# Detection of Interest Points Using Symmetry

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**In order to reduce the amount of information involved in visual processing we seek a low level operator, which can be used to direct the computational resources toward "interesting" points of an image. We are introducing an operator based on the intuitive notion of symmetry, which effectively locates interest points in real time, and can be incorporated also in active visual systems. The results of its operation agree with some psychophysical evidence concerning symmetry as well as evidence concerning fixation points. The operator can be applied successfully without a priori knowledge of the world. Combining the operator with some preconceptions about the image is a powerful tool for feature detection in intricate natural scenes. We demonstrate the localization of faces and facial features in real time on detailed and noisy pictures.**

## 1 Introduction

Biological and machine vision tasks involve the processing of an enormous amount of information. In order to analyze it under plausible time and space constraints, this amount of information has to be reduced. A solution suggested by primate vision, is based on foveated vision: the field of view is selectively processed, and computational resources are directed toward "interesting" areas or points. These points are natural *attention points*, and provide a basis for a more thorough investigation by higher processes. When a primate focuses his attention on a location, events occurring at that location are responded to more rapidly, give rise to enhanced electrical activity, and can be reported at a lower threshold [17]. Interest points can serve as a guide for shifting the eyes to new *fixation points* – points that are projected on the fovea. In this paper we suggest an operator based on the intuitive notion of symmetry, which effectively locates interest point of a picture in real time, and can be incorporated in passive, as well as active, visual systems. The results of its operation are consistent with psychophysical evidence concerning symmetry as well as evidence concerning fixation points. The operator can be applied successfully without a priori knowledge of the world. Combining the operator with some preconceptions can provide a powerful tool for finding features in intricate natural scenes. We demonstrate, in this article, the localization of faces and facial features in real time on detailed and noisy pictures.

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Natural objects often give rise to the human sensation of symmetry. Our sense of symmetry is so strong that most man-made objects are symmetric, and the Gestalt school considered symmetry as a fundamental principle of perception [24]. This sensation is more general than the strict mathematical notion. For instance, a picture of a human face is considered highly symmetric by the layman, although it is not symmetric in the mathematical sense. The presented operator is inspired by the intuitive notion of symmetry, and assigns a "symmetry value" to every point in a picture at a very low level vision stage. In this respect, points with high symmetry value are natural interest points.

Interest points are regarded by many as points of high curvature (e.g. [2, 11, 26]). Others suggest to measure busyness – the smoothed absolute value of the Laplacian of the data [16], or rapid changes in the gray levels [21]. We argue that symmetry is a more general and powerful concept, since it more closely fits psychophysical evidence, and it is more useful in detecting interesting features in complex scenes.

Symmetry is being widely used in computer vision [1, 3, 5, 4, 8, 9, 14, 15, 13, 25, 27] (additional comprehensive bibliography can be found in [25, 27]). However, it is used as a mean of convenient representation, characterization, shape simplification, or approximation of objects, whose existence is already assumed. A typical vision task consists of edge detection, followed by segmentation, followed by recognition. A symmetry operator is usually applied after the segmentation stage. Our symmetry operator can be applied immediately after the stage of edge detection, where there is absolutely no knowledge regarding the objects in the scene. Moreover, the output of the symmetry operator can be used effectively to direct higher level processes, such as segmentation and recognition, and can serve as a guide for locating the interesting objects. We shall demonstrate these ideas on complex natural scenes.

More detailed description can be found in [18].

## 2 Defining the Operator

In the usual mathematical notion, an object is regarded as symmetric if the application of certain transformations, called symmetry operations, leaves it unchanged while permuting its parts. In order to use these symmetry operations it is necessary to know the shape of an object before we can predicate whether it is symmetric or not. However, the process of finding interest points must precede complex processes of detecting the objects in the scene. Even if the objects' shapes are known,



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truly symmetric objects are rare in natural scenes, and therefore any attempt to formulate an interest operator based on the strict mathematical notion of symmetry is doomed to fail.

