Compilation

Lecture 8a

Code generation for procedure calls

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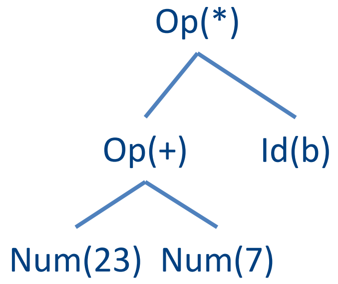
A Short Reminder

IR Generation

…



Valid Abstract Syntax Tree

Symbol Table

Verification (possible runtime)
Errors/Warnings

Intermediate Representation (IR)

input → Executable Code → output

TAC generation

- At this stage in compilation, we have
  - an AST
  - annotated with scope information
  - and annotated with type information
- To generate TAC for the program, we do recursive tree traversal
  - Generate TAC for any subexpressions or substatements
  - Using the result, generate TAC for the overall expression

cgen for binary operators

```
cgen($e_1 + e_2$) = {
    Choose a new temporary $t$
    Let $t_1$ = cgen($e_1$)
    Let $t_2$ = cgen($e_2$)
    Emit( $t = t_1 + t_2$ )
    Return $t$
}
```

cgen for statements

- We can extend the **cgen** function to operate over statements as well
- Unlike **cgen** for expressions, **cgen** for statements does not return the name of a temporary holding a value.
  - *(Why?)*

cgen for `if-then-else`

cgen(if (e) s_1 else s_2)

```
Let _t = cgen(e)
Let $L_{true}$ be a new label
Let $L_{false}$ be a new label
Let $L_{after}$ be a new label
Emit( IfZ _t Goto $L_{false}$; )
cgen($s_1$)
Emit( Goto $L_{after}$; )
Emit( $L_{false}$: )
cgen($s_2$)
Emit( Goto $L_{after}$;)
Emit( $L_{after}$: )
```

cgen for **`while`** loops

cgen(while (*expr) stmt)*

```
Let $L_{before}$ be a new label.
Let $L_{after}$ be a new label.
Emit( $L_{before}$: )
Let t = cgen(expr)
Emit( IfZ t Goto Lafter; )
cgen(stmt)
Emit( Goto $L_{before}$; )
Emit( $L_{after}$: )
```

Temporaries

Weighted register allocation

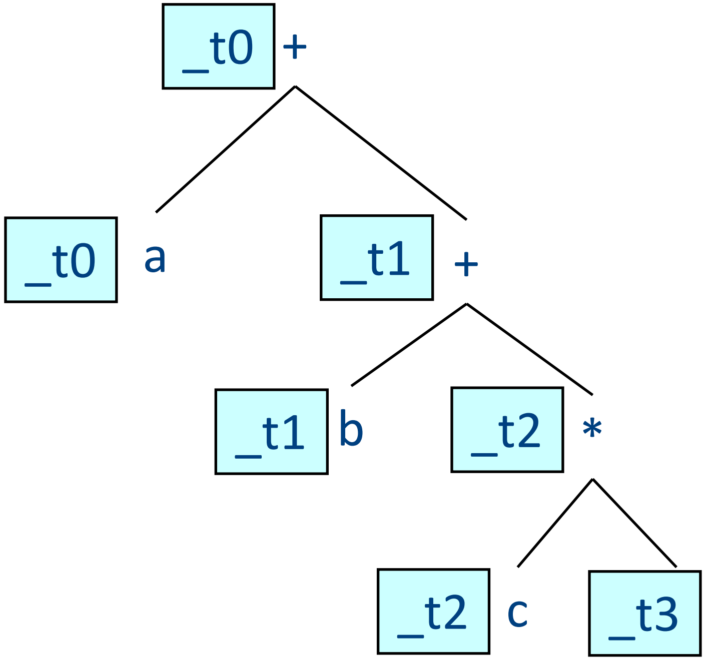
- Suppose we have expression e_1 *op* e_2
  - e_1, e_2 without side-effects
    - That is, no function calls, memory accesses, ++x
  - **cgen**(e_1 *op* e_2) = **cgen**(e_2 *op* e_1)
  - *Does order of translation matter?*
- Sethi & Ullman’s algorithm translates heavier sub-tree first
  - Optimal local (per-statement) allocation for side-effect-free statements

Example

_t0 = **cgen**(a+(b+(c*d)))

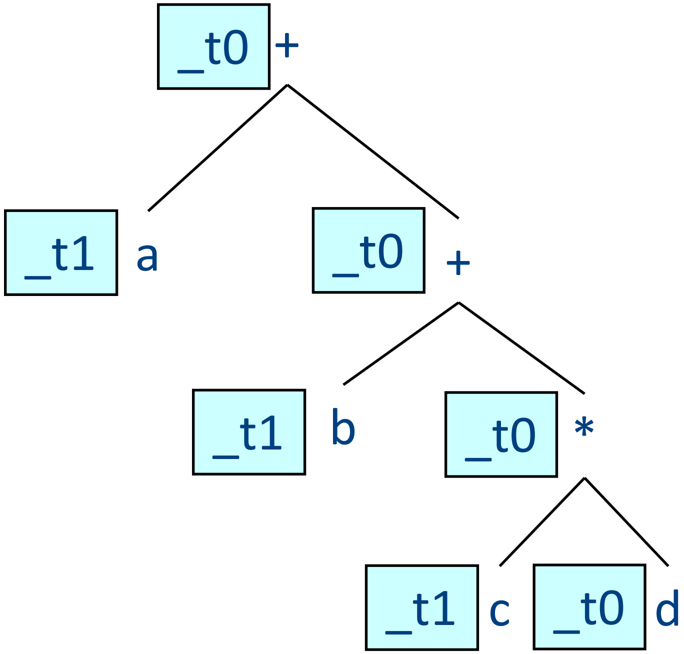
*+ and * are commutative operators*

left child first



4 temporaries

right child first



2 temporary

Code generation for procedure calls (+ a few words on the runtime system)

Code generation for procedure calls

- Compile time generation of code for procedure invocations

- Activation Records (aka Stack Frames)

Supporting Procedures

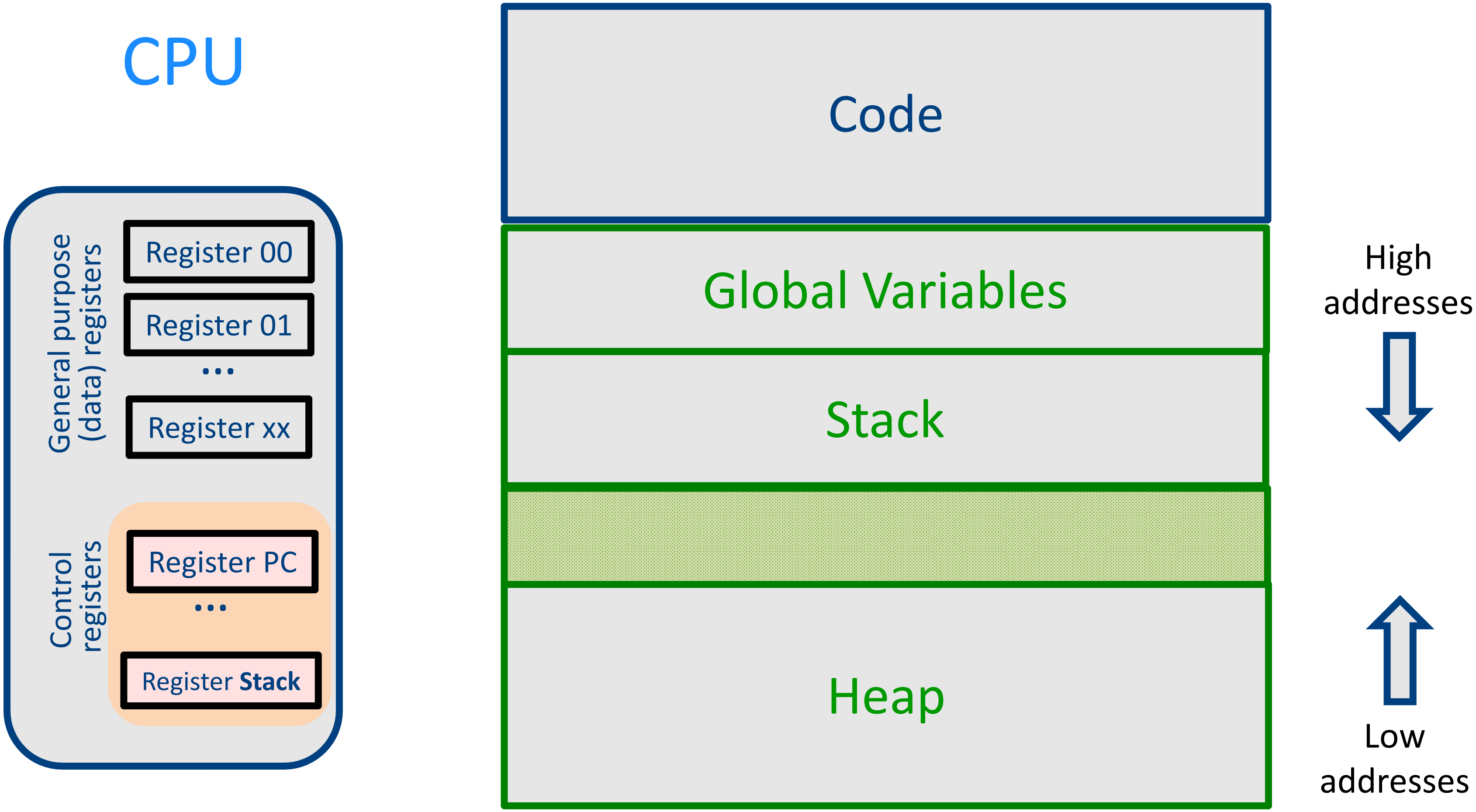
- **Stack**: a new computing environment
  - e.g., temporary memory for **local variables**
- Passing information into the new environment
  - **Parameters**
- **Transfer** of **control** to/from procedure
- Handling return values

Calling Conventions

- In general, compiler can use any convention to handle procedures

- In practice, CPUs specify standards
    - Aka calling conventios
  - Allows for compiler interoperability
    - Libraries!

Abstract Register Machine



CPU

General purpose (data) registers

Register 00

Register 01

...

Register xx

Control registers

Register PC

...

Register **Stack**

Code

Global Variables

Stack

Heap

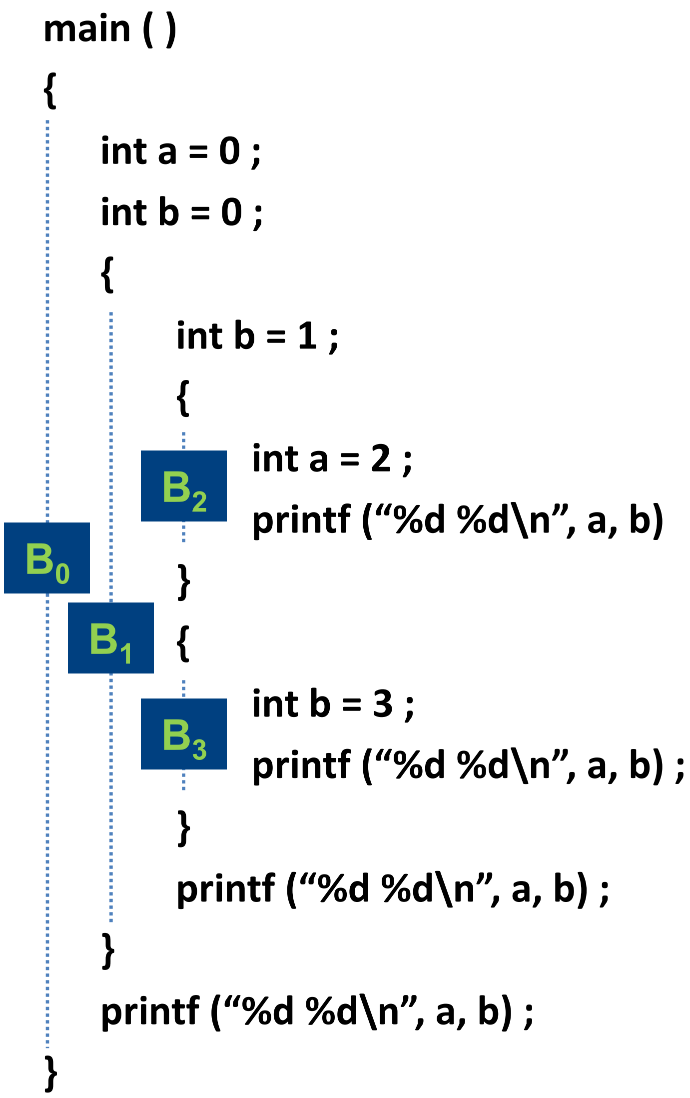
High addresses

Low addresses

Design Decisions

- Scoping rules
  - Static scoping vs. dynamic scoping
- Caller/callee conventions
  - Parameters
  - Who saves register values?
- Allocating space for local variables

Static (lexical) Scoping



```
main ( )
{
    int a = 0 ;
    int b = 0 ;
    {
        int b = 1 ;
        {
            int a = 2 ;
            printf ("%d %d\n", a, b)
        }
        {
            int b = 3 ;
            printf ("%d %d\n", a, b) ;
        }
        printf ("%d %d\n", a, b) ;
    }
    printf ("%d %d\n", a, b) ;
}
```

a name refers to its (closest) enclosing scope

known at compile time

Declaration	Scopes
a=0	B0,B1,B3
b=0	B0
b=1	B1,B2
a=2	B2
b=3	B3

Dynamic Scoping

- Each identifier is associated with a global stack of bindings
- When entering scope where identifier is declared
  - push declaration on identifier stack
- When exiting scope where identifier is declared
  - pop identifier stack
- **Evaluating the identifier in any context binds to the current top of stack**
- Determined **at runtime**

Example

```
int x = 42;

int f() { return x; }
int g() { int x = 1; return f(); }
int main() { return g(); }
```

- What value is returned from main?
  - Static scoping?
  - Dynamic scoping?

Why do we care?

- We need to generate code to access variables
- Static scoping
  - Identifier binding is known at compile time
  - “Address” of the variable is known at compile time
  - Assigning addresses to variables is part of code generation
  - No runtime errors of “access to undefined variable”
  - Can check types of variables

Variable addresses for static scoping: first attempt

```
int x = 42;

int f() { return x; }
int g() { int x = 1; return f(); }
int main() { return g(); }
```

identifier	address
x (global)	0x42
x (inside g)	0x73

Variable addresses for static scoping: first attempt

```
int a [11] ;

void quicksort(int m, int n) {
  int i;
  if (n > m) {
    i = partition(m, n);
    quicksort (m, i-1) ;
    quicksort (i+1, n) ;
  }

main() {
...
 quicksort (1, 9) ;
}
```

what is the address of the variable "i" in the procedure quicksort?

