Simple Sybil-Proof Mechanisms for Multi-Level Marketing *

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Multi-level marketing refers to a marketing approach in which buyers are encouraged to take an active role in promoting the product. This is done by offering them a reward for each successful referral of the product to other prospective buyers. To encourage potential customers to buy early and to give referrals to influential people, these mechanisms also reward *indirect* referrals — direct referrals linked to the buyer through other direct referrals. We model referrals using a directed *referral tree* T. Each node in T corresponds to a buyer. T has an edge from v to u if u buys the product as a result of a referral from v. A *reward mechanism* determines the reward, $R(v)$, that each node $v \in T$ receives for its direct and indirect referrals.

One potential drawback to multi-level mechanisms is that they could allow for sybil attacks. A *sybil attack* consists of a user purchasing multiple copies of the product under false identities to increase its reward. For example, a user can create two identities, the second with a referral from the first. Then, for each referral made through the second identity both identities would capture some reward. The additional reward captured could be greater than the cost of buying additional product. A sybil attack is undesirable because it reduces the profits of the seller, since buyers are getting higher rewards than what the seller intended for them to receive. After a sybil attack, an ancestor of the attacker is farther in the referral tree from the children of the attacker. In many reward mechanisms, this means that the ancestor gets less reward from each child, reducing his incentive to make referrals.

For simplicity, we assume, each potential buyer has intrinsic value for at most a single unit of product and purchases additional units only to benefit by receiving additional rewards. Let π be the price of a unit of product and let the *profit* of a set of nodes be the total reward received minus the cost of purchasing the product incurred by those nodes. Agents may buy a single unit of product regardless of their profit. However, they have no incentive to buy additional units unless doing so increases their profit. A sybil attack is *profitable* if it increases the profit of the node performing it. A reward mechanism is *split proof* if no sybil attack can be profitable under it.

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The following are some natural properties of reward mechanisms: (1) *Subtree constraint*: $R(u)$ should depend only on the subtree rooted at u, so u has no incentive to delay buying the product to hold out for a referral in a more rewarded position in the tree. (2) *Budget constraint*: The seller is only willing to spend a given portion, ϕ, of her revenue on rewards. Therefore, the total reward should satisfy $R(T) \leq |T|\phi\pi$. (3) *Monotonicity*: For all $u \in T_v$, adding a child to u increases v's reward at least as much as adding a child to a descendant of u. Thus, direct referrals are rewarded more than indirect referrals. (4) *Anonymity*: If a node v is replaced in T by a node u, u gets the same reward v was getting; the reward of all other nodes is unchanged. We also discuss the following properties: (5) *Unbounded rewards*: Potential rewards, even given a limit d in the number of referrals each person can make, should be unbounded. This gives buyers incentive to refer influential people. (6) *Summing contributions*: There exists a sequence $\{c_k\}_{k\geq 1}$ such that given any tree T_v rooted at v, $R(v) = \sum_{u \in T_v} c_{\mathrm{dist}(v,u)}$, where $\mathrm{dist}(v,u)$ is the length of the shortest path from v to u.

A mechanism that gives no rewards is sybil proof. However, this provides no incentive for referrals. Previous work provides a sybil proof mechanism that satisfies the subtree constraint, the budget constraint, and unbounded rewards [1]. However, [1] does not satisfy monotonicity and may not have a poly-time implementation.

We give a simple way of modifying any anonymous reward mechanism that satisfies summing contributions, monotonicity, and the budget constraint to obtain an anonymous mechanism that satisfies the subtree, monotonicity, and the budget constraint, and is split proof, while rewarding each subtree at least as much. If the original mechanism satisfies unbounded rewards, so does the modified mechanism. Our modification caps the reward a node v can get through any single tree rooted at a child u of v. It then recursively reassigns the extra reward to the tree rooted at u. This prevents sybil attacks while rewarding each subtree at least as much as the original mechanism. We give a concrete example of one such mechanism that is simple to implement. Finally, we show that there is no anonymous split proof mechanism that satisfies unbounded rewards and summing contributions.

[1] Y. Emek, R. Karidi, M. Tennenholtz, and A. Zohar. Mechanisms for multi-level marketing. In EC 2011.