Critical CS Questions

What is a computer?
And What is a Computation?

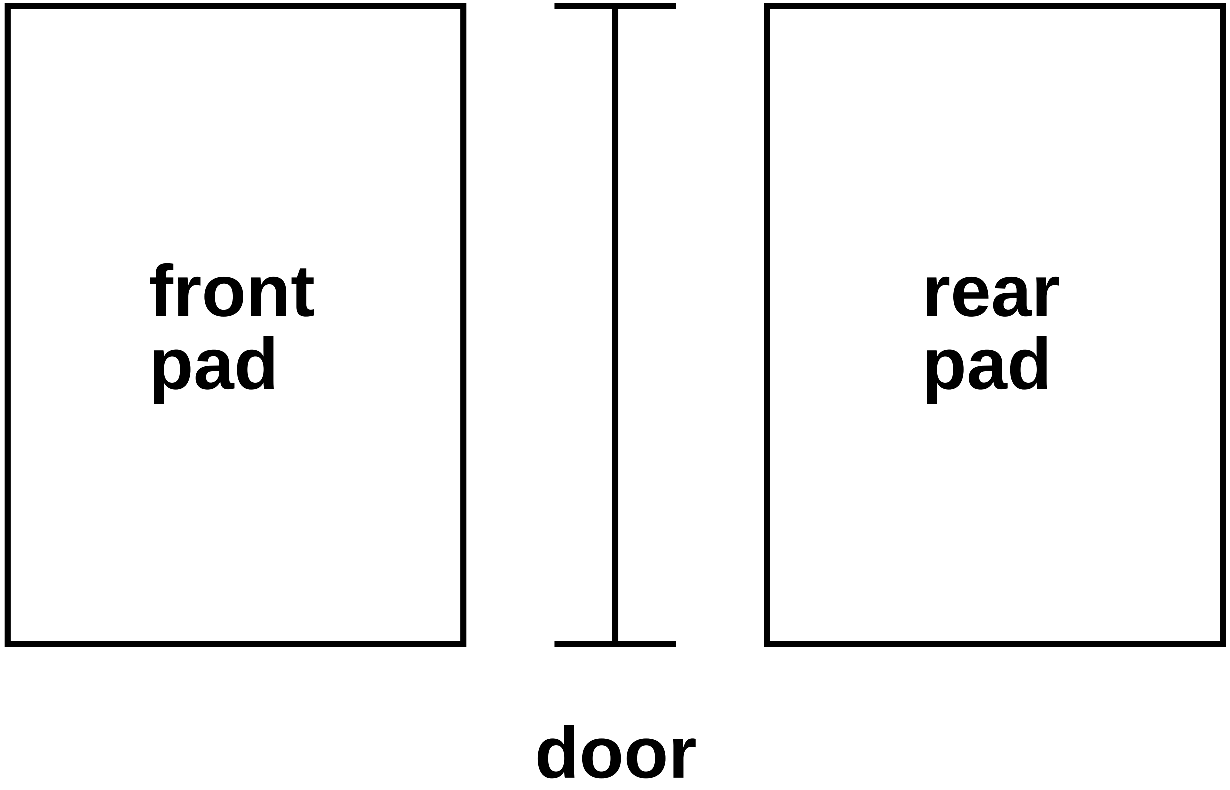
- real computers too complex for any theory
- need manageable mathematical abstraction
- idealized models: accurate in some ways, but not in all details

Slides modified by Benny Chor, based on original slides by Maurice Herlihy, Brown University.