In this section we define a symmetry measure for each point and direction. Let $p_k = (x_k, y_k)$ be any point $(k = 1, \ldots, K)$, and denote by $\nabla p_k = \left(\frac{\partial}{\partial_x} p_k, \frac{\partial}{\partial_y} p_k\right)$ the gradient of the intensity at point $p_k$. We assume that a vector $v_k = (r_k, \theta_k)$ is associated with each $p_k$ such that $r_k = \log(1 + \|\nabla p_k\|)$ and $\theta_k = \arctan\left(\frac{\partial}{\partial_y} p_k / \frac{\partial}{\partial_x} p_k\right)$. For each two points $p_i$ and $p_j$, we denote by $l$ the line passing through them, and by $\alpha_{ij}$ the angle counterclockwise between $l$ and the horizon.

We define the set $\Gamma(p, \psi)$, a distance weight function $D_\sigma(i,j)(\sigma)$, and a phase weight function $P(i,j)$ as

$$\Gamma(p,\psi) = \left\{(i,j) \,\middle|\, \frac{p_i + p_j}{2} = p\,, \;\; \frac{\theta_i + \theta_j}{2} = \psi\right\}$$

$$D_\sigma(i,j) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\|p_i - p_j\|}{2\sigma}}$$

$$P(i,j) = (1 - \cos(\theta_i + \theta_j - 2\alpha_{ij}))(1 - \cos(\theta_i - \theta_j))$$

The symmetry measure $S_\sigma(p, \psi)$ of each point $p$ in direction $\psi$ is defined as

$$S_\sigma(p,\psi) = \sum_{(i,j) \in \Gamma(p,\psi)} D_\sigma(i,j) P(i,j) r_i r_j$$

The rationale of this formula can be understood by decomposition and explanation of the operands:

$D_\sigma(ij)$ : The symmetry induced by two points decreases as their distance increases, thus the operator has a local nature. Different values for $\sigma$ enable different scales.

$r_i r_j$ : This term is high when there is a good correlation between two large gradients. We use gradients rather than intensities since meaningful information is usually connected with changes of intensity. For instance, a uniform intensity wall is highly symmetric but probably not very interesting. In natural scenes we prefer to use the logarithm of magnitude instead of the magnitude itself, since it reduces the differences between high gradients, and therefore the correlation measure is less sensitive to very strong edges.

$1 - \cos(\theta_i + \theta_j - 2\alpha_{ij})$ : A maximum symmetry measure is achieved when $(\theta_i - \alpha_{ij}) + (\theta_j - \alpha_{ij}) = \pi$, i.e. when the gradients at $p_i$ and $p_j$ are oriented in the same direction towards each other. This is consistent with the intuitive notion of symmetry. The expression $1 - \cos(\theta_i + \theta_j - 2\alpha_{ij})$ decreases continuously as the situation deviates from the ideal one. Notice that the same measure is achieved when the gradients are oriented towards each other or against each other. The first situation corresponds to symmetry within a dark object on a light background, and the second corresponds to symmetry within a light object on a dark background. It is easy to distinguish between the two cases.

$1 - \cos(\theta_i - \theta_j)$ : The previous expression attains its maximum whenever $(\theta_i - \alpha_{ij}) + (\theta_j - \alpha_{ij}) = \pi$, and includes the case $\theta_i - \alpha_{ij} = \theta_j - \alpha_{ij} = \pi/2$, which occures on a straight edge, which we do not regard as interesting. The current expression compensates for this situation.

$S_\sigma$ can be implemented either as an array of a fixed number of discrete angle bins (typically $n = 1, 2, 4, 8$ or 16) or as a dynamic data structure containing all the nonzero symmetry value directions. Representing in a discrete number of bins is faster and has a natural interpretation. We shall denote by $S_n$ the symmetry operator implemented by using $n$ bins as follows:

$$S_n(p,i) = \int_{\psi \in bin(i)} S_\sigma(p,\psi)\, d\psi$$

where $bin(i)$ is defined for $i = 1, \ldots, n$ as

$$bin(i) = \bigcup_{k=0,1} \left[k\,\pi + \frac{(i-1)\,\pi}{n} - \frac{\pi}{2\,n},\; k\,\pi + \frac{i\,\pi}{n} - \frac{\pi}{2\,n}\right)$$

An important special case is $S_1$:

$$S_1(p,1) = \int_0^{2\pi} S_\sigma(p,\psi)\, d\psi$$

which is the isotropic symmetry operator. It is interesting to note, in that regard, that if the image is considered to be the cotangent field of the intensity image, then the isotropic operator will detect the singularities of the principle direction fields [20]. $S_2(p,i)$, for example, classifies symmetry points as horizontal – $S_2(p,1)$ and vertical symmetry points – $S_2(p,2)$.