Compile-Time Information on Variables

- Name
- Type
- Scope
  - when is it recognized
- Duration
  - Until when does its value exist
- Size
  - How many bytes are required at runtime
- Address
  - Fixed
  - Relative
  - Dynamic

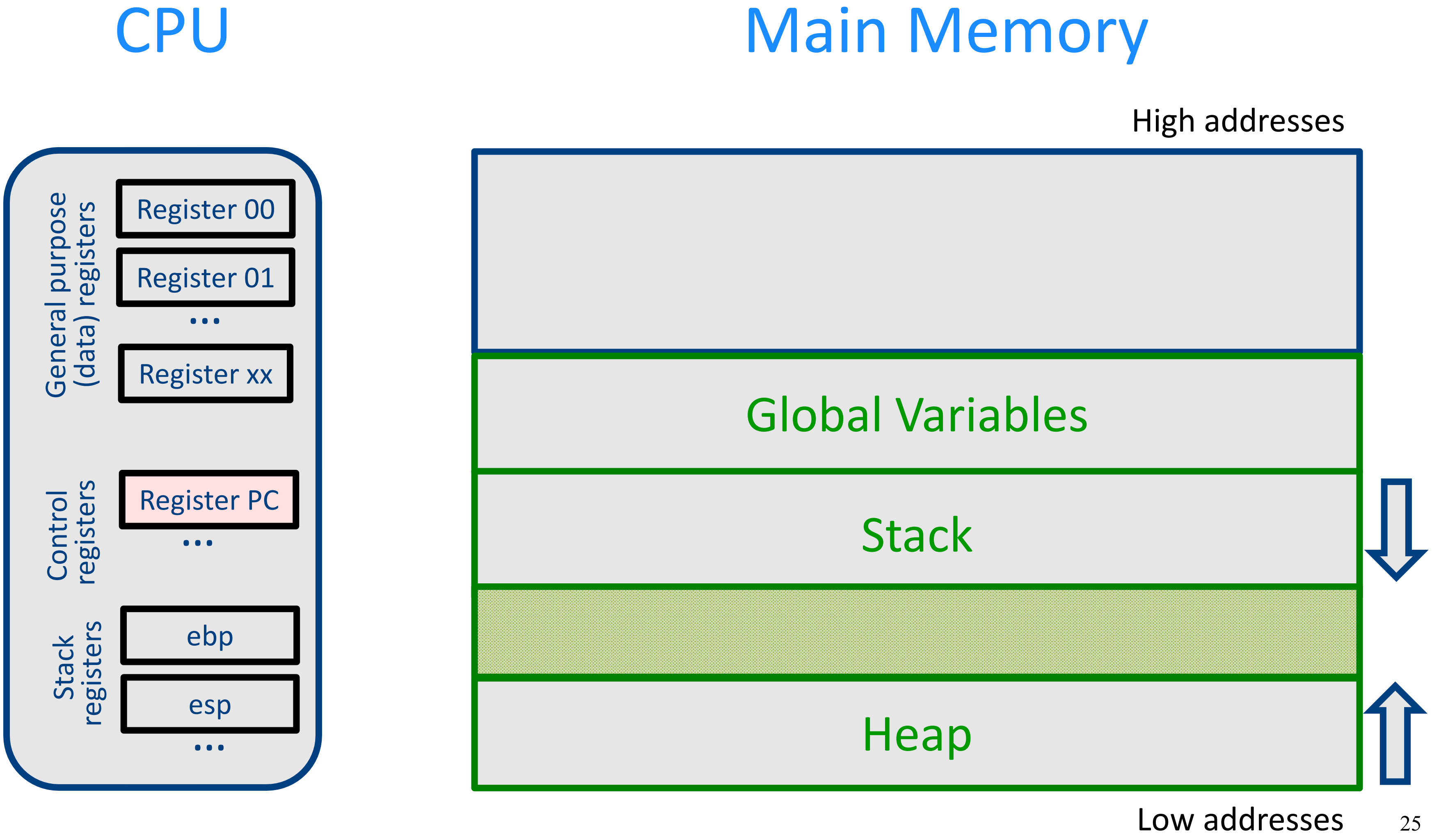
Activation Record (Stack Frames)

- separate space for each procedure invocation

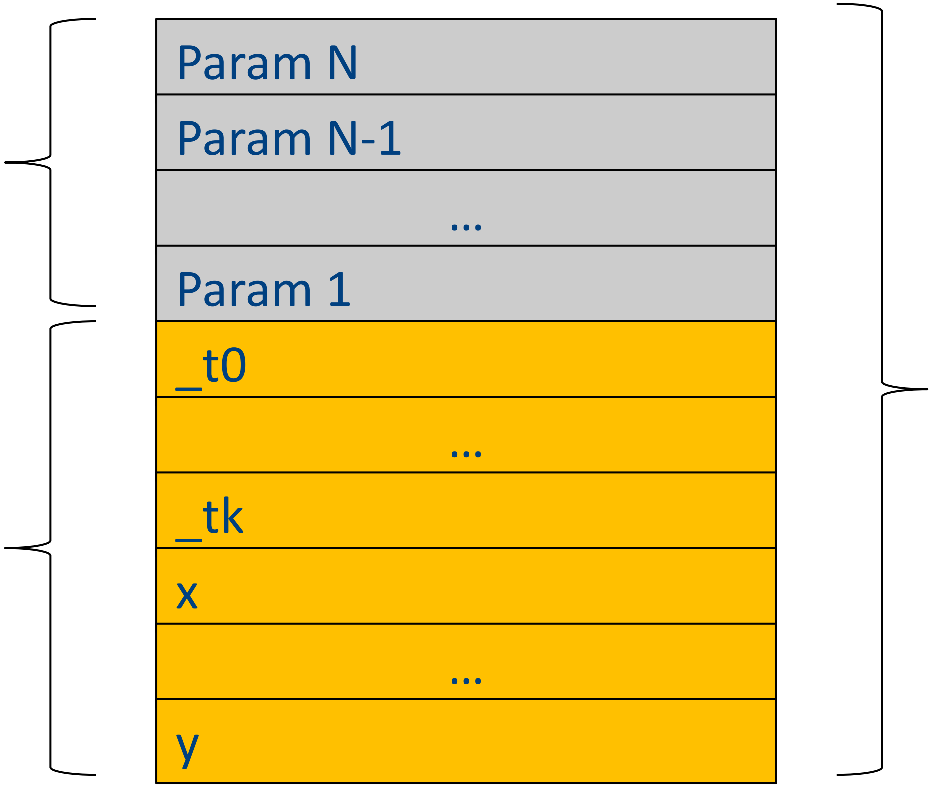
- **managed at runtime**
  - **code for managing it generated by the compiler**

- desired properties
  - efficient allocation and deallocation
    - procedures are called frequently
  - variable size
    - different procedures may require different memory sizes

Semi-Abstract Register Machine



A Logical Stack Frame (Simplified)



Parameters (actual arguments)

Param N

Param N-1

…

Param 1

Locals and temporaries

_t0

…

_tk

x

…

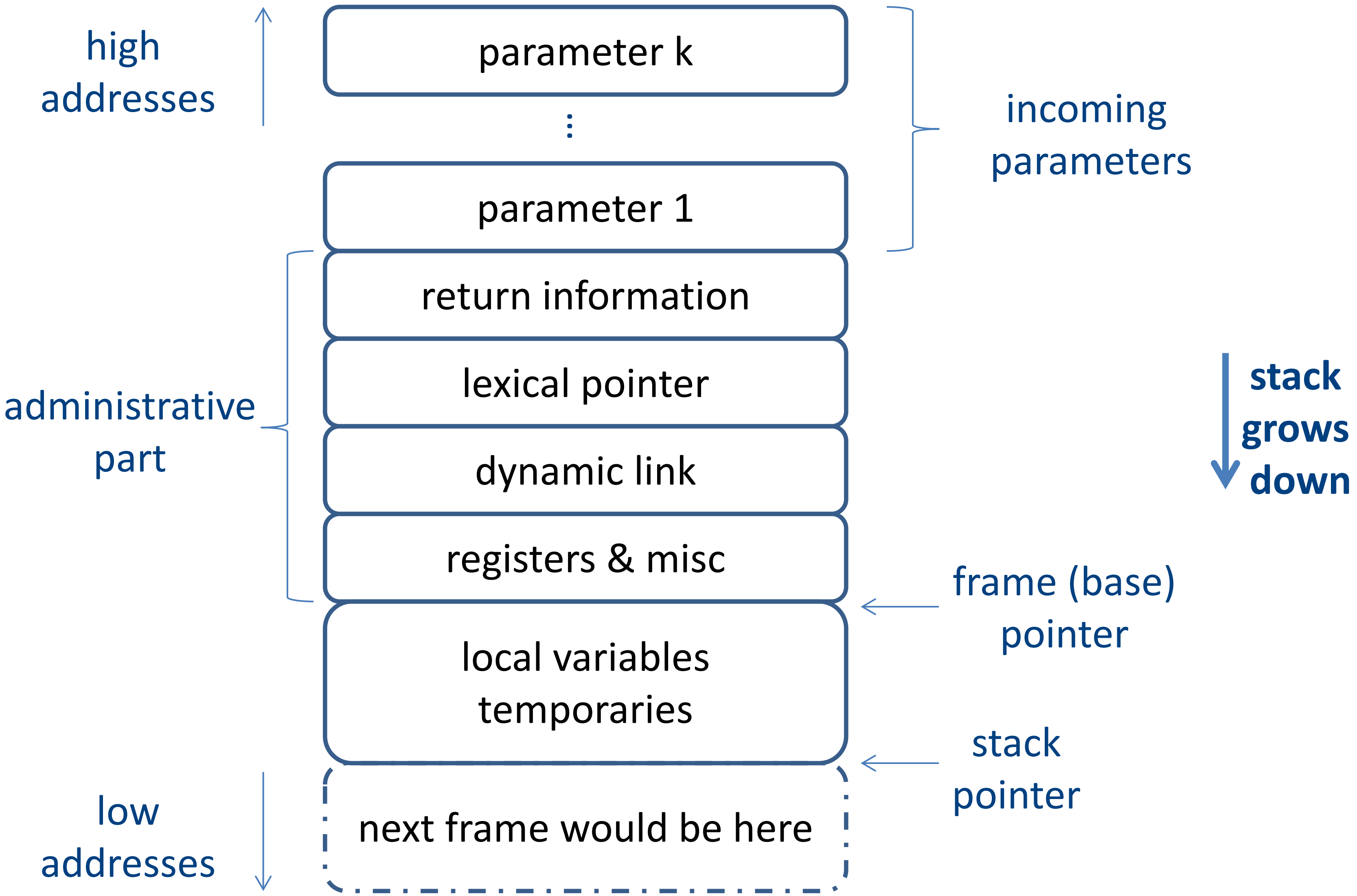
y

Stack frame for function f(a1,…,aN)

Runtime Stack

- Stack of activation records
- Call = push new activation record
- Return = pop activation record
- Only one “active” activation record – top of stack
- How do we handle recursion?

Activation Record (frame)



high addresses

parameter k

⋮

parameter 1

incoming parameters

administrative part

return information

lexical pointer

dynamic link

registers & misc

stack grows down

frame (base) pointer

local variables
temporaries

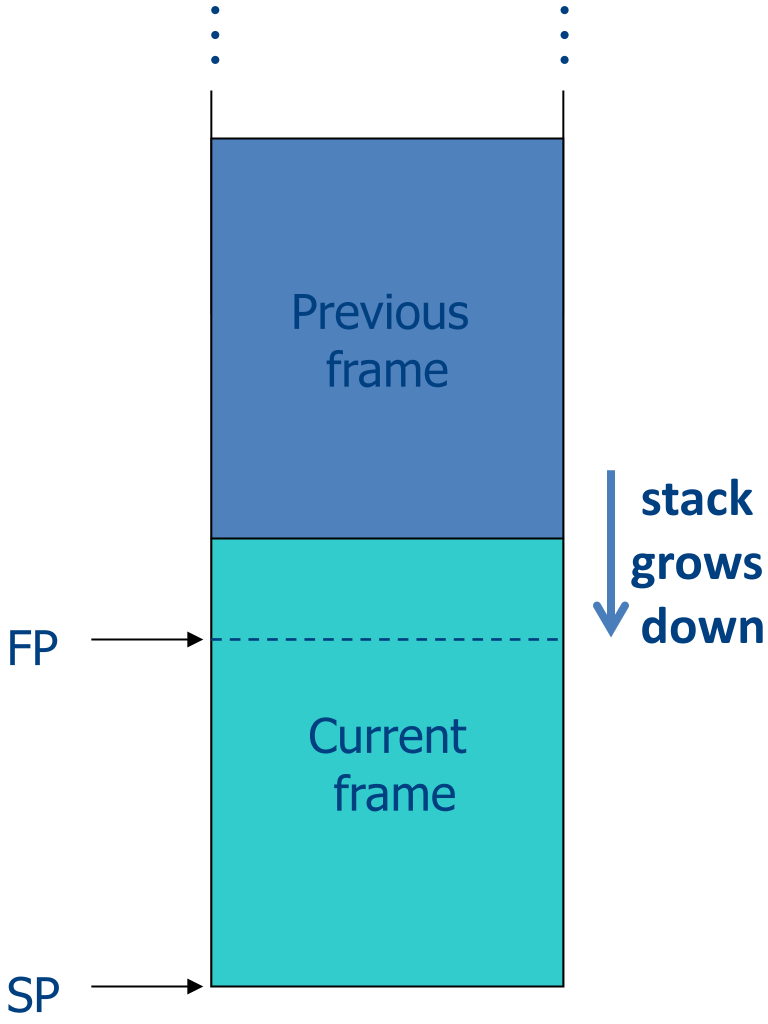
stack pointer

low addresses

next frame would be here

Runtime Stack

- SP – stack pointer
  – top of current frame
- FP – frame pointer
  – base of current frame
  - Sometimes called BP (base pointer)
  - Usually points to a "fixed" offset from the "start" of the frame



Code Blocks

- Programming language provide code blocks

```
void foo()
{
 int x = 8 ; y=9;//1
   { int x = y * y ;//2 }
   { int x = y * 7 ;//3}
      x = y + 1;
 }
```

adminstrative
x1
y1
x2
x3
…

L-Values of Local Variables

- The offset in the stack is known at compile time
- L-val(x) = FP+offset(x)
- x = 5 $\Rightarrow$ Load_Constant 5, R3
  Store R3, offset(x)(FP)

Pentium Runtime Stack

Register	Usage
ESP	Stack pointer
EBP	Base pointer

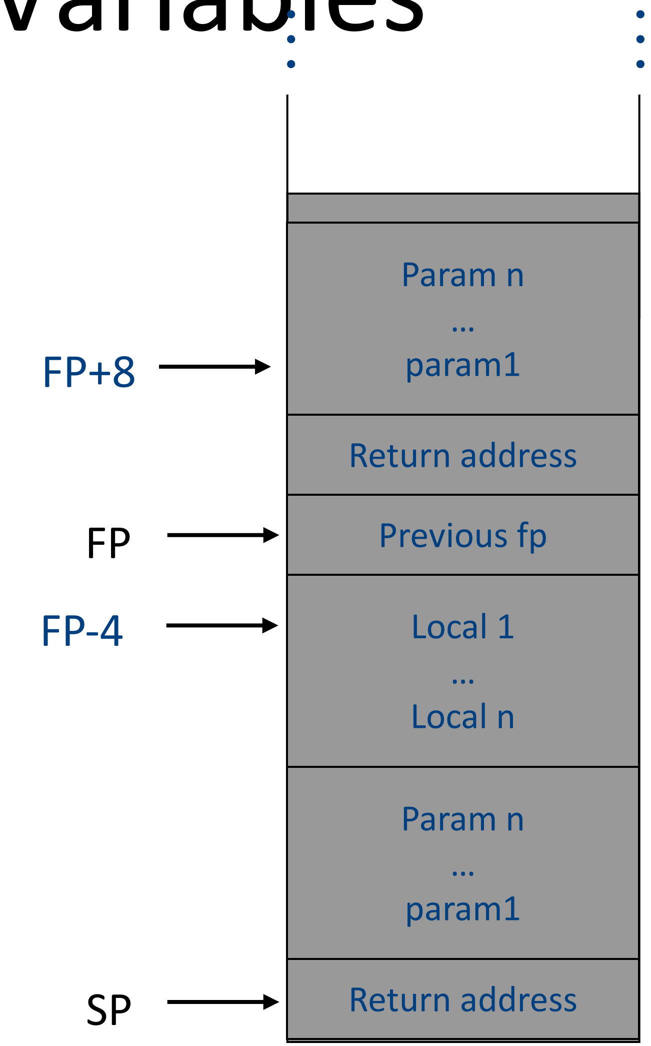
Pentium stack registers

Instruction	Usage
push, pusha,…	push on runtime stack
pop,popa,…	Base pointer
call	transfer control to called routine
return	transfer control back to caller