Finite Automata

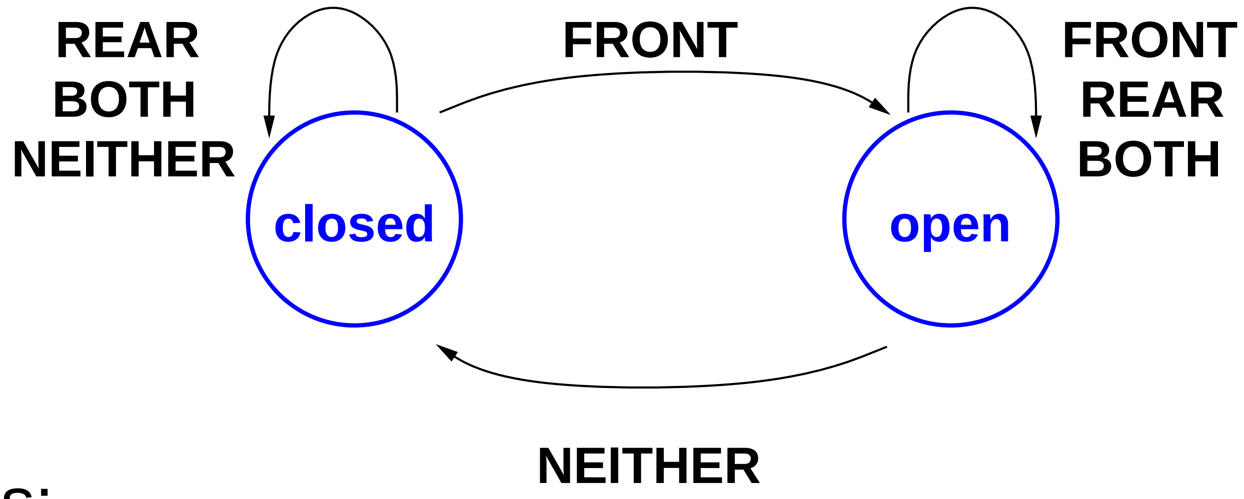
- formal definition of finite automata
- deterministic vs. non-deterministic finite automata
- regular languages
- operations on regular languages
- regular expressions
- pumping lemma

Example: A One-Way Automatic Door



- open when person approaches
- hold open until person clears
- don't open when someone standing behind door

The Automatic Door as DFA



- States:
  - OPEN
  - CLOSED
- Sensor:
  - FRONT: someone on front pad
  - REAR: someone on rear pad
  - BOTH: someone(s) on both pads
  - NEITHER no one on either pad.

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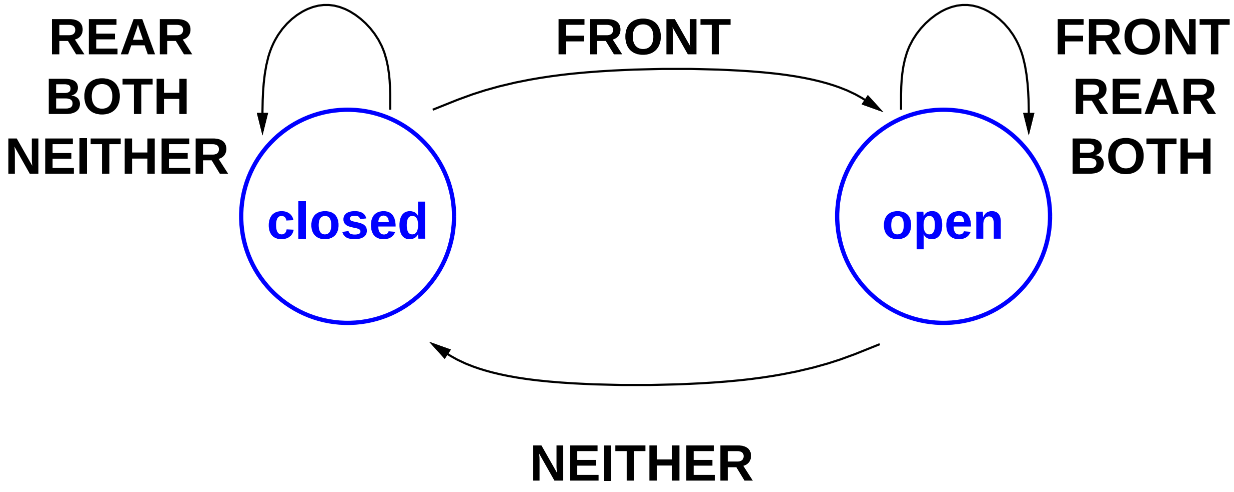
The Automatic Door as DFA

DFA is Deterministic Finite Automata



	neither	front	rear	both
closed	closed	open	closed	closed
open	closed	open	open	open

