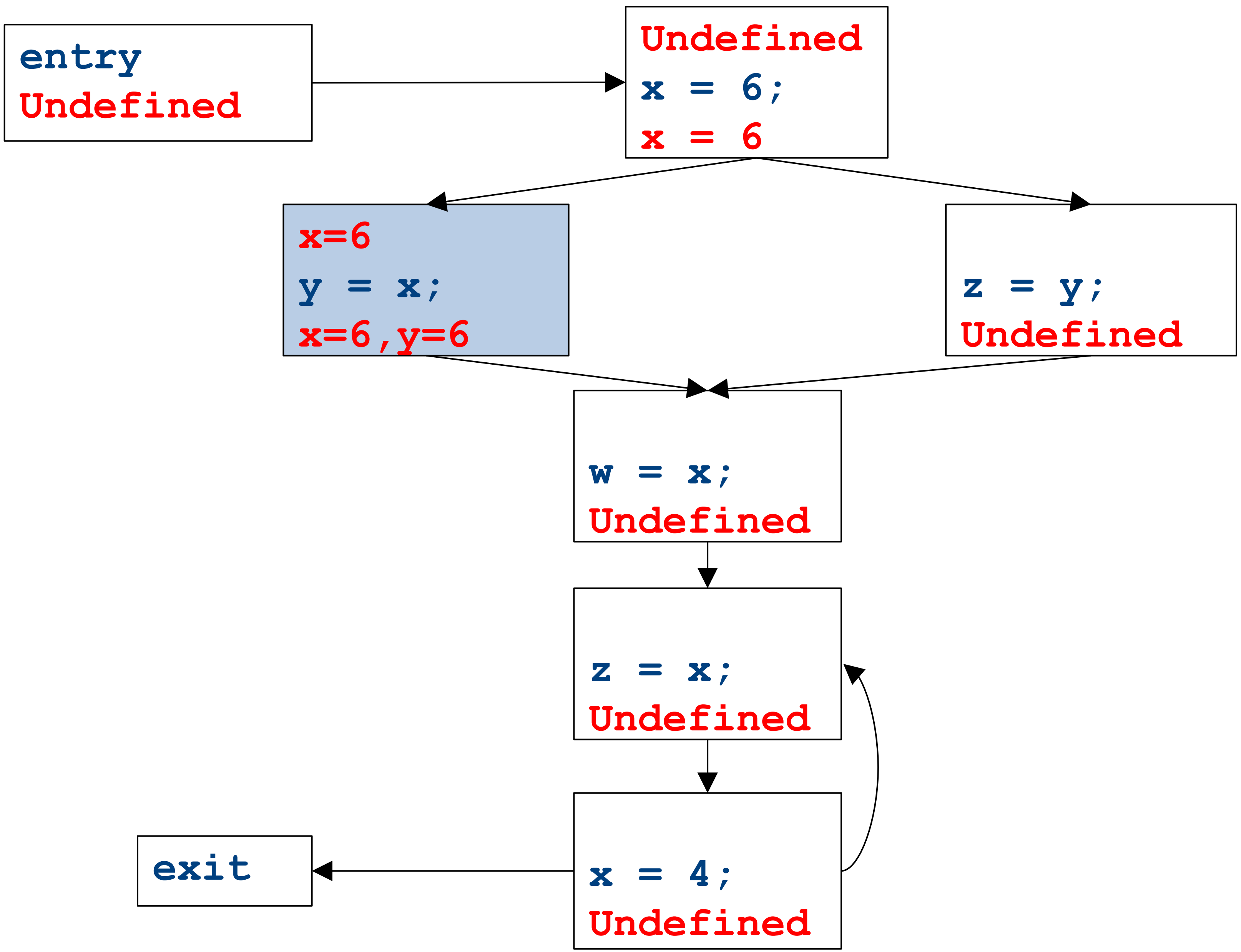

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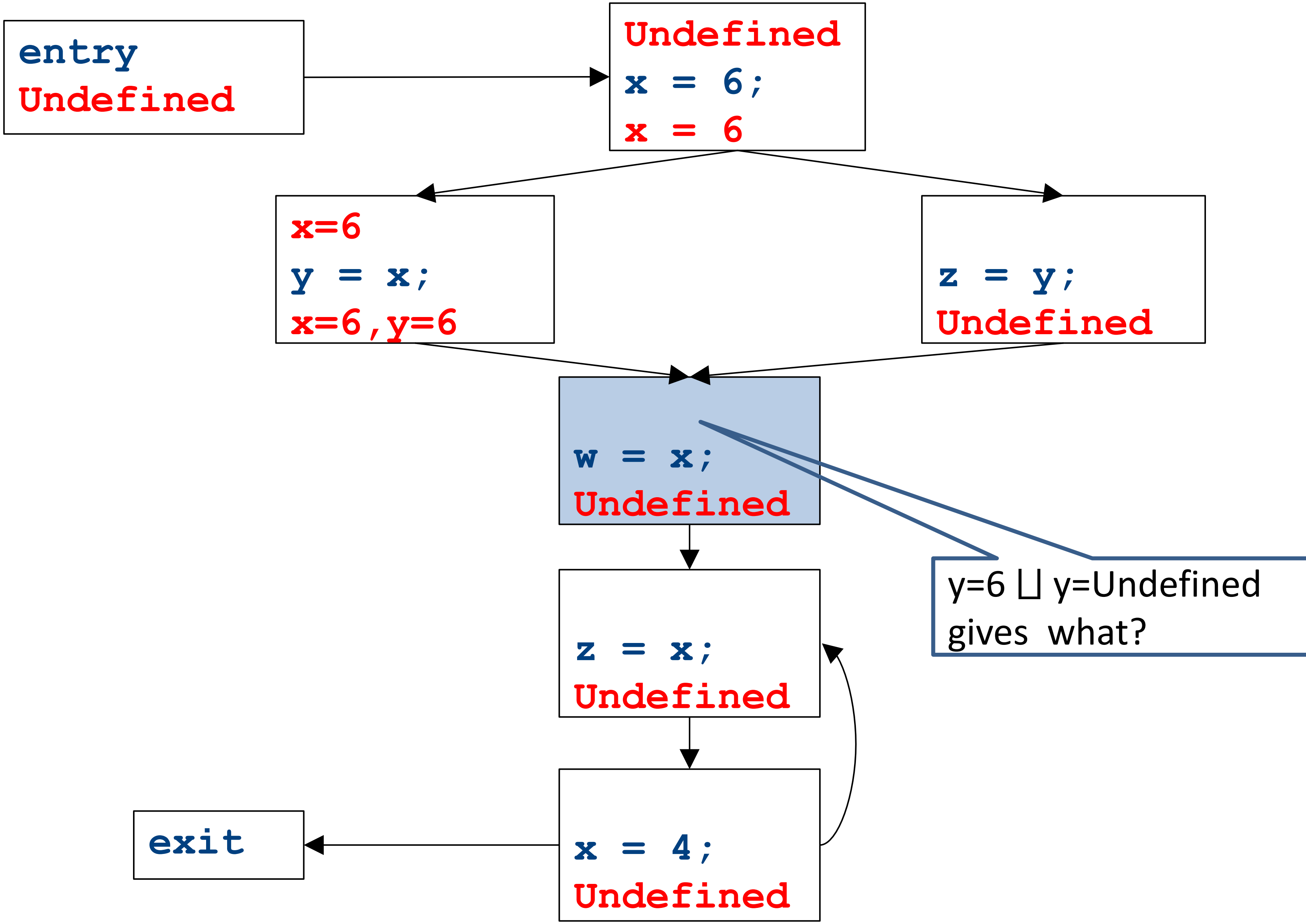

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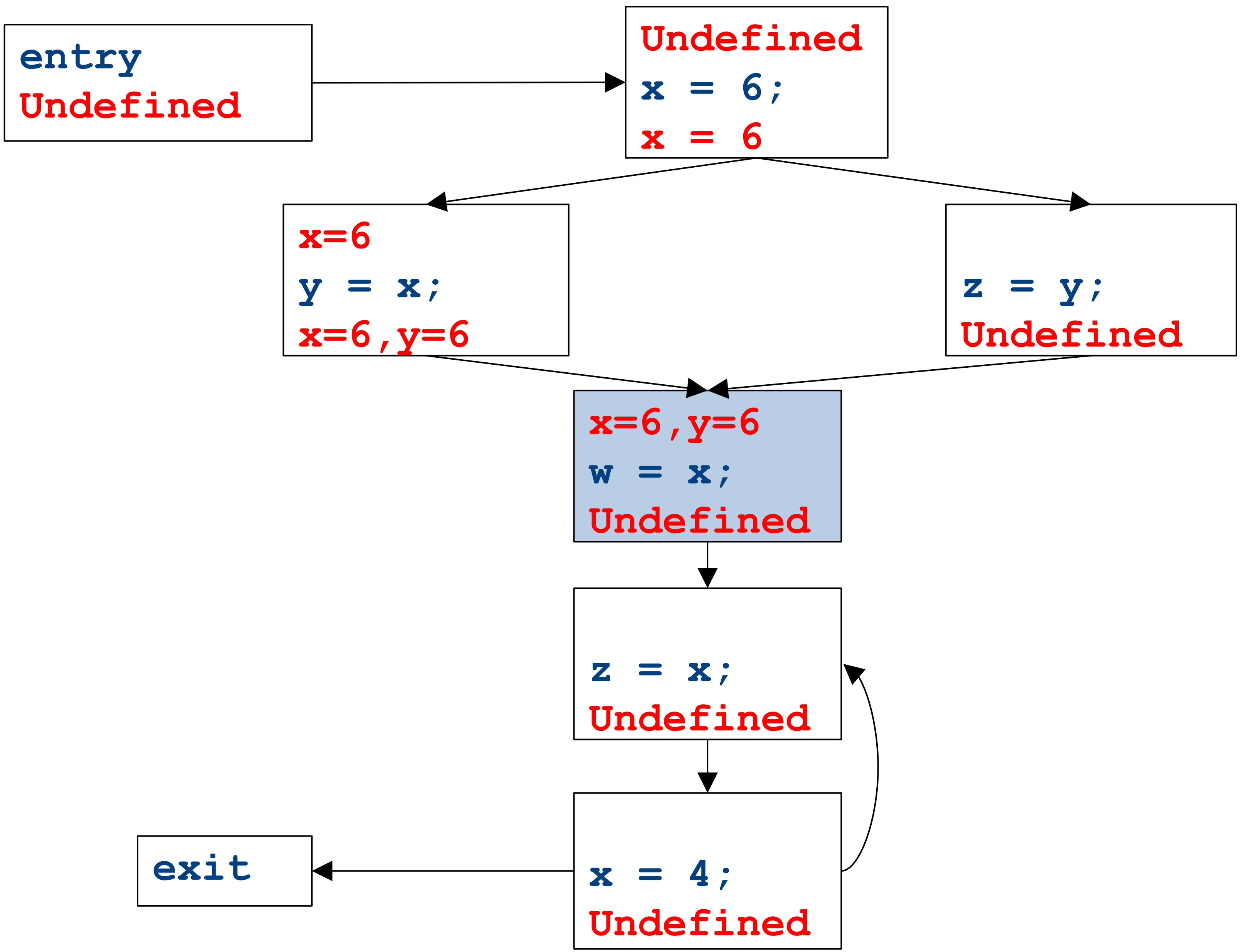