Pentium stack and call/ret instructions

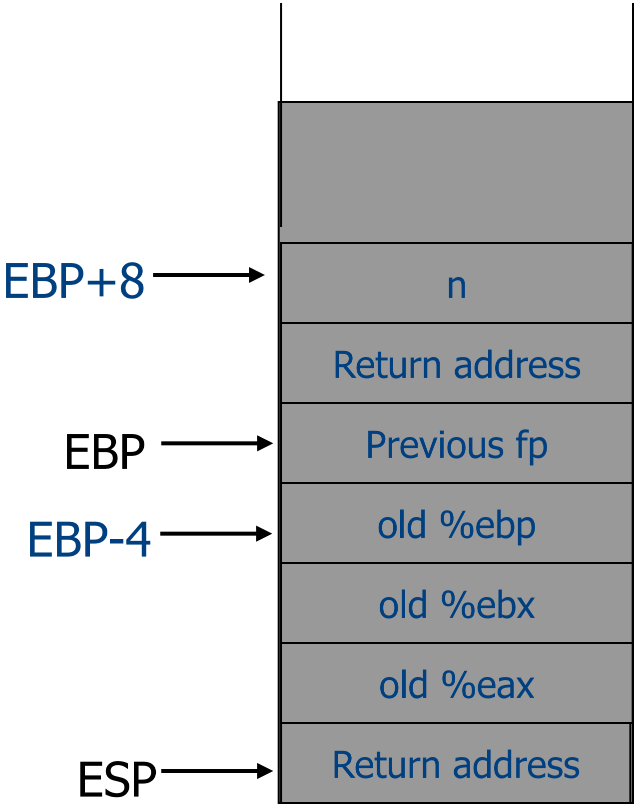
Accessing Stack Variables

- Use offset from FP (%ebp)
  - Remember: stack grows downwards
- Above FP = parameters
- Below FP = locals
- Examples
  - %ebp + 4 = return address
  - %ebp + 8 = first parameter
  - %ebp – 4 = first local



Factorial – `fact(int n)`

```
fact:
pushl %ebp              # save ebp
movl %esp,%ebp          # ebp=esp
pushl %ebx              # save ebx
movl 8(%ebp),%ebx       # ebx = n
cmpl $1,%ebx            # n = 1 ?
jle .lresult            # then done
leal -1(%ebx),%eax      # eax = n-1
pushl %eax              #
call fact               # fact(n-1)
imull %ebx,%eax         # eax=retv*n
jmp .lreturn            #
.lresult:
movl $1,%eax            # retv
.lreturn:
movl -4(%ebp),%ebx      # restore ebx
movl %ebp,%esp          # restore esp
popl %ebp               # restore ebp
```



EBP+8 → n

Return address

EBP → Previous fp

EBP-4 → old %ebp

old %ebx

old %eax

ESP → Return address

(stack in intermediate point)

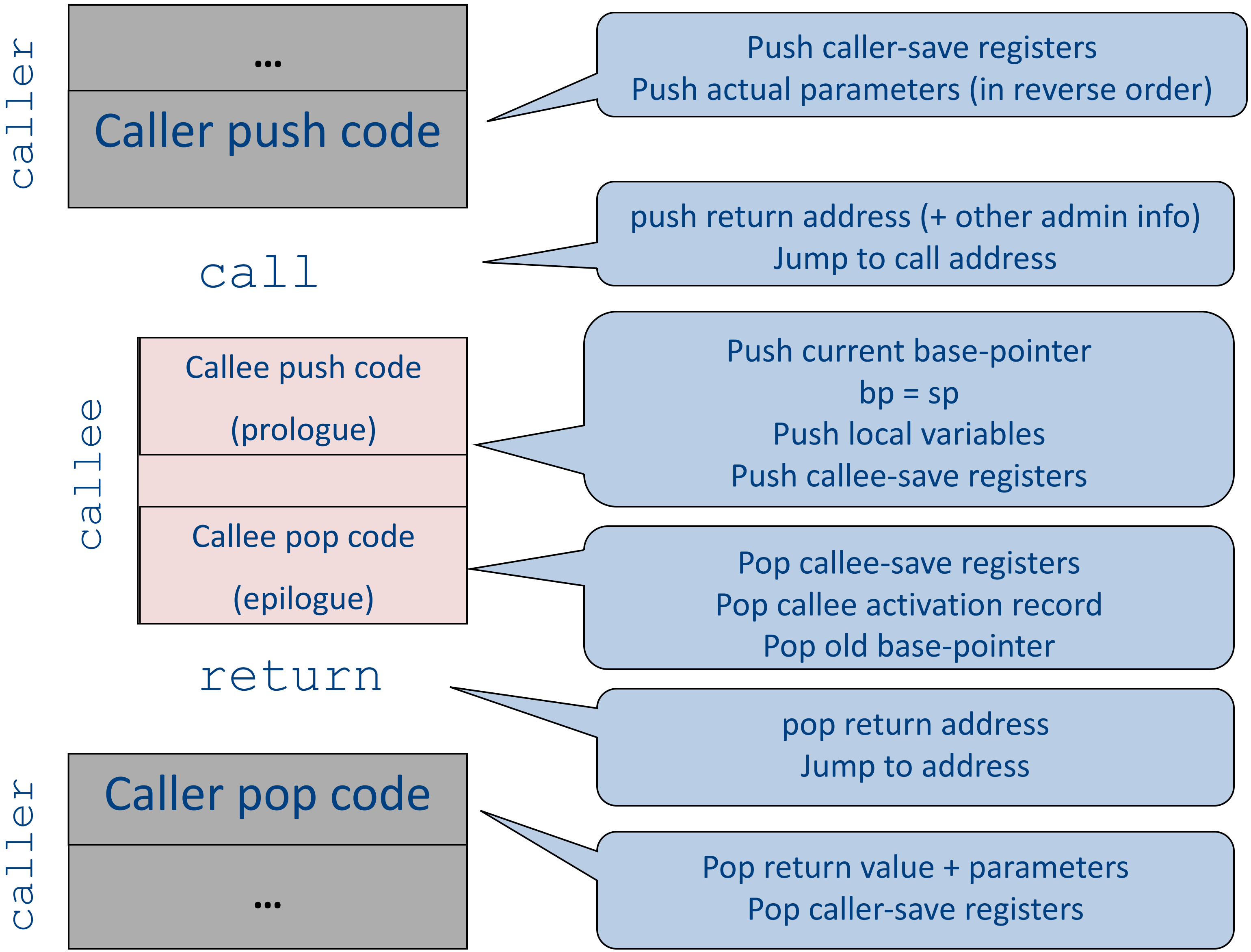
(disclaimer: real compiler can do better than that)

Call Sequences

- The **processor** **does not save** the content of **registers** on procedure calls

- So who will?
  - Caller saves and restores registers
  - Callee saves and restores registers
  - But can also have both save/restore some registers

Call Sequences



caller

...

Caller push code

Push caller-save registers
Push actual parameters (in reverse order)

`call`

push return address (+ other admin info)
Jump to call address

callee

Callee push code

(prologue)

Push current base-pointer
bp = sp
Push local variables
Push callee-save registers

Callee pop code

(epilogue)

Pop callee-save registers
Pop callee activation record
Pop old base-pointer

`return`

pop return address
Jump to address

caller

Caller pop code

...

Pop return value + parameters
Pop caller-save registers

“To Callee-save or to Caller-save?”

- Callee-saved registers need only be saved when callee modifies their value
- Some heuristics and conventions are followed

Caller-Save and Callee-Save Registers

- Callee-Save Registers
  - Saved by the callee before modification
  - Values are automatically preserved across calls
- Caller-Save Registers
  - Saved (if needed) by the caller before calls
  - Values are not automatically preserved across calls
- Usually the architecture defines caller-save and callee-save registers

- Separate compilation
- Interoperability between code produced by different compilers/languages
- But compiler writers decide when to use caller/callee registers

Callee-Save Registers

- Saved by the callee before modification
- Usually at procedure prolog
- Restored at procedure epilog
- Hardware support may be available
- Values are automatically preserved across calls

```
int foo(int a)    {
        int b=a+1;
        f1();
        g1(b);
        return(b+2);
}
```

```
.global _foo
              Add_Constant -K, SP //allocate space for foo
              Store_Local  R5, -14(FP) // save R5
              Load_Reg  R5, R0; Add_Constant R5, 1
              JSR f1 ; JSR g1;
              Add_Constant R5, 2; Load_Reg R5, R0
             Load_Local -14(FP), R5 // restore R5
             Add_Constant K, SP; RTS // deallocate
```

Caller-Save Registers

- Saved by the caller before calls when needed
- Values are not automatically preserved across calls

```
void bar (int y) {
	int x=y+1;
	f2(x);
	g2(2);
	g2(8);
}
```

```
.global _bar
	Add_Constant -K, SP //allocate space for bar
	Add_Constant R0, 1
	JSR f2
	Load_Constant  2, R0  ;       JSR g2;
	Load_Constant 8, R0 ;         JSR g2
	Add_Constant K, SP // deallocate space for bar
	 RTS
```

Parameter Passing

- 1960s
  - In memory
    - No recursion is allowed
- 1970s
  - In stack
- 1980s
  - In registers
  - First k parameters are passed in registers (k=4 or k=6)
  - Where is time saved?

- Most procedures are leaf procedures
- Interprocedural register allocation
- Many of the registers may be dead before another invocation
- Register windows are allocated in some architectures per call (e.g., sun Sparc)

Activation Records & Language Design

Compile-Time Information on Variables

- Name, type, size
- Address kind
  - Fixed (global)
  - Relative (local)
  - Dynamic (heap)

- Scope
  - when is it recognized
- Duration
  - Until when does its value exist

Scoping

```
int x = 42;

int f() { return x; }
int g() { int x = 1; return f(); }
int main() { return g(); }
```

- What value is returned from main?
- Static scoping?
- Dynamic scoping?

Nested Procedures

- For example – Pascal
- Any routine can have sub-routines
- Any sub-routine can access anything that is defined in its containing scope or inside the sub-routine itself
  - “non-local” variables

Example: Nested Procedures

```
program p(){
   int x;
  procedure a(){
     int y;
    procedure b(){ … c() … };
    procedure c(){
       int z;
      procedure d(){
         y := x + z
       };
       … b() … d() …
    }
     … a() … c() …
  }
   a()
}
```

Possible call sequence:
P→a → a → c → b → c → d

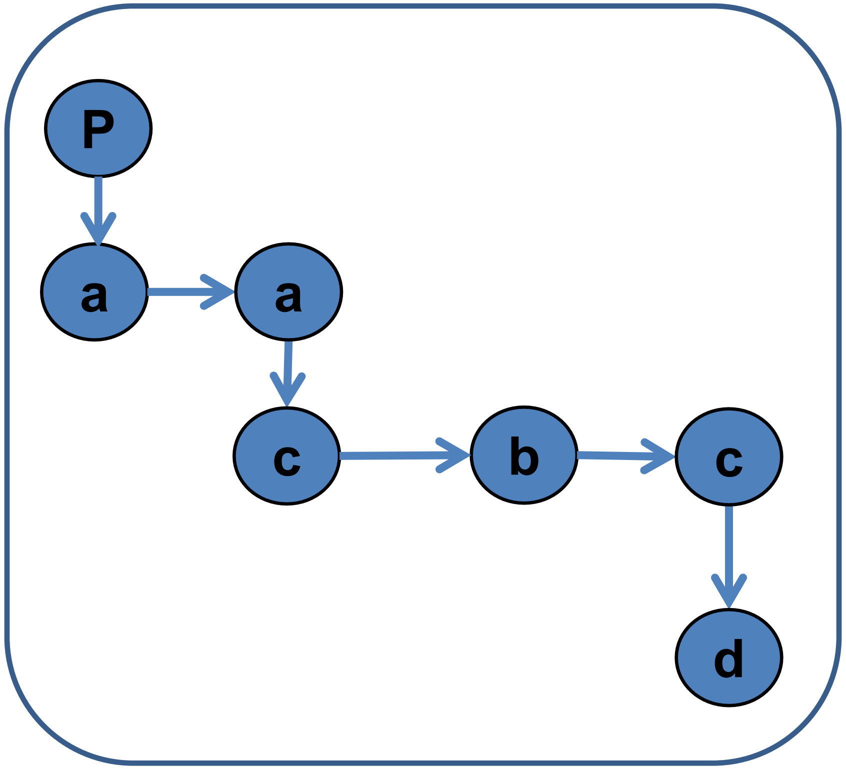
what are the addresses of variables “**x**,” “**y**” and “**z**” in procedure d?

Nested Procedures

- **can call a sibling, ancestor**
- when “c” uses (non-local) variables from “a”, which instance of “a” is it?

- how do you find the right activation record at runtime?

