Boolean Set-Operations on X-monotone Curve Bounded Point-Sets

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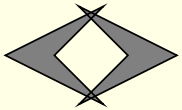
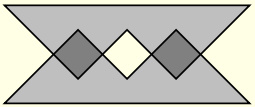
Overview

- What are Boolean Set Operations?
- Regularized versus Ordinary Operations
- General Polygons and such
- Interface
- Internal Representation
- The Traits Concept

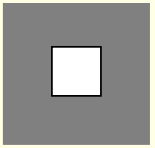
Boolean Set Operations



Union



Intersection



Complement

For two point sets P and Q:

Intersection predicate	$P \cap Q \neq \emptyset$, overlapping cell(s) are not explicitly computed.
Intersection	$R = P \cap Q$
Union	$R = P \cup Q$
Difference	$R = p \setminus Q$
Symmetric Difference	$R = (P \setminus Q) \cup (Q \setminus P)$
Complement	$R = \overline{P}$

Regularized versus Ordinary Operations

- Regularized Boolean-set operations (op*)
  - P op* Q = closure(interior(P op Q))
  - Appear in Constructive Solid Geometry (CSG).
  - Used to avoid isolated elements and open boundaries.
  - Operate on, and result with, closed non-degenerate polygons of area greater than 0.
- Ordinary Boolean-set operations (op)
  - Not implemented yet.
  - Distinguish between interior and boundary of sets.
  - `Nef_2` package supports these operation on linear point sets.

General Polygons and such

A polygon P is *simple* (or Jordan) if the only points of the plane belonging to two polygon edges of P are the polygon vertices of P.

A polygon is *strictly simple* if (in addition) its boundary does not contain the same vertex twice.

A *general polygon* is a connected point set whose boundary edges are weekly x-monotone.

A *general polygon with holes* is a general polygon that contains holes, which are general polygons.

Types and Interface

`General_polygon_2` — represents a simple general polygon.

`General_polygon_with_holes_2` — represents a simple general polygon with holes.

`General_polygon_set_2<General_polygon,Traits>` — represents a set of general polygons with holes. Can be constructed from:

- a single general polygons or a set of them.
- a single general polygons with holes or a set of them.
- the application of one of the regularized operations.

Internal Representation of `General_polygon_set_2`

- Represented by an `Arrangement_2` instance.
- Parameterized with a traits class.
- Supports a single unbounded general polygon.
  - represented as a polygon with holes with an empty outer boundary.

Global functions that operate on, and result with, general polygons (resp. general polygons with holes) directly are available.

When several operations are performed in a sequence, it is more efficient to use `General_polygon_set_2` methods.

