1. Show that P is closed under polynomial-time Cook reductions.

2. Prove that the following problems are self reducible by a (direct) polynomial Cook reduction from the search version to the decision version of the same problem.
   (a) Clique = \{ (G, k) | G contains a clique of size k \}.\footnote{The decision version is “Given a pair (G, k) does G contain a clique of size k?” and the search version is “Given a pair (G, k) find a clique of size k in G if exists, and reject otherwise”.
   (b) GraphIsomorphism = \{ (G_1, G_2) | G_1 and G_2 are isomorphic \}.\footnote{Two graphs are isomorphic if there is a way to label the vertices of one graph, such that the two graphs become identical.}

3. Let UpToOneSat be the following language:
   UpToOneSat = \{ \phi | \phi \text{ is a CNF formula that has at most one satisfying assignment} \}. Prove that UpToOneSat ∈ NP if and only if NP = coNP.

4. Let \( A \subseteq \{0, 1\}^n \) be a language which satisfies \( |A \cap \{0, 1\}^n| = n^3 \) for all \( n \geq 10 \). Prove that \( A \in NP \) implies \( A \in coNP \).

5. The class DP is defined as the set of all languages \( L \) for which there are two languages \( L_1 \in NP \) and \( L_2 \in coNP \) such that \( L = L_1 \cap L_2 \).
   (a) Show that \( P \subseteq DP \).
   (b) Is \( DP = NP \cap coNP \)? (prove or disprove or show that it is equivalent to a well-known open question).
   (c) Let SAT-UNSAT be the language of all the pairs \( (\phi_1, \phi_2) \) such that \( \phi_1 \) and \( \phi_2 \) are CNF formulas, \( \phi_1 \) is satisfiable and \( \phi_2 \) is not. Show that SAT-UNSAT is DP-complete, i.e., SAT-UNSAT ∈ DP and every language in DP is polynomial-time reducible to it.
   (d) Bonus: Show that MAX-IS is DP-complete.