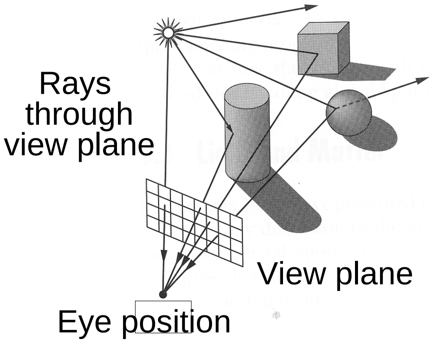
Ray Casting

Based on slides of Thomas
Funkhouser

3D Rendering

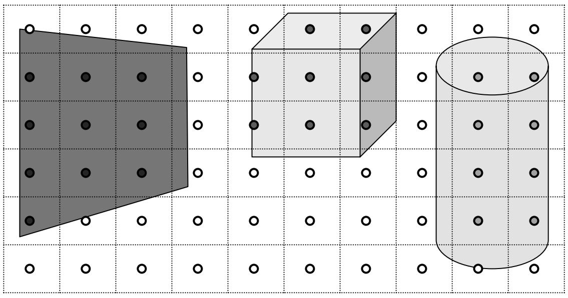
- The color of each pixel on the view plane depends on the radiance emanating from visible surfaces

Simplest method
is ray casting



Ray Casting

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color sample based on surface radiance



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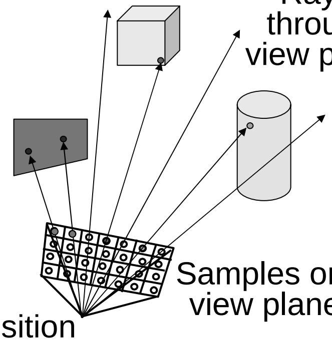

