# Exercise 6 (beta version) – Computational Models

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1. We define the class $\mathcal{EXP}$: $L \in \mathcal{EXP}$ if there is a constant $c$ and a deterministic TM $M$, such that $M$ decides $L$ in $2^{O(|x|^c)}$ steps, for any $x \in L$.

   (a) Prove that $Clique \in \mathcal{EXP}$ by giving an exponential algorithm that solves $Clique$.

   (b) prove that $NP \subseteq EXP$.

2. A triangle in an undirected graph is a clique of size 3. We define the problem $Triangle{-}Free{-}VC = \{\langle G, k\rangle | G$ is an undirected graph with no triangles that has a VC of size $\leq k\}$. Prove that $Triangle - Free - VC \in NPC$.

3. We define the class $coNPC$: $L \in coNPC$ if $L \in coNP$ and $L \in coNP{-}hard$, namely $\forall L' \in coNP, L' \leq_p L$. Prove that $L \in NPC$ if and only if $\overline{L} \in coNPC$.

4. We define the problem: $Max - Clique =$
   $\{\langle G, k\rangle | G$ is an undirected graph whose largest clique size is exactly $k\}$.

   (a) Prove that $Max - Clique \in NP - hard$.

   (b) Prove that $Max - Clique \in co - NP - hard$.

   (c) Assume $P \neq NP$. Is it possible that $Max - Clique \in NPC$? Explain!