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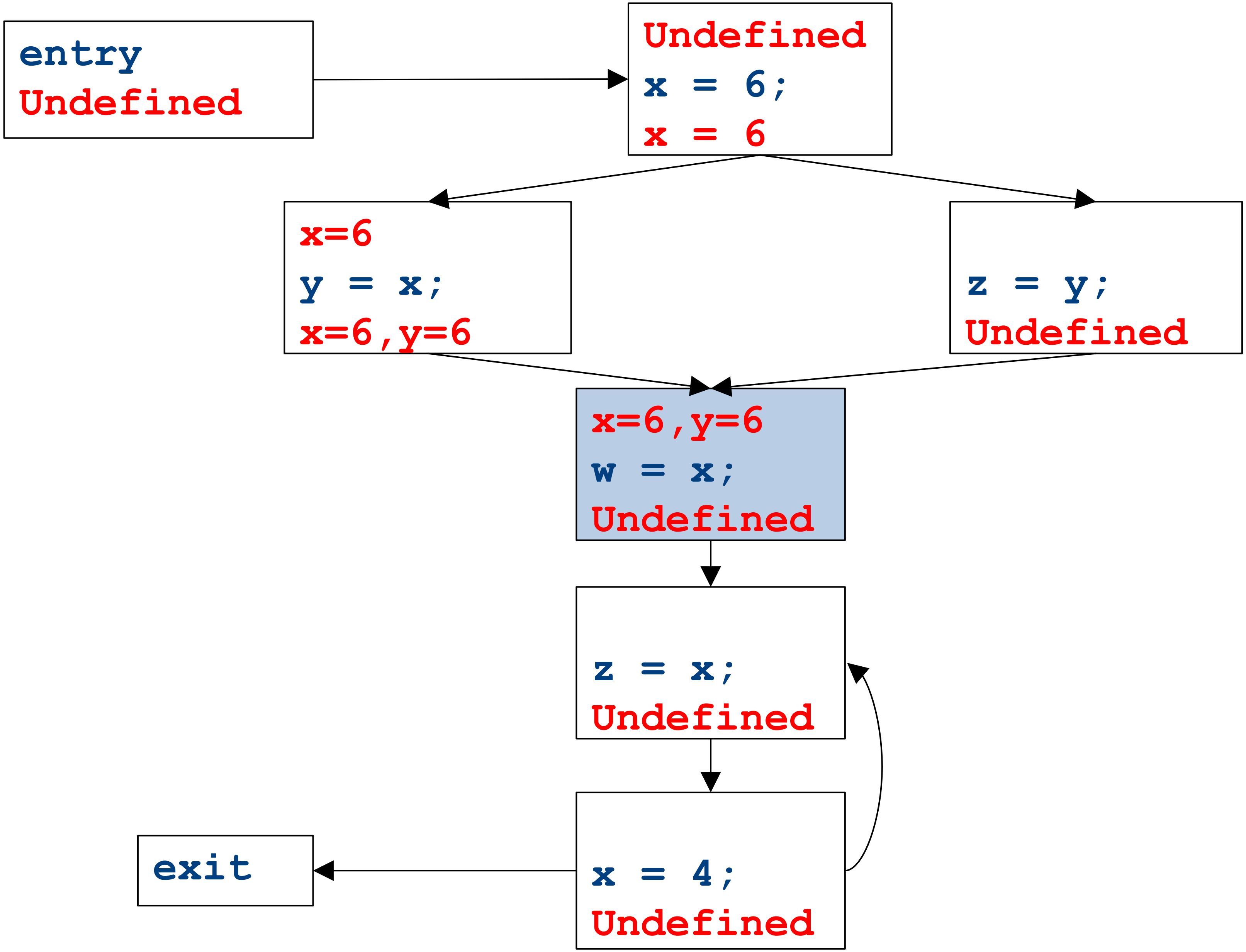

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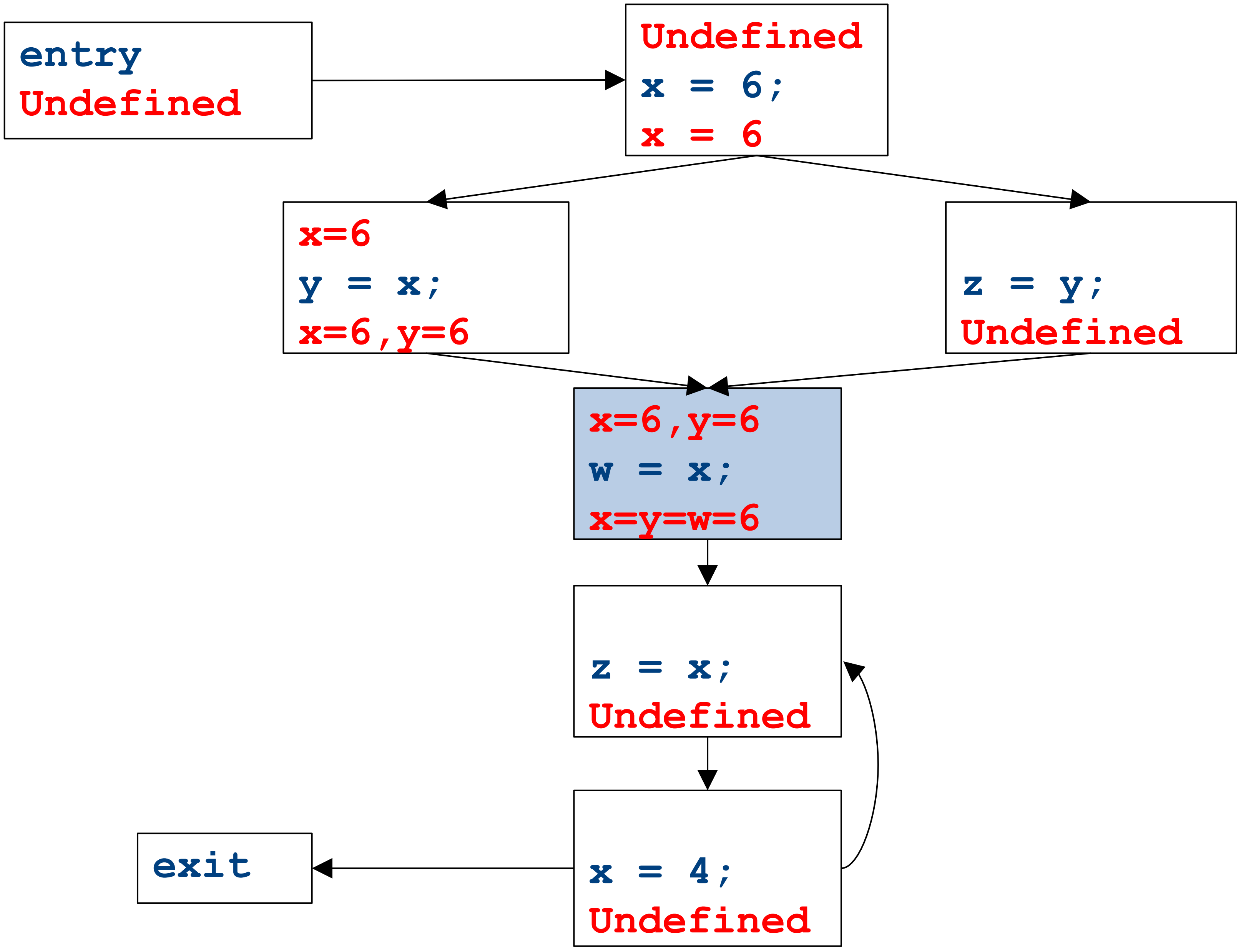

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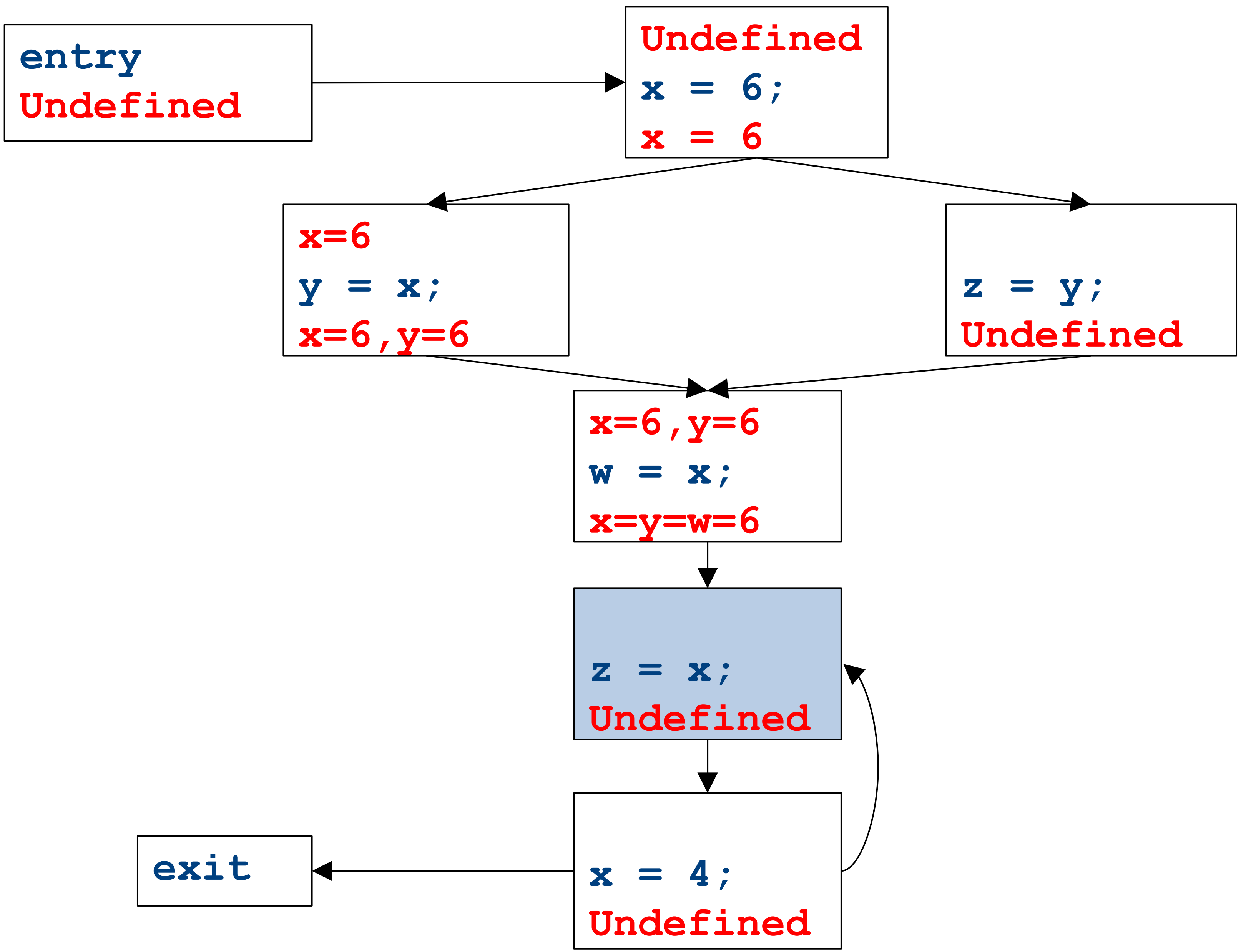