Rays through view plane

Samples on view plane

Eye position

Ray Casting

- Simple implementation:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
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            image[i][j] = GetColor(hit);
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    return image;
}
```

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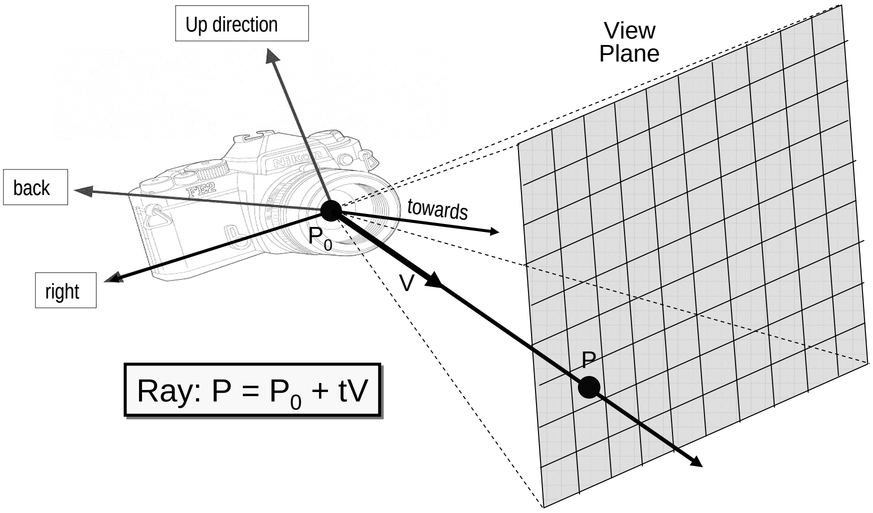
Constructing Ray Through a Pixel



Ray: $P = P_0 + tV$

Constructing Ray Through a Pixel

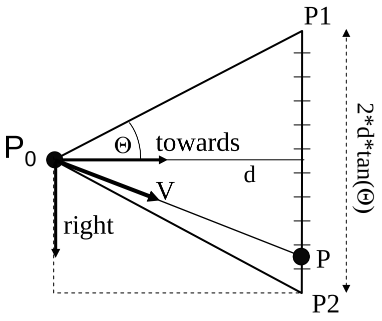
- 2D Example

Θ = frustum half-angle
d = distance to view plane

right = towards x up

$P1 = P_0 + d*towards - d*tan(\Theta)*right$
$P2 = P_0 + d*towards + d*tan(\Theta)*right$

$P = P1 + (i/width + 0.5) * 2*d*tan (\Theta)*right$
$V = (P - P_0) / ||P - P_0 ||$



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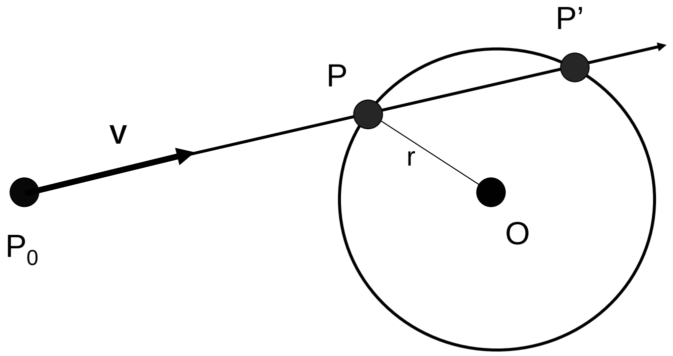
Ray-Scene Intersection

- Intersections with geometric primitives
    - Sphere
    - Triangle
    - Groups of primitives (scene)
- Acceleration techniques
    - Bounding volume hierarchies
    - Spatial partitions
        - Uniform grids
        - Octrees
        - BSP trees

Ray-Sphere Intersection

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$



Ray-Sphere Intersection I

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$

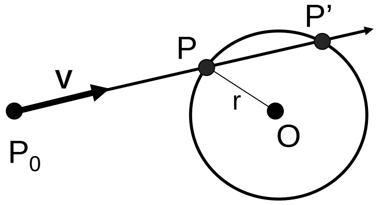
Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

$$a = 1$$
$$b = 2\, V \bullet (P_0 - O)$$
$$c = |P_0 - C|^2 - r^2 = 0$$

$P = P_0 + tV$



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

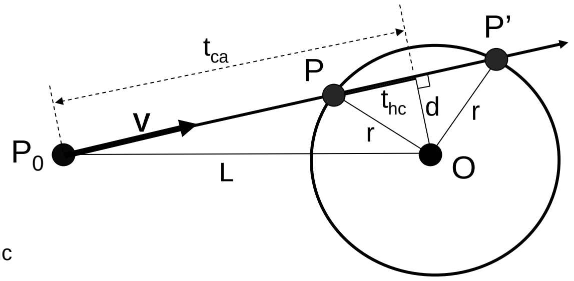
$L = O - P_0$

$t_{ca} = L \bullet V$
if ($t_{ca} < 0$) return 0

$d^2 = L \bullet L - t_{ca}^2$
if ($d^2 > r^2$) return 0

$t_{hc} = \mathrm{sqrt}(r^2 - d^2)$
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$

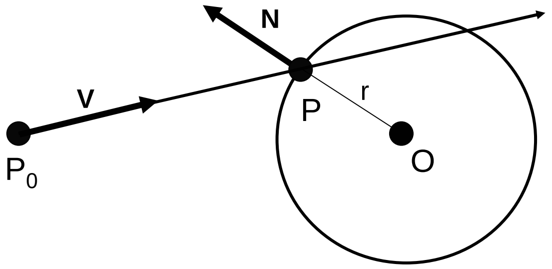
$P = P_0 + tV$



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations

$N = (P - O) / ||P - O||$



Ray-Scene Intersection

- Intersections with geometric primitives
    - Sphere
    - » **Triangle**
    - Groups of primitives (scene)
- Acceleration techniques
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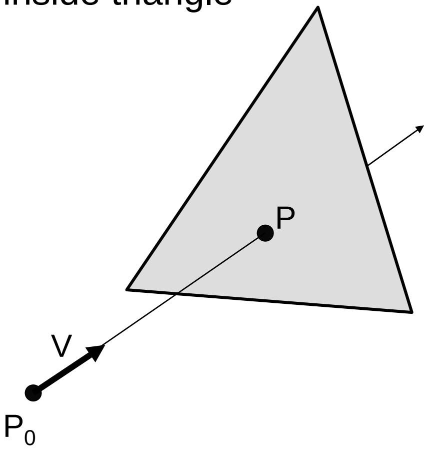
Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if point is inside triangle



Ray-Plane Intersection

Ray: $P = P_0 + tV$
Plane: $P \bullet N + d = 0$

Algebraic Method

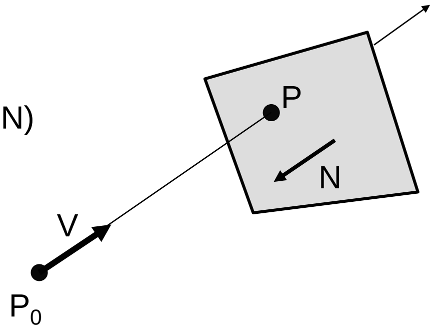
Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \bullet N + d = 0$$

Solution:

$$t = -(P_0 \bullet N + d) / (V \bullet N)$$

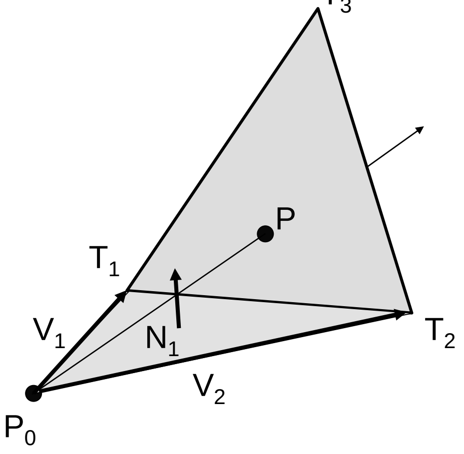
$$P = P_0 + tV$$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

