Striving for Social Status

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In this paper, we consider a network of agents producing a positional good; an agent's utility for the good is a function of both his intrinsic value for the good and his status, or production level relative to the production levels of his neighbors. We demonstrate that the set of Nash equilibria of our game form a complete lattice, and that natural best-reply dynamics from certain initial configurations converge to either the minimum or maximum elements of the lattice. We then study the effects of status and network structure on the total production of the good and the social welfare. We find that production increases and welfare decreases with status effects. Network structure, on the other hand, can have arbitrary impacts on both production and welfare.

1. INTRODUCTION

Since at least Veblen's (1899) classic work on conspicuous consumption, economists and social scientists have recognized that social comparisons can influence individual decisions. People compare their production, their income, and their belongings to those of people around them, and they strive to maintain their position within their community. We study social comparisons and striving for status in a network context. As in the real social world, in this paper people do not live in isolated and closed communities. People have a variety of social contacts, and these contacts can overlap. People are indirectly connected, and therefore indirectly influenced, by people who may be quite distant in social and geographic space. We study how the status considerations and network structure influences individual outcomes and aggregate provision of goods. Prominent scholars, such as Robert Frank (1985, 2000), argue that increasing inequality has led to excessive spending, as people try to emulate and compete with the rich. This process accelerates as more people are exposed to the lives of the rich and what they consume. We demonstrate these phenomena in a theoretical setting.

To study these phenomena we consider a game where individuals exert effort to produce a good. This good can be thought of as a private consumption good with status implications, like cars or designer clothing. Alternatively, the good can be thought of as a public good – charity organizations, city opera houses, and the like – where individuals receive recognition in proportion to their contribution to the good in the form of published donor lists, for example. Agents incur individual costs as a function of their effort level. They also experience benefits from their own effort levels and, in public good settings, those of others.

Agents additionally compare themselves in both relative and absolute terms to their neighbors in a network. We adopt the status loss model[1] of Stark and Wang [2005] where agents suffer a loss as they compare to themselves to those with higher consumption in their reference group. This utility specification finds its basis in the work of classic sociology and social psychology (Merton [1938])and captures the idea that humans tend to look to those doing better than themselves in order to form a view of their relative position. Mathematically, the specification has features of both cardinal and ordinal social comparison. Namely, agents suffer both from the degree to which their neighbors out-perform them and from the number of neighbors that out-perform them. If an agent owns a Toyota Corolla (cheap car), then he prefers that his rich neighbor own a Ford Mustang (medium-priced car) to a Ferrari (expensive car) holding all other consumption fixed, demonstrating sensitivity to cardinal comparisons. Furthermore, he prefers that his poor neighbor

[1]Called *relative deprivation* by Stark.

also own a Toyota Corolla as opposed to a Toyota Camry (a slightly more expensive car than a Corolla), demonstrating sensitivity to ordinal comparisons.

In our model, networks play a fundamental role as an agent's social comparison is only with his neighbors in the network. Our model is the first to introduce social comparison in such a network setting. This paper tackles the primary theoretical challenges of this endeavor. Our primary objective is to solve for and analyze the Nash equilibria of the model. We find that the best-response function is isotone in the lattice structure induced by effort profiles, and so the equilibria of our game exist and themselves form a complete lattice. Furthermore, sequential best-response dynamics from either the minimum or maximum effort profiles converges in polynomial time to either the minimum-producing or maximum-producing equilibria.

We then study how the production and welfare of equilibria change as agents care more about status. We find that production at both the minimum-producing and maximum-producing equilibria increases with status considerations, indicating that external organizations like charities or luxury good retailers have incentives to perpetuate the perception of their goods as status symbols. Welfare, on the other hand, decreases with status concerns. Starting from any equilibrium and increasing status concerns causes agents to converge to a new equilibrium (according to sequential best-response dynamics) in which every agent has (weakly) lower welfare. This supports the intuition that attempts to "keep up with the Joneses," as people start to care more and more about the Joneses, cause agents to over-spend. This happens for all agents, even those that don't even know the Joneses, demonstrating the spillover effects of the network structure. We conclude with a study of the effect of network structure on production and welfare. We characterize the set of supportable equilibria of a network by a notion of connectivity of groups of nodes, called *cohesion*: namely for agents to exert high effort in equilibrium, they must be well-connected to other high-effort agents. Using this characterization, we are able to show that adding edges to the network can either increase or decrease production (and also welfare) as it alters the connectivity of groups of nodes.

2. RELATED WORK

While the study of status-seeking behavior is mostly new to computer science, there is a variety of previous work on social status in economics. By and large, this work considers agents who compare themselves to the same fixed reference group. The objective of this work is to show how agents strive to conform to the behavior of their people in that reference group. Particular assumptions are made about the form of utility for social comparisons and trade-offs between positional and non-positional goods. The objective is to show how changes in these tradeoffs affect various economic outcomes, such as consumption and immigration.[2] For our purposes, the important commonality of the work is the assumption that all agents' status is jointly determined in one group - be it a closed social community or the whole society. We study overlapping social links. Individuals have their own reference group, but these reference groups overlap with the reference groups of others.

In terms of theory, the work most relevant to our model of competitive status is the study of *social context games* in Ashlagi et al. [2008]. This setting specifies a graph G, an underlying game H, and some aggregation function, such as max/min/mean. The payoff for an agent i depends on both the payoffs from H and the aggregation of payoffs for i's neighbors in H. By contrast, our work assumes no distinction between the payoffs due to the game and the payoffs due to social comparisons. Brandt et al. [2009] study the complexity of computing equilibria in the social context of ranking functions, where the underlying game produces some ordinal ranking of players and each player's utility weakly increases as he improves his position in the ranking.

