Applications of DFS
DFS(G)
1. For each vertex $u \in G.V$
2. $u.\text{color} = \text{WHITE}$
3. $u.\pi = \text{NIL}$
4. time = 0
5. For each vertex $u \in G.V$
6. if $u.\text{color} == \text{WHITE}$
7. DFS-Visit(G,u)

DFS-Visit(G,u)
1. time = time + 1
2. $u.d = \text{time}$
3. $u.\text{color} = \text{GRAY}$
4. For each vertex $v \in G.\text{Adj}[u]$
5. if $v.\text{color} == \text{WHITE}$
6. $v.\pi = u$
7. DFS-Visit(G,v)
8. $u.\text{color} = \text{BLACK}$
9. time = time + 1
10. $u.f = \text{time}$

Each vertex has two timestamps:

$u.d$ : First discovered and GRAYed

$u.f$ : Finished and Blackened

$\forall u: 1 \leq u.d < u.f \leq 2n$
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10. \( u.f = \text{time} \)

Thm 2: \( [v.d, v.f] \subset [u.d, u.f] \) iff \( v \) is a descendant of \( u \) in the DFS-forest,
\( [v.d, v.f] \cap [u.d, u.f] = \emptyset \) iff \( u \) and \( v \) are unrelated in the DFS-forest \( \square \)
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DFS-Visit(G,u)
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10. \( u.f = \text{time} \)

**Thm 3**: \( v \) is a descendant of \( u \) in the DFS-forest iff when we invoke DFS-Visit(u) there is a white path from \( u \) to \( v \)
Edge classification

- **Tree edges**: \((u,v)\), DFS-Visit\(v\) was called from DFS-visit\(u\)
- **Back edges**: \((u,v)\) such that \(v\) is an ancestor of \(u\) in the DFS-forest
- **Forward edges**: nontree edges \((u,v)\) such that \(v\) is a descendant of \(u\)
- **Cross edges**: \((u,v)\) such that \(v.f < u.d\)

Observe, among non-tree edges:
1. (Only) backward edges go to a vertex with a later finish time
2. (Only) forward edges go to a vertex with later discovery time
Is my directed graph acyclic?

**Thm 5:** $G$ is acyclic iff DFS on $G$ does not produce back edges
Topological sort

- **Input:** $G = (V, E)$ a directed, acyclic graph (DAG)
Topological sort

- **Input:** $G = (V, E)$ a directed, acyclic graph (DAG)
- **Output:** Ordering of the vertices such that if $(u, v) \in E$ then $u$ precedes $v$
Recall:

Observe, among non-tree edges:
1. (Only) backward edges go to a vertex with a later finish time
2. (Only) forward edges go to a vertex with later discovery time
Topological sort

• Lets run DFS
Topological sort

- Lets run DFS
Topological sort

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• Lets run DFS
Topological sort

• Order the vertices by reverse finishing times
Topological sort

• Order the vertices by reverse finishing times