1. (a) Show that for every \( A \in \text{BPP} \), \( \text{BPP}^A = \text{BPP} \).
   
   (b) Show that if \( \text{SAT} \in \text{BPP} \) then \( \text{PH} = \text{NP}^\text{SAT} \).

2. Call a CNF formula over \( n \) variables simple if it is either unsatisfiable or it has at least \( \frac{2^n}{n^3} \) satisfying assignments. Show a randomized algorithm for checking the satisfiability of simple formulas, that outputs the correct answer with probability at least \( \frac{2}{3} \).

3. Let PP be the set of languages for which there exists a probabilistic polynomial-time Turing machine \( M \), such that for every \( x \in L \) the machine \( M \) accepts \( x \) with probability greater than \( 1/2 \), and for every \( x \notin L \) the machine \( M \) accepts \( x \) with probability at most \( 1/2 \).
   
   (a) Show that BPP is closed under union and intersection and explain why your argument fails for PP.
   
   (b) Show that \( \text{NP} \subseteq \text{PP} \subseteq \text{PSpace} \).

4. Define ZPP as the class of all languages decided by a probabilistic Turing machine running in expected polynomial time. That is, for a language \( L \in \text{ZPP} \) there is a probabilistic Turing machine \( M(x, y) \) with the following behavior: on input \( x \in L \), \( M \) always accepts, on input \( x \notin L \), \( M \) always rejects, and for every \( x \),
   
   \[
   \mathbb{E}_{y}[\text{number of steps before } M(x, y) \text{ halts}] \leq |x|^c
   \]
   
   for some fixed \( c > 0 \). Prove that \( \text{ZPP} = \text{RP} \cap \text{coRP} \).