[2]For example, Frank (1985) considers a model where agents produce a positional and non-positional good. When agents care about their position within society, agents will under produce the non-positional goods. [Clark and Oswald 1998]Oded Stark and You Qiang Wang (2005) [Stark and Wang 2005] considers the case of a finite set of players in but in a setting where consumption levels are fixed and agents are free to choose the set of agents that they associate with and thus base their relative status on a fixed number of full connected agents. [Stark and Wang 2005]

More widely, our work contributes to the research on games played on a fixed network. The game in our paper has several interpretations, including provision of public goods and charitable contributions, and thus we share applications with other network literature.[3]

3. THE MODEL

There are n agents $V = \{1, 2, ..., n\}$ working to procure a good (e.g., income, luxury goods like automobiles, or public goods like charity contributions). Each agent i chooses an effort level $e_i \in [0, \infty]$, representing the degree to which he produces the good. Let $\mathbf{e}$ denote the effort profile, and $\mathbf{e}_{-i}$ denote the effort profile of all agents other than i. Agents are arranged in a network according to a connected, undirected graph $G = (V, E)$. An edge between agents i and j indicates that each agent's payoff is directly affected by the other's effort. We will denote the neighbors of agent i by $N_i = \{j \in V | (i, j) \in E\}$. The payoffs consist of two components:

(1) **Economic Costs and Benefits.** We specify a simple form of costs and benefits that captures the basic tradeoffs between individual costs and benefits and possibly positive consumption externalities. We suppose the production of the good is proportional to the effort level, with an agent-specific proportionality factor $\alpha_i > 0$, referred to as the agent's *type*. Some goods (e.g., contributions to a charity) exhibit a positive externality, thus causing an agent's payoff to increase with the effort level of his neighbors[4] as well as his own effort level. We model the externality with a tunable parameter $\delta \in [0, 1]$. Thus, given effort profile $\mathbf{e}$, the contribution to the payoff of agent i is $\alpha_i \cdot (e_i + \delta(\sum_{j \in N_i} e_j))$. As is standard in economics literature, we assume the cost of effort is quadratic in effort level contributing $-\frac{1}{2}e_i^2$ to the payoff of agent i. With this specification, the network costs and benefits remain simple enough so we can engage our main interest which is social status.

(2) **Social Status.** Status concerns of agents cause them to experience a disutility from being less productive than their neighbors. Given effort profile $\mathbf{e}$, we posit a status loss function $S(e_i, \mathbf{e}_{-i}; G) = -\sum_{j \in N_i} \frac{1}{|N_i|+1} \max\{e_j - e_i, 0\}$, adopted from that of Stark and Wang [2005]. The status loss function contributes $\beta \cdot S(e_i, \mathbf{e}_{-i}; G)$ to agent i's payoff, where $\beta > 0$ controls the extent of the status effect on agents' payoffs. Note this implies agent i has lower status when he produces a lower level of good relative to his neighbors, and this status loss is higher as the gap between the quantity of i's good and his neighbors' goods increases.

Thus, in summary, given effort profile $\mathbf{e}$, social network $G = (V, E)$, and parameters β, δ, an agent i of type α_i has a payoff of:

$$u_i(\mathbf{e}; \alpha_i, \beta, \delta, G) = \alpha_i \left(e_i + \delta(\sum_{j \in N_i} e_j) \right) - \frac{1}{2}e_i^2 - \beta \sum_{j \in N_i} \frac{1}{|N_i| + 1} \max\{e_j - e_i, 0\}.$$

When it is clear from the context, we drop the parameters and graph specification from the utility function.

[3]Bramoulle and Kranton [2007] introduced public goods game and study the characteristics of the Nash equilibrium. Galeotti et al. [2009] study the effect of change in the degree of a node under the various assumptions of the positive or negative externalities imposed by the actions of neighbors. Conitzer and Sandholm [2004] consider a model of charitable contributions where agents specify amounts that they are willing to contribute, contingent upon other agents contributing certain amounts. They introduce a bidding language for agents to express their preferences and provide optimal algorithms for determining clearing contribution levels for a restricted class of bids. Ghosh and Mahdian [2008] give a similar model where an individuals value for contribution to a charity is equal to the sum of contribution levels of all of the agent's neighbors minus his own contribution. They give a mechanism which has the maximum aggregate contribution as an equilibrium under the condition that the graph is strongly connected.

[4]In full generality, we could define the effect of j's effort level on i's payoff to be $\delta_{ij} e_j$ for fixed parameters δ_{ij}; all results follow. We restrict attention to a homogenous δ for neighbor's efforts for ease of presentation.

We define the *status game* as follows. Given the graph structure $G = (V, E)$, agents simultaneously choose effort levels $e_i \in [0, \infty)$ and receive payoffs $u_i(\mathbf{e}; G)$. We are interested in the Nash equilibria of the game. An effort profile $\mathbf{e}^*$ is Nash equilibrium vector if and only if for all agents i, $u_i(e_i^*, \mathbf{e}_{-i}^*; G) \geq u_i(e_i, \mathbf{e}_{-i}^*; G)$ for all e_i.

4. EQUILIBRIUM ANALYSIS

Our game supports a potentially infinite set of Nash equilibria $\{\mathbf{e}\}$. In this section, we argue that these equilibria form a complete lattice[5] with respect to the partial order $\succeq$ defined by $\mathbf{e} \succeq \mathbf{e}'$ if $e_i \geq e_i'$ for all i, and that the maximum element $\mathbf{e}^{\max}$ and minimum element $\mathbf{e}^{\min}$ of the lattice can be reached in polynomial time by a natural best-response dynamic from particular initial configurations.

4.1. Best-Response Functions

In order to prove that the set of Nash equilibria form a complete lattice, we will define a best-response function whose fixed points are Nash equilibria and argue that this is an isotone function on the lattice of effort profiles defined by partial order $\succeq$.

The best-response of agent i to effort profile $\mathbf{e}$ is $B_i(\mathbf{e}_{-i}) \equiv \arg\max_e u_i(e, \mathbf{e}_{-i}; G)$. In solving this maximization problem, agent i must weigh the marginal economic benefits less the costs of his effort, which is simply $\alpha_i - e_i$, against the marginal effect of his action on his status loss. To see the effect of e_i on agent i' s status, consider the following reformulation of the status loss function $S(e_i, \mathbf{e}_{-i}; G)$:

$$\begin{aligned} S(e_i, \mathbf{e}_{-i}; G) &= -\sum_{j \in N_i} \frac{1}{|N_i| + 1} \max\{e_j - e_i, 0\} \\ &= \frac{-1}{|N_i| + 1} \sum_{j | e_j > e_i} e_j + \frac{1}{|N_i| + 1} \sum_{j | e_j > e_i} e_i \\ &= \frac{-1}{|N_i| + 1} \sum_{j | e_j > e_i} e_j + e_i \frac{1}{|N_i| + 1} \sum_{j | e_j > e_i} 1 \end{aligned}$$