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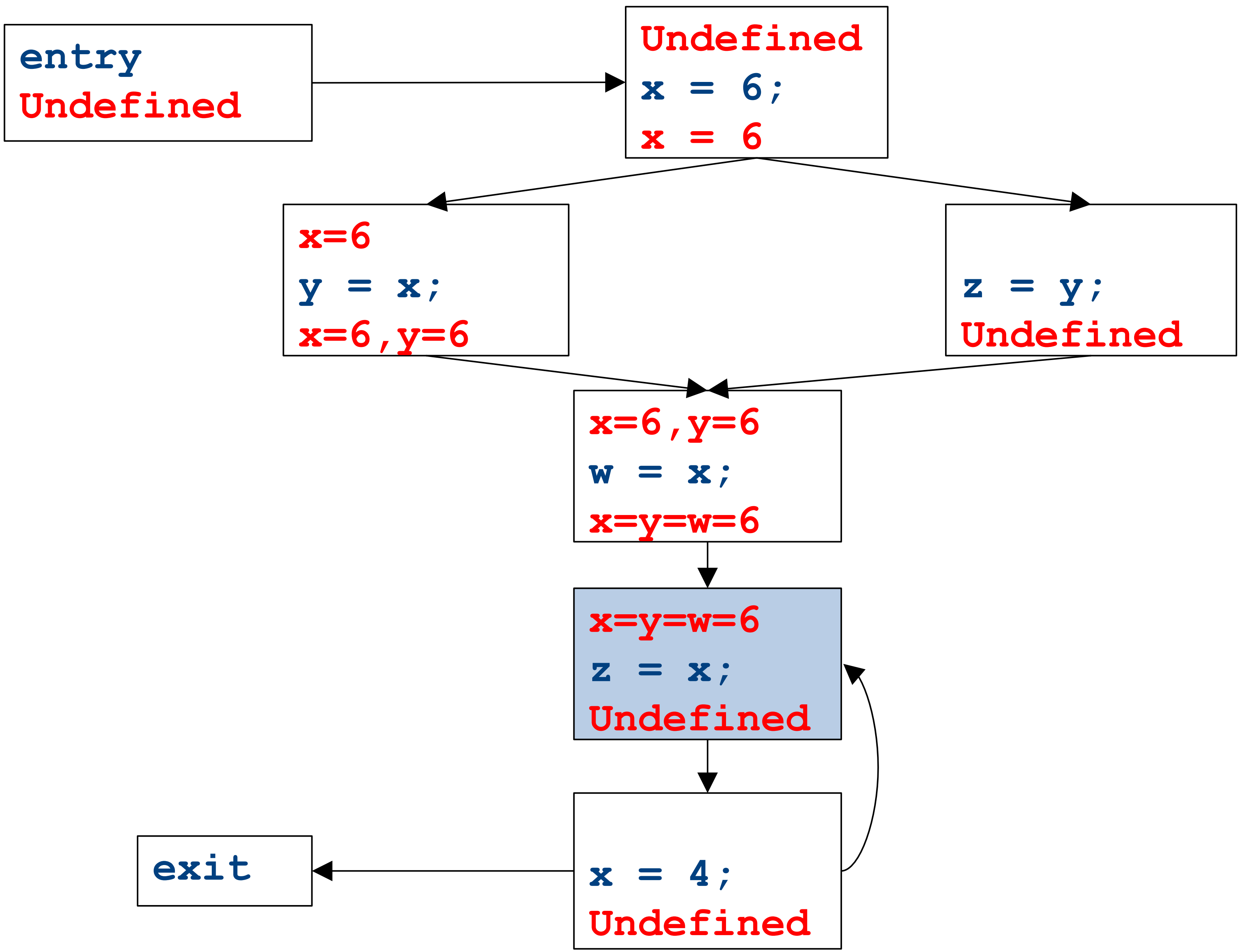

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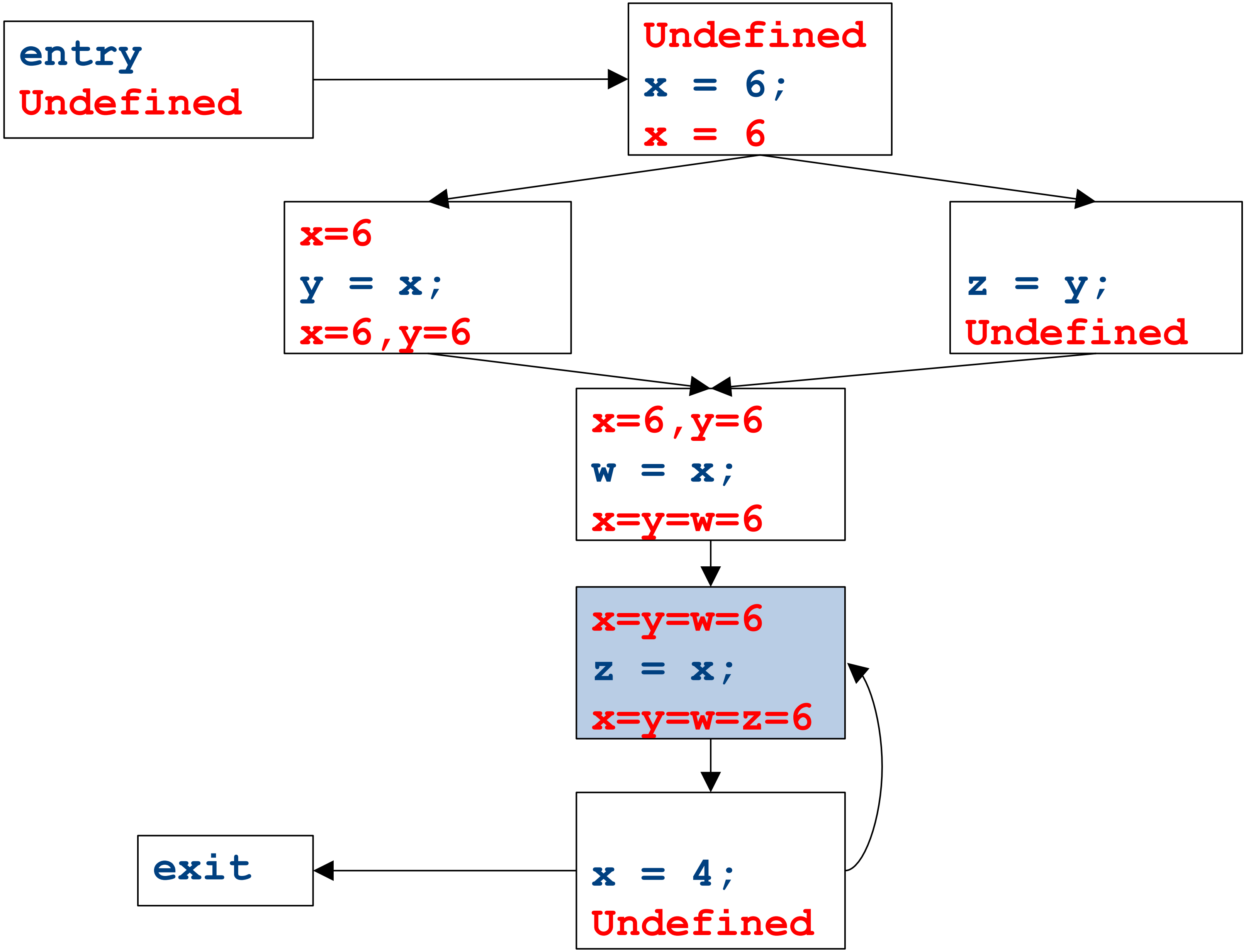

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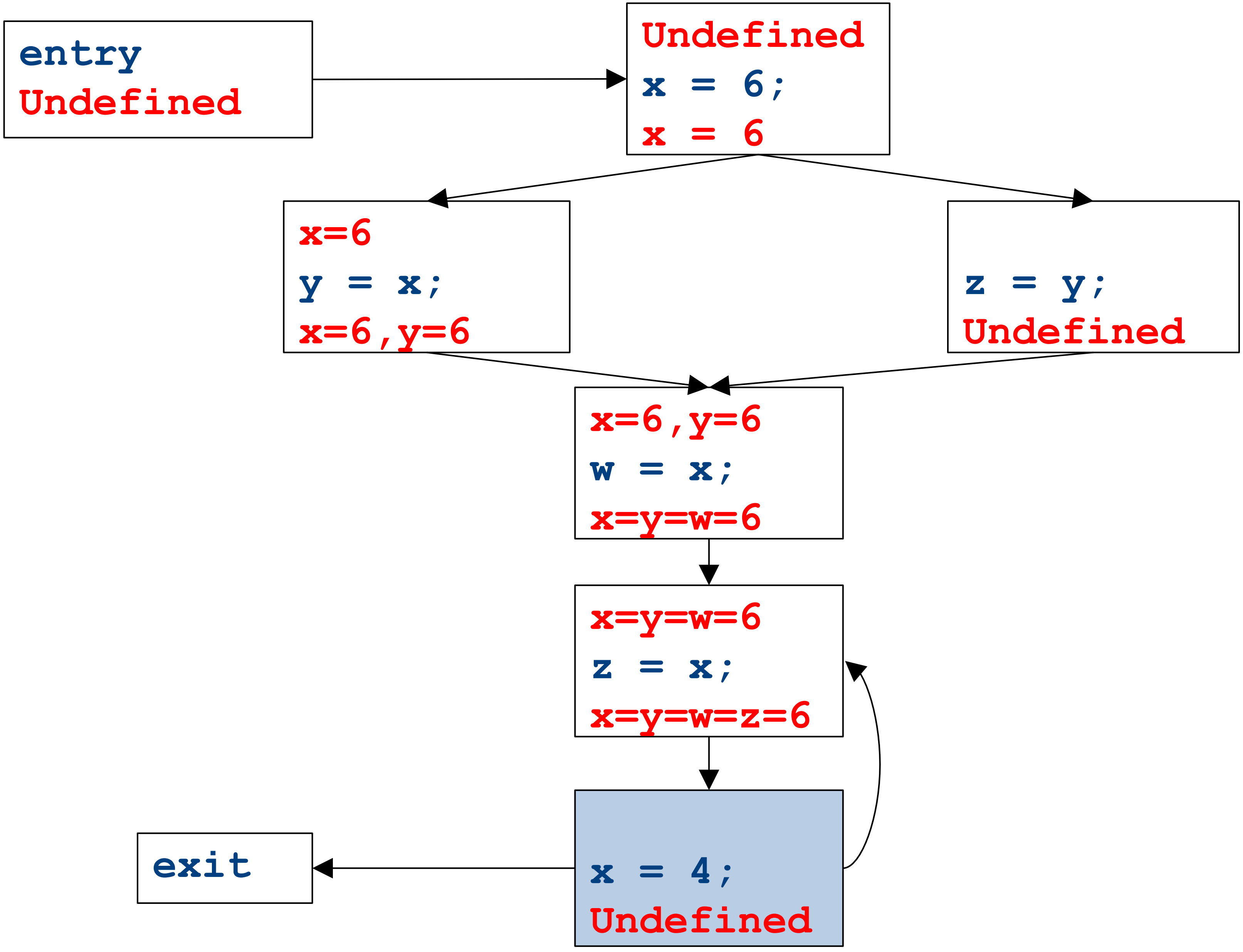