```
For each side of triangle
    $V_1 = T_1 - P$
    $V_2 = T_2 - P$
    $N_1 = V_2 \times V_1$
    Normalize $N_1$
    $d_1 = -P_0 \bullet N_1$
    if $((P \bullet N_1 + d_1) < 0)$
        return FALSE;
end
```



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

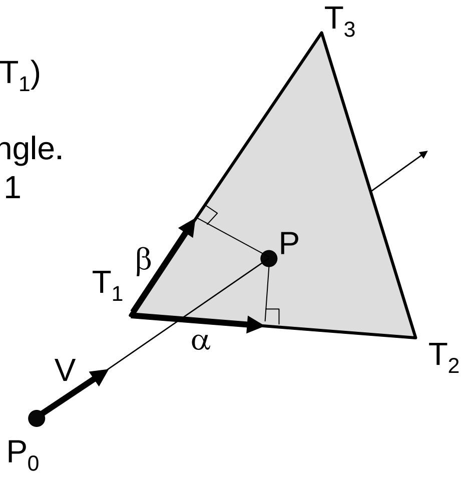
Compute α, β:

$P = \alpha (T_2 - T_1) + \beta (T_3 - T_1)$

Check if point inside triangle.

$0 \le \alpha \le 1$ and $0 \le \beta \le 1$

$\alpha + \beta \le 1$



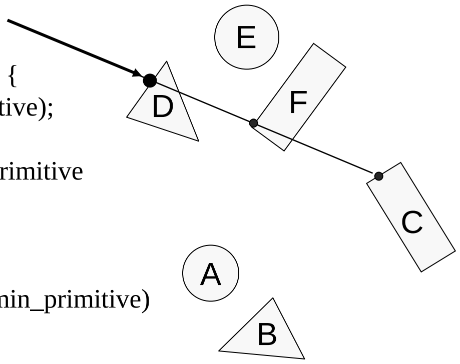
Other Ray-Primitive Intersections

- Cone, cylinder, ellipsoid:
  - Similar to sphere
- Box
  - Intersect 3 front-facing planes, return closest
- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
- Concave polygon
  - Same plane intersection
  - More complex point-in-polygon test

Ray-Scene Intersection

- Find intersection with front-most primitive in group