The second term is e_i times the proportion of agents in i's neighborhood that are playing a higher action. This number $\frac{1}{|N_i|+1} \sum_{j|e_j > e_i} 1$ is key to our analysis, as it indicates an agent's relative rank in his neighborhood. We will denote the proportion of higher-action players as $p(e_i, \mathbf{e}_{-i}; G) \equiv \frac{1}{|N_i|+1} \sum_{j \in N_i | e_j > e_i} 1$. Now the status loss term can be written as

$$S(e_i, e_{-i}; G) = \frac{-1}{|N_i| + 1} \sum_{j \in N_i | e_j > e_i} e_j + e_i \cdot p(e_i, \mathbf{e}_{-i}; G)$$

where $p(e_i, \mathbf{e}_{-i}; G)$ is piece-wise linear. Figure 1 in Appendix A.1 provides an example of the change in agent i's status loss as agent i changes his action.

Except at points of discontinuity, small positive change in effort level also corresponds to a reduction in status loss of $\beta \cdot p(e_i, \mathbf{e}_{-\mathbf{i}}; G)$. Intuitively, any best-response effort level should be at a point where the marginal cost of effort is equal to the marginal gain in value for the good plus the marginal reduction in status loss (when well-defined). We formalize this intuition in the following proposition. The proof is deferred to Appendix A.2.

[5]Recall a complete lattice is a partially ordered set $(L, \succeq)$ in which each non-empty subset A of L has a greatest lower and least upper bound.In particular, the lattice itself has a maximum and minimum element.

Proposition 4.1. *Fix the effort profile* $\mathbf{e_{-i}}$ *of other agents and let* $B_i(\mathbf{e_{-i}}) = \arg\max_{e_i} u_i(e_i, \mathbf{e_{-i}})$ *be the best-response of agent* i *to* $\mathbf{e_{-i}}$*. Then*

$$B_i(\mathbf{e_{-i}}) = \arg\min_{e_i}\{e_i \; |e_i \geq \alpha_i + \beta \cdot p(e_i, \mathbf{e_{-i}}; G)\}.$$

Note B_i is technically a function of the parameters α_i, β, δ and the network structure G. As before, when these are clear from the context, we omit them from the function.

From a graphical perspective, the best-response for agent i is the minimum effort level e_i such that the line $y = x$ lies weakly above the graph of $\alpha_i + \beta \cdot p(e_i, e_{-i}; G)$. Figure 2 provides examples which demonstrate this intuition. Figure 2a in Appendix A.1 shows a case where there is an exact effort level e_i where the marginal cost of effort is equal to the marginal gain plus the marginal reduction in status loss, i.e., $e_i = \alpha_i + \beta \cdot p(e_i, \mathbf{e_{-i}}; G)$. In the case shown in Figure 2b, there's no exact effort level which balances the marginal costs and benefits because $p(e_i, \mathbf{e_{-i}}; G)$ exhibits a discontinuity when agent i increases his effort level. To get a feel for why this is the best-response, playing any effort level less than the crossing point causes a large increase in loss of status compared with the small decrease in cost of effort while playing anything greater than this point has a larger marginal cost than marginal gain.

We next show that the best-response function obeys a monotonicity property.

Proposition 4.2. *Let* e_{-i} *and* e'_{-i} *be effort level profiles such that* $e_{-i} \geq e'_{-i}$ *for all* i*. Then* $B_i(\mathbf{e_{-i}}) \geq B_i(\mathbf{e'_{-i}})$

Proof. If $e_{-i} \geq e'_{-i}$ at all points, then $p(e_i, \mathbf{e_{-i}}; G) \geq p(e_i, \mathbf{e'_{-i}}; G)$ at all effort levels e_i. Let $B_i(\mathbf{e_{-i}}) = e_i^*$ and let $B_i(\mathbf{e'_{-i}}) = e'_i$. Then

$$e_i^* \geq \alpha_i + \beta p(e_i^*, \mathbf{e_{-i}}; G) \geq \alpha_i + \beta p(e_i^*, \mathbf{e'_{-i}}; G)$$

Where the first inequality is from theorem 4.1. Then $e_i^* < e'_i$ would contradict $e'_i = \arg\min_{e_i}\{e_i | e_i \geq \alpha_i + \beta p(e_i, \mathbf{e'_{-i}}; G)\}$, so we conclude $e_i^* \geq e'_i$. □

We are now ready to show that the set of Nash equilibria form a complete lattice. We will use Tarski's fixed point theorem [Tarski 1955].

Theorem 4.3. (Tarski [1955]). *Let* $(L, \succeq)$ *be any complete lattice. Suppose* $f : L \to L$ *is monotone increasing (or isotone), i.e., for all* $x, y \in L$, $x \succeq y$ *implies* $f(x) \succeq f(y)$*. Then the set of all fixed points of* f *is a non-empty complete lattice with respect to* $\succeq$*.*

Define the function $B : \mathbb{R}^n \to \mathbb{R}^n$ as $B(\mathbf{e}) = (B_1(\mathbf{e}_{-1}), \ldots, B_n(\mathbf{e}_{-n}))$. By definition, the fixed points of B are Nash equilibria of our game. Furthermore, by Proposition 4.2, B is isotone with respect to partial order $\succeq$ where $\mathbf{e} \succeq \mathbf{e}'$ if and only if $e_i \geq e'_i$ for all i. To apply Tarski's fixed point theorem, we need to argue that the effort levels in a Nash equilibrium are bounded in a convex set (and hence the relevant effort profiles form a complete lattice). This follows from an immediate corollary of Proposition 4.1.

Corollary 4.4. *For any equilibrium action level* e_i^* *of agent* i, $e_i^* \in [\alpha_i, \alpha_i + \frac{|N_i|}{|N_i+1|}\beta]$.

In conclusion, we have proved the following theorem.