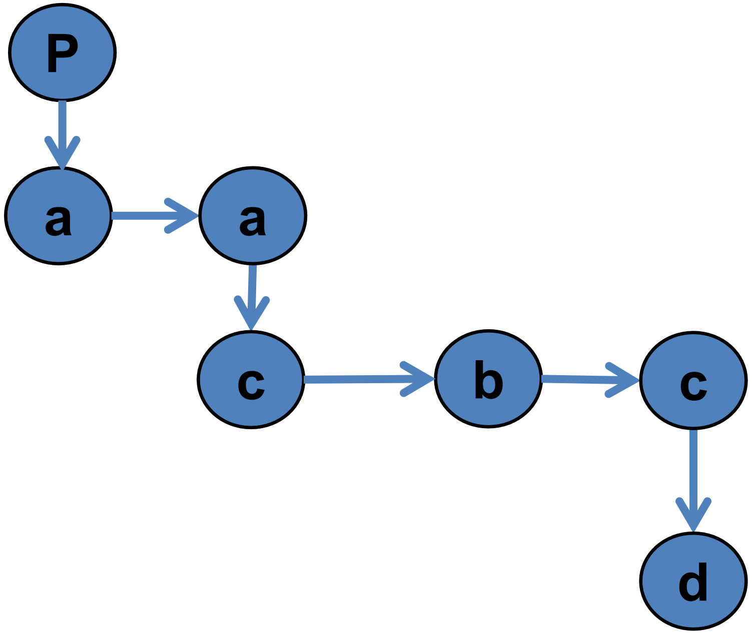
Possible call sequence:
P→a → a → c → b → c → d



Nested Procedures

- goal: **find the closest routine in the stack from a given nesting level**
- if we reached the same routine in a sequence of calls
  - routine of level k uses variables of the same nesting level, it uses its own variables
  - if it uses variables of nesting level j < k then it must be the last routine called at level j
- If a procedure is last at level j on the stack, then it must be ancestor of the current routine

Possible call sequence:
P→a → a → c → b → c → d



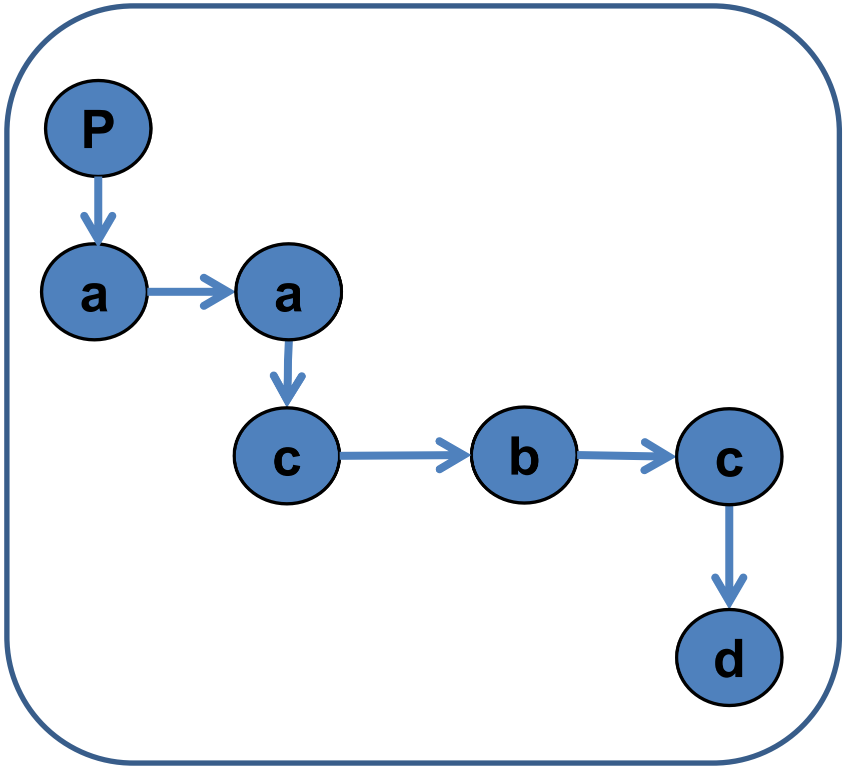
Nested Procedures

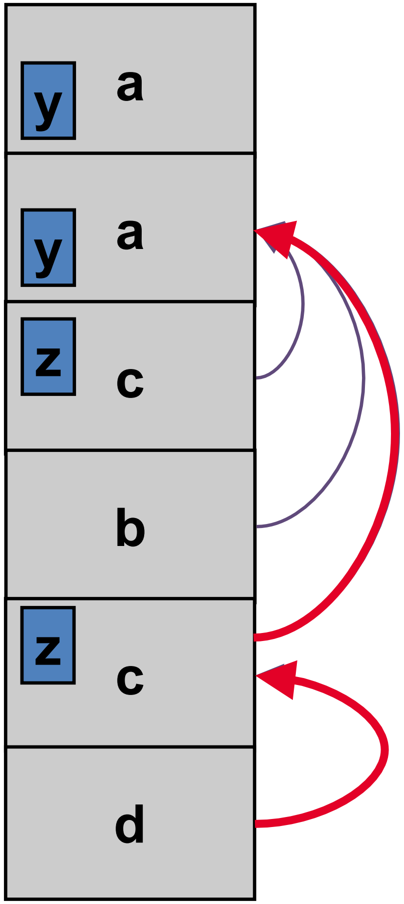
- problem: a routine may need to access variables of another routine that contains it statically
- solution: **lexical pointer** (a.k.a. **access link**) in the activation record
- lexical pointer points to the last activation record of the nesting level above it
  - in our example, lexical pointer of d points to activation records of c
- lexical pointers created at runtime
- number of links to be traversed is known at compile time

Lexical Pointers

```
program p(){
   int x;
  procedure a(){
     int y;
    procedure b(){ c() };
     procedure c(){
        int z;
       procedure d(){
          y := x + z
        };
        … b() … d() …
     }
    … a() … c() …
  }
  a()
}
```

Possible call sequence:
P→a → a → c → b → c → d

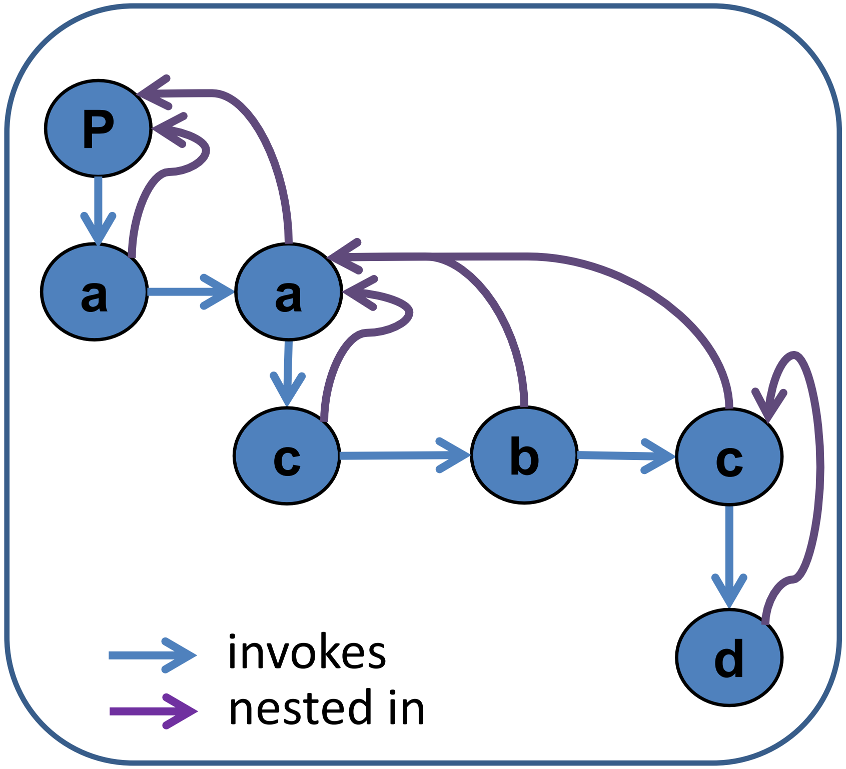


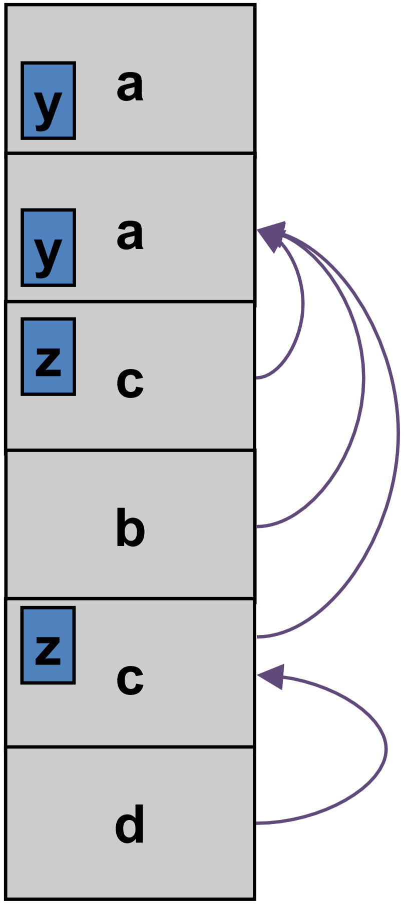


Lexical Pointers

```
program p(){
   int x;
   procedure a(){
      int y;
     procedure b(){ c() };
      procedure c(){
         int z;
        procedure d(){
           y := x + z
         };
         … b() … d() …
      }
      … a() … c() …
   }
   a()
}
```

Possible call sequence:
P→a → a → c → b → c → d



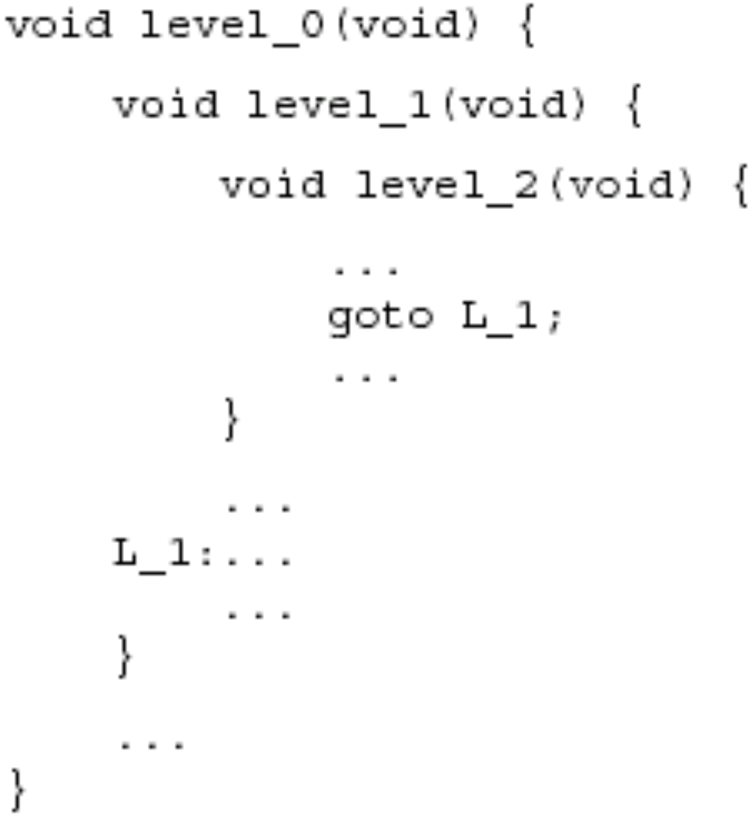


Activation Records: Remarks

Stack Frames

- Allocate a separate space for every procedure incarnation
- Relative addresses
- Provide a simple mean to achieve modularity
- Supports separate code generation of procedures
- Naturally supports recursion
- Efficient memory allocation policy
  - Low overhead
  - Hardware support may be available
- LIFO policy
- Not a pure stack
  - Non local references
  - Updated using arithmetic

Non-Local goto in C syntax

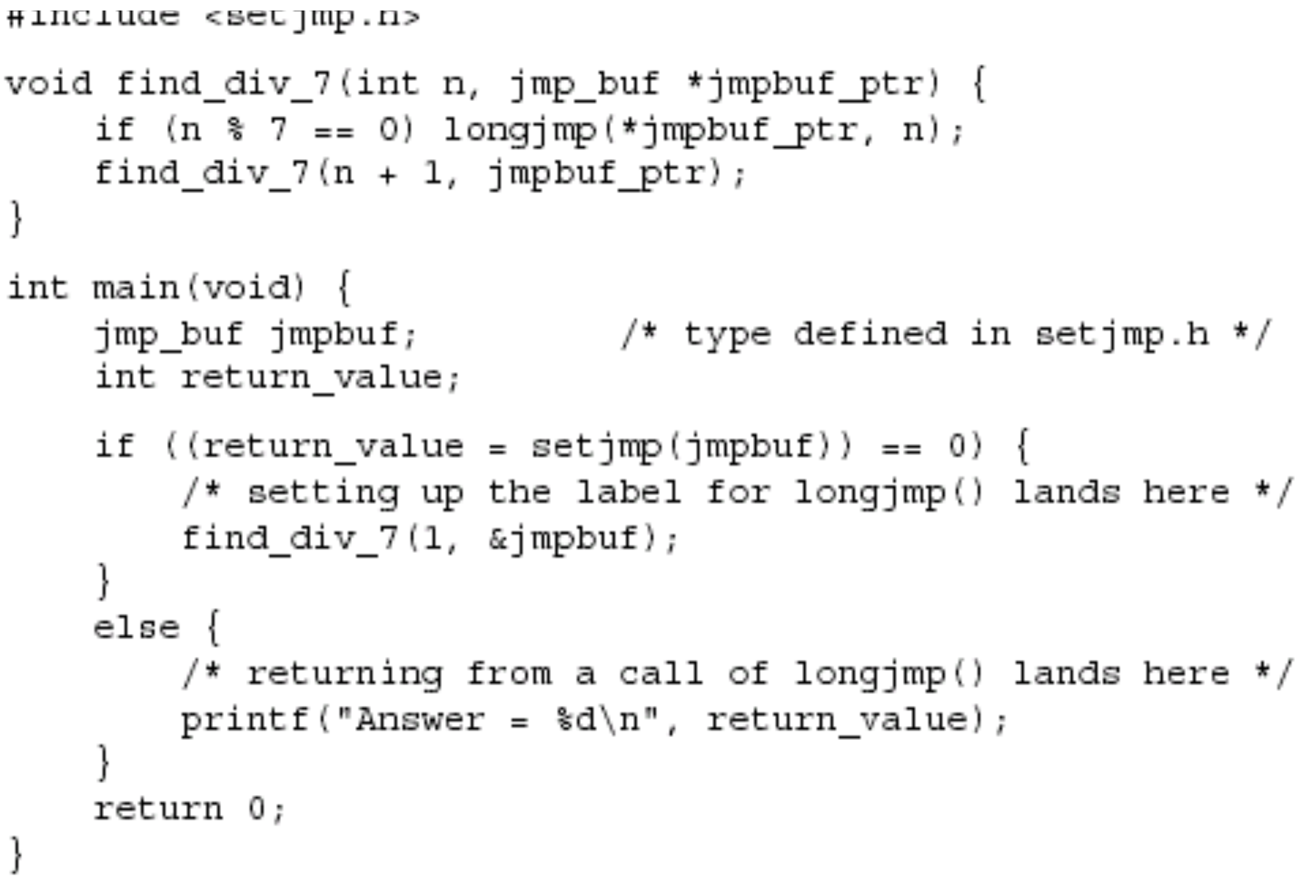


```
void level_0(void) {
    void level_1(void) {
        void level_2(void) {
            ...
            goto L_1;
            ...
        }
        ...
    L_1:...
        ...
    }
    ...
}
```

Non-local gotos in C

- setjmp remembers the current location and the stack frame
- longjmp jumps to the current location (popping many activation records)

Non-Local Transfer of Control in C



```
#include <setjmp.h>

void find_div_7(int n, jmp_buf *jmpbuf_ptr) {
    if (n % 7 == 0) longjmp(*jmpbuf_ptr, n);
    find_div_7(n + 1, jmpbuf_ptr);
}

int main(void) {
    jmp_buf jmpbuf;          /* type defined in setjmp.h */
    int return_value;

    if ((return_value = setjmp(jmpbuf)) == 0) {
        /* setting up the label for longjmp() lands here */
        find_div_7(1, &jmpbuf);
    }
    else {
        /* returning from a call of longjmp() lands here */
        printf("Answer = %d\n", return_value);
    }
    return 0;
}
```

Variable Length Frame Size

- C allows allocating objects of unbounded size in the stack

```
 void p() {
   int i;
   char *p;
   scanf(“%d”, &i);
   p = (char *) alloca(i*sizeof(int));
  }
```

- Some versions of Pascal allows conformant array value parameters

Limitations

- The compiler may be forced to store a value on a stack instead of registers
- The stack may not suffice to handle some language features

Frame-Resident Variables

- A variable x cannot be stored in register when:
  - x is passed by reference
  - Address of x is taken (&x)
  - is addressed via pointer arithmetic on the stack-frame (C varags)
  - x is accessed from a nested procedure
  - The value is too big to fit into a single register
  - The variable is an array
  - The register of x is needed for other purposes
  - Too many local variables
- An escape variable:
  - Passed by reference
  - Address is taken
  - Addressed via pointer arithmetic on the stack-frame
  - Accessed from a nested procedure

