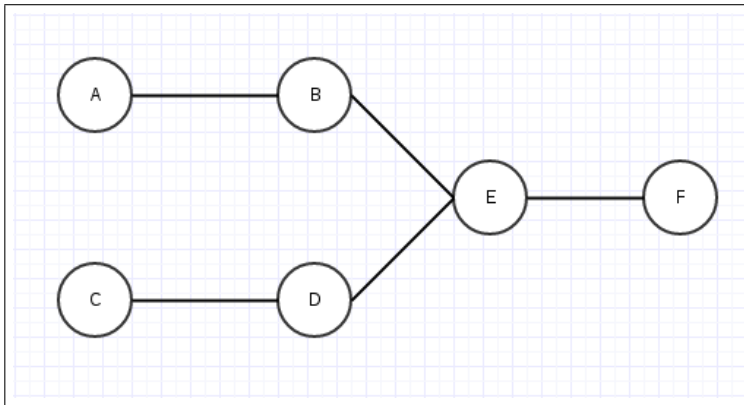


Week 5: Inference with Clique Trees

Background - Variable Elimination

$$\mathcal{X} = \{A, B, C, D, E, F\}$$

$$\Phi = \left\{ \phi_1(A, B), \phi_2(C, D), \phi_3(B, D, E), \phi_4(E, F) \right\}$$



Goal: compute $\tilde{P}_\Phi(F)$.

Solution 1 - Direct calculation

$$\tilde{P}_\Phi(F) = \sum_{\mathcal{X} \setminus F} \prod_{\phi \in \Phi} \phi$$

Solution 2 - VE

$$\tilde{P}_\Phi(F) = \sum_E \left[\phi_4(E, F) \cdot \sum_{B,D} \left[\phi_3(B, D, E) \cdot \sum_C \left[\phi_2(C, D) \cdot \sum_A \left[\phi_1(A, B) \right] \right] \right] \right]$$

$\psi_1 = \phi_1(A, B)$	$\tau_1 = \sum_A \psi_1$
$\psi_2 = \phi_2(C, D)$	$\tau_2 = \sum_C \psi_2$
$\psi_3 = \phi_3(B, D, E) \cdot \tau_1 \cdot \tau_2$	$\tau_3 = \sum_{B,D} \psi_3$
$\psi_4 = \phi_4(E, F) \cdot \tau_3$	

$$\tilde{P}_\Phi(F) = \sum_E \psi_4$$

Algorithm 10.1 Upward pass of variable elimination in clique tree

```

Procedure CTree-SP-Upward (
     $\Phi$ , // Set of factors
     $\mathcal{T}$ , // Clique tree over  $\Phi$ 
     $\alpha$ , // Initial assignment of factors to cliques
     $C_r$  // Some selected root clique
)
1 Initialize-Cliques
2 while  $C_r$  is not ready
3   Let  $C_i$  be a ready clique
4    $\delta_{i \rightarrow p_r(i)}(\mathcal{S}_{i,p_r(i)}) \leftarrow \text{SP-Message}(i, p_r(i))$ 
5    $\beta_r \leftarrow \psi_r \cdot \prod_{k \in \text{Nb}_{C_r}} \delta_{k \rightarrow r}$ 
6   return  $\beta_r$ 

Procedure Initialize-Cliques (
)
1 for each clique  $C_i$ 
2    $\psi_i(C_i) \leftarrow \prod_{\phi_j : \alpha(\phi_j)=i} \phi_j$ 
3

Procedure SP-Message (
     $i$ , // sending clique
     $j$  // receiving clique
)
1  $\psi(C_i) \leftarrow \psi_i \cdot \prod_{k \in (\text{Nb}_i - \{j\})} \delta_{k \rightarrow i}$ 
2  $\tau(\mathcal{S}_{i,j}) \leftarrow \sum_{C_i - \mathcal{S}_{i,j}} \psi(C_i)$ 
3 return  $\tau(\mathcal{S}_{i,j})$ 

```

Algorithm 10.2 Calibration using sum-product message passing in a clique tree

```

Procedure CTree-SP-Calibrate (
     $\Phi$ , // Set of factors
     $\mathcal{T}$  // Clique tree over  $\Phi$ 
)
1 Initialize-Cliques
2 while exist  $i, j$  such that  $i$  is ready to transmit to  $j$ 
3    $\delta_{i \rightarrow j}(\mathcal{S}_{i,j}) \leftarrow \text{SP-Message}(i, j)$ 
4   for each clique  $i$ 
5      $\beta_i \leftarrow \psi_i \cdot \prod_{k \in \text{Nb}_i} \delta_{k \rightarrow i}$ 
6   return  $\{\beta_i\}$ 

```

Algorithm 10.3 Calibration using belief propagation in clique tree

```

Procedure CTree-BU-Calibrate (
     $\Phi$ , // Set of factors
     $\mathcal{T}$  // Clique tree over  $\Phi$ 
)
1 Initialize-CTree
2 while exists an uninformed clique in  $\mathcal{T}$ 
3   Select  $(i-j) \in \mathcal{E}_{\mathcal{T}}$ 
4   BU-Message( $i, j$ )
5   return  $\{\beta_i\}$ 

Procedure Initialize-CTree (
)
1 for each clique  $C_i$ 
2    $\beta_i \leftarrow \prod_{\phi : \alpha(\phi)=i} \phi$ 
3 for each edge  $(i-j) \in \mathcal{E}_{\mathcal{T}}$ 
4    $\mu_{i,j} \leftarrow \mathbf{1}$ 

Procedure BU-Message (
     $i$ , // sending clique
     $j$  // receiving clique
)
1  $\sigma_{i \rightarrow j} \leftarrow \sum_{C_i - S_{i,j}} \beta_i$ 
2 // marginalize the clique over the sepset
3  $\beta_j \leftarrow \beta_j \cdot \frac{\sigma_{i \rightarrow j}}{\mu_{i,j}}$ 
4  $\mu_{i,j} \leftarrow \sigma_{i \rightarrow j}$ 
    
```