Theorem 4.5. *The set of Nash equilibria of the status game form a non-empty complete lattice.*

4.2. Best-Responses Dynamics

In this section we consider best-response dynamics in our setting. It is well-known that iterating an isotone function on a lattice results in a fixed point. In our setting, this corresponds to a best-response dynamic in which agents update strategies simultaneously in each round. Arguably more natural is a sequential best-response dynamic in which, in each round each agent i sequentially computes a best-response to the efforts of other agents. We first formally define the sequential best-response dynamics. Label agents in an arbitrary order $\{1, \ldots, n\}$. We allow agents to update efforts in round-robin fashion according to that ordering as shown in the pseudocode of Algorithm 1.[6]

Algorithm 1 Best-Response($\mathbf{e}^0$)

```
t ← 0
repeat
  e ← e^t
  for i = 1 to n do
    e_i ← arg max_e u_i(e, e_{-i}; G)
  end for
  e^{t+1} ← e
  t ← t + 1
until e^t = e^{t-1}
```

Here we prove via a standard potential function argument that these dynamics also converge and moreover do so in polynomial time.

THEOREM 4.6. *For an effort level profile* $\mathbf{e}^0$ *where* $e_i^0 = \alpha_i + \frac{|N_i|}{|N_i|+1}\beta$*, Algorithm 1 on input* $\mathbf{e}^0$ *converges in time* $O(n^3)$ *to the maximum Nash equilibrium* $\mathbf{e}^{\max}$*. Similarly, for an effort level profile* $\mathbf{e}^0$ *where* $e_i^0 = \alpha_i$*, Algorithm 1 on input* $\mathbf{e}^0$ *converges in time* $O(n^3)$ *to the minimum Nash equilibrium* $\mathbf{e}^{\min}$*.*

PROOF. We prove the first claim, that Algorithm 1 on input $e_i^0 = \alpha_i + \frac{|N_i|}{|N_i|+1}\beta$ converges to $\mathbf{e}^{\max}$ in polynomial time (the proof of the second claim is similar). We first show by induction that at every round $t \geq 1$ until the last round of Algorithm 1, the effort profile $\mathbf{e}^t$ is strictly dominated by $\mathbf{e}^{t-1}$ according to partial order $\succeq$ and so the effort profiles computed by the algorithm form a chain in the lattice. Note that in $\mathbf{e}^0$, each agent exerts an effort at least as large as his effort in any equilibrium by Corollary 4.4. Thus after the first round, each agent's effort weakly decreases and at least one agent's effort strictly decreases (otherwise we have reached an equilibrium). Now consider an arbitrary round t. In this round, by induction each agent best-replies to an intermediate effort profile that is weakly dominated by the one he considered in his last best-response computation. Hence, by Theorem 4.2, each agent's effort weakly decreases and at least one agent's effort strictly decreases (otherwise we have reached an equilibrium).

We conclude by noting that due to the form of the best-response function as stated in Proposition 4.1, each agent's effort in any best-response is a constant plus one of $|N_i|$ multiples of $p(e_i, \mathbf{e}^{-i}; G) = \frac{1}{|N_i|+1} \sum_{j \in N_i | e_j > e_i} 1$. Thus each best-response function B_i takes on at most n distinct values. Therefore, the maximum length of a chain in the lattice is n^2. Therefore the algorithm can take at most n^2 rounds, and so converges in time $O(n^3)$ as claimed. □

5. ECONOMIC RESULTS

In this section, we explore various properties of equilibrium effort levels, including the amount of production of the good and the social welfare of the agents. We are particularly interested in the effects of status considerations on these properties. In absence of status effects (i.e., $\beta = 0$), an agent's utility-maximizing effort level is equal to his type, $e_i = \alpha_i$, regardless of the efforts of others $\mathbf{e}_{-i}$ and the extent of the positive externality δ. This is because at effort level $e_i = \alpha_i$, the marginal

[6] Actually, the order of updates is not important; as can be seen from the following proof, so long as each agent updates at least every d steps, the dynamic converges in time polynomial in d and n.

contribution of effort $-e_i + \alpha_i$ is zero (when $\beta = 0$). Thus these effort levels define the unique Nash equilibrium when $\beta = 0$ and will be our baseline for comparison.

Definition 5.1. The *status-free effort profile* $\mathbf{e}^{sf}$ is defined by $e_i^{sf} = \alpha_i$.

We will compare the properties of this status-free effort profile to the properties of the minimum equilibrium $\mathbf{e}^{\min}$ and the maximum one $\mathbf{e}^{\max}$ for various settings of the parameters and network structures.

5.1. Production

We first study the effects of the parameters on production in equilibrium. We define the *production* of the good at profile $\mathbf{e}$ to be $\sum_{i=1}^{n} e_i$. The level of production is of particular interest when the good has some public benefit, e.g., contribution to a charity. Naturally production increases with type and status concerns.

THEOREM 5.2. *The minimum-producing equilibrium is* $\mathbf{e}^{\min}$ *and the maximum-producing one is* $\mathbf{e}^{\max}$. *Furthermore,*

— *production at* $\mathbf{e}^{\min}$ *is increasing in* α_i *and weakly increasing with respect to* β,
— *and production at* $\mathbf{e}^{\max}$ *is increasing in both* α_i *and* β.

The degree of positive externality, δ, *has no effect on production.*

PROOF. By the definition of the partial order, it is clear that for any fixed setting of parameters, $\mathbf{e}^{\min}$ and $\mathbf{e}^{\max}$ are the minimum-producing and maximum-producing equilibria, respectively.

To prove the claims about the effects of α_i and β on the equilibria, we show that the best-response function $B_i(\cdot; \alpha_i, \beta)$ is increasing in these parameters and then use the analysis of the best-response dynamics to conclude the result. First recall Proposition 4.1, which states that $B_i(\mathbf{e}_{-i}; \alpha_i, \beta) = \min\{e_i | e_i \geq \alpha_i + \beta p(e_i, \mathbf{e_{-i}}; G)\}$. From this form, it is clear that if we hold fixed $\mathbf{e_{-i}}$, increasing α_i and/or β can only (weakly) increase i's best-response. Let α_i', β' be parameters such that $\alpha_i' \geq \alpha_i$ for all i and $\beta' \geq \beta$. Then for any effort profile $\mathbf{e}$ and agent i, we have that $B(\mathbf{e}_{-i}; \alpha_i', \beta') \geq B(\mathbf{e}_{-i}; \alpha_i, \beta)$. In particular, fixing $\mathbf{e}^{\min}$ to be the minimum equilibrium with parameters α_i, β, we see that $B(\mathbf{e}^{\min}; \alpha_i', \beta')$ is weakly increasing for any i and hence, by the proof of Theorem 4.6, sequential best-response dynamics from $\mathbf{e}^{\min}$ computes the minimum equilibrium $\mathbf{e'}^{\min}$ for parameters α_i', β', and $\mathbf{e'}^{\min} \succeq \mathbf{e}^{\min}$. A similar argument holds for the maximum equilibrium.