The Frames in Different Architectures

g(x, y, z) where x escapes

	Pentium	MIPS	Sparc
x	InFrame(8)	InFrame(0)	InFrame(68)
y	InFrame(12)	InReg(X_{157})	InReg(X_{157})
z	InFrame(16)	InReg(X_{158})	InReg(X_{158})
View Change	M[sp+0]←fp fp ←sp sp ←sp-K	sp ←sp-K $M[sp+K+0] \leftarrow r_2$ $X_{157} \leftarrow$ r4 $X_{158} \leftarrow$ r5	save %sp, -K, %sp $M[fp+68] \leftarrow i_0$ $X_{157} \leftarrow i_1$ $X_{158} \leftarrow i_2$

Limitations of Stack Frames

- A local variable of P cannot be stored in the activation record of P if its duration exceeds the duration of P
- Example 1: Static variables in C
  (own variables in Algol)

```
void p(int x)
{
   static int y = 6 ;
   y += x;
 }
```

- Example 2: Features of the C language

```
int * f()
{ int x ;
   return &x ;
}
```

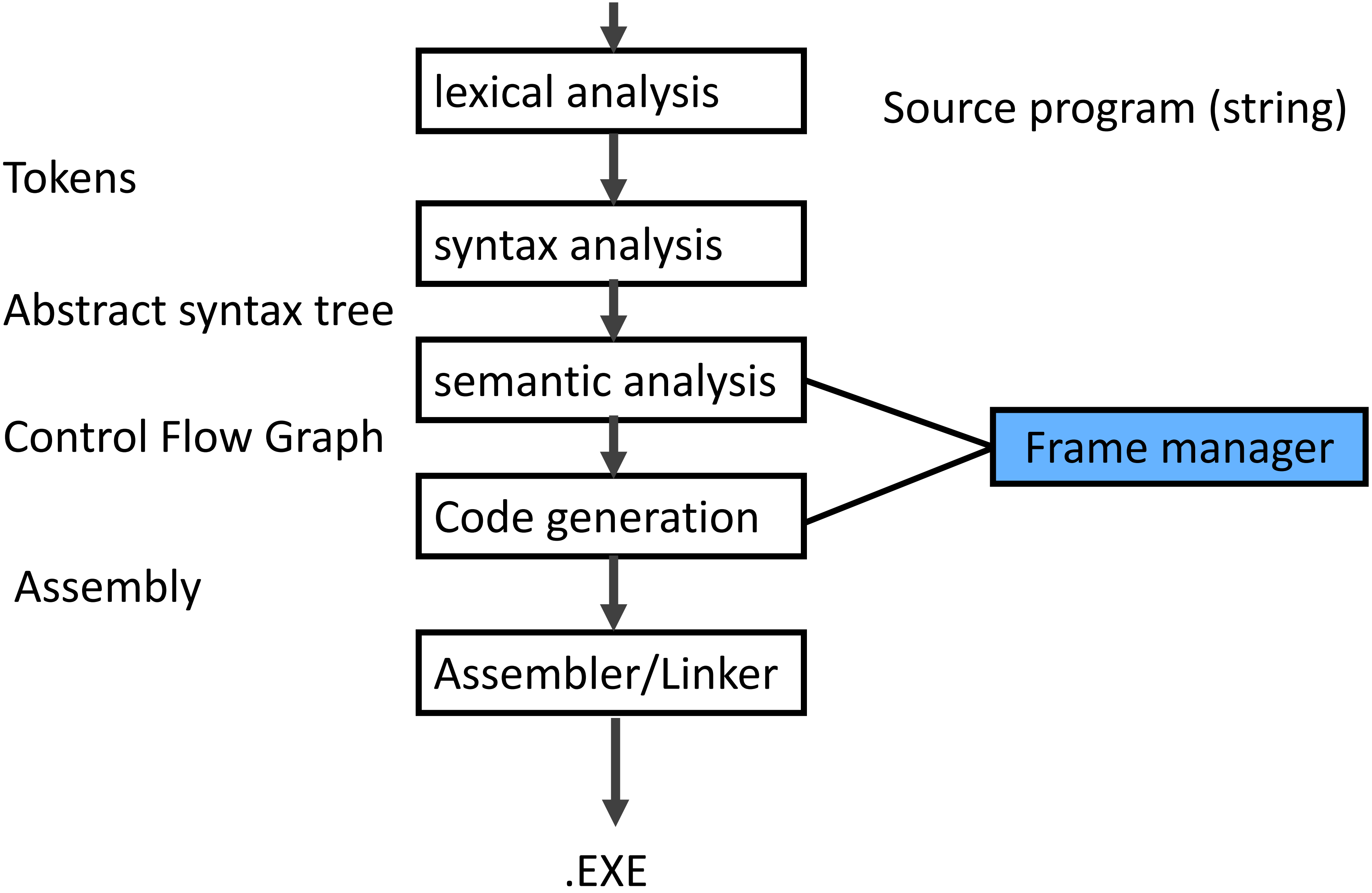
- Example 3: Dynamic allocation

```
int * f()  { return (int *)
malloc(sizeof(int)); }
```

Compiler Implementation

- Hide machine dependent parts
- Hide language dependent part
- Use special modules

Basic Compiler Phases



Source program (string)

lexical analysis

Tokens

syntax analysis

Abstract syntax tree

semantic analysis

Control Flow Graph

Code generation

Assembly

Assembler/Linker

.EXE

Frame manager

Hidden in the frame ADT

- Word size
- The location of the formals
- Frame resident variables
- Machine instructions to implement “shift-of-view” (prologue/epilogue)
- The number of locals “allocated” so far
- The label in which the machine code starts

Activation Records: Summary

- compile time memory management for procedure data
- works well for data with well-scoped lifetime
  - deallocation when procedure returns

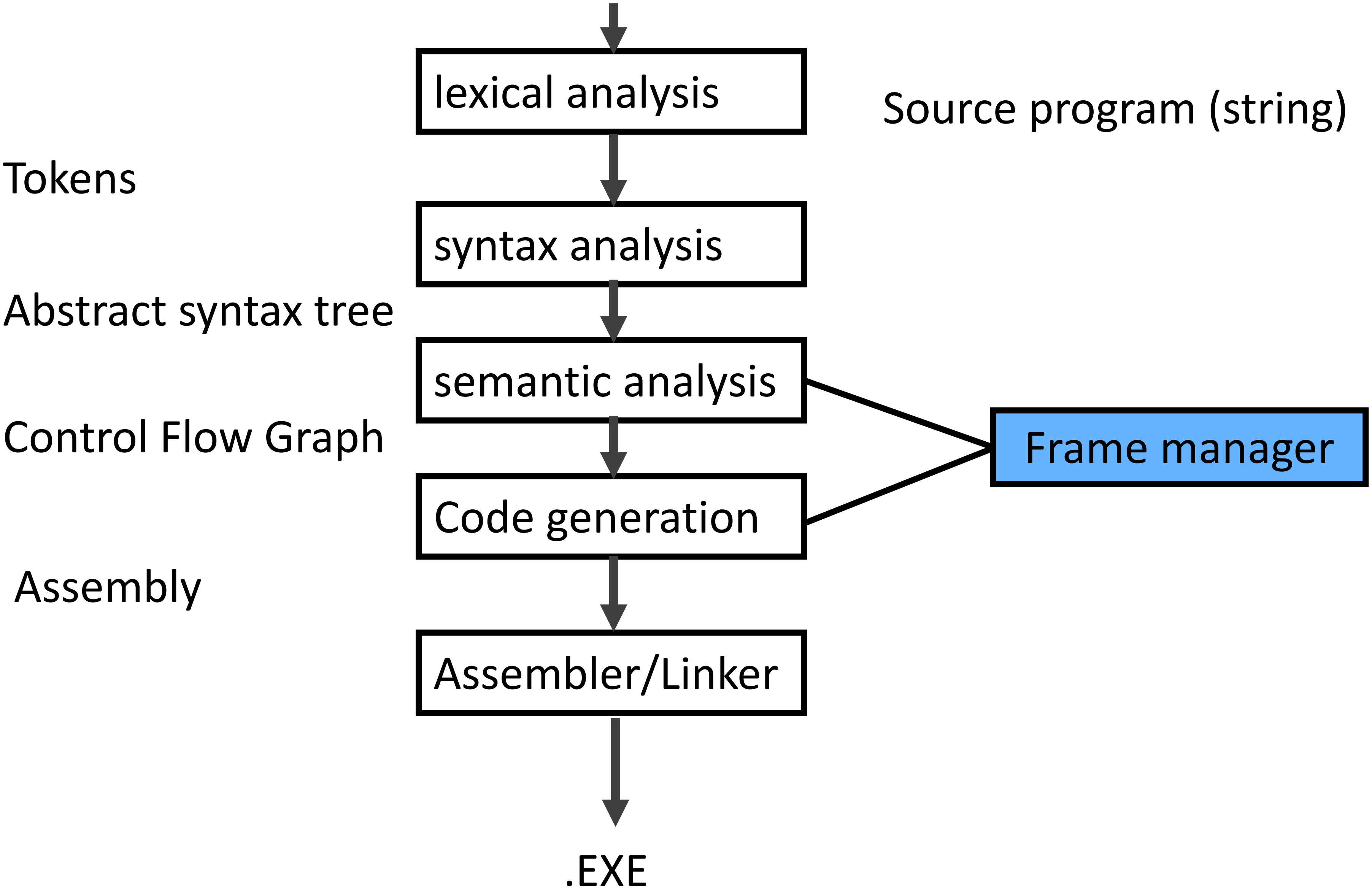
Compilation

Lecture 8b

Optimizations

Noam Rinetzky

Basic Compiler Phases



Source program (string)

lexical analysis

Tokens

syntax analysis

Abstract syntax tree

semantic analysis

Control Flow Graph

Code generation

Frame manager

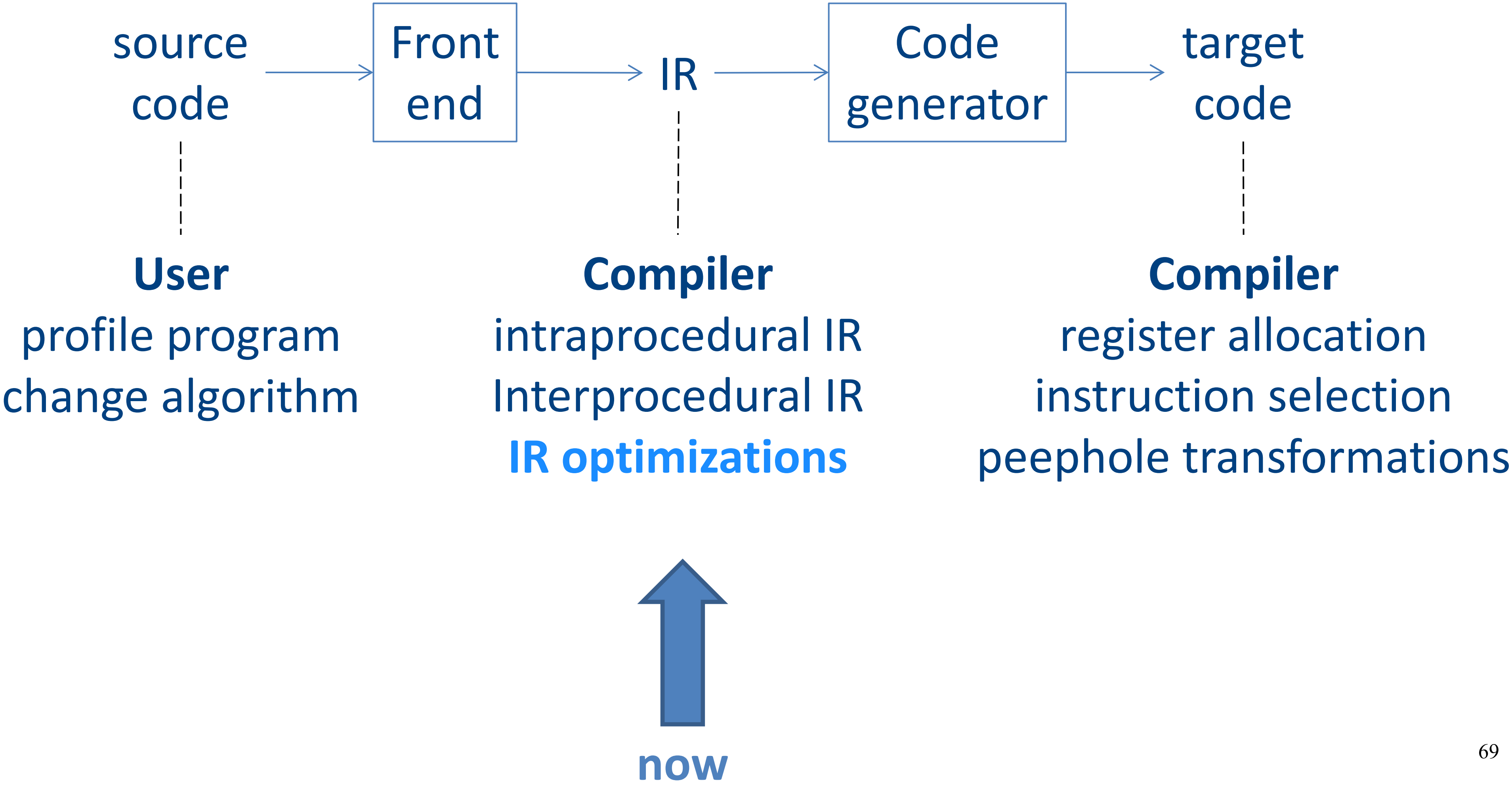
Assembly

Assembler/Linker

.EXE

IR Optimization

Optimization points



source code → Front end → IR → Code generator → target code

User
profile program
change algorithm

Compiler
intraprocedural IR
Interprocedural IR
IR optimizations

Compiler
register allocation
instruction selection
peephole transformations

now

IR Optimization

- Making code better

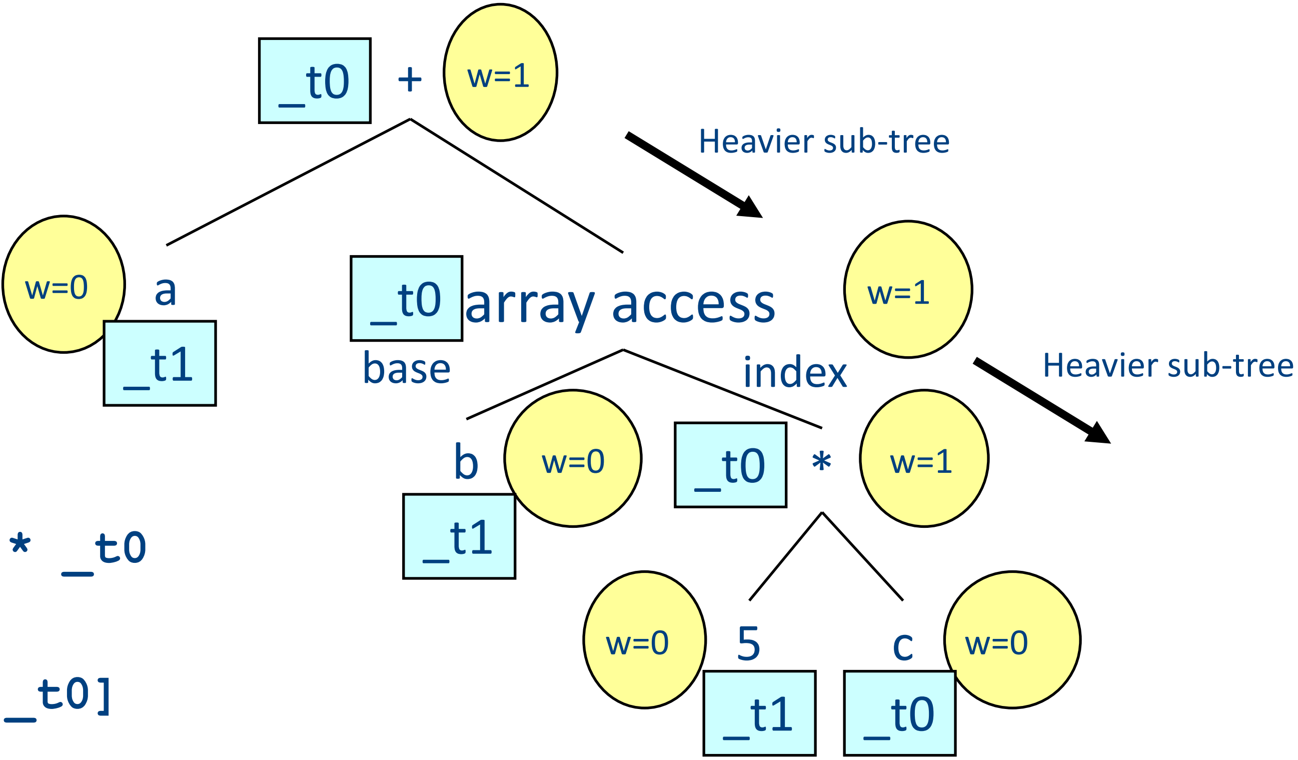
IR Optimization

- Making code “better”

