0368.4511 PGM Seminar
Fall 2015

Jonathan Shafer, 200364792
November 21, 2015

# Problem Set 4: Inference with Clique Trees

Let $\Phi$ be a set of factors over a family $\mathcal{X}$ of random variables and let $\mathcal{T}$ be a valid clique tree for $\Phi$. Assume we have run the belief update algorithm (Algorithm 10.3) and obtained the beliefs $\beta_i$ and $\mu_{i,j}$ for all cliques $C_i$ and sepsets $S_{i,j}$ in $\mathcal{T}$ respectively.

## Question 1 – Queries outside a clique

Let $\mathbf{Y} \subseteq \mathcal{X}$. We would like to compute $\tilde{P}_\Phi(\mathbf{Y}) = \sum_{\mathcal{X}\setminus\mathbf{Y}} \tilde{P}_\Phi(\mathcal{X})$.

1. Explain in at most two sentences why the belief update algorithm does not directly provide the desired output.
2. Let $\mathcal{T}'$ be a subtree of $\mathcal{T}$ such that $\mathbf{Y} \subseteq scope[\mathcal{T}']$. Provide an algorithm that computes $\tilde{P}_\Phi(\mathbf{Y})$ using only $\mathcal{T}'$ and the beliefs $\beta_i$ and $\mu_{i,j}$ for the cliques and sepsets that belong to $\mathcal{T}'$.

**Hint:** If you get stuck or are short on time, you can find the solution in section 10.3.3.2.

## Question 2 – Multiple queries

We would now like to compute $\tilde{P}_\Phi(X, Y)$ for all distinct pairs $\{X, Y\} \subseteq \mathcal{X}$. Construct a dynamic programming algorithm that achieves this using $\mathcal{T}$, $\beta_i$ and $\mu_{i,j}$ as following:

1. Show how to compute $\tilde{P}_\Phi(C_i \cup C_j)$ for all adjacent clusters $C_i, C_j$.
2. Let $d(v, u)$ be the number of edges on the path between vertices $u, v$ in $\mathcal{T}$. Assume $\tilde{P}_\Phi(C_i \cup C_j)$ is known for all clusters $C_i, C_j$ such that $d(C_i, C_j) \leq n$ and show how to compute this factor for all clusters $C_i, C_j$ such that $d(C_i, C_j) = n + 1$.
3. Compute the desired output using $\tilde{P}_\Phi(C_i \cup C_j)$ for all $i, j$.

**Hint:** If you get stuck or are short on time, you can find the solution in section 10.3.3.3.