PGM Seminar - Inference as Optimization HW

November 29, 2015

0.1 Definitions:

Given a set of calibrated beliefs $\mathbf{Q} = \{\beta_i\}_{i \in V_\tau} \cup \{\mu_{ij}\}_{(i,j) \in E_\tau}$ for the clusters and sepsets of a clique tree τ,

we defined the distribution Q induced by them as $Q(\chi) = \frac{\Pi_{i \in V} \beta_i(C_i)}{\Pi_{(i,j) \in E} \mu_{ij}(S_{ij})}$ (where $\frac{0}{0} = 0$).

Moreover, we defined the following two “functionals”:

a) The Energy Functional:

$F[\tilde{P}_\Phi, Q] = \sum_{\phi \in \Phi} E_Q[ln(\phi)] + H(Q)$.

Where $\tilde{P}_\Phi(\chi) = \prod_{\phi \in \Phi} \phi(scope[\phi])$ is the original unnormalized distribution,

$ln(.)$ means the natural logarithm, and $H(Q)$ is the entropy of the distribution Q, using the natural logarithm.

b) The Factored Energy Functional:

$\tilde{F}[\tilde{P}_\Phi, \mathbf{Q}] = \sum_{i \in V_\tau} E_{C_i \sim \beta_i}[ln(\psi_i)] + \sum_{i \in V_\tau} H(C_i \sim \beta_i) - \sum_{(i,j) \in E_\tau} H(S_{ij} \sim \mu_{ij})$.

Where $C_i \sim \beta_i$ means the distribution over the variables of the node C_i , induced by the belief β_i (remember that every belief β_i of a node C_i is a distribution over the variables of that node, and same for μ_{ij}),

$S_{ij} \sim \mu_{ij}$ means the distribution on the variables of the sepset $S_{ij} = C_i \cap C_j$ induced by μ_{ij},

and $\psi_i = \prod_{\phi \; s.t \; \alpha(\phi)=i} \phi$, where α is the mapping from factors in Φ to clusters in τ, which exists since τ is a cluster tree.

0.2 Questions

0.2.1 Equivalence of functionals

Show that if $\mathbf{Q}$ is a set of calibrated beliefs for a clique tree τ, then $F[\tilde{P}_\Phi, Q] = \tilde{F}[\tilde{P}_\Phi, \mathbf{Q}]$.

(Hint: what is the relation between a belief β_i (or μ_{ij}) and the distribution Q?)

0.2.2 A bound on the Energy functional for normalized distributions

Suppose our original distribution is normalized ($Z = 1$).

Show that for every distribution Q, we have $F[\tilde{P}_\Phi, Q] \leq 0$.

(Hint: what is the relation between the energy functional and the relative entropy $D(Q|P_\Phi)$?)