“Optimized” evaluation

_t0 = **cgen**(a+b[5*c])

Phase 2: - use weights to decide on order of translation



```
_t0 = c
_t1 = 5
_t0 = _t1 * _t0
_t1 = b
_t0 = _t1[_t0]
_t1 = a
_t0 = _t1 + _t0
```

But what about…

```
a := 1 + 2;
y := a + b;
x := a + b  + 8;
z := b + a;

a := a + 1;
w:= a + b;
```

Overview of IR optimization

- Formalisms and Terminology
  - Control-flow graphs
  - Basic blocks
- Local optimizations
  - Speeding up small pieces of a procedure
- Global optimizations
  - Speeding up procedure as a whole
- The dataflow framework
  - Defining and implementing a wide class of optimizations

Program Analysis

- In order to optimize a program, the compiler has to be able to reason about the properties of that program
- An analysis is called **sound** if it never asserts an incorrect fact about a program
- All the analyses we will discuss in this class are sound
  - *(Why?)*

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

"At this point in the program, **`x`** holds some integer value"

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

"At this point in the program, **x** is either 137 or 42"

(Un)Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

“At this point in the program, **`x`** is 137”

Soundness & Precision

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

"At this point in the program, **x** is either 137, 42, or 271"

Semantics-preserving optimizations

- An optimization is semantics-preserving if it does not alter the semantics of the original program
- Examples:
  - Eliminating unnecessary temporary variables
  - Computing values that are known statically at compile-time instead of runtime
  - Evaluating constant expressions outside of a loop instead of inside
- Non-examples:
  - Replacing bubble sort with quicksort (why?)
  - The optimizations we will consider in this class are all semantics-preserving

A formalism for IR optimization

- Every phase of the compiler uses some new abstraction:
  - Scanning uses regular expressions
  - Parsing uses CFGs
  - Semantic analysis uses proof systems and symbol tables
  - IR generation uses ASTs
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization

Visualizing IR

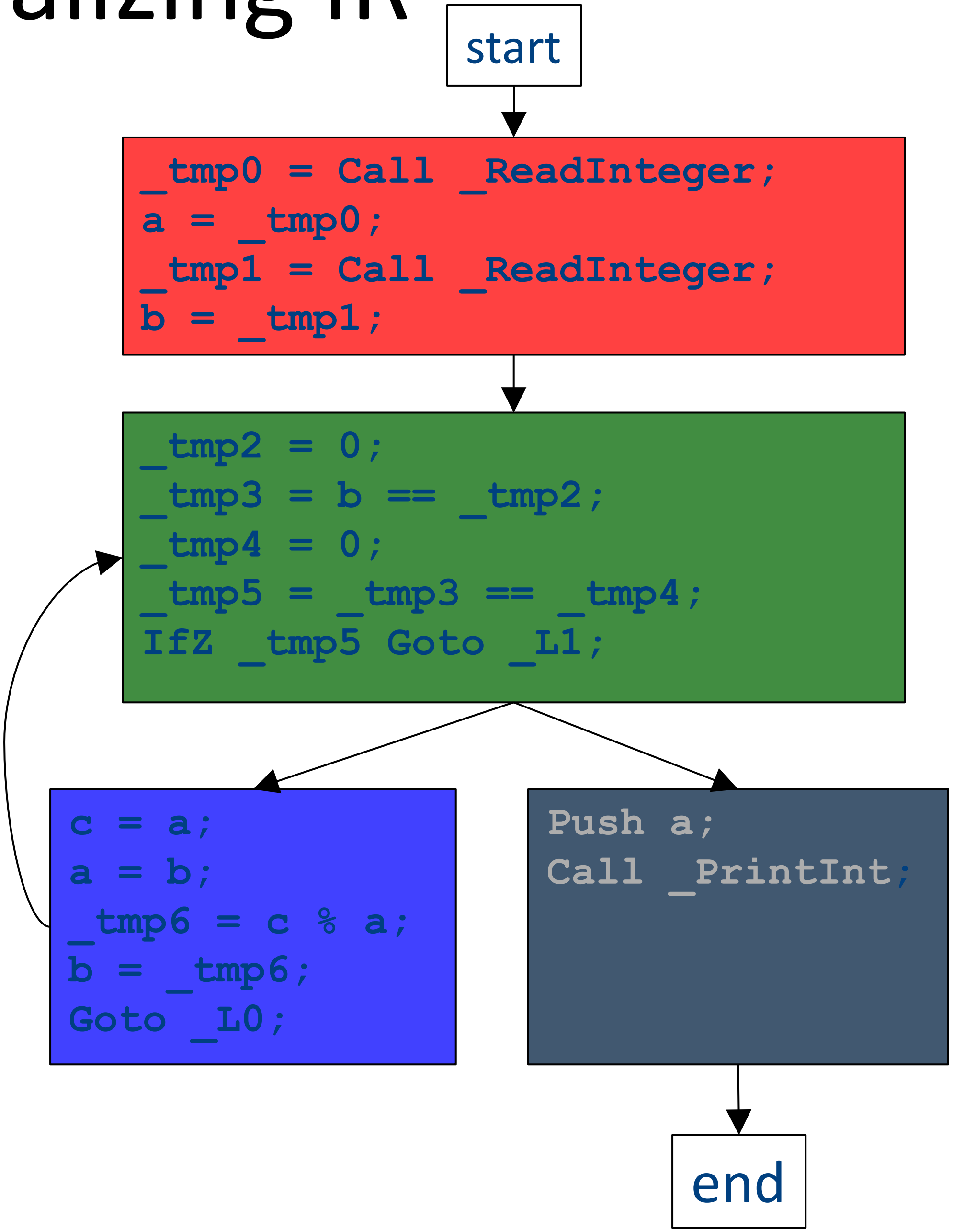
```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```



start

```
_tmp0 = Call _ReadInteger;
a = _tmp0;
_tmp1 = Call _ReadInteger;
b = _tmp1;
```

```
_tmp2 = 0;
_tmp3 = b == _tmp2;
_tmp4 = 0;
_tmp5 = _tmp3 == _tmp4;
IfZ _tmp5 Goto _L1;
```

```
c = a;
a = b;
_tmp6 = c % a;
b = _tmp6;
Goto _L0;
```

```
Push a;
Call _PrintInt;
```

end

Basic blocks

- A basic block is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence
- Informally, a sequence of instructions that always execute as a group

Control-Flow Graphs

- A control-flow graph (CFG) is a graph of the basic blocks in a function
- The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context free grammar”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block
- There is a dedicated node for the start and end of a function

Types of optimizations

- An optimization is local if it works on just a single basic block
- An optimization is global if it works on an entire control-flow graph
- An optimization is interprocedural if it works across the control-flow graphs of multiple functions
  - We won't talk about this in this course

Basic blocks exercise

```
int main() {
   int x;
   int y;
   int z;

   y = 137;
   if (x == 0)
      z = y;
   else
      x = y;
}
```

```
START:
      _t0 = 137;
      y = _t0;
      IfZ x Goto _L0;
      t1 = y;
      z = _t1;
      Goto END:
_L0:
      _t2 = y;
      x = _t2;
END:
```

Divide the code into basic blocks

Control-flow graph exercise

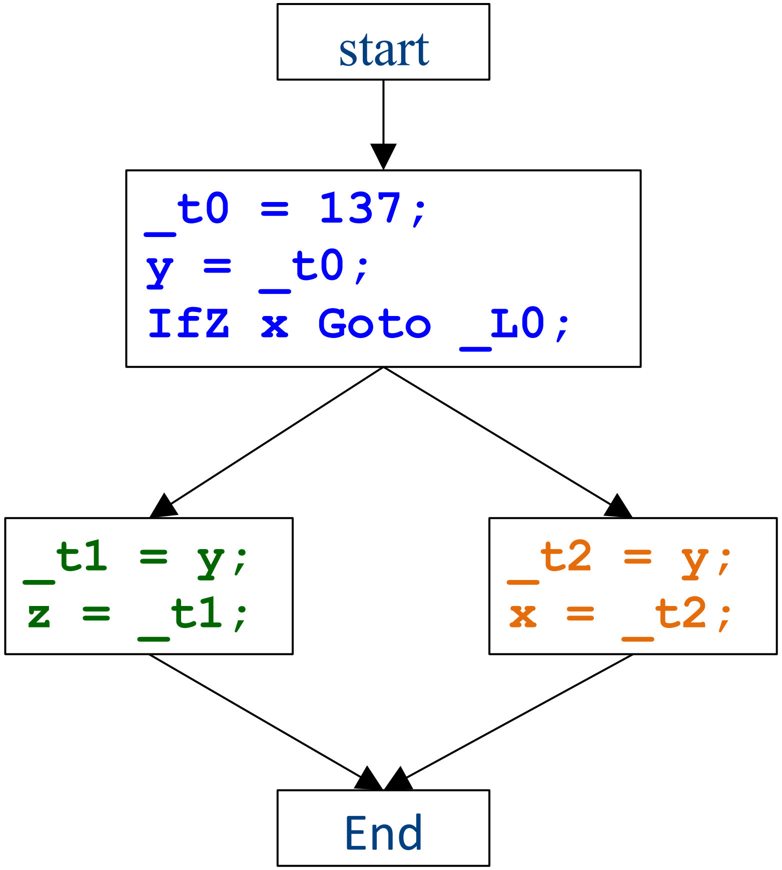
```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```

```
START:
        _t0 = 137;
        y = _t0;
        IfZ x Goto _L0;
        t1 = y;
        z = _t1;
        Goto END:
_L0:
        _t2 = y;
        x = _t2;
END:
```

Draw the control-flow graph

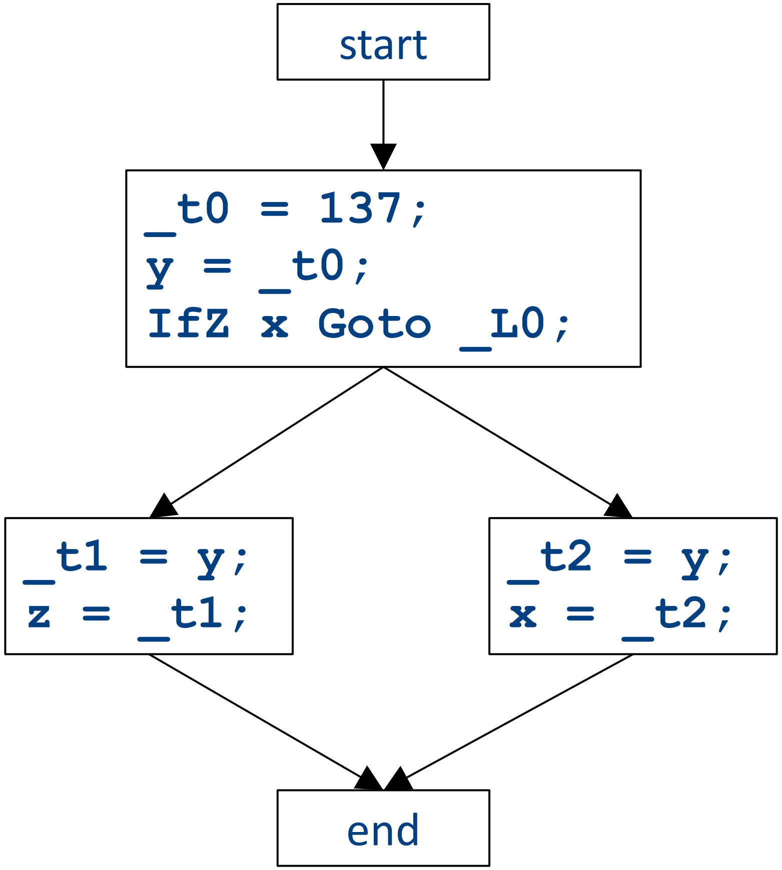
Control-flow graph exercise

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



Local optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



start

```
_t0 = 137;
y = _t0;
IfZ x Goto _L0;
```

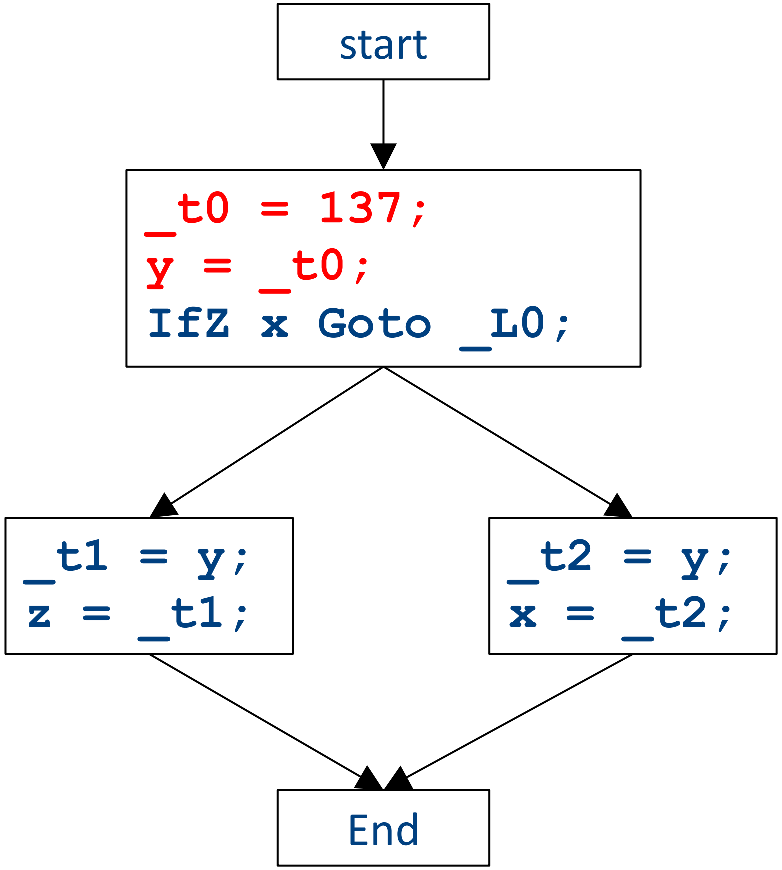
```
_t1 = y;
z = _t1;
```

```
_t2 = y;
x = _t2;
```