Sometimes it is necessary to detect points that are highly symmetric in several distinct directions. We call such a symmetry a *circular symmetry* – $CS(p)$ and its value can be evaluated using the formula:

$$CS_\sigma(p) = \iint_{\zeta < \eta} S_\sigma(p,\zeta)\, S_\sigma(p,\eta) \sin\left(\frac{\eta - \zeta}{2}\right) d\zeta\, d\eta$$

Note that the term $\sin\left(\frac{\eta - \zeta}{2}\right)$ reaches its peak when $\eta$ and $\zeta$ are in opposite directions, and decreases monotonically until they are identical.

If the operator is implemented in a small number of discrete bins, a simpler definition for circular symmetry suffices:

$$CS_n(p) = \prod_{i=1}^{n} (1 + S_n(p,i))$$

# 3 Cognitive Correlates

There are many psychophysical works investigating humans ability to detect bilateral symmetry (For example

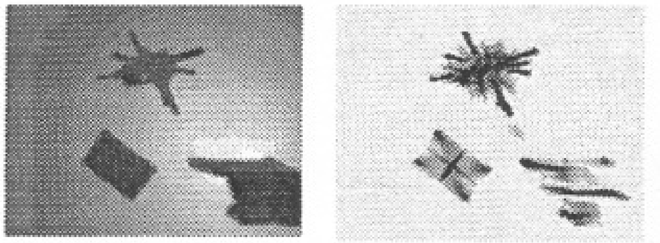

Figure 1: Still life (left) and the application of $S_1(p, 1)$ on it (right).

Figure 2: The strongest four peaks of $CS_8(p)$ are marked by crosses on the original picture.

[6]). It is accepted that human are able to detect mirror symmetry and that symmetry detection has a local nature. However, there is no link suggested between these capabilities and the choosing of attention points. Kaufman and Richards [11] studied spontaneous fixation tendencies of the eyes when they are confronted with very simple forms. Their results are in intriguing agreement with the results of applying the symmetry operator to a similar image – the operator $S_1(p, 1)$ attains its peaks at the same points for all the figures reported.

# 4 Operation on Natural scenes

The symmetry operator can be applied successfully on intricate natural scenes. Figure 1 demonstrates that the operator performance is not affected by the existence of several objects in the scene.

Figure 2 is a picture of three people on a noisy background. The strongest four peaks of $CS_8(p)$ are marked by crosses and are located on the three faces.

Another intricate scene is demonstrated in Figure 3. The location of the two strongest peaks of $CS_8(p)$ are marked by crosses and are located on the eyes. The next highest peaks (not marked) are located on the reflections of the face.

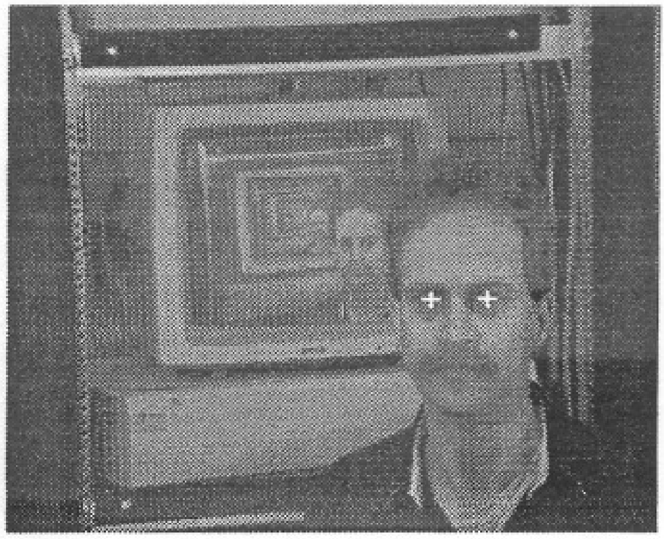

Figure 3: An image – the two highest peaks of $CS_8(p)$ are marked by crosses (on the eyes). The next peaks (unmarked) lay on the reflections of the faces.