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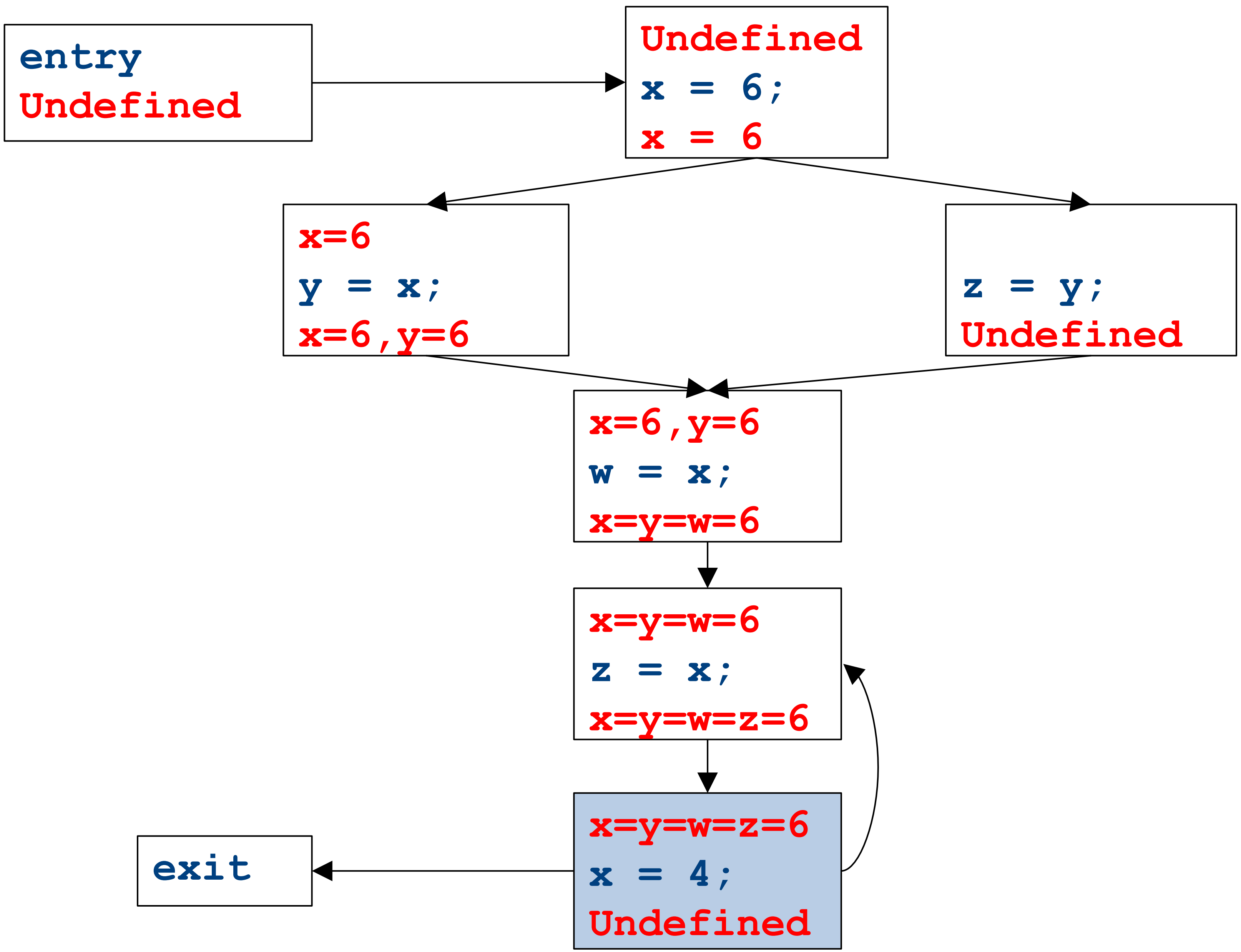

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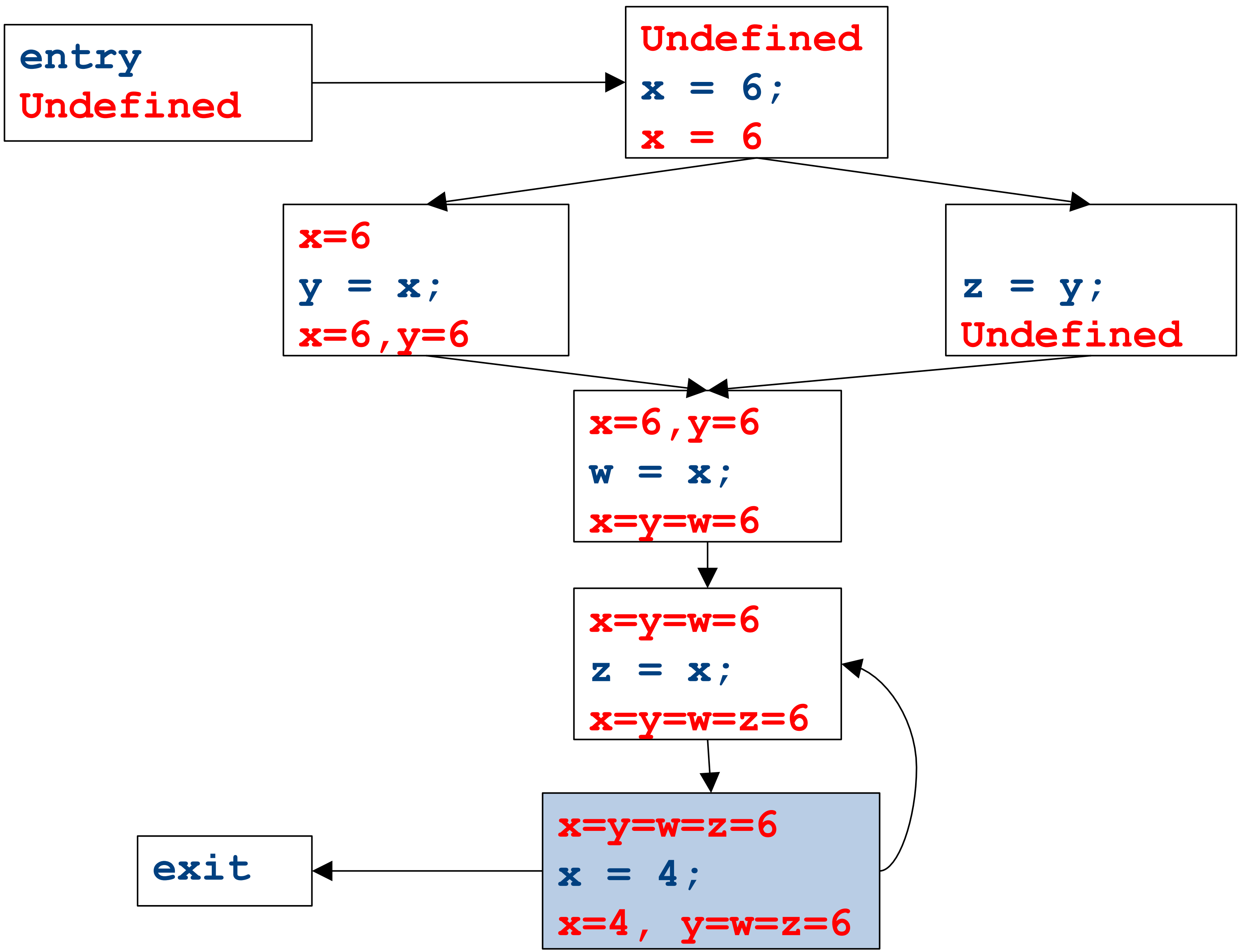

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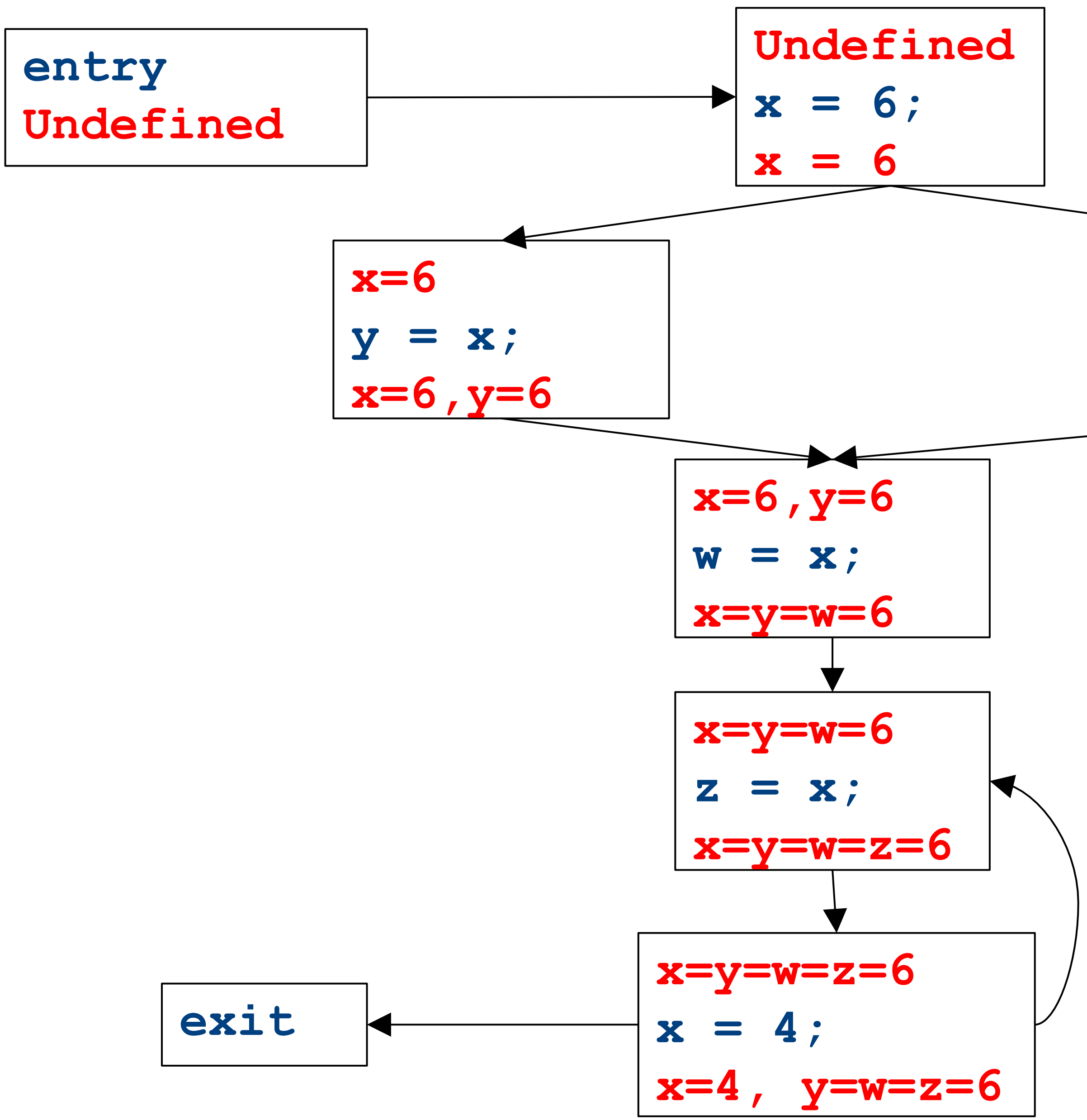