end

Local optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



start

```
_t0 = 137;
y = _t0;
IfZ x Goto _L0;
```

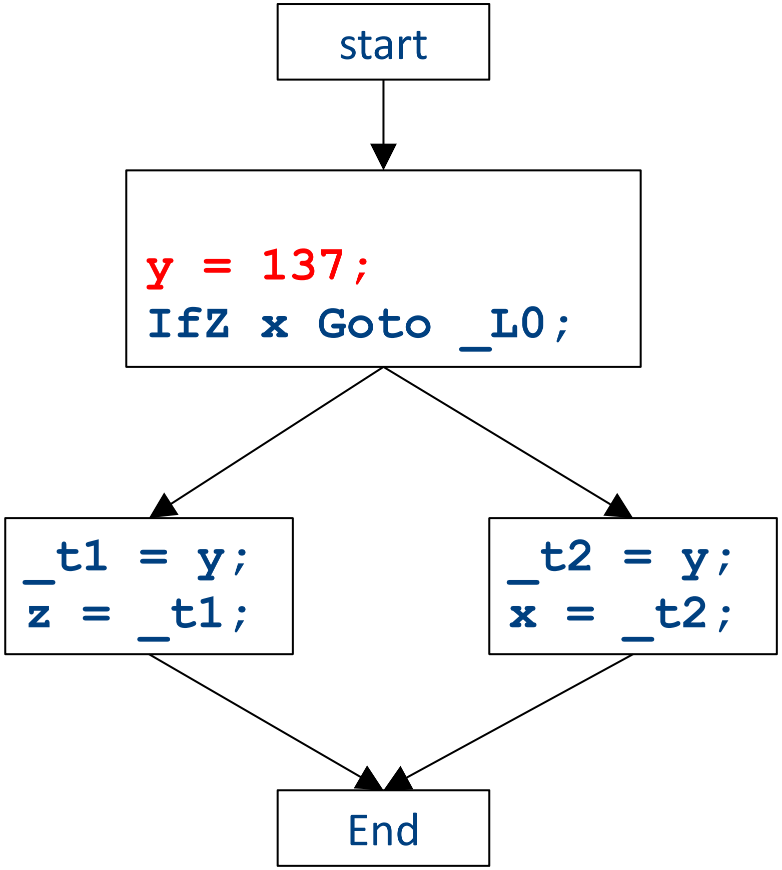
```
_t1 = y;
z = _t1;
```

```
_t2 = y;
x = _t2;
```

End

Local optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



start

```
y = 137;
IfZ x Goto _L0;
```

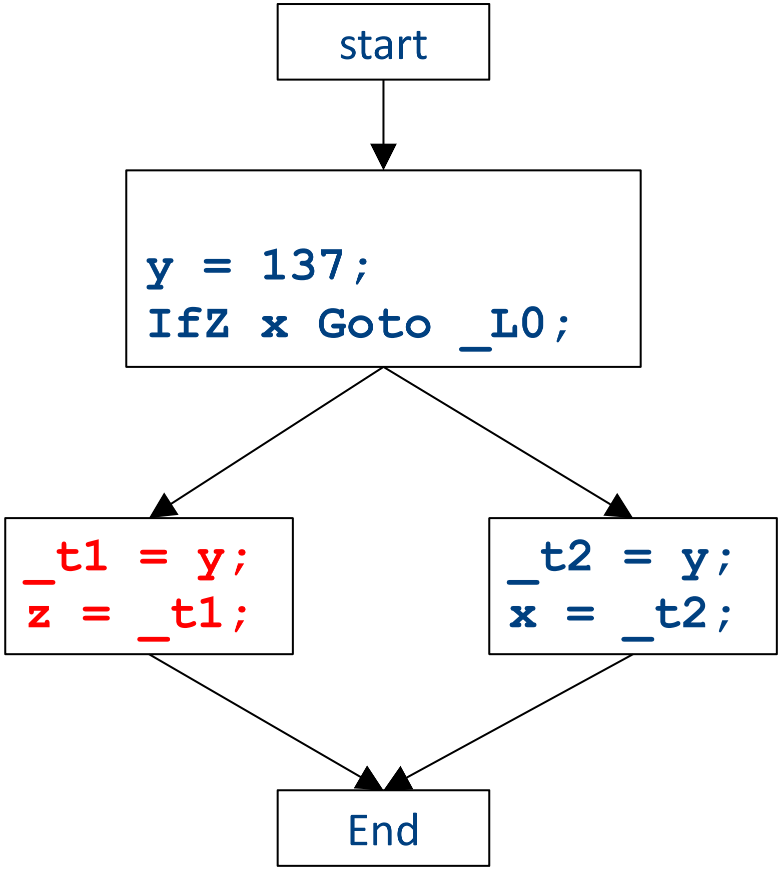
```
_t1 = y;
z = _t1;
```

```
_t2 = y;
x = _t2;
```

End

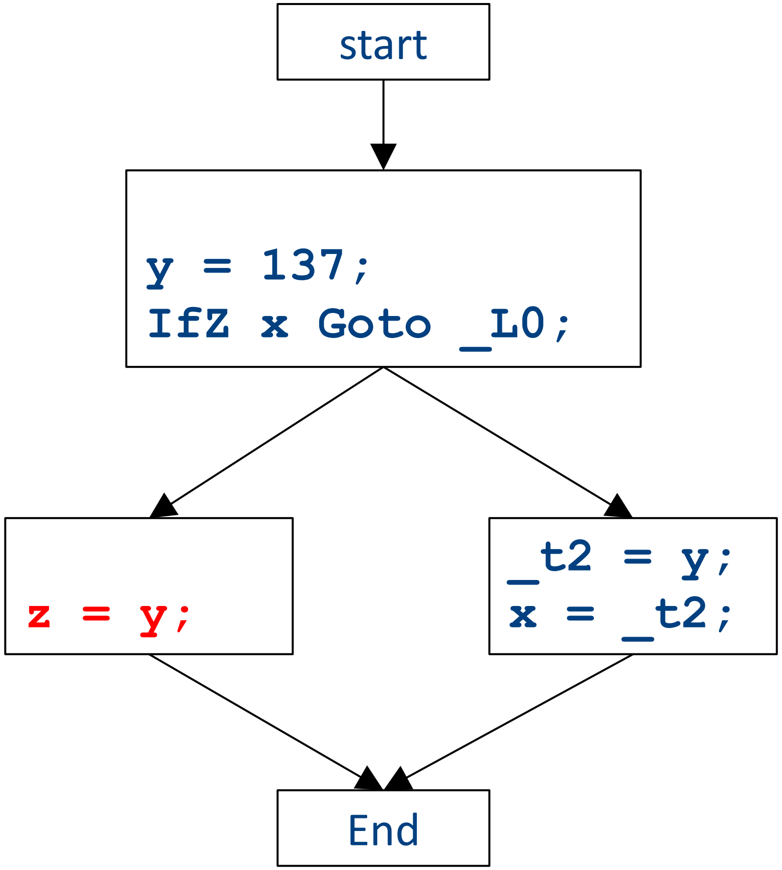
Local optimizations

```
int main() {
   int x;
   int y;
   int z;

   y = 137;
   if (x == 0)
      z = y;
   else
      x = y;
}
```



Local optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



start

```
y = 137;
IfZ x Goto _L0;
```

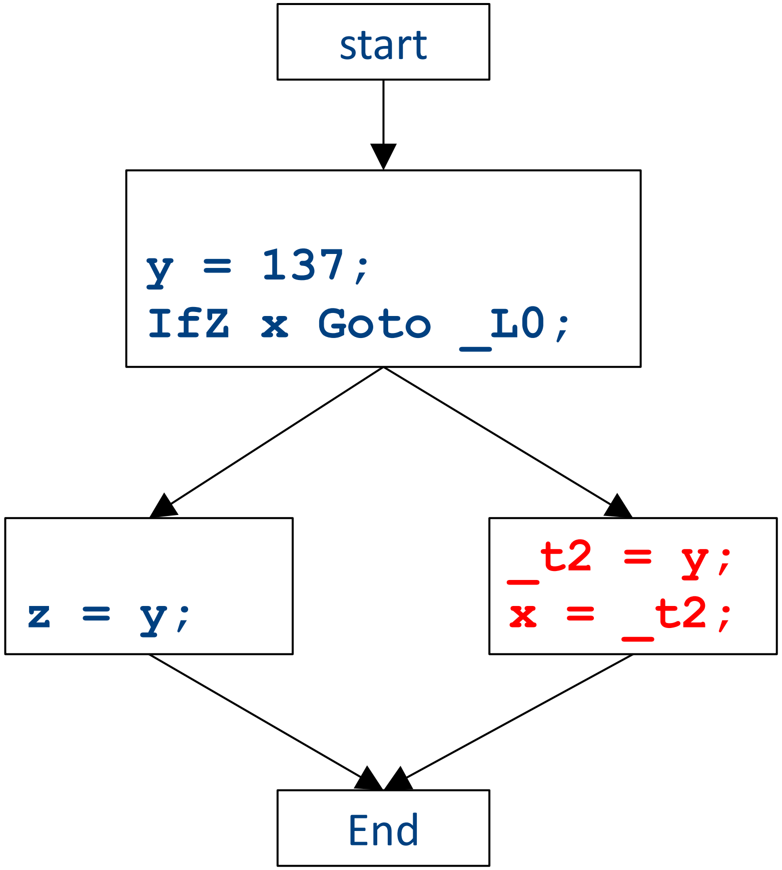
```
z = y;
```

```
_t2 = y;
x = _t2;
```

End

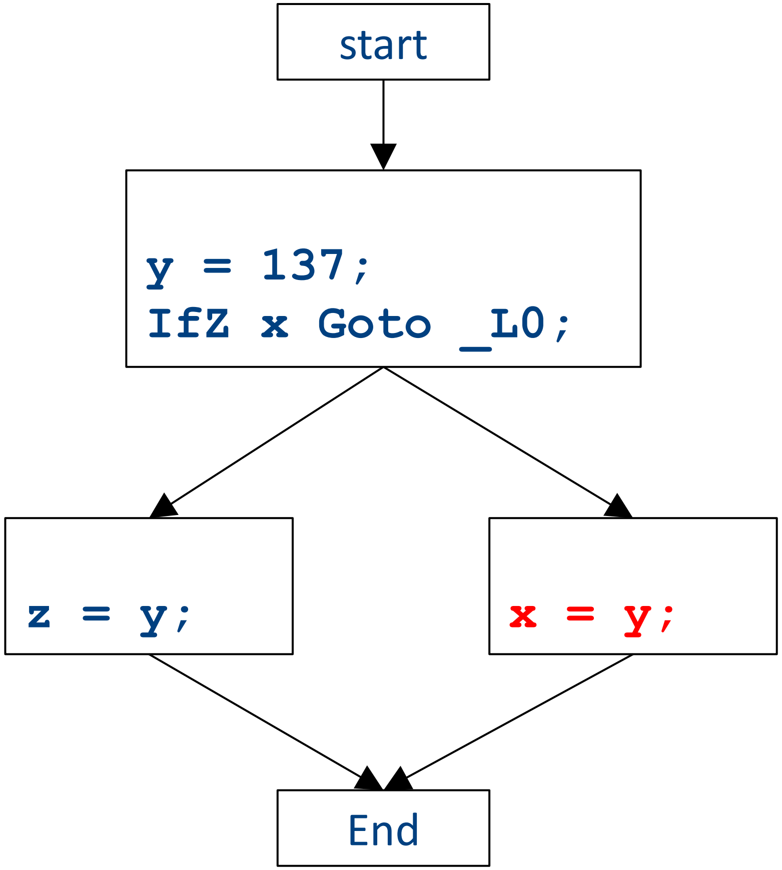
Local optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



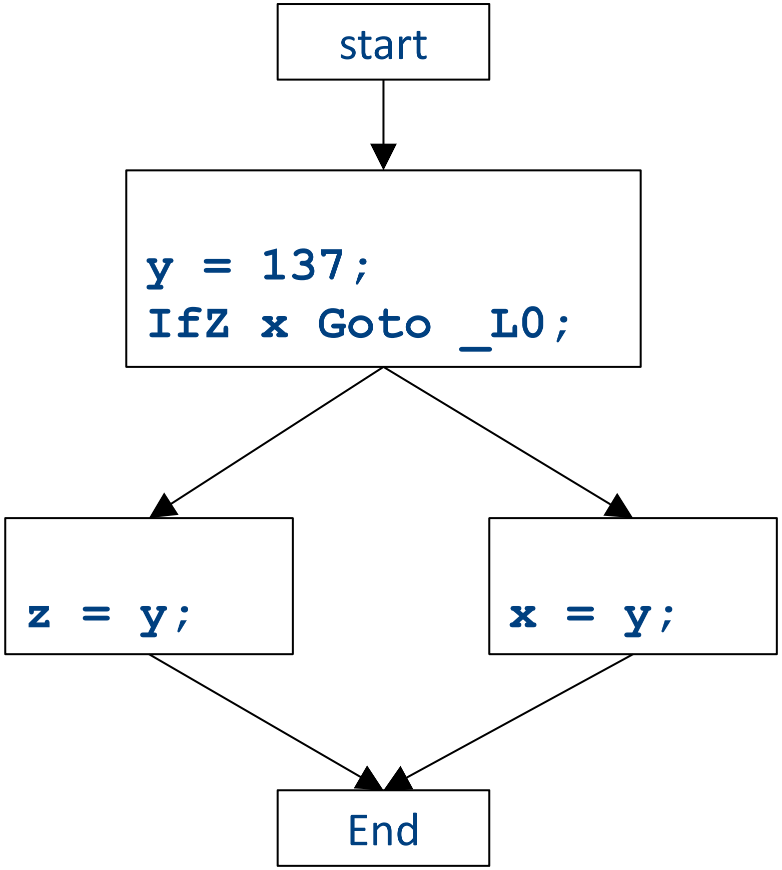
Local optimizations

```
int main() {
   int x;
   int y;
   int z;

   y = 137;
   if (x == 0)
      z = y;
   else
      x = y;
}
```