```
Intersection FindIntersection(Ray ray, Scene scene)
{
    min_t = infinity
    min_primitive = NULL
    For each primitive in scene {
        t = Intersect(ray, primitive);
        if (t < min_t) then
            min_primitive = primitive
            min_t = t
        }
    }
    return Intersection(min_t, min_primitive)
}
```



Ray-Scene Intersection

- Intersections with geometric primitives
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    - Groups of primitives (scene)

» Acceleration techniques
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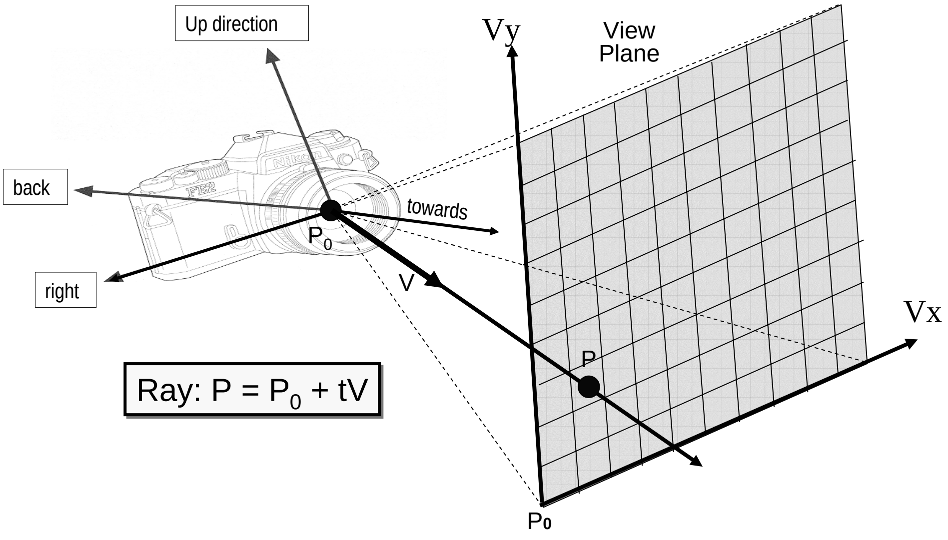
Next Time!

Summary

- Writing a simple ray casting renderer is easy
  - Generate rays
  - Intersection tests
  - Lighting calculations

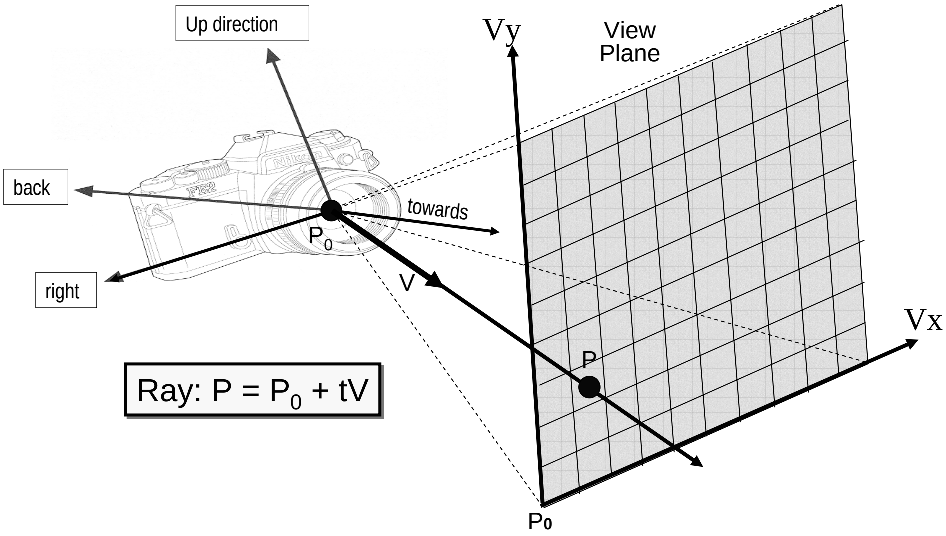
```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
        Image image = new Image(width, height);
        for (int i = 0; i < width; i++) {
                for (int j = 0; j < height; j++) {
                        Ray ray = ConstructRayThroughPixel(camera, i, j);
                        Intersection hit = FindIntersection(ray, scene);
                        image[i][j] = GetColor(hit);
                }
        }
        return image;
}
```

Constructing Ray Through a Pixel



Ray: $P = P_0 + tV$

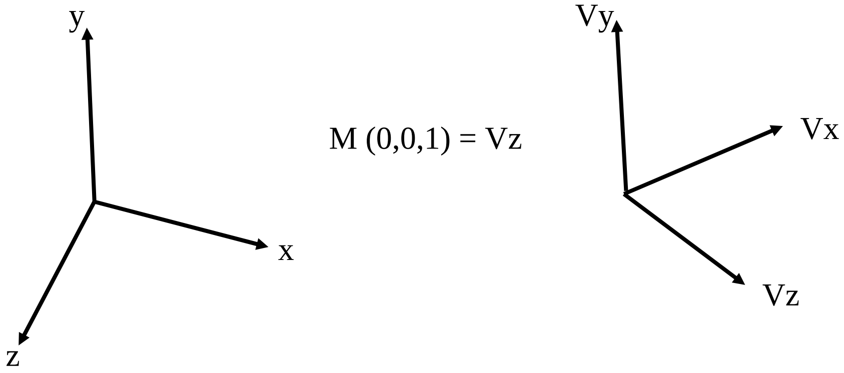
We need to determine Vx and Vy



Ray: $P = P_0 + tV$

Camera Coordinate System

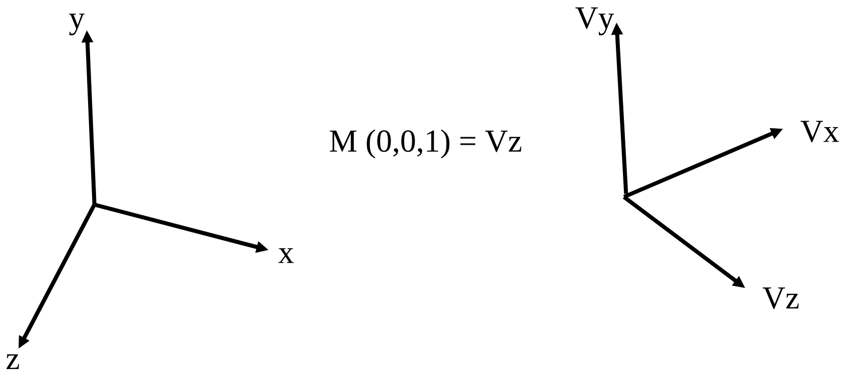
- Find the transformation matrix M that rotate the world coordinate system to the camera coordinate system (Vx,Vy,Vz) (normalized)



$M\,(0,0,1) = Vz$

Camera Coordinate System

- The vector X and Y are rotated by M



$M\ (0,0,1) = Vz$

The definition of the Matrix M