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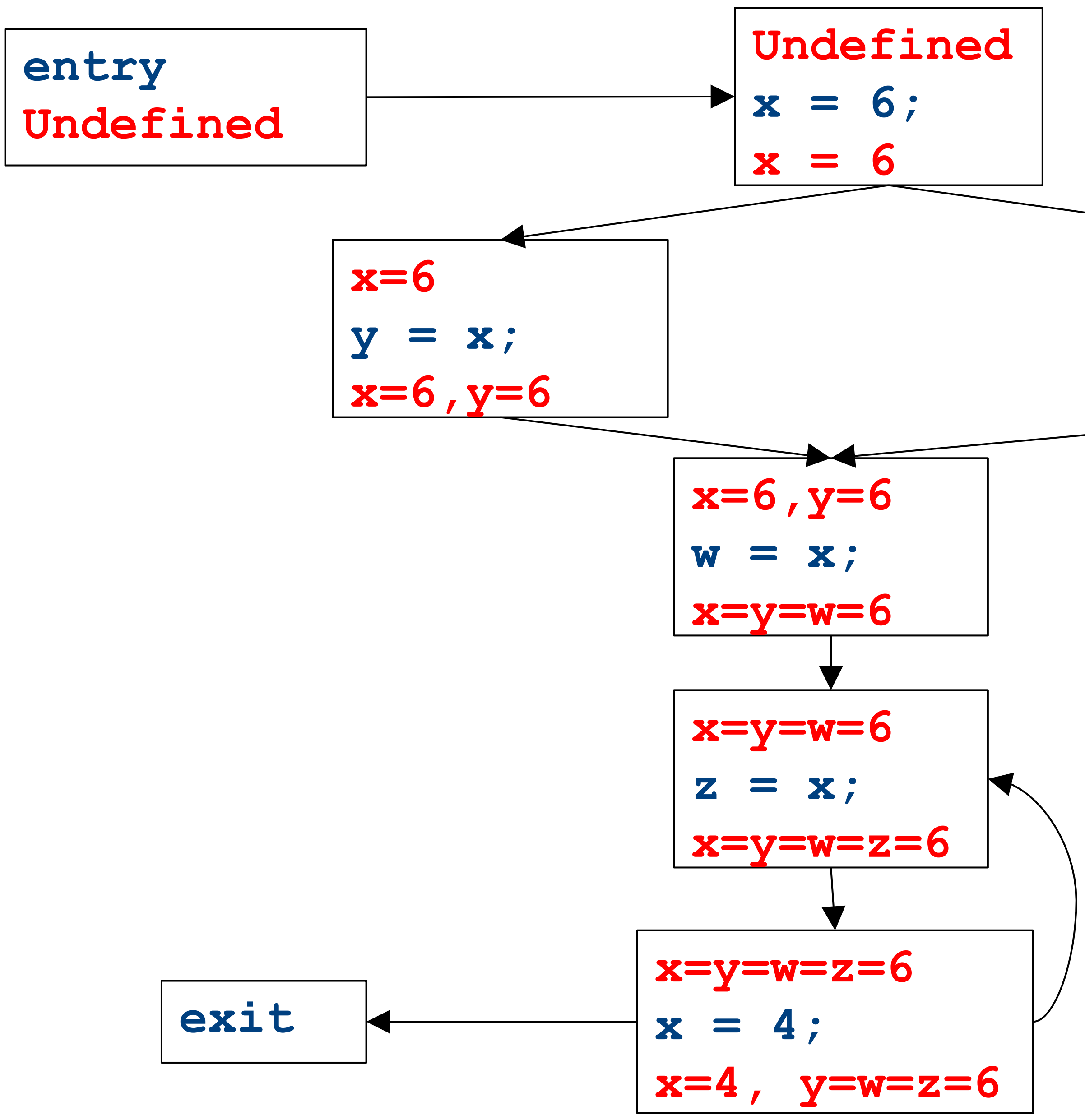

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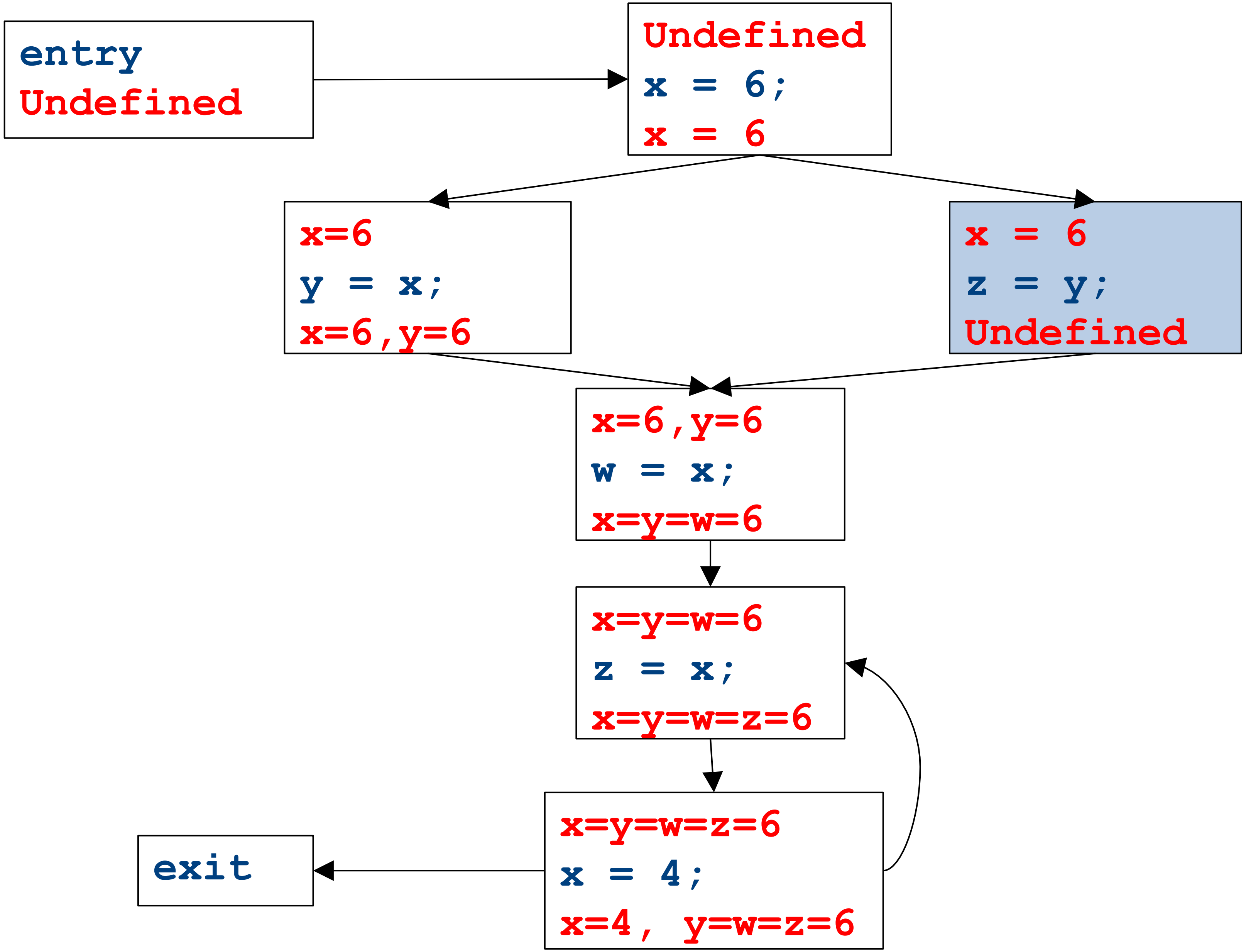

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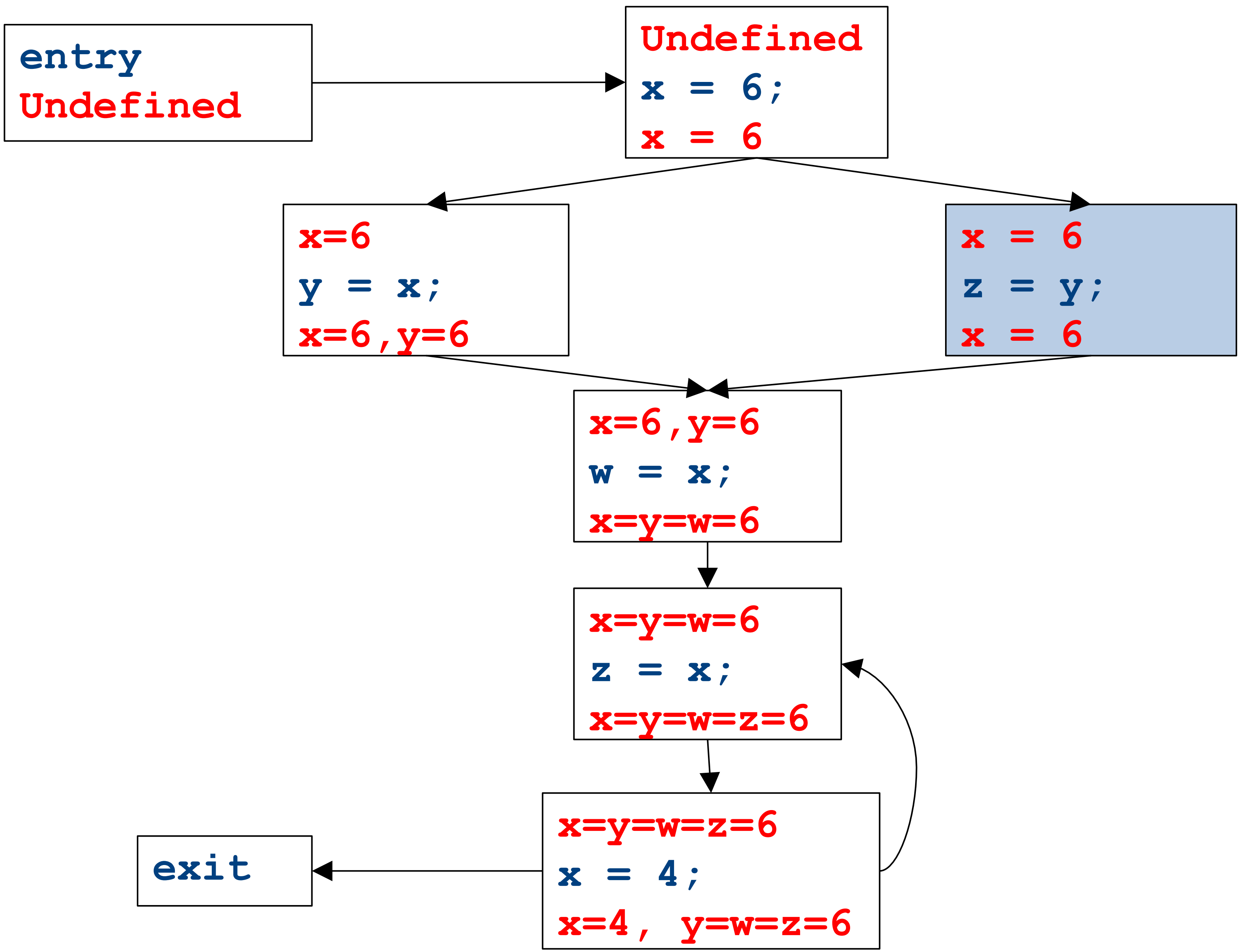