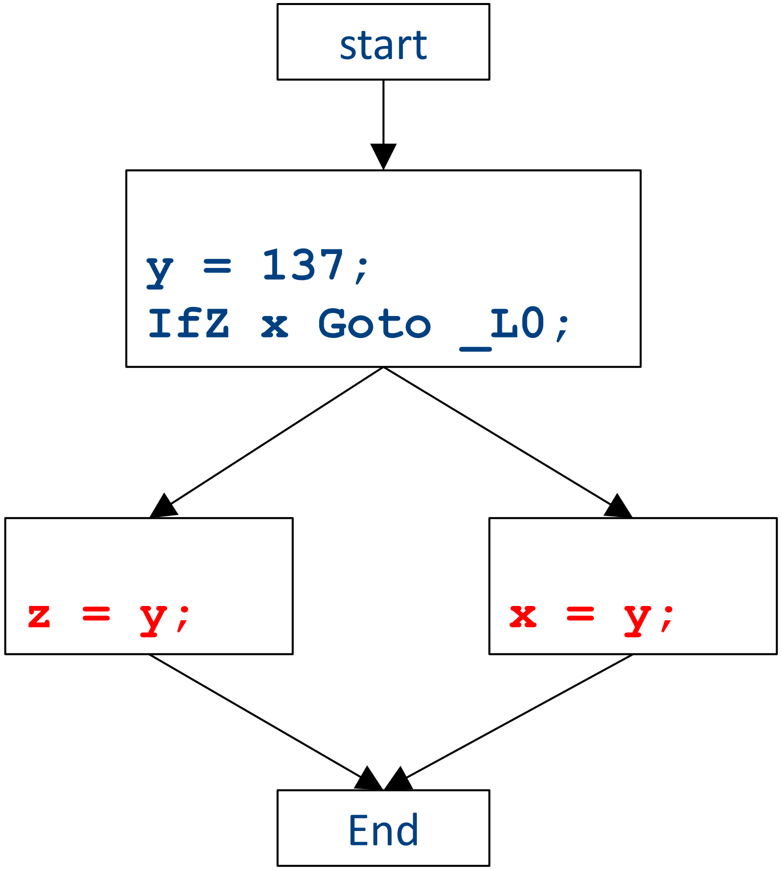
Global optimizations

```
int main() {
   int x;
   int y;
   int z;

   y = 137;
   if (x == 0)
      z = y;
   else
      x = y;
}
```



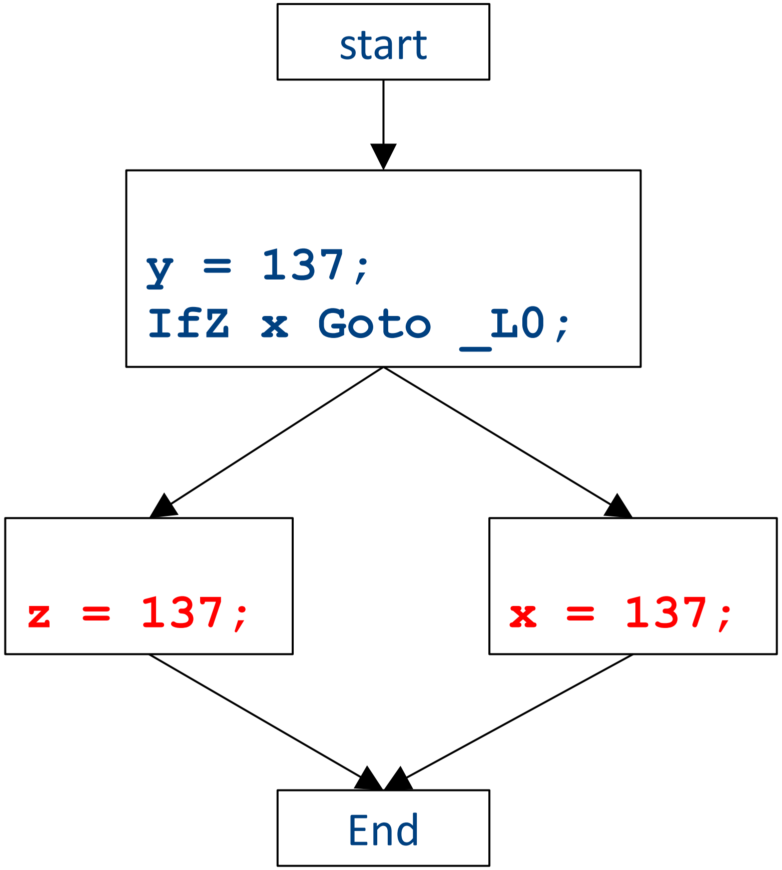
Global optimizations

```
int main() {
   int x;
   int y;
   int z;

   y = 137;
   if (x == 0)
      z = y;
   else
      x = y;
}
```



Global optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



start

```
y = 137;
IfZ x Goto _L0;
```

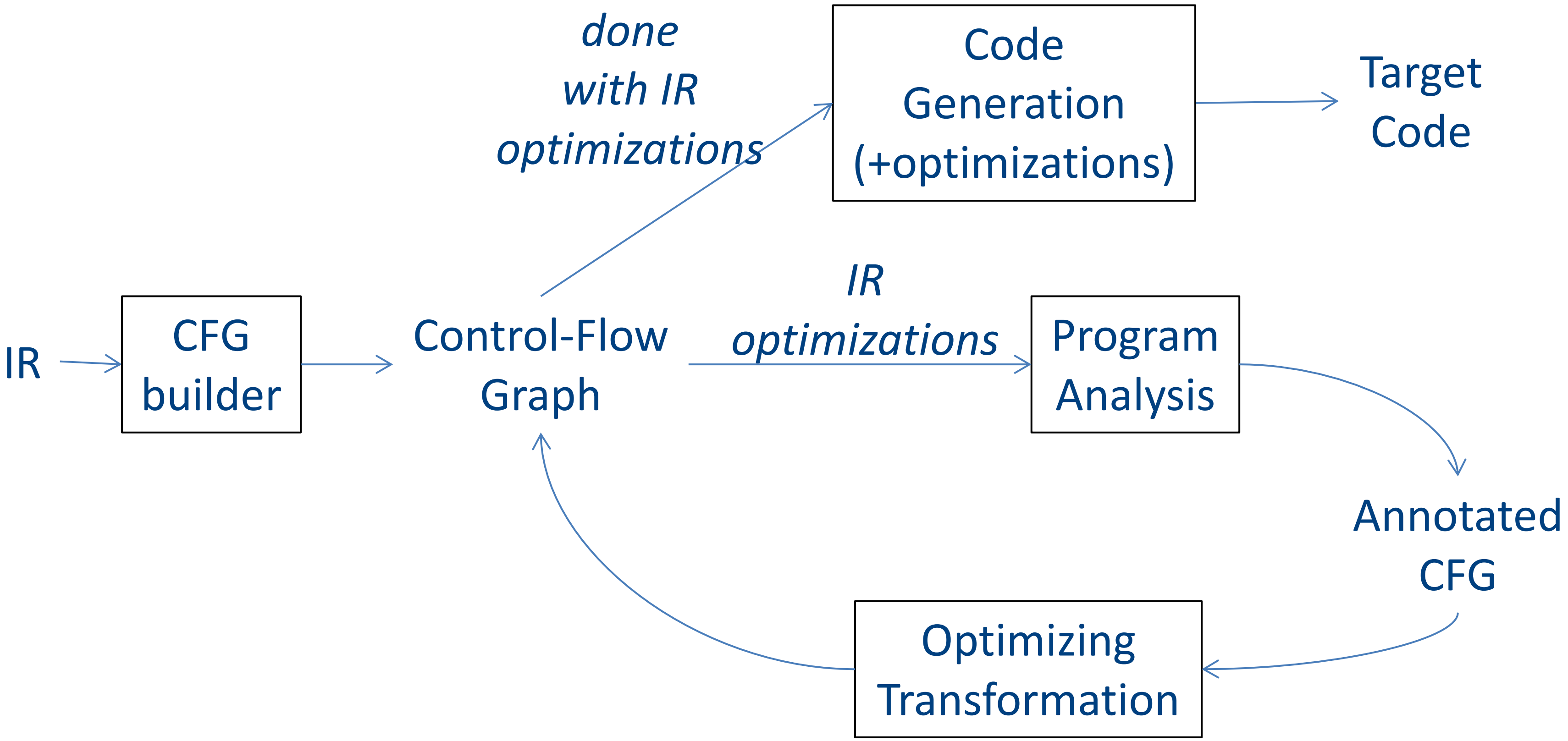
```
z = 137;
```

```
x = 137;
```

End

Local Optimizations

Optimization path



IR

CFG builder

Control-Flow Graph

done with IR optimizations

Code Generation (+optimizations)

Target Code

IR optimizations

Program Analysis

Annotated CFG

Optimizing Transformation

Example

```
Object x;
int a;
int b;
int c;

x = ne
a = 4;
c = a
x.fn(a
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
```

For brevity:

Simplified IR for procedure returns

```
+ b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
Class Object {
  method fn(int);
}
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Explaining the program

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

Class Object {
method fn(int);
}

For simplicity, ignore Popping return value, parameters etc.

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Size of Object

Object Class

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
Class Object {
  method fn(int);
}
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Explaining the program

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
Class Object {
  method fn(int);
}
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Explaining the program

Example

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

Class Object {
  method fn(int);
}

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Points to ObjectC

Start of `fn`

Explaining the program

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b
  …
  v2 = a op b
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common **S**ubexpression **E**limination

- If we have two variable assignments
  v1 = a op b [or: v1 = a]
  …
  v2 = a op b [or: v2 = a]
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b [or: v1 = a]
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = a + b;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = _tmp4;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = 4;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = _tmp4;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = _tmp4;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = _tmp4;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Common subexpression elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Copy Propagation

- If we have a variable assignment
  v1 = v2
  then as long as v1 and v2 are not reassigned, we can rewrite expressions of the form
  a = … v1 …
  as
  a = … v2 …
  provided that such a rewrite is legal

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
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*(_tmp1) = _tmp2;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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_tmp0 = 4;
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_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
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a = 4;
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a = _tmp3;
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c = _tmp4;
_tmp5 = c;
_tmp6 = *(x);
_tmp7 = *(_tmp6);
Push _tmp5;
Push x;
Call _tmp7;
```

Copy Propagation

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int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push _tmp5;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = a + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push _tmp5;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
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_tmp0 = 4;
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c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push _tmp5;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
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_tmp0 = 4;
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_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
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a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push _tmp5;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
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_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
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_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push c;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = *(_tmp1);
_tmp7 = *(_tmp6);
Push c;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(_tmp6);
Push c;
Push _tmp1;
Call _tmp7;
```

Is this transformation OK?
What do we need to know?

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(_tmp6);
Push c;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp3;
_tmp4 = _tmp3 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
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_tmp3 = _tmp0;
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c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp0;
_tmp4 = _tmp0 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Dead Code Elimination

- An assignment to a variable v is called dead if the value of that assignment is never read anywhere
- Dead code elimination removes dead assignments from IR
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned

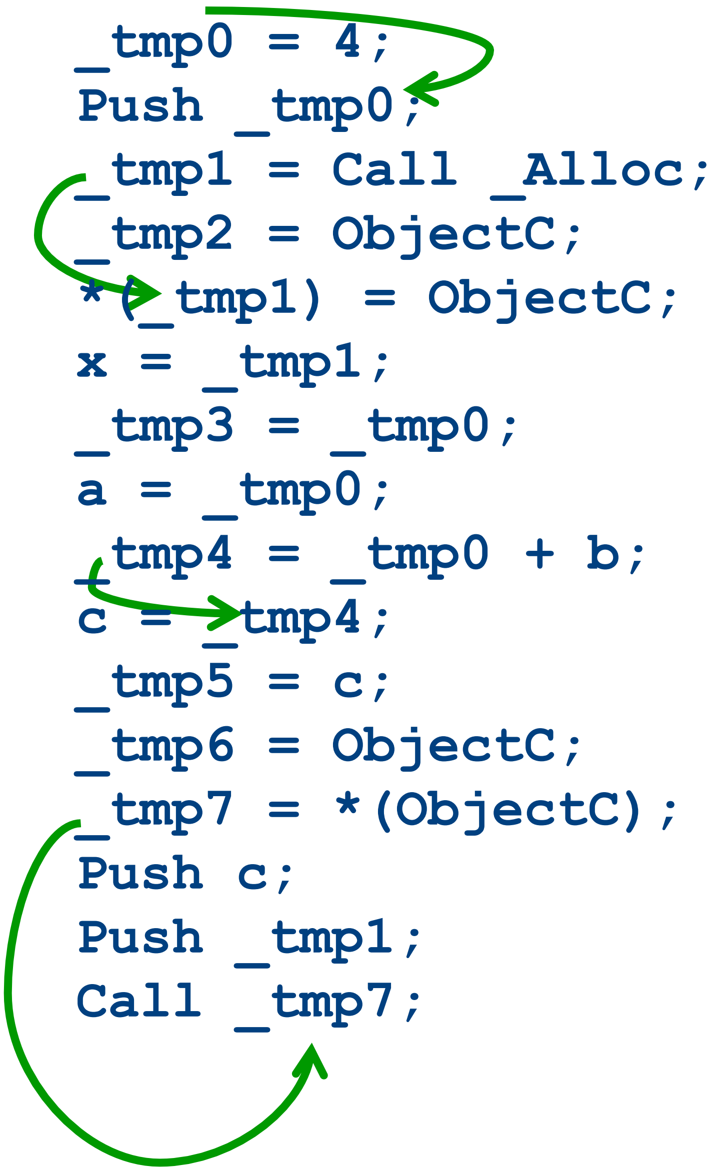
Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp0;
_tmp4 = _tmp0 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```



```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp0;
_tmp4 = _tmp0 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new
Object;
a = 4;
c = a + b;
x.fn(a + b);
```

values never read

values never read

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;
_tmp2 = ObjectC;
*(_tmp1) = ObjectC;
x = _tmp1;
_tmp3 = _tmp0;
a = _tmp0;
_tmp4 = _tmp0 + b;
c = _tmp4;
_tmp5 = c;
_tmp6 = ObjectC;
_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new
Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4;
Push _tmp0;
_tmp1 = Call _Alloc;

*(_tmp1) = ObjectC;




_tmp4 = _tmp0 + b;
c = _tmp4;



_tmp7 = *(ObjectC);
Push c;
Push _tmp1;
Call _tmp7;
```

Applying local optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization
- Common subexpression elimination eliminates unnecessary statements
- Copy propagation helps identify dead code
- Dead code elimination removes statements that are no longer needed
- To get maximum effect, we may have to apply these optimizations numerous times

Applying local optimizations example

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying local optimizations example

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Common sub-expression elimination

Applying local optimizations example

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Which optimization should we apply here?

Copy propagation

Applying local optimizations example

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Which optimization should we apply here?

Applying local optimizations example

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Which optimization should we apply here?

Common sub-expression elimination (again)

Other types of local optimizations

- Arithmetic Simplification
  - Replace “hard” operations with easier ones
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- Constant Folding
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite `x = 4 * 5;` as `x = 20;`

Optimizations and analyses

- Most optimizations are only possible given some analysis of the program's behavior
- In order to implement an optimization, we will talk about the corresponding program analyses

Available expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the available expressions in a program
- An expression is called available if some variable in the program holds the value of that expression
- In common subexpression elimination, we replace an available expression by the variable holding its value
- In copy propagation, we replace the use of a variable by the available expression it holds

Finding available expressions

- Initially, no expressions are available
- Whenever we execute a statement **a = b *op* c**:
  - Any expression holding **a** is invalidated
  - The expression **a = b *op* c** becomes available
- **Idea:** Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable

Available expressions example

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
 { b = x, d = x, e = a + b }
f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {   }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = d;
 { b = x, d = a + b, e = a + b }
d = b;
 { b = x, d = x, e = a + b }
f = e;
 { b = x, d = x, e = a + b, f = a + b }
```

Common sub-expression elimination

```
 {  }
a = b + 2;
 { a = b + 2}
b = x;
 { b = x}
d = a + b;
 { b = x, d = a + b }
e = a + b;
 { b = x, d = a + b, e = a + b }
d = x;
 { b = x, d = x, e = a + b }
f = a + b;
 { b = x, d = x, e = a + b, f = a + b }
```