Assignment no. 3

http://www.cs.tau.ac.il/∼danha/courses/robotics07.html

due date: Monday, January 1st, 2007

Exercise 3.1 Consider the motion-planning problem of a line segment (the *rod*) translating and rotating in the plane among polygonal obstacles with a total of n vertices. In class we saw a presentation of the free space the complexity of which is $O(n^5)$. The actual complexity of the free space in this motion planning instance is much lower. **(a)** Show that the maximum combinatorial complexity of the free configuration space in this case is $O(n^2)$. To show this bound you have to bound the number of semi-free triple contacts (namely placements of the rod where it touches the obstacles boundaries in three points without penetrating into the obstacles). *Hint:* Use the result for the single-segment "robot arm" in Exercise 2.2. **(b)** Show that the above bound is tight in the worst case. That is, describe a scene where the complexity of the free space is $\Omega(n^2)$.

Exercise 3.2 Let S be a set of n pairwise disjoint segments in the plane. In class we saw that S admits a separation sequence along any given direction $\vec{d}$, that is, there is an ordering of the segments in S: $s_1, s_2, \ldots, s_n$ such that the segment s_i can be translated to infinity in direction $\vec{d}$ without hitting any segment $s_j, j > i$. Given a set S of segments and a direction $\vec{d}$ as above, design an *efficient* algorithm that will determine a separation sequence for S.

Exercise 3.3 (p) (bonus) Write a program that solves the partition problem for a collection of polygons in the plane under infinitesimal translations. Namely, given a collection A of pairwise interior-disjoint polygons in the plane, the program should determine whether there exists a proper subset $S \subset A$ and a direction $\vec{d}$ such that S can be translated as a rigid body an arbitrary small distance in direction $\vec{d}$ without hitting the polygons in $A \setminus S$. If no such subset+direction exist, the program should output INTERLOCKED.

Exercise 3.4 (p) (bonus) Same as Exercise 3.3, only for infinite translations in the plane.