Finally, note that δ has no effect on the best-reply dynamics and thus doesn't affect $\mathbf{e}^{\max}$ or $\mathbf{e}^{\min}$. □

The above theorem implies that production in equilibria weakly dominates production in the status-free effort profile. In fact, it is not hard to see that in heterogenous settings, this comparison is strict: the production of the status-free effort profile is strictly less than any equilibrium.

5.2. Welfare

We next study the effects of the parameters on welfare in equilibrium. We define the *social welfare* of profile $\mathbf{e}$ to be $\sum_{i=1}^{n} u_i(\mathbf{e}; G)$. We focus on the case of private goods, i.e., $\delta = 0$ (for $\delta > 0$ the welfare comparisons of various equilibria depend on the specific setting and so are of less interest). Naturally, in this case, the status-free effort profile maximizes welfare, the minimum-equilbrium $\mathbf{e}^{\min}$ is the maximum-welfare equilibrium, and the maximum-equilibrium $\mathbf{e}^{\max}$ is the minimum-welfare one. This is because profiles with higher production can only decrease each agent's payoff as $u_i(e_i, \mathbf{e}_{-i}; G)$ is maximized at $e_i = \alpha_i$ and decreasing in e_j for $j \neq i$.

Somewhat more surprising is that, when $\delta = 0$, welfare decreases with β. In other words, for any fixed equilibrium, increasing status concerns and allowing agents to best-respond causes agents to over-exert effort and results in a net loss to their welfare in the new equilibrium.

We first show that holding an effort profile fixed, increasing status concerns and letting one agent best-respond causes that agent to lose welfare.

PROPOSITION 5.3. *Consider a fixed setting of parameters* α_i, β *and* $\delta = 0$*, and fix any effort profile* $\mathbf{e}$ *for these parameters. Let* $\beta' > \beta$*, choose an arbitrary agent* i*, and let* $e_i = B(\mathbf{e}_{-i}; \alpha_i, \beta, G)$ *and* $e_i' = B(\mathbf{e}_{-i}; \alpha_i, \beta', G)$ *be agent* i*'s best-response to the old and new parameters, respectively. Then* $e_i' > e_i$ *and* $u_i(e_i, \mathbf{e}_{-i}; \alpha_i, \beta, G) \geq u_i(e_{-i}', \mathbf{e}_{-i}; \alpha_i, \beta', G)$*. Furthermore, the inequality is strict for any agent* i *such that* $S(e_i', \mathbf{e_{-i}} : G) < 0$*.*

PROOF. Note by the form of the best-response function given in Proposition 4.1, B_i is weakly increasing in β and so $e_i' \geq e_i$. Let $\sigma = e_i' - e_i$, let $\epsilon = \beta' - \beta$, and note both σ and ϵ are non-negative. Consider the utility of agent i in the two settings:

$$u_i(e_i; \alpha_i, \beta, G) = \alpha_i e_i + \beta S(e_i) - \frac{1}{2} e_i^2$$

and

$$u_i(e_i'; \alpha_i, \beta', G) = \alpha_i(e_i + \sigma) + (\beta + \epsilon) S(e_i') - \frac{1}{2}(e_i + \sigma)^2.$$

Subtract the first from the second

$$\alpha_i \sigma + \epsilon S(e_i + \sigma) + \beta(S(e_i + \sigma) - S(e_i)) - \sigma(e_i + \frac{\sigma}{2}).$$

By Lemma A.1, $\sigma p(e_i + \sigma) \geq S(e_i + \sigma) - S(e_i)$ and since e_i was a best-response, $e_i \geq \alpha_i + \beta p(e_i)$. Substituting in these two inequalities gives that the above equation is at most

$$\alpha_i \sigma + \epsilon S(e_i + \sigma) + \beta(\sigma p(e_i + \sigma)) - \sigma(\alpha_i + \beta p(e_i) + \frac{\sigma}{2})$$

$$= \beta \sigma (p(e_i + \sigma) - p(e_i)) + \epsilon S(e_i + \sigma) - \frac{\sigma^2}{2} \leq 0.$$

The function p is non-increasing in its argument, so the first term is non-positive; the status loss function S is non-positive by definition; and the last term is non-positive as $\sigma \geq 0$. Hence the welfare of agent i weakly decreased, and strictly decreased if any of the terms (e.g., the status) is negative. □

We next show that, starting from an equilibrium, increasing status concerns, and then letting agents best-respond to reach a new equilibrium causes every agent to have (weakly) less welfare.

PROPOSITION 5.4. *Consider a fixed setting of parameters* α_i, β *and* $\delta = 0$*, and fix any equilibrium* $\mathbf{e}$ *for these parameters. Let* $\beta' > \beta$*. Then there is a new equilibrium* $\mathbf{e}'$ *for parameters* α_i, β' *and* $\delta = 0$ *such that for each agent* i*,* $u_i(\mathbf{e}; \alpha_i, \beta, G) \geq u_i(\mathbf{e}'; \alpha_i, \beta', G)$*. Furthermore, the new equilibrium* $\mathbf{e}'$ *can be reached from the old equilibrium by sequential best-response dynamics.*

PROOF. We prove by induction that after each best-response by an agent, his utility decreases and his effort level increases.

Consider the first agent i to best-respond by playing $B_i(\mathbf{e}_{-i}; \alpha_i, \beta', G)$. By Proposition 5.3, $B_i(\mathbf{e}_{-i}; \alpha_i, \beta', G) \geq B_i(\mathbf{e}_{-i}; \alpha_i, \beta, G)$ and $u_i(e_i, \mathbf{e}; \alpha_i, \beta, G) \geq u_i(e_{-i}', \mathbf{e}_{-i}; \alpha_i, \beta', G)$.