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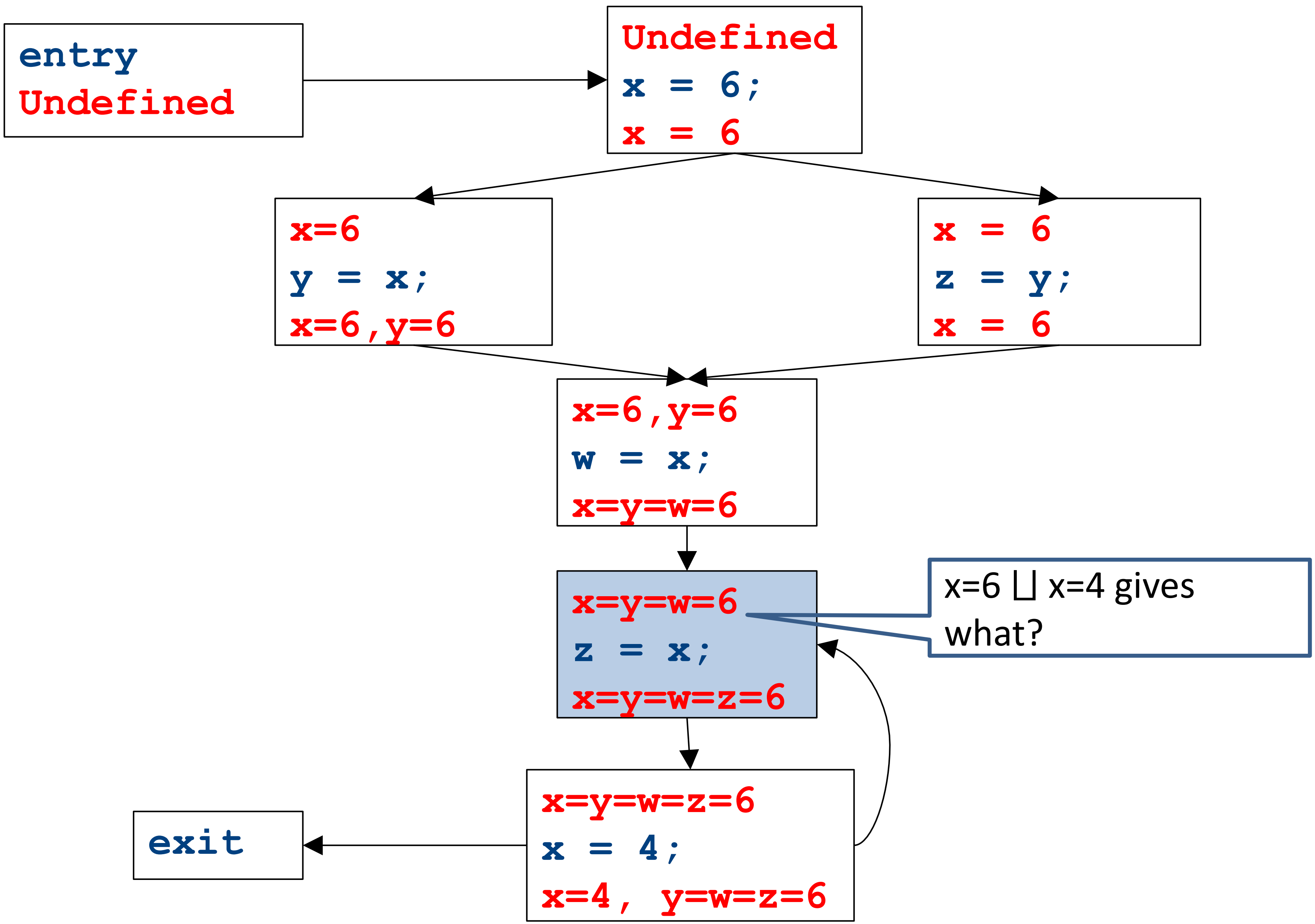

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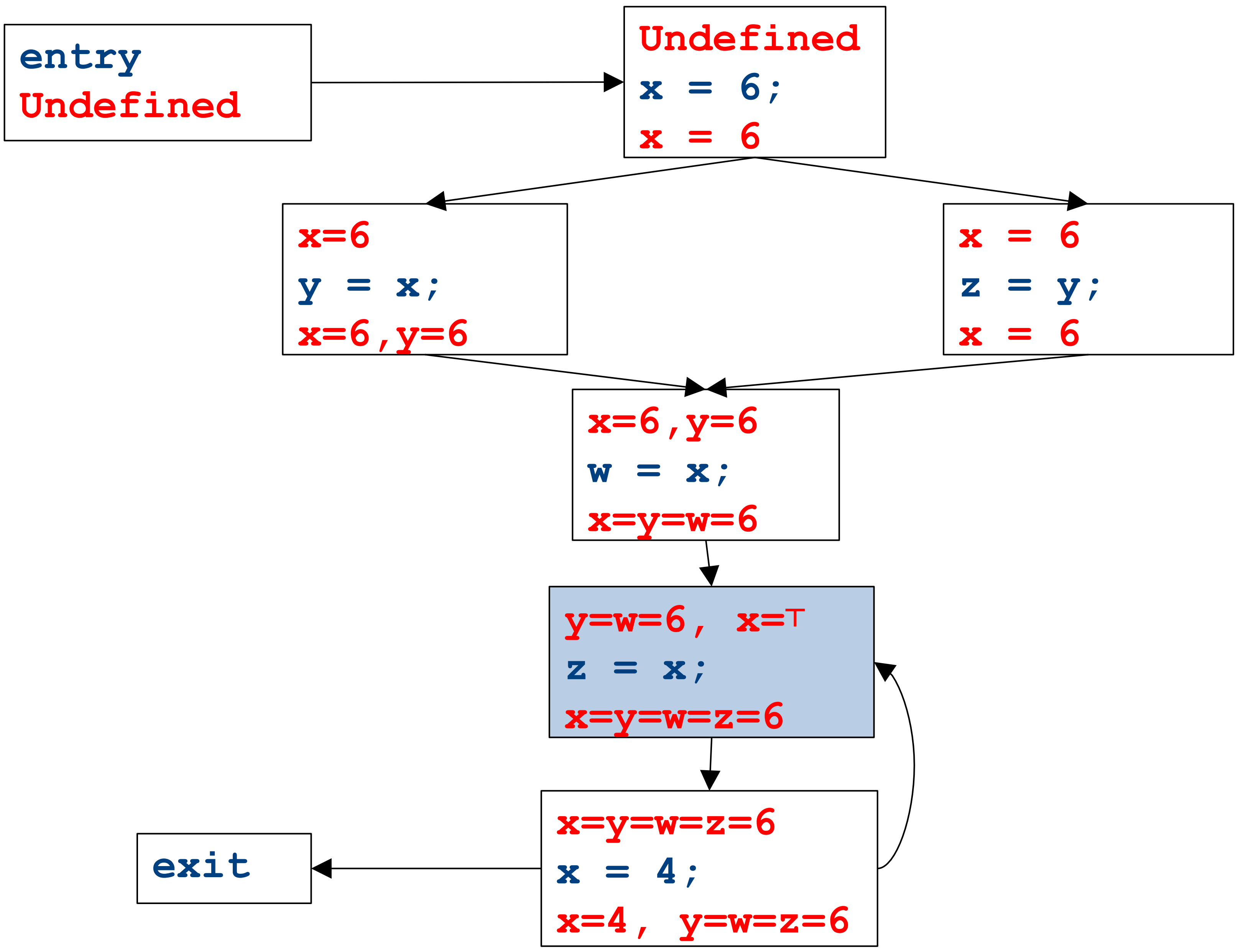

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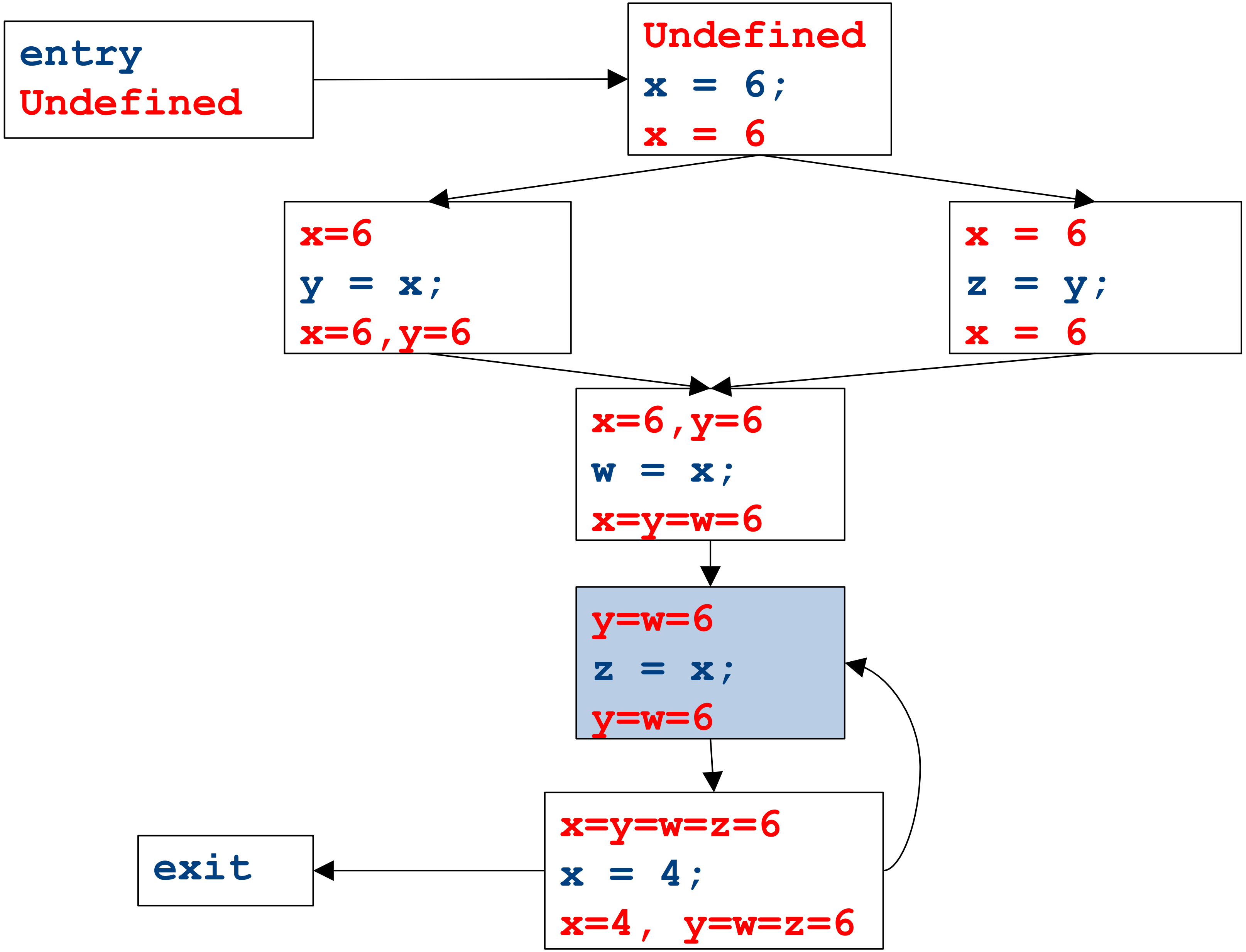