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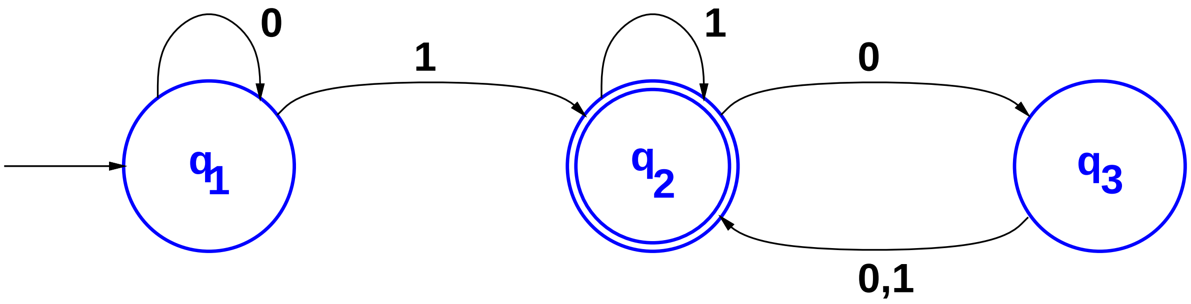
DFA: Informal Definition



The machine M_1:

- states: q_1, q_2, and q_3.
- start state: q_1 (arrow from “outside”).
- accept state: q_2 (double circle).
- state transitions: arrows.

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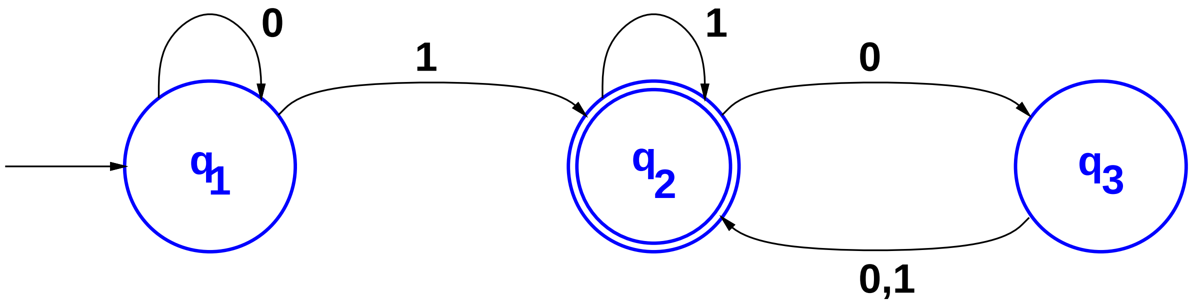
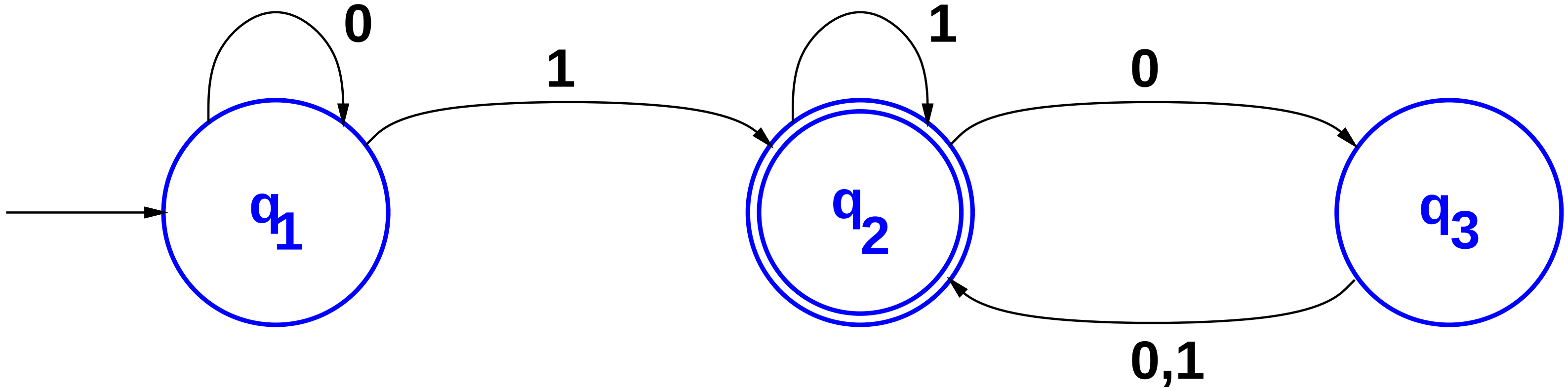
DFA: Informal Definition (cont.)