Now consider any subsequent agent i to move. Denote the current effort profile of others before his best-response by $\mathbf{e}_{-i}^t$. By induction, $\mathbf{e}_{-i}^t \succeq \mathbf{e}_{-i}$ and so, by monotonicity of best-responses, $e_i^t \equiv B_i(\mathbf{e}_{-i}^t; \beta) \geq B_i(\mathbf{e}_{-i}; \beta) = e_i$. Furthermore, $u_i(\mathbf{e}) \geq u_i(e_i^t, \mathbf{e}_{-i}) \geq u_i(\mathbf{e}^t)$ where the first inequality holds by definition of equilibrium and the second since, holding all else fixed, increasing efforts of others decreases welfare of i as it only increases his status loss. Now let $e_i' = B(\mathbf{e}_{-i}^t; \alpha_i, \beta', G)$ and note by Proposition 5.3, $e_i' > e_i^t$ and $u_i(e_i^t, \mathbf{e}_{-i}^t; \alpha_i, \beta, G) \geq u_i(e_i', \mathbf{e}_{-i}^t; \alpha_i, \beta', G)$. Combining inequalities, we see agent i's welfare decreases from his initial welfare level after each best-response (i.e.,

$u_i(\mathbf{e};\beta) \geq u_i(B(\mathbf{e}^t_{-i};\beta'), \mathbf{e}^t_{-i};\beta'))$ and furthermore, his effort increased, so the inductive hypothesis holds.

As we noted above, effort is weakly increasing throughout this process. Thus this best-responding forms a monotonic function on our polynomial-sized lattice, which converges in polynomial time to an equilibrium $\mathbf{e}'$ for status concerns β in which each agent has (weakly) less welfare. □

Note that these theorems were stated for a homogenous setting of β. However, the theorems and proofs still hold in settings where agents have heterogenous concerns for status so long as each agent's status concerns weakly increase.

5.3. Network Effects

We conclude by studying the effect of the social network structure on both production and welfare. In particular, we discuss the implications of adding or deleting links from the network. For clarity, we focus on the setting where agents have a homogenous type $\alpha_i = \alpha$ (in which case $\mathbf{e}^{sf} = \mathbf{e}^{\min}$). In such settings, we can derive the following general characterization of effort levels sustainable in equilibrium in terms of the *cohesion* of nodes in the network.

Definition 5.5. For a given set of nodes T, we define the *cohesion* of T to be the smallest number ρ such that for any node $i \in T$, at least a ρ fraction of $N_i \cup \{i\}$ are also in T. We say a set of nodes T is *ρ-cohesive* if the cohesion of T is at least ρ. Formally, T is ρ-cohesive if $\forall i \in T$, $\frac{|N_i \cap T|}{|N_i|+1} \geq \rho$.

THEOREM 5.6. *Given effort profile $\mathbf{e}$, define f_i such that $e_i = \alpha + f_i\beta$ for all i. Let $S_f = \{i \in N | f_i \geq f\}$. If $\mathbf{e}$ is an equilibrium, then S_f is f-cohesive for all $f \in [0, \frac{|N|}{|N|+1}]$.*

PROOF. Pick an arbitrary $f \in [0, \frac{|N}{|N|+1}]$ and let agent i be a member of S_f. For all agents j, define f_j such that $\mathbf{e_j} = \alpha + f_j\beta$. Note that in this setting, $\mathbf{e_i} > \mathbf{e_j}$ iff $f_i > f_j$. Let $g(f, \mathbf{e_{-i}}; G) = \frac{|N_i \cap S_f|}{|N_i+1|}$ be the fraction of i's neighborhood that is playing $f_j \geq f$ in $\mathbf{e_{-i}}$. For the sake of contradiction, assume that agent i's neighborhood is not $f-$cohesive, i.e.

$$f > g(f, \mathbf{e_{-i}}; G)$$

Then we will show that there exists a small negative deviation for agent i which satisfies the conditions of theorem 4.1, contradicting that $\mathbf{e_i}$ is the best-response for player i (and that $\mathbf{e}$ is an equilibrium).
Then let $\sigma = f - g(f, \mathbf{e_{-i}}; G)$, let $\epsilon = \min_{j \in N_i | f_j < f} f - f_j$, and let $\theta = \min\{\sigma, \epsilon\}$. Because θ is small, any agent j such that $f_j < f$ also satisfies $f_j < f - \frac{\theta}{2}$. This implies

$$g(f, \mathbf{e_{-i}}; G) = g(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)$$

This yields the following

$$f - \frac{\theta}{2} > g(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G) \geq p(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)$$

Where the last inequality holds because $g()$ gives the fraction i's neighbors that are playing a weakly higher effort level while $p()$ gives the fraction of i's neighbors that are playing a strictly higher effort level. Finally the above inequality gives, $\alpha + (f - \frac{\theta}{2})\beta \geq \alpha + p(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)\beta$, so it satisfies the conditions in theorem 4.1 for a best-response. This contradicts that e_i was the best-response for player i.

So our assumption that player i's neighborhood was not f-cohesive is false. Then for any agent in S_f, their neighborhood must be at least f-cohesive in S_f. Thus S_f is f-cohesive. □

From the above theorem, it is clear that adding or deleting links will change the set of supportable equilibria as it changes the cohesion of subsets of nodes. Indeed, the following examples show that these effects can have arbitrary consequences for production and welfare.

Example 5.7. Consider the homogenous setting where $\alpha_i = \alpha$ and agents arranged according to the complete graph on 10 nodes, K_{10}, and a complete graph on 3 nodes K_3 such that there's an edge between node $i \in K_{10}$ and $j \in K_3$. This graph is shown in 3a. The maximum effort level of node j is $\alpha + \frac{3}{4}\beta$. Thus for any $e > \alpha + \frac{3}{4}\beta$, at most a $\frac{9}{11}$ fraction of i's neighborhood can play e in equilibrium (the i's 9 neighbors in K_{10} can play a high effort level but j cannot). Then, by theorem 5.6, i can play at most $\alpha + \frac{9}{11}\beta$ in equilibrium. Indeed, $\mathbf{e}^{\max}$ on this graph is $e = \alpha + \frac{9}{11}\beta$ for all the nodes in K_{10} and $e = \alpha + \frac{2}{3}\beta$ for all nodes in K_3. However, if we add the edge between agent i and another node $k \in K_3$, then for any $e > \alpha + \frac{3}{4}\beta$, the cohesion in i's neighborhood drops to $\frac{9}{12}$, implying that agent i can play at most $\alpha + \frac{9}{12}\beta$ in $\mathbf{e}^{\max}$. Additionally, this edge didn't raise the $\mathbf{e}^{\max}$ effort levels for the nodes in K_3 because the cohesion is bounded at $\frac{2}{3}$ due to the node in K_3 that doesn't have an edge with i. Thus adding this edge strictly decreased the production of the good.