Figure 4:
$S_8(p, 1)$ result on a face – prominent peaks at the facial features and some additional noise.

Combination of semantics with the symmetry operator can serve for feature detection. An example is facial feature detection. We have already demonstrated that $CS_8(p)$ applied to a face attains high peaks at the eyes, since the eyes are symmetric along various directions and scales. In addition, various facial features have high symmetry values along vertical lines, and therefore we expect $S_8(p, 1)$ to highly respond to the eyes, mouth and nose. Figure 4 shows the result of the application of $S_8(p, 1)$ on a face image.

# 5 Complexity

We first discuss serial implementation. Suppose the operator is applied to a picture composed of $n$ pixels, and the gaussian in the weight function almost vanish in radius $r < \sqrt{n}$. Each two pixels, whose distance is less than or equal to $2r$, contribute a value which can be computed using look-up tables. Therefore the time complexity is $O\left(n\,r^2\right)$. In practice it took a few seconds of a Sun3 machine to process the pictures presented in this article.

Space complexity is small too. We can either represent the symmetry of different phases in a discrete (and small) number of bins, or alternatively, use a linked list (or any other form of dynamic memory) to hold the symmetry measures at any precision. The worst case space complexity is $O\left(n\,r^2\right)$. It arises when we hold symmetry measures in maximal phase precision, and the sum of gradient phase of each two pixels, whose distance is less or equal $2\,r$, is different. For the implementation in a small number, $k$, of bins the space needed is no more than $k\,n$ words.

The complexity of a parallel implementation depends on the architecture. For an architecture where there is a processor allocated for each pixel and each processor is connected to its neighbors up to radius r, we can achieve maximal speed-up of the algorithm and reduce time complexity to $O\left(r^2\right)$ (with $O\left(n\,r^2\right)$ messages if no global memory is available). If we have at our disposal a sufficiently large neural network, we can easily implement the look-up table and summation operations needed and then perform the operation in constant time.

## 6 Conclusion and Further Work

We have introduced an operator which can serve as an efficient low level process for indexing attention to the regions that are likely to be of high interest in a picture. Other processes can then move the attention to these regions and interpret the data in them. The symmetry operator agrees with some psychophysical data and can be further investigated to find its relation to models for indexing attention in biological vision, such as the models used by Posner and Peterson [17] and by Ullman [23].

The symmetry operator value is high near points of high curvature and the location of its peaks is more consistent with psychophysical evidence than the location of points based on high-curvature per-se. It is also more general since it locates other classes of interest points. Busyness measures can also be regarded as a rough estimate of the symmetry operator, since both are influenced by edge intensities, although busyness does not account for their relative orientations. Thus, the busyness measure will yield poor results in noisy scenes.

A powerful tool for object recognition can be designed using semantics in combination with the symmetry operators. For example, we can locate a face using the circular symmetry operator, $CS_n\,(p)$ applied to a rough resolution of the picture, and then apply both circular symmetry, $CS_n\,(p)$, and symmetry along horizontal lines, $S_n\,(p,1)$, on smaller focused regions of the picture in higher resolution. We have already obtained preliminary results locating faces and their features using the symmetry operator along with the gaussian pyramid [7, 19].

Recent object recognition paradigms [12, 22, 10] have shown that recognition can be performed using interest points in unsegmented scenes. Such paradigms may use as input the symmetry operator output. Moreover, the symmetry operator produces more than interest points. One may view the computation of the symmetry measure $S\,(p,\psi)$ as a "symmetry edge" extraction procedure. For every point in the image one gets the strength of the response in given directions. An appealing consequence of this approach is in the observation that one may apply most of the "standard" computer vision operations to this map exactly in the same way that we apply it to a standard edge map. For example, by applying a Hough transform for line detection, one detects significant straight symmetry axes. By applying any standard edge linking procedure, one obtains all the symmetry axes in the image, which may be curved. All this can be carried out without prior segmentation of the original image. As a consequence, a finer symmetry measure based on the intersection of symmetry lines rather than the original symmetry points can be established.

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