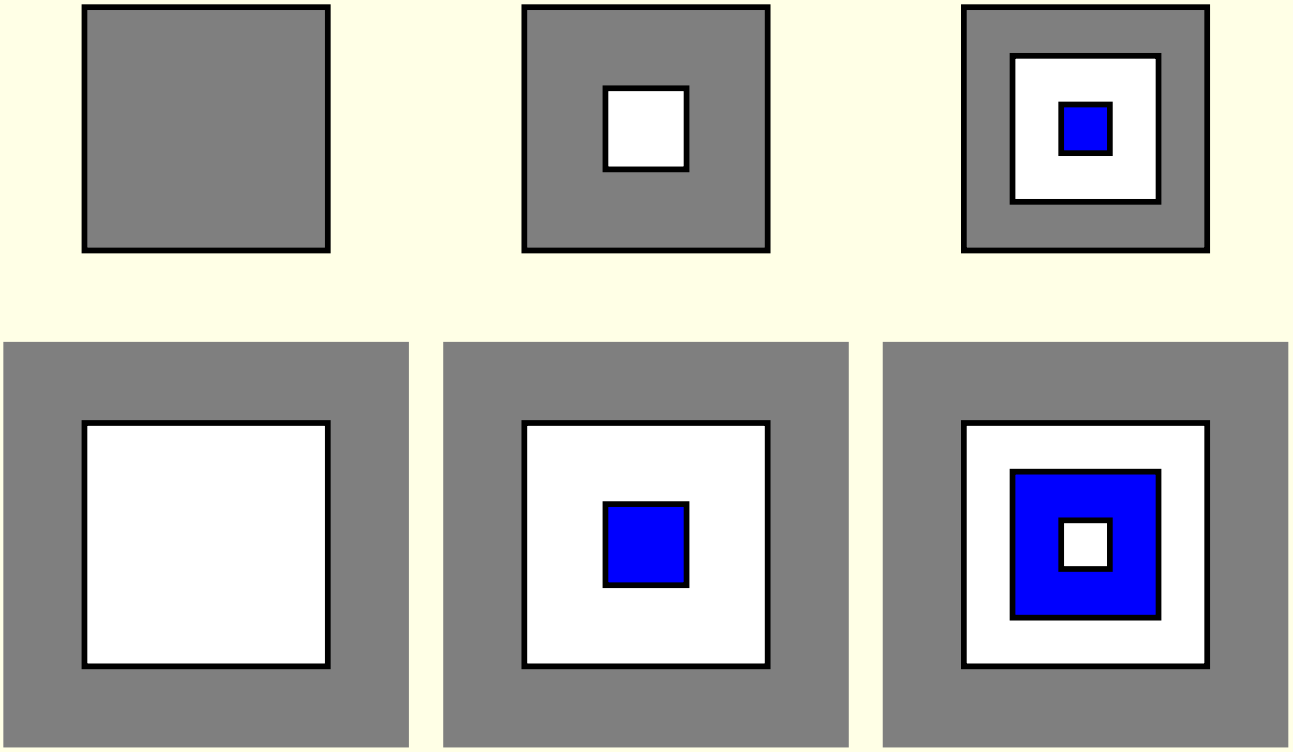
The Traits Concept

- Tailored to handle a specific family of curves.
- Refines the `ArrangementXMonotoneTraits_2` concept.
- Defines the types:
  - `Point_2`
  - `X_monotone_curve_2`
  - `General_polygon_2`
  - `General_polygon_with_holes_2`
- Contains predicates on objects of these types sufficient to enable the boolean set-operations.
- Inexact arithmetic + (nearly) degenerate configurations $\Rightarrow$ abnormal termination or incorrect result.

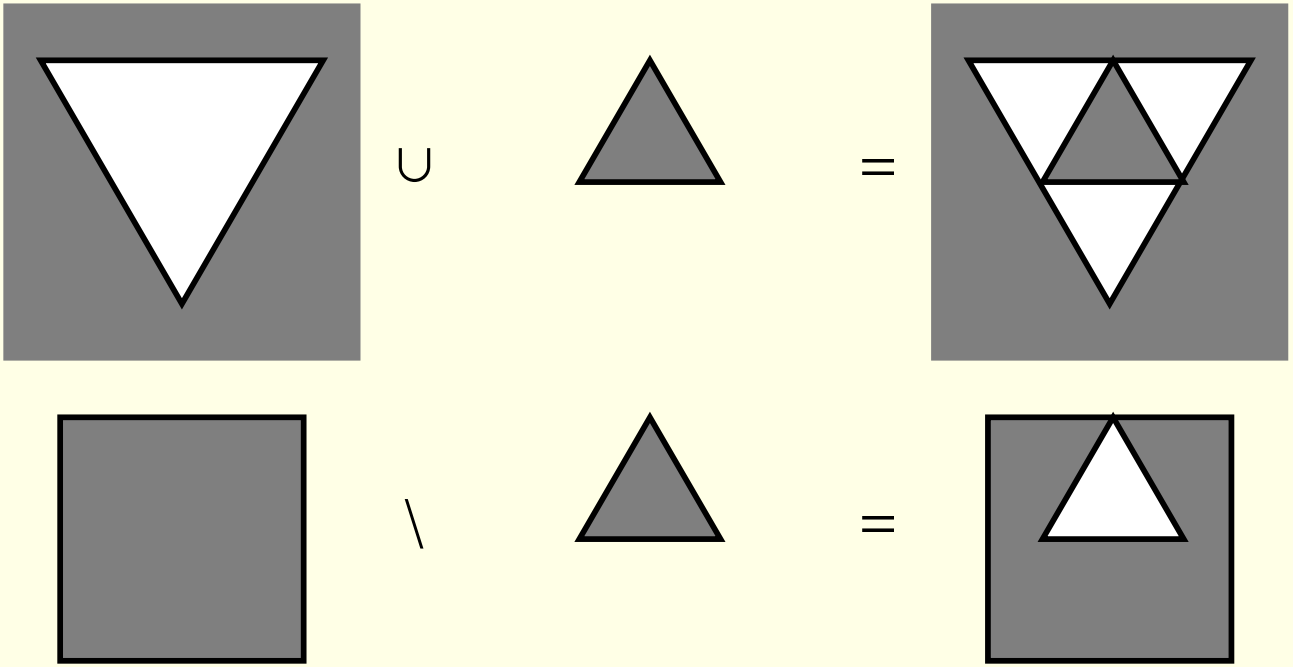
Issues

- Names
  - Package: `Boolean_set_operations_2`.
  - Types: `General_polygon`, `Curved_polygon`, `Polygonal_region`.
- Affinity between Minkowski sums and Boolean set-operations.
- Handling `Triangle_2`, `Iso_rectangle_2`, and `Polygon_2`.
- Interface
  - Consistency with `Nef_2` based Boolean set-operations.
  - `General_polygon_set_2<GeneralPolygon,Traits>`
- Exact definition of obtained general polygons with holes.

Boolean Set Operations



Boolean Set Operations



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