6. CONCLUSION

This paper presents a network of social status. Agents have economic costs and benefits of providing a good and they compare their production to those in their neighborhood. When a graph is connected but not complete, agents' incentives are affected by not only by their neighbors, but by actions of agents at a distance in the network. We study the Nash equilibria of the game; we find the equilibrium set forms a lattice. Best response dynamics naturally converge to either the minimum or maximum equilibria. We compare equilibrium outcomes for different types of agents and show that striving for status has significant impact on agents' production and welfare. While status concerns increase aggregate production, they decrease social welfare (even pointwise) as agents over-produce the good relative to the economic costs and benefits. Furthermore, adding or deleting links can affect production (and also welfare) in arbitrary ways.

We leave open for future work the obvious and interesting network design questions. Given a fixed network and initial agent types α_i, how should a designer with limited resources increase types to maximize production? Such a strategy amounts to marketing the good to particular agents in the network. Alternatively, Given fixed types α_i and an initial network, how should a designer with limited resources introduce edges to maximize production? This strategy amounts to introducing particular agents in the network, e.g., through charity balls.

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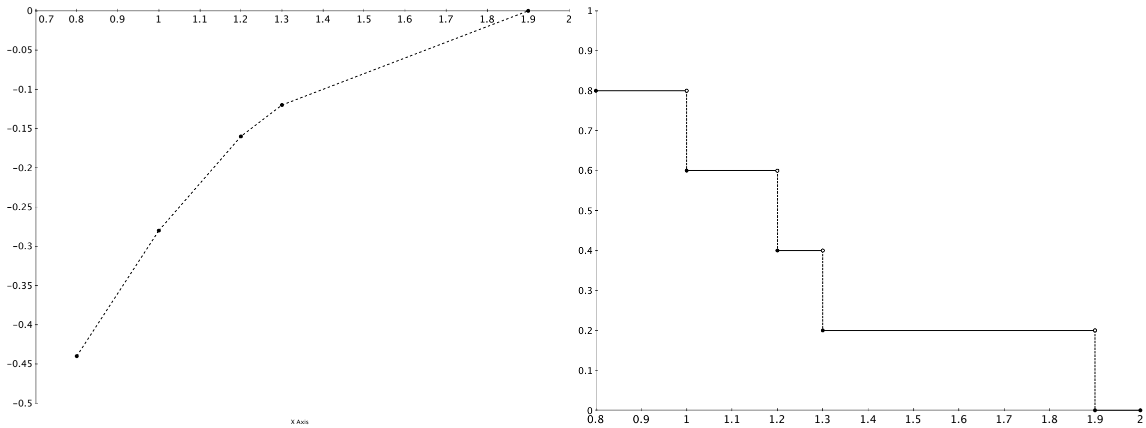
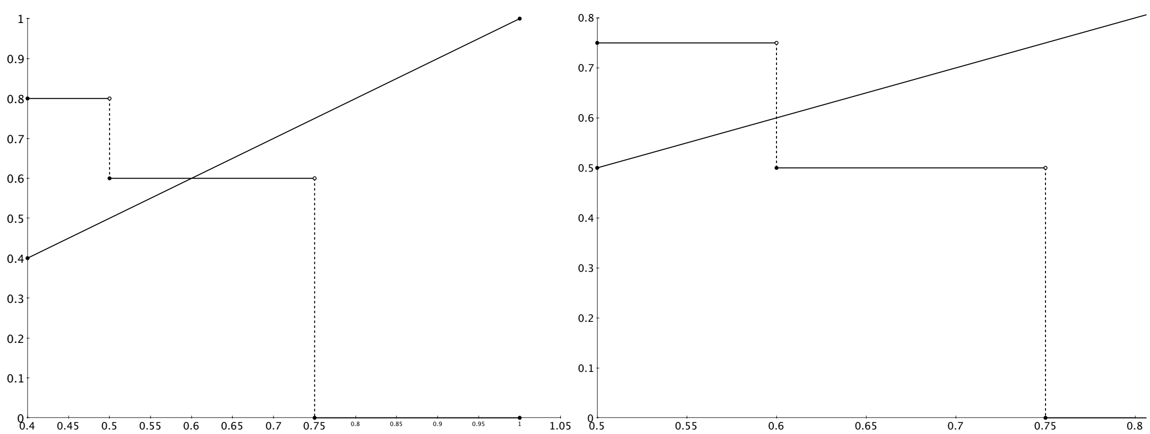
A. APPENDIX

A.1. Missing Figures



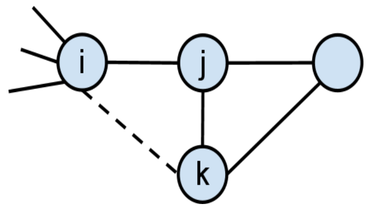
(a) Status Loss Curve (b) Marginal Status Loss Curve

Fig. 1: Figure 1a plots the status loss curve as function of effort of an agent whose neighbors' effort levels are $(1, 1.2, 1.3, 1.9)$. Figure 1b plots the marginal loss in status as a function of effort when faced with the same profile.



(a) Best-Response Curves (b) Best-Response Curve

Fig. 2: Given fixed effort levels $\mathbf{e}_{-i}$, we plot the curve $\alpha_i + \beta \cdot p(e_i; \mathbf{e}_{-i}; G)$ as a function of effort level e_i. The intersection point between this curve and the line $y = x$ is the best-response of agent i. Figure $2a$ (respectively $2b$) depicts the curve when the effort levels of i's neighbors are $(0.75, 0.75, 0.75, 0.5)$ (respectively $(0.6, 0.75, 0.75)$).



(a) Homogenous Case

Fig. 3: Figure 3a shows a graph for which adding an edge decreases $\mathbf{e}^{\max}$, as in example 5.7. The entire K_{10}, with the exception of node i, is not shown. In this example, adding the edge represented by the dotted line decreases the effort level that agent i plays in $\mathbf{e}^{\max}$ and in fact causes aggregate production to decrease on this graph.

