1. Prove that every simple graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

2. Prove that the number of graphs on $n$ vertices with all degrees even is $2^{\binom{n-1}{2}}$.

3. Let $0 < d_1 \leq d_2 \leq \ldots \leq d_n$ be integers. Prove that there exists a tree with degrees $d_1, \ldots, d_n$ if and only if

\[ d_1 + \ldots + d_n = 2n - 2. \]

4. Let $G = (V, E)$ be a connected graph with an even number of vertices. Prove that $G$ contains a spanning subgraph $G' = (V, F)$ with all degrees odd.

5. Prove that every graph $G = (V, E)$ with $|E| = m$ edges has a bipartition $V = V_1 \cup V_2$ such that the number of edges of $G$ crossing between $V_1$ and $V_2$ is at least $m/2$.

6. (a) Let $G$ be a graph with all degrees at least three. Prove that $G$ contains a cycle with a chord.

(b) Let $G$ be a graph on $n \geq 4$ vertices with $2n - 3$ edges. Prove that $G$ contains a cycle with a chord.

7. Prove that every graph $G$ with minimal degree $d$ contains every tree on $d + 1$ vertices as a subgraph.

8. Compute the number of spanning trees in the complete bipartite graph $K_{m,n}$. 