Improved Lower Bounds for Non-Utilitarian Truthfulness

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Outline

1. Introduction
   - Algorithmic Mechanism Design
   - Our Results

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2. Unrelated Machines Scheduling
   - The Problem
   - The Lower Bound

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3. Concluding Remarks

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Algorithmic mechanism design

- Studies algorithmic problems in scenarios where the input is presented by strategic agents.
- Focuses on the development of truthful mechanisms.

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May declare any fallacious input in order to manipulate the algorithm in a way that will maximize its own utility.

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Truthful mechanism

A way to motivate the agents to truthfully report their inputs.

- Allocation algorithm – attends to the algorithmic issue (solves the underlying algorithmic problem).
- Payment scheme – addresses the issue of truthfulness (compensates the agents for revealing the truth).

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- There are non-utilitarian goal functions, which cannot be optimally implemented in a truthful manner.

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Given any optimal allocation algorithm, there is no payment scheme that makes the mechanism truthful.

Truthful implementation of non-utilitarian functions is

- not a computational difficulty.
- (in some sense) like an information theoretic limitation.

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Tru

Understand the inherent limitations in the infrastructure of non-utilitarian truthfulness.

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Workload minimization in inter-domain routing problem

- Lower bound of 2 for any truthful deterministic mechanism, and any universal truthful randomized mechanism.
- Improve the lower bounds of 1.618 and 1.309, due to Mu'alem and Schapira [SODA '07].

We study two non-utilitarian multi-parameter problems:

Workload minimization in inter-domain routing problem

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Unrelated machines scheduling problem

- Lower bound of $1 + \sqrt{2}$ for any truthful deterministic mechanism when the number of machines is at least 3.
- Comparable to a result by Christodoulou, Koutsoupias and Vidali [SODA '07]. Our approach is considerably simpler.

Outline

Input:

- n machines, and m tasks.
- The execution time of task j on machine i is t_{ij}.

$$t = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1m} \\ t_{21} & t_{22} & \dots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \dots & t_{nm} \end{pmatrix}$$

- Machine 2's execution times.
- Task 1's execution times.

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- Machine 2's execution times.
- Task 1's execution times.

$$x = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{pmatrix}$$

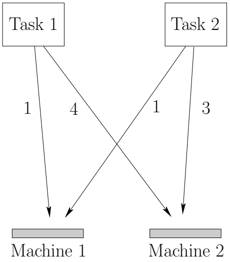
- $x_{ij} \in \{0, 1\}$ and $\sum_{i \in [n]} x_{ij} = 1$.
- $\max_{i \in [n]} \sum_{j \in [m]} x_{ij} t_{ij}$ is minimized.

Objective:

Allocate the tasks to the machines to minimizes the makespan (maximum completion time).

Example

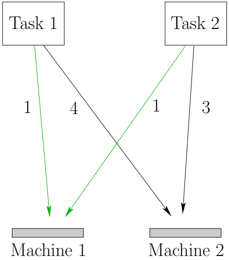
- 2 machines and 2 tasks.



$$t = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}$$

Example

- 2 machines and 2 tasks.
- Assign both tasks to machine 1 (makespan value of 2).



$$t = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}, x = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Mechanism design variant

Machines correspond to agents, which may be untruthful about their execution times vector.

Example

Machine 2 may be dishonest about the execution times of $t_2 = \langle t_{21}, t_{22}, \dots, t_{2m} \rangle$.

$$t = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1m} \\ t_{21} & t_{22} & \dots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \dots & t_{nm} \end{pmatrix}$$

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Weak monotonicity (Bikhchandani et al.)

- A property that every truthful mechanism must satisfy.
- Favorably, this property conditions the allocation algorithm (no need to care about the payments).

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Weak monotonicity for unrelated machines scheduling

Suppose t and t' differ only in the execution times of machine i. The associated allocations x and x' (of every truthful allocation algorithm) must satisfy

$$\sum_{j \in [m]} (x_{ij} - x'_{ij})(t_{ij} - t'_{ij}) \leq 0 .$$

Using the weak monotonicity property

Allows us to claim that if we change the execution times of one machine in a particular way then the allocation remains (almost) the same.

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Claim 1 by example

$$t = \begin{pmatrix} 1 & 1 & 2 \\ \infty & 1 & 3 \\ \infty & 2 & 2 \end{pmatrix} \quad \rightarrow \quad t' = \begin{pmatrix} 100 & 1 - \epsilon_1 & 2 + \epsilon_2 \\ \infty & 1 & 3 \\ \infty & 2 & 2 \end{pmatrix}$$

Using the weak monotonicity property

Allows us to claim that if we change the execution times of one machine in a particular way then the allocation remains (almost) the same.

Claim 1 by example

$$t = \begin{pmatrix} \boxed{1} & 1 & 2 \\ \infty & 1 & 3 \\ \infty & 2 & 2 \end{pmatrix} \quad \rightarrow \quad t' = \begin{pmatrix} \boxed{100} & 1-\epsilon_1 & 2+\epsilon_2 \\ \infty & 1 & 3 \\ \infty & 2 & 2 \end{pmatrix}$$

Can only be allocated to this machine.

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Allocated w.r.t. t, and have a lower execution time w.r.t. t'.

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Allows us to claim that if we change the execution times of one machine in a particular way then the allocation remains (almost) the same.