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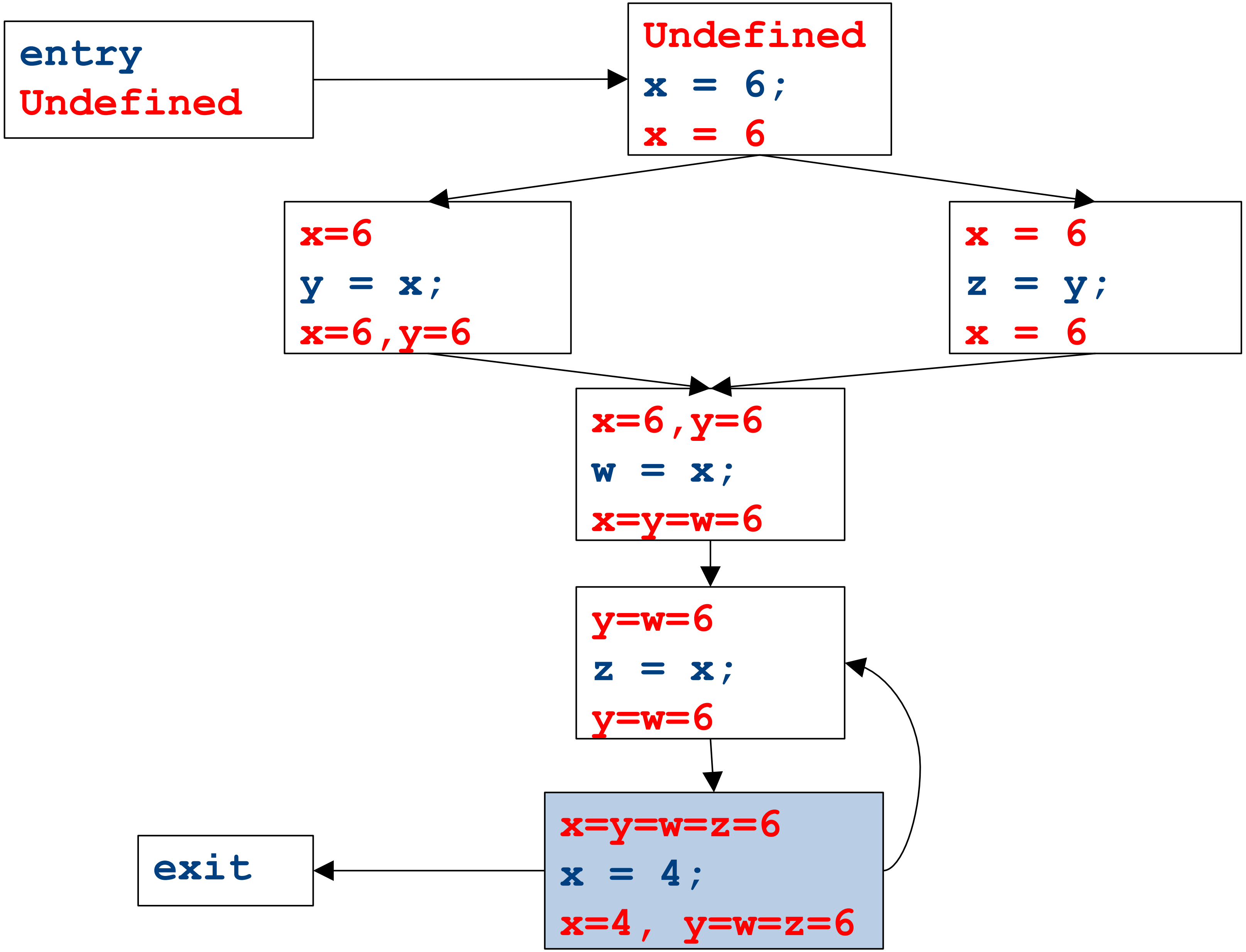

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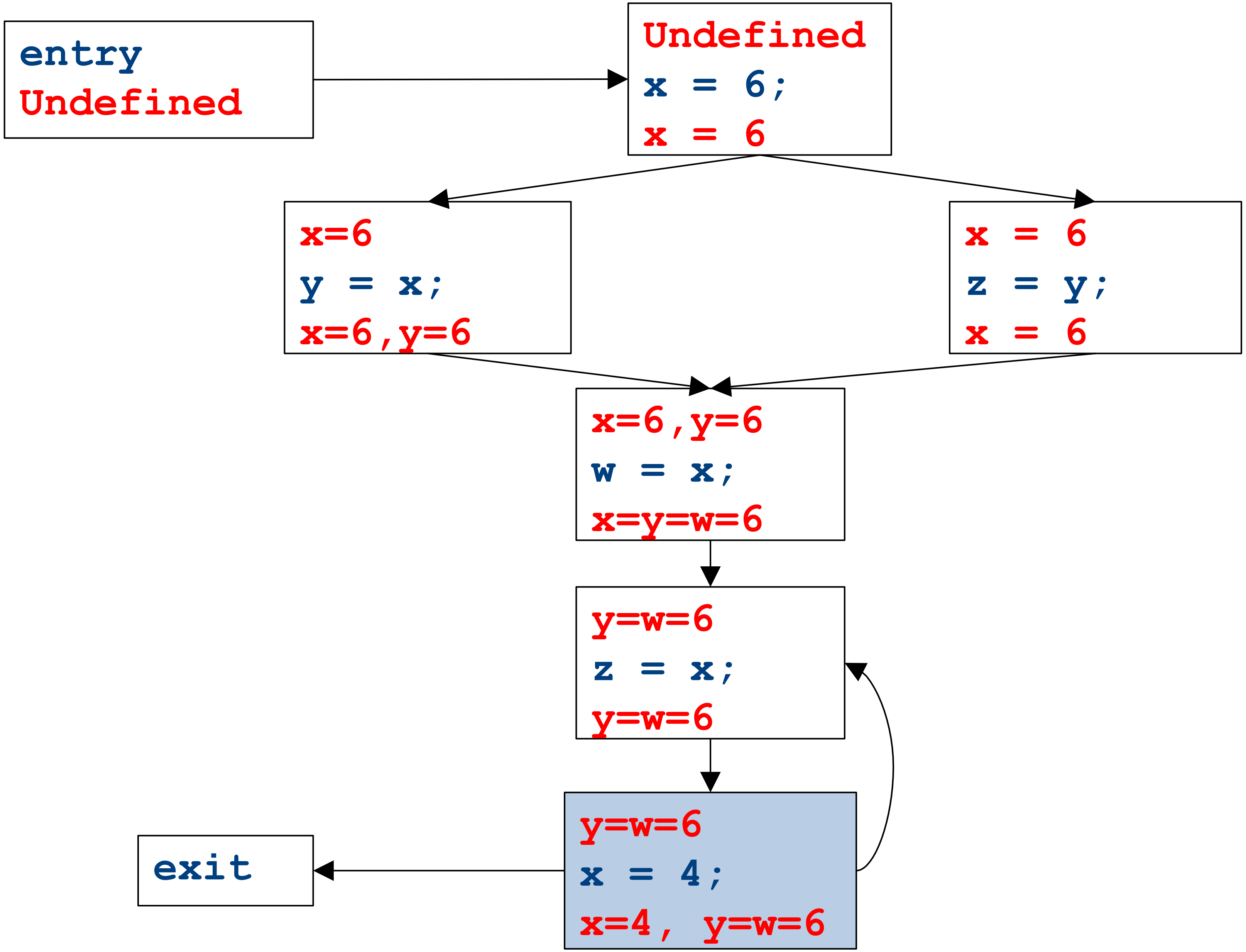

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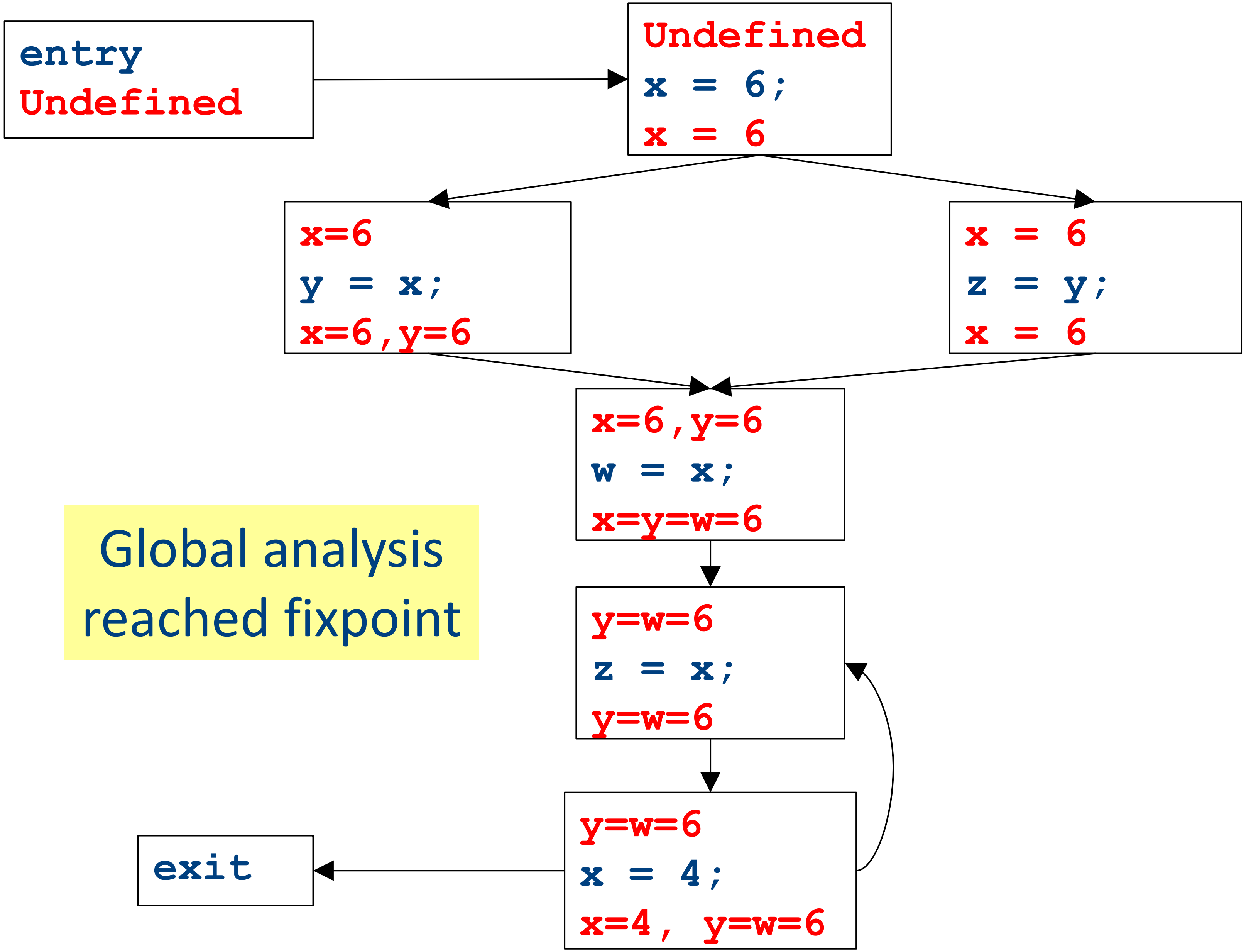