Let Cx and Sx denote sin(x), cos(x), respectively

Rotate around z

$$(0,0,1) \bullet \begin{vmatrix} Cz & Sz & 0 \\ -Sz & Cz & 0 \\ 0 & 0 & 1 \end{vmatrix} = (0,0,1)$$

Rotate around x   Rotate around y

$$M = \begin{vmatrix} 1 & 0 & 0 \\ 0 & Cx & Sx \\ 0 & -Sx & Cx \end{vmatrix} \bullet \begin{vmatrix} Cy & 0 & Sy \\ 0 & 1 & 0 \\ -Sy & 0 & Cy \end{vmatrix} = \begin{vmatrix} Cy & 0 & Sy \\ -SxSy & Cx & SxCy \\ -CxSy & -Sx & CxCy \end{vmatrix}$$

The definition of the Matrix M

Since:

(0,0,1) • M = (-CxSy, -Sx, CxCy) =
(Vz.x,Vz.y,Vz.z) = Vz = (a,b,c).

We get:

a = -CxSy; b = -Sx; c = CxCy,

or

Sx = -b; Cx = sqrt(1 - sqr(Sx));
Sy = -a/Cx;Cy = c/Cx;

Compute the Camera Coordinate System

Now, use M to rotate the world coordinate vectors:

Vx = (1,0,0) • M

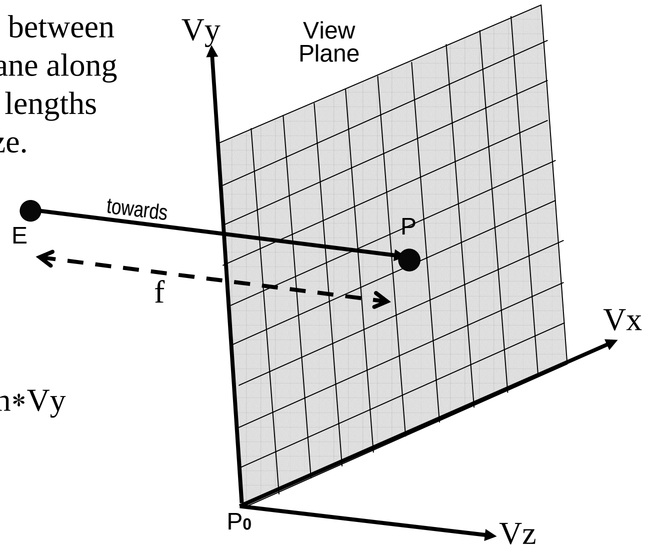
Vy = (0,1,0) • M

Vz = (0,0,1) • M

Note that the vector V is normalized.

We need to determine Vx and Vy

Let f be the distance between the eye E and the plane along Vz, and w and h the lengths of half the screen size.



$P = E + Vz * f$

$P_0 = P - w*Vx - h*Vy$

The main loop

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
        Image image = new Image(width, height);
        Set P0 (as in the previous slide);
        for (int i = 0; i < height; i++) {
                p = P0;
                for (int j = 0; j < width; j++) {
                        Ray ray = E + t * (p – E);
                        Intersection hit = FindIntersection(ray, scene);
                        image[i][j] = GetColor(hit);
                        p += Vx; // move one pixel along the vector Vx
                }
        P0 += Vy; // move one pixel along the vector Vy
        }
        return image;
}
```