A.2. Missing Proofs

Here we prove Proposition 4.1. We begin with the following lemma.

LEMMA A.1. *For any e_i and $x > 0$, the difference in social status effects $S(e_i + x; \mathbf{e_{-i}}; G) - S(e_i, \mathbf{e_{-i}}; G)$ is bound by following inequalities*

$$x \cdot p(e_i, \mathbf{e_{-i}}; G) \geq S(e_i + x; \mathbf{e_{-i}}; G) - S(e_i, \mathbf{e_{-i}}; G) \geq x \cdot p(e_i + x, \mathbf{e_{-i}}; G)$$

PROOF. By definition $S(e_i, \mathbf{e_{-i}}; G) = -\frac{1}{|N_i|+1} \sum_{j \in N_i} \max\{e_j - e_i, 0\}$. For any agent j such that $e_j > e_i + x$, increasing effort from e_i to $e_i + x$ reduces the status loss that j places on i by $\frac{x}{|N_i|+1}$ and thus the total reduction in status loss is at least $\sum_{j|e_j > e_i + x} \frac{x}{|N_i|+1} = xp(e_i + x, \mathbf{e_{-i}}; G)$. Similarly, for each agent j such that $e_j > e_i$, the reduction in status loss is at most $\frac{x}{|N_i|+1}$ (it may be less in the case where $e_i + x > e_j$). Therefore the total reduction in status is at most $\sum_{j|e_j > e_i} \frac{x}{|N_i|+1} = xp(e_i, \mathbf{e_{-i}}; G)$ □

PROOF. of 4.1 Define the set $Y = \{e_i | e_i \geq \alpha_i + \beta p(e_i, \mathbf{e_{-i}}; G)\}$ and let $e_i^* = \min_{e_i \in Y} e_i$. This proof is broken down into two parts; First, we show that e_i^* exists, then show that $u_i(e_i^*, \mathbf{e_{-i}}; G) > u_i(e_i', \mathbf{e_{-i}}; G)$ for all effort levels $e_i' \neq e_i^*$.

First we must establish the existence of a minimum element of Y. Fix the vector $\mathbf{e_{-i}}$ and reorder such that $e_1 \leq e_2$.... Then for any $e_j \neq e_{j+1}$, any $e_i \in [e_j, e_{j+1})$ satisfies $p(e_i, \mathbf{e_{-i}}; G) = p(e_j, \mathbf{e_{-i}}; G)$ because $p(e_i, \mathbf{e_{-i}}; G)$ is defined as the proportion of i's neighbors that are playing a strictly higher effort level than e_i.

We now use this claim to show that e_i^* is the unique maximizer of $u_i(\cdot, \mathbf{e_{-i}}; G)$.

— e^* is better than any lower effort level. For any $x > 0$, consider $u_i(e_i^*) - u_i(e_i^* - x) =$

$$\alpha_i \cdot (e_i^*) - S(e_i^*, \mathbf{e_{-i}}; G) - \frac{1}{2} e_i^{*2} - \left(\alpha_i \cdot (e_i^* - x) - S(e_i^* - x, \mathbf{e_{-i}}; G) - \frac{1}{2} (e_i^* - x)^2 \right)$$

$$= \alpha_i \cdot x - x(e_i^* - \frac{x}{2}) + \beta(S(e_i^*) - S(e_i^* - x))$$

By Lemma A.1, $S(e_i^*) - S(e_i^* - x) \geq xp(e_i^* - x)$. Plugging that in yields

$$\geq \alpha_i \cdot x - x(e_i^* - \frac{x}{2}) + \beta xp(e_i^* - x)$$

$e_i^* - \frac{x}{2} < e_i^*$ implies $e_i^* - \frac{x}{2} \notin Y$ and thus $e_i^* - \frac{x}{2} < \alpha_i + \beta p(e_i^* - \frac{x}{2})$ Then the above equation is at least

$$\alpha_i \cdot x - x(e_i^* - \frac{x}{2}) + \beta xp(e_i^* - x) > \alpha_i \cdot x + \beta xp(e_i^* - x) - x(\alpha_i + \beta p(e_i^* - \frac{x}{2}))$$

$$= \beta x(p(e_i^* - x) - p(e_i^* - \frac{x}{2})) \geq 0$$

Where the last inequality holds because $p()$ is a decreasing function. Therefore $u_i(e_i^*, \mathbf{e_{-i}}; G) > u_i(e_i^* - x, \mathbf{e_{-i}}; G)$.

— e^* is better than any higher effort level. For any $x > 0$, consider $u_i(e_i^*) - u_i(e_i^* + x) =$

$$\alpha_i \cdot (e_i^*) - S(e_i^*, \mathbf{e_{-i}}; G) - \frac{1}{2}e_i^{*2} - \Big(\alpha_i \cdot (e_i^* + x) - S(e_i^* + x, \mathbf{e_{-i}}; G) - \frac{1}{2}(e_i^* + x)^2\Big)$$

$$= -\alpha_i \cdot x + x(e_i^* + \frac{x}{2}) - \beta(S(e_i^* + x) - S(e_i^*))$$

Since $e_i^* + \frac{x}{2} > e_i^*$, it is in the set Y, so $e_i^* + \frac{x}{2} > \alpha_i + \beta p(e_i^* + \frac{x}{2}, \mathbf{e_{-i}}; G)$. Notice this inequality is strict since the only element where this can hold with equality is at e_i^*.

$$> -\alpha_i \cdot x + x(\alpha_i + \beta p(e_i^*, \mathbf{e_{-i}}; G) + \frac{x}{2}) - \beta(S(e_i^* + x) - S(e_i^*))$$

$$= x(\beta p(e_i^*), \mathbf{e_{-i}}; G) + \frac{x}{2}) - \beta(S(e_i^* + x) - S(e_i^*))$$

By Lemma A.1 , $S(e_i^* + x) - S(e_i^*) \leq xp(e_i^*, \mathbf{e_{-i}}; G)$. Plugging that into the above equation yields

$$> x(\beta p(e_i^*, \mathbf{e_{-i}}; G) + \frac{x}{2}) - \beta(xp(e_i^*, \mathbf{e_{-i}}; G)) = \frac{x^2}{2} > 0$$

□