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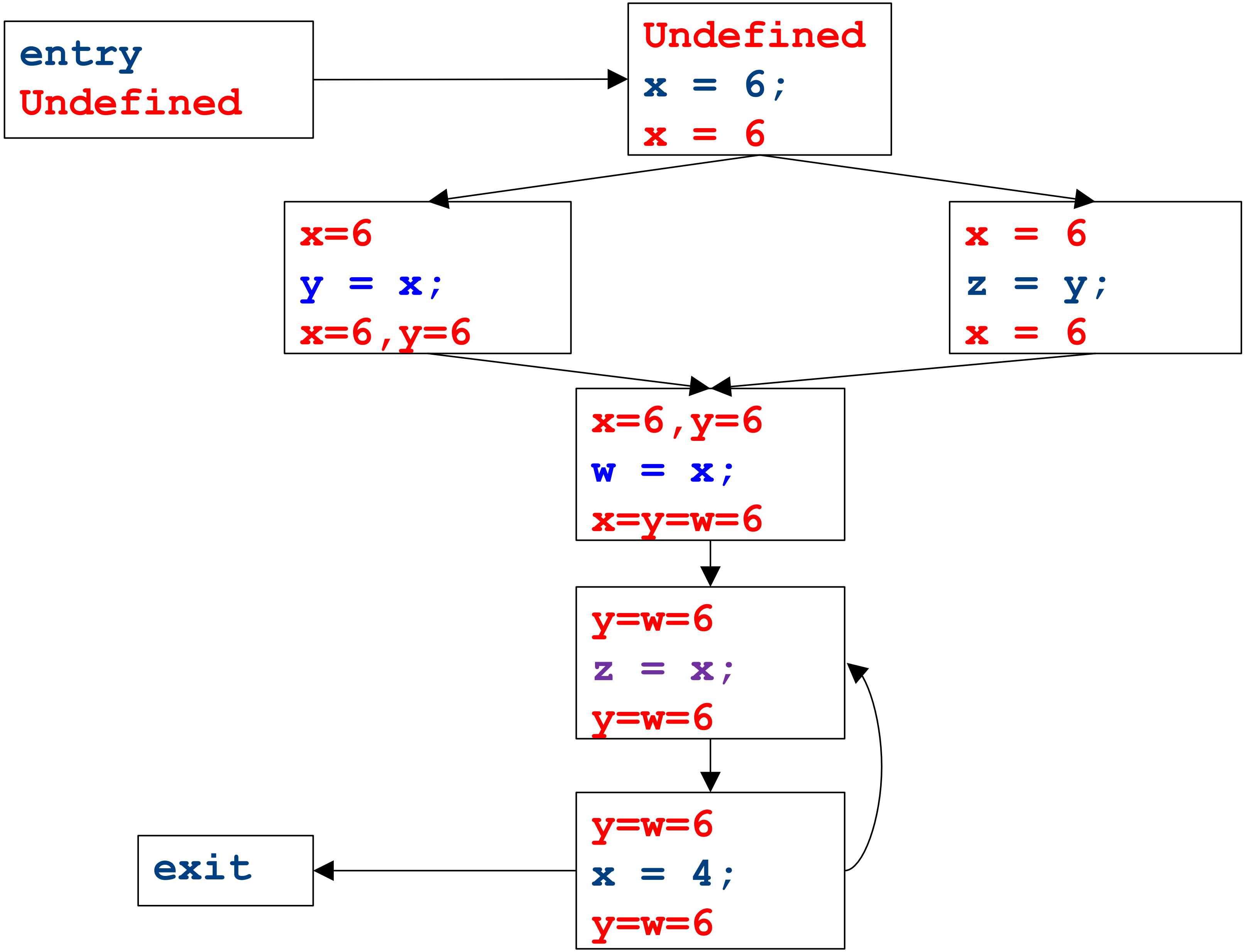

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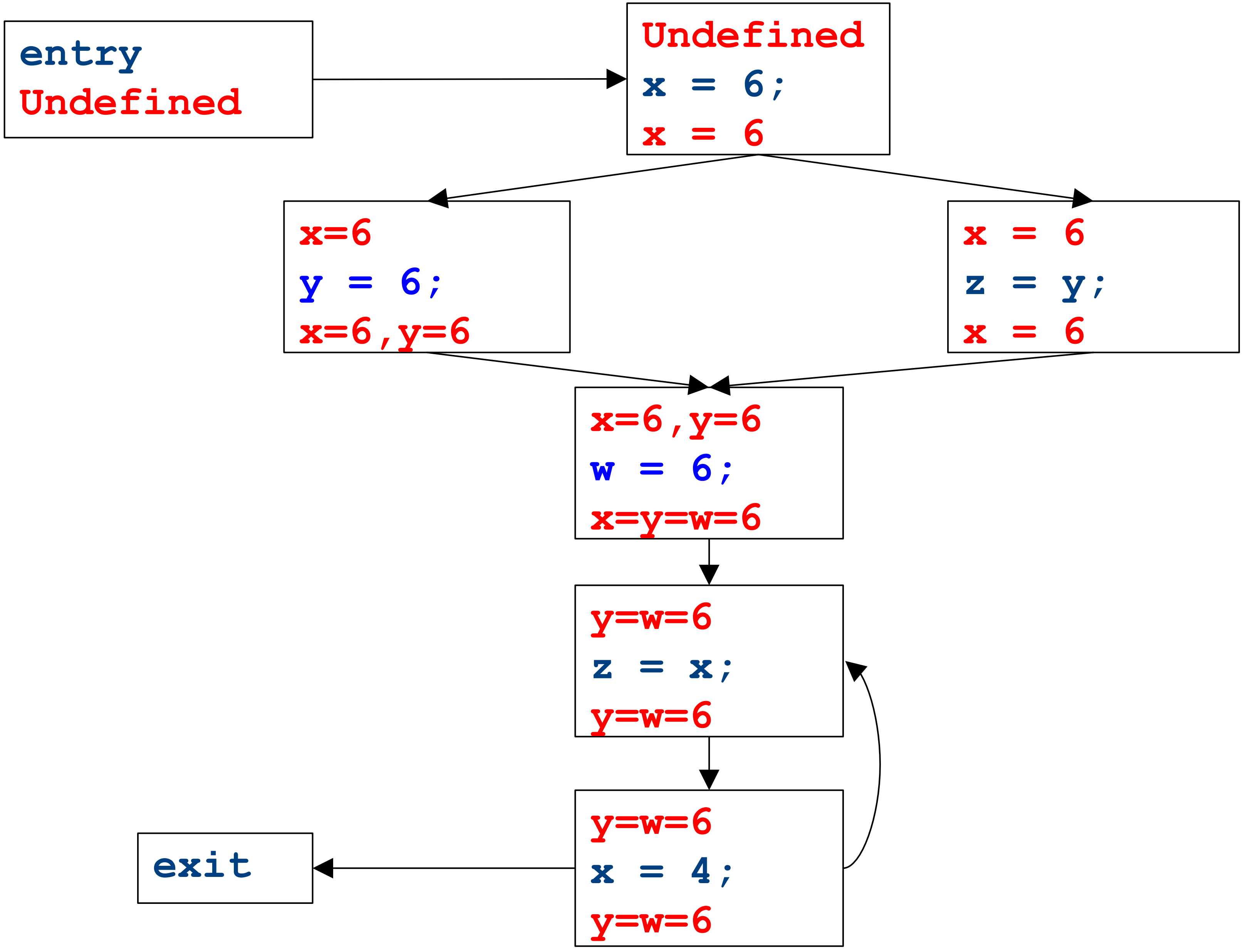

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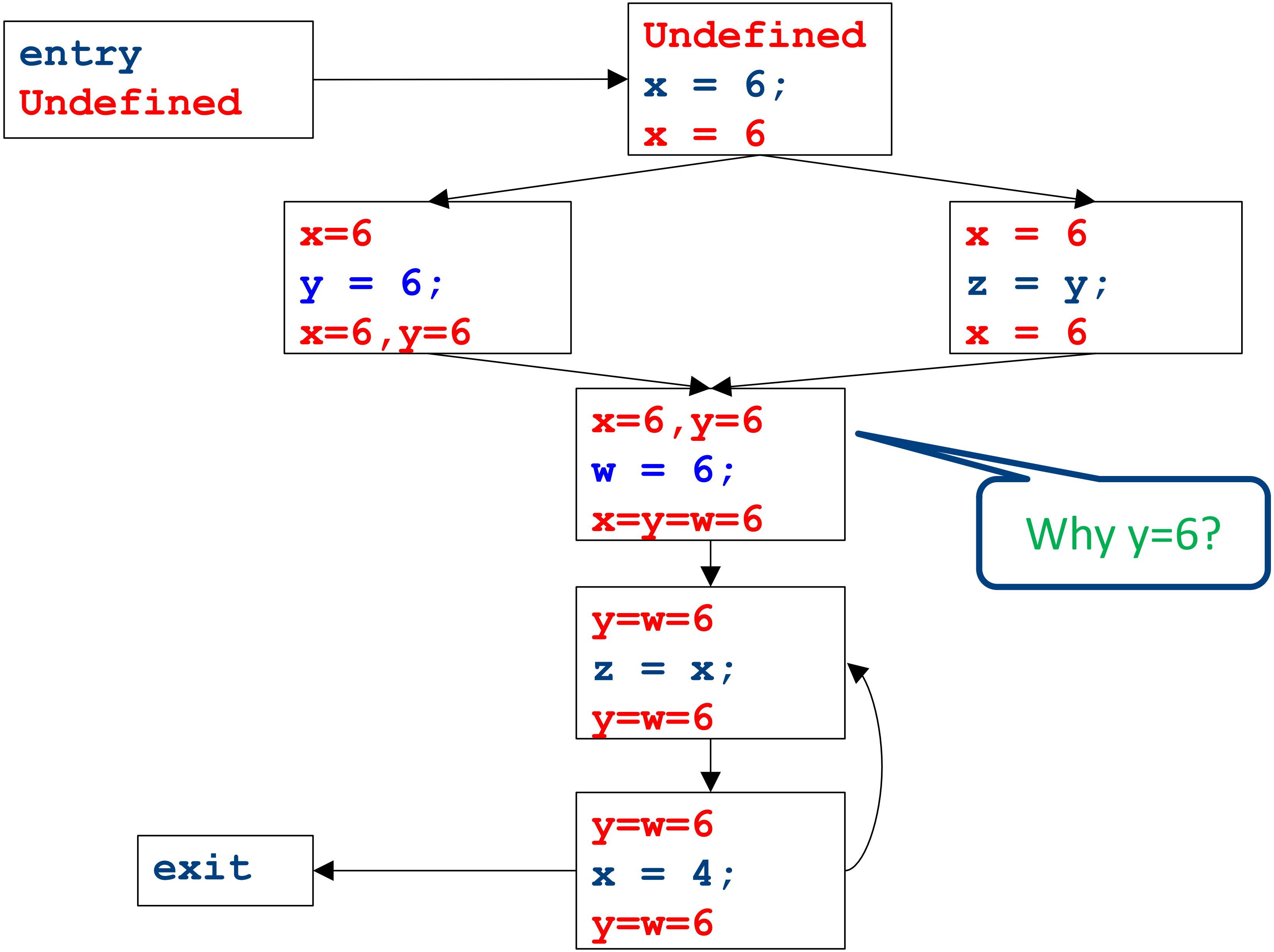

Dataflow for constant propagation

- Direction: **Forward**
- Semilattice: Vars→ {Undefined, 0, 1, -1, 2, -2, …, Not-a-Constant}
  - Join mapping for variables point-wise
    $\{x \mapsto 1, y \mapsto 1, z \mapsto 1\} \sqcup \{x \mapsto 1, y \mapsto 2, z \mapsto \text{Not-a-Constant}\} =$
    $\{x \mapsto 1, y \mapsto \text{Not-a-Constant}, z \mapsto \text{Not-a-Constant}\}$
- Transfer functions:
  - $f_{\mathtt{x=k}}(V) = V|_{x \mapsto k}$ *(update V by mapping x to k)*
  - $f_{\mathtt{x=a+b}}(V) = V|_{x \mapsto \text{Not-a-Constant}}$ *(assign Not-a-Constant)*
- Initial value: **x is Undefined**
  - (When might we use some other value?)

Proving termination

- Our algorithm for running these analyses continuously loops until no changes are detected
- Given this, how do we know the analyses will eventually terminate?
  - In general, **we don‘t**

Terminates?