**Peer-to-Peer Papers Introduction:**

We distinguish between 3 main peer-to-peer applications types, different from one another in the requirements they put on the system.

1. **File Sharing:**
   a. Not real-time (RT)
   b. All or nothing – if the file lacks 1 byte transmission fails.

2. **Video On Demand (VOD):**
   a. Usually bigger files than those shared by the users
   b. Not RT – yet should be as close as possible to RT in the sense of transmission times. Note that unlike RT – the wanted video is taken from an archive unlike VOIP/live conference traffic.
   c. Mostly All or nothing – we usually want the whole video, incomplete video is mostly worthless.

3. **Streaming:**
   a. Mostly live broad-cast.
   b. Mostly RT (delay of few seconds at most)

All 3 Applications types needs have different tools to support their needs. The course papers will cover the different ways to cope with each need.

The full description of the course papers can be found in the web site:

Rehearsal and introduction

Last lecture we’ve introduced Markovian chains discrete in both time and space. Today we close this subject and move to continuous Markovian chains.

\( \pi_n \) is a vector size \( k \), defining the probability to be in states 1..k on time \( n \). \( \pi_0 \) for example is the probability to start at each state. Transition between different times is done by multiplying \( \pi \) with transition matrix \( P : \pi_i = \pi_0 \cdot P \ldots \pi_n = \pi_0 \cdot P^n \)

If \( \pi \) converges, the following holds: \( \pi = \lim_{n \to \infty} \pi_n \Rightarrow \pi \cdot P = \pi \).

Usually convergence is very fast.

Recall Pij definition: \( P_{ij} \triangleq \Pr[X_n = j \mid X_{n-1} = i] \)

Therefore \( P_{ii} = \Pr[\text{stay at the same place}] \).

The return-time distribution \( \Pr[\text{remains in E, exactly n times}] = P_{ii}^n \cdot (1 - P_{ii}) \)

The distribution is geometric.

2.5: Death Birth Process:

We now present the birth-death chains. In birth-death chains, transitions can be made only to neighbors \( \pi_j \) there’s no edge going from \( E_i \to E_j \) if \( j > i + 1 \).

Moving ‘forward’ called birth and moving backwards called death.

To understand the process one can think of the participants’ number in a class: either a student leaves the class or a new student enters the class. Each leaving represents a death and each enter represents a birth.

Examine the matrix created by birth-death chains. Denote \( b_i \) to be the probability of birth and \( d_i \) the probability of death. The probability to stay at the same state is therefore \( 1 - b_i - d_i \).

\[
P = \begin{pmatrix}
1-b_0 & b_0 & 0 & 0 & \cdots & \cdots & 0 \\
0 & 1-b_1-d_1 & b_1 & 0 & \cdots & b_{i-1} & \cdots & 0 \\
0 & 0 & 1-b_i-d_i & b_i & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 1-b_{i+1}-d_{i+1} & b_{i+1} & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 1-b_{i+1}-d_{i+1} & b_{i+1} & 0 & \cdots & 0 \\
\end{pmatrix}
\]
2.4: Continuous time Markov Chains:

In this chapter the states remained discrete, same as what we had till now. On the other hand times are now continuous. Due to this change we need to Markovity.

Markovity: a system is Markovian if the next state depends on the current state and not on what happened before t. when n describes a arbitrary time on the time axis, the following holds:

\[
\Pr[X(t_{n+1}) = j \mid X(t_1) = j_1, X(t_2) = j_2,...X(t_n) = j_n] = \Pr[X(t_{n+1}) = j \mid X(t_n) = j_n]
\]

Markovity represents memorylessness in continuous time as well.

Note: remaining time is distribution is memoryless exponential – the continuous case of geometrical distribution.

Back to the ‘jumping man’ at the continuous case: each index change is a jumping point.

\[\tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4 \quad \ldots\ldots\]

\[t_0 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad \ldots\ldots \]

\[\Pr[\tau_i > s + t \mid \tau_i > s] = h(t)\]

Meaning that the probability of a ‘jump’ depends solely on t, s does not affect h – it doesn’t matter how much time the person stayed at the same point.

A short math would result:

\[\Pr[\tau_i > s + t] = \Pr[\tau_i > s] \cdot \Pr[\tau_i > t]\]

P continuous equivalent - Q:
We’d like to find a continuous equivalent to the transition matrix P.
Definitions:
Define: \(P_{ij}(s,t) \triangleq \Pr[X(t) = j \mid X(s) = i]\)

This definition is possible thanks to memorylessness of the system.
Note: we don’t restrict the number of transitions during (s,t) tie period.

Define: \(q_{ij} = \lim_{\Delta t \to 0} \frac{P_{ij}(t,t+\Delta t)}{\Delta t}\) an infinitesimal transition (where to be in \(\Delta t\) time). It represents the rate of arrival from i to j when i≠j.

Define: \(g_{ii} = \lim_{\Delta t \to 0} \frac{P_{ii}(t,t+\Delta t) - 1}{\Delta t}\) an infinitesimal staying.

Note: if we didn’t add (-1) we would always get \(\infty\). To retrieve a meaningful number on the staying rate we must subtract 1. Therefore \(g_{ii}\) is a negative number.

Note: do not confuse between rate and probability: both q and g are rates and not probabilities. It is more notable when considering the measure units: g and q is measured at [1/sec] while probabilities are pure numbers.
Easy to see that:
\[
\sum_j q_{ij}(t) = 0
\]

Finally we can define a transition rate matrix \( Q \) equivalent to \( P \) using \( q \) as the elements. 

**Redefine:** \( \pi_j(t) = \Pr[X(t) = j]; \quad \pi(t) = [\pi_0, \pi_1, \ldots, \pi_k] \)

The transition function does not depend on \( t \) therefore it is easy to define \( Q \) as a function of \( i \) and \( j \).

The matrix \( Q \) is constant in time, only the vector \( \pi \) is time-dependent. Though we can explore the changes in \( \pi \) through time, at this course we’re only interested in equilibrium when \( n \) goes to \( \infty \) with no indices on \( t \).

\[
\pi = \lim_{t \to \infty} \pi(t), \quad \pi(i) = \lim_{t \to \infty} \pi[i](t)
\]

**Relations between \( Q \) and \( \pi \):**

**Define:** \( \pi_j \cdot q_{ij} + \sum_{k \neq j} \pi_k \cdot q_{kj} = 0 \) or matrix wise \( \pi \cdot Q = 0 \).

**Intuitively:** \( Q = P - I \) cause of the definition of \( g \) which affects the diagonal. Recall that: \( \pi(P - I) = 0 \), therefore: \( \pi Q = 0 \).

Usually \( Q \) is given and we want to find \( \pi \). We need another equation to solve the above equation set: \( \sum \pi_i = 0 \).

**Back to Birth-death continuous systems:**

As defined before, transitions can be made only to neighbor states.

We will define **transition rates** equivalent to transition probabilities.

Denote \( \lambda_k \) birth-rate, and \( \mu_k \) death rate.

The \( Q \) matrix will look like this:

\[
Q = \begin{pmatrix}
-\lambda_0 & \lambda_0 & 0 & 0 \\
\mu_1 & -\lambda_1 - \mu_1 & \lambda_1 \\
0 & \mu_2 & - \lambda_2 & \mu_{i-1} \\
. & . & . & . \\
0 & . & . & \mu_i \\
. & . & . & . \\
. & . & . & . \\
0 & . & . & \lambda_{i+1} \\
\end{pmatrix}
\]