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Not allocated w.r.t. t, and have a higher execution time w.r.t. t'.

Using the weak monotonicity property

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Application – a lower bound of 2

- We begin with the following 2-machines 3-tasks instance.
  - It has one possible allocation (up to symmetries).

$$\begin{pmatrix} 0 & \infty & 1 \\ \infty & 0 & 1 \end{pmatrix}$$

Application – a lower bound of 2

- We begin with the following 2-machines 3-tasks instance.
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  - We neglect the ϵ-changes of t_{12}, t_{13} for simplicity.

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Solution value $= 2$, Optimal value $= 1$

Claim 2 by example

$$t = \begin{pmatrix} a & a & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}_{(a>1)} \quad \rightarrow \quad t' = \begin{pmatrix} 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Claim 2 by example

$$t = \begin{pmatrix} a & a & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}_{(a>1)} \quad \rightarrow \quad t' = \begin{pmatrix} 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

The "new" allocation is one of the following...

$$\begin{pmatrix} 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Theorem

No truthful deterministic mechanism for unrelated machines scheduling problem has approximation ratio better than $1 + \sqrt{2}$.

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Theorem

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Proof.

- Follows similar approach as the lower bound of 2.
- Clearly, it will also employ the second claim.
- We begin with the following 3-machines 5-tasks instance.

$$\begin{pmatrix} 0 & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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Solution $= 3\sqrt{2}$, Optimal $= \sqrt{2}$

$$\begin{pmatrix} 0 & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix} \xrightarrow{(t_{11}\Uparrow)} \begin{pmatrix} \sqrt{2} & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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Solution $= 3\sqrt{2}$, Optimal $= \sqrt{2}$

$\downarrow$ $(t_{14}, t_{15}\Downarrow)$ $\searrow$

$$\begin{pmatrix} 0 & \infty & \infty & 1 & 1 \\ \infty & 0 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix} \qquad \begin{pmatrix} 0 & \infty & \infty & 1 & 1 \\ \infty & 0 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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$$\xrightarrow{(t_{11}\Uparrow)} \begin{pmatrix} \sqrt{2} & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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Solution $= 2 + \sqrt{2}$, Optimal $= \sqrt{2}$

$$\begin{pmatrix} 0 & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix} \xrightarrow{(t_{11}\Uparrow)} \begin{pmatrix} \sqrt{2} & \infty & \infty & \sqrt{2} & \sqrt{2} \\ \infty & 0 & & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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$(t_{14}\Downarrow)\downarrow$ $\qquad$ $(t_{11}\Uparrow)\downarrow$

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$$\begin{pmatrix} 0 & \infty & \infty & 1 & 1 \\ \infty & 0 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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$(t_{22}\Uparrow)\downarrow$

$$\begin{pmatrix} 0 & \infty & \infty & 0 & 1 \\ \infty & 1 & \infty & \sqrt{2} & \sqrt{2} \\ \infty & \infty & 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

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$(t_{11}\Uparrow)\downarrow$

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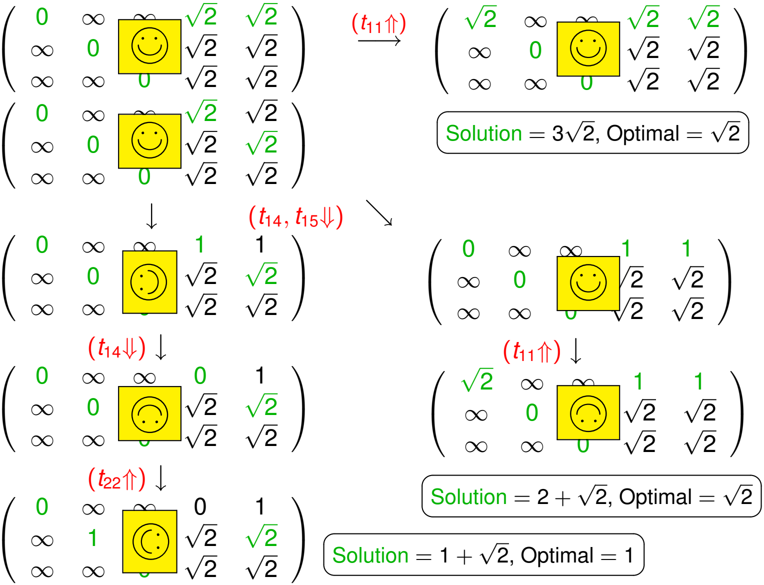
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$(t_{22}\Uparrow)$
Solution $= 2 + \sqrt{2}$, Optimal $= \sqrt{2}$
Solution $= 1 + \sqrt{2}$, Optimal $= 1$

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  - Workload minimization – between 2 and n.

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  - Unrelated machines scheduling – between $1+\sqrt{2}$ and n.
  - Koutsoupias and Vidali [MFCS '2007]

Open Questions

- Close the gaps for both problems...
  - Workload minimization – between 2 and n.
  - Unrelated machines scheduling – between $1 + \sqrt{2}$ and n.
  - Koutsoupias and Vidali [MFCS '2007]
- Study variants of these problems – fractional version, domain restricted version, etc.
  - Christodoulou, Koutsoupias and Kovács [ICALP '2007]
  - Lavi and Swamy [EC '2007]

Thank You

Slides will be available at my home page

`http://www.cs.tau.ac.il/~iftgam`