4-Points Congruent Sets for Robust Pairwise Surface Registration

Dror Aiger  Niloy J. Mitra  Daniel Cohen-Or
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Dror Aiger
Ben Gurion University

Niloy J. Mitra
IIT, Delhi

Daniel Cohen-Or
Tel Aviv University
Problem Statement

Given: Two models (P and Q)
- corrupted with noise and outliers
- in arbitrary initial poses
Problem Statement

Goal: *Automatically* align the models
Aligning with Feature Points
Aligning with Feature Points

3 corresponding point-pairs! solve for aligning rigid transform
Aligning with Feature Points

Data + holes + outliers + noise! unreliable feature points (FP)
Why not De-noise Scans?

de-noise, compute FP-s, align

align with 4PCS, de-noise
Insight #1

A pair of triples (from $P$ and $Q$) is enough to uniquely define a **rigid transform**! $O(n^3)$

Surprisingly, a *special* set of 4-points, **congruent sets**, makes problem simpler! $O(n^2)$
Affine Invariance
Affine Invariance

$S_1$

Points: a, b, c, d
Affine Invariance

4 coplanar points
Affine Invariance
Affine Invariance

\[ r_1 = \frac{\|a - e\|}{\|a - b\|} \]
\[ r_2 = \frac{\|c - e\|}{\|c - d\|} \]
Affine Invariance

$S_1$  

$S_2$
Affine Invariance
Affine Invariance
Affine Invariance

\[ \frac{\|a' - e'\|}{\|a' - b'\|} = r_1 \]
\[ \frac{\|c' - e'\|}{\|c' - d'\|} = r_2 \]
Extracting Congruent 4-points

\[ r_1 = \frac{|a-e|}{|a-b|} \]

\[ r_2 = \frac{|c-e|}{|c-d|} \]
Extracting Congruent 4-points

\[ r_1 = \frac{|a-e|}{|a-b|} \]

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Extracting Congruent 4-points

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Extracting Congruent 4-points

\[ r_1 = \frac{||a-e||}{||a-b||} \]

\[ r_2 = \frac{||c-e||}{||c-d||} \]
What if $e_1 \neq e_2$?

$r_1 = \frac{|a-e|}{|a-b|}$

$r_2 = \frac{|c-e|}{|c-d|}$

typical scenario
What if $e_1 = e_2$?

$r_1 = \frac{|a-e|}{|a-b|}$

$r_2 = \frac{|c-e|}{|c-d|}$

typical scenario

congruent 4-points!
Extracting Congruent 4-points

$q_1$  

$q_2$  

$q_3$  

$q_4$  

$q_5$
Extracting Congruent 4-points
Extracting Congruent 4-points
Extracting Congruent 4-points
Extracting Congruent 4-points

\{a, b, c, d\} \equiv \{q_1, q_2, q_3, q_4\}
Wide-base! more stable

narrow base! unstable

wide base (4-points set)! increased stability
Result: Gallery