1. (a) Show that PAL, the language of all palindromes over \{0, 1\}, can be decided using logarithmic space. What is the running time of your Turing machine?

(b) Show that there is a non-deterministic Turing machine that decides PAL in linear time and logarithmic space.

2. In the problem OddCycle the input is a directed graph \(G = (V, E)\), and the question is “Does \(G\) contain a directed cycle of odd length?”. Show that OddCycle is NL-complete.

3. Consider the problem CYCLE-1 defined as follows:
Input: A directed graph with out-degree at most one.
Question: Does it contain a cycle?
Show that CYCLE-1 \(\in L\).

4. In the problem UstCON the input is an undirected graph \(G = (V, E)\), and two vertices \(s, t \in V\), and the question is “Is \(s\) connected to \(t\) in \(G\)?”.

(a) Show that UstCON \(\in NL\).

Remark: UstCON is known to be in L and is complete for a class called SL.

(b) Show that \(\overline{2-Col} \in NL\).

(c) Show that \(\overline{2-Col} \leq_L UstCON\) and that \(UstCON \leq_L \overline{2-Col}\).

5. The following problem is complete for a certain complexity class; which one?
Input: A deterministic Turing machine \(M\) that has only one Read-Write tape (on which it gets the input) and an input \(x\) for \(M\).
Question: Does \(M\) accept \(x\) without leaving the first \(|x| + 1\) cells of the tape?

6. Prove that for any \(c \geq 2\), \(SPACE(\log^c n)\) is closed under the Kleene star.\(^1\)

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\(^1\)Recall that a graph is \(k\)-colorable if its vertices can be colored using up to \(k\) different colors in such a way that any two adjacent vertices have different colors. For any \(k \in \mathbb{N}\) we define the language \(k-Col = \{ G \mid G \text{ is } k\text{-colorable} \}\).

\(^2\)That is, if \(A \in SPACE(\log^c n)\) then \(A^* \in SPACE(\log^c n)\).