1. Show that if
\[ 3 \binom{n}{k} 3^{-\binom{k}{2}} < 1 \]
then there is a coloring of the edges of the complete graph on \( n \) vertices by 3 colors with no monochromatic clique of size \( k \).

2. Let \( A \) be a set of \( 3m \) points in the Euclidean plane, and suppose that the distance between any two of these points is smaller than \( \sqrt{2} \). Prove that the number of pairs \( P, Q \) of points of \( A \) so that the distance between \( P \) and \( Q \) is at least 1 does not exceed \( 3m^2 \).

3. Show that if the edges of a graph \( G \) can be covered by two trees then its chromatic number is at most 4.

4. Let \( G \) be a simple graph with maximum degree 7 containing no clique of size 4. Prove that the chromatic number of \( G \) is at most 6.
Hint: Show first that one can delete from \( G \) a bipartite graph leaving each degree in what’s left at most 3.

5. Let \( G \) be a graph with chromatic number \( \chi(G) = 11 \) and with no cycle of length at most 20. Show that the number of vertices of \( G \) exceeds the world population (which is close to 7,000,000,000).

6. Let \( G \) be a 2-connected, simple 5-regular planar graph drawn in the plane so that every face contains the same number of edges. What is the number of vertices of \( G \)? Prove your claim.