- On an input string
  - DFA begins in start state q_1
  - after reading each symbol, DFA makes state transition with matching label.
- After reading last symbol, DFA produces output:
  - accept if DFA is an accepting state.
  - reject otherwise.

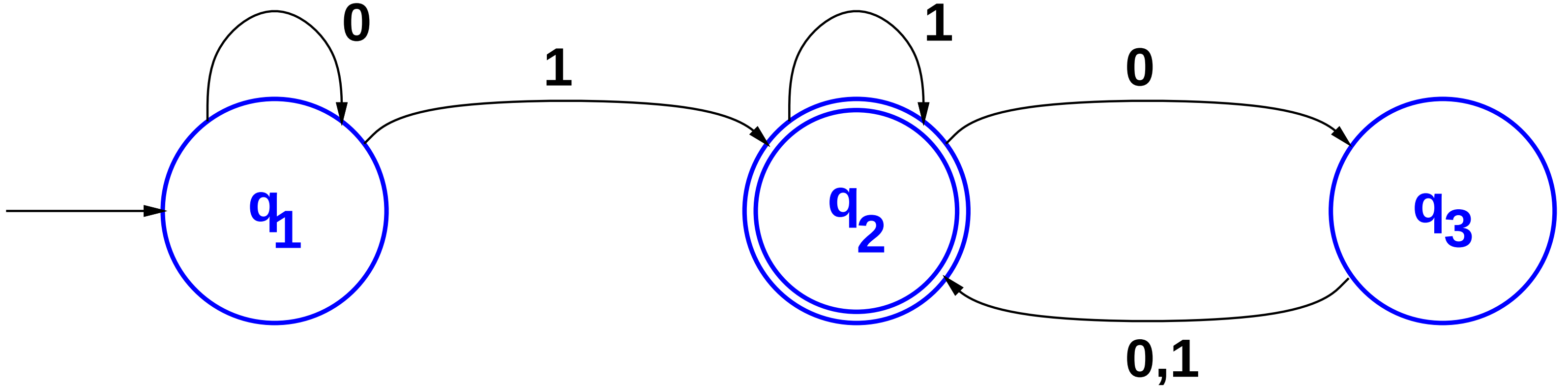
Informal Definition - Example



What happens on input strings

- 1101
- 0010
- 01100

Informal Definition



This DFA accepts

- all input strings that end with a 1
- all input strings that contain at least one 1, and end with an even number of 0's
- no other strings

Languages and Alphabets

An alphabet Σ is a finite set of letters.

- $\Sigma = \{a, b, c, \ldots, z\}$ – the English alphabet.
- $\Sigma = \{\alpha, \beta, \gamma, \ldots, \zeta\}$ – the Greek alphabet.
- $\Sigma = \{0, 1\}$ – the binary alphabet.
- $\Sigma = \{0, 1, \ldots, 9\}$ – the digital alphabet.

The collection of all strings over Σ is denoted by Σ^*.

For the binary alphabet, ε, 1, 0, 000000000, 1111111000 are all members of Σ^*.

Languages and Examples

A language over Σ is a subset $L \subseteq \Sigma^*$. For example

- Modern English.
- Ancient Greek.
- All prime numbers, written using digits.
- $A = \{w | w \text{ has at most seventeen 0's}\}$.
- $B = \{0^n 1^n | n \geq 0\}$.
- $C = \{w | w \text{ has an equal number of 0's and 1's}\}$.

Languages and DFA

Definition: $L(M)$, the language of a DFA M, is the set of strings L that M accepts, $L(M) = L$.

Note that

- M may accept many strings, but
- M accepts only one language.

What language does M accept if it accepts no strings?

A language is called regular if some deterministic finite automaton accepts it.

Formal Definitions

A deterministic finite automaton (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set called the states,
- Σ is a finite set called the alphabet,
- $\delta : Q \times \Sigma \to Q$ is the transition function,
- $q_0 \in Q$ is the start state, and
- $F \subseteq Q$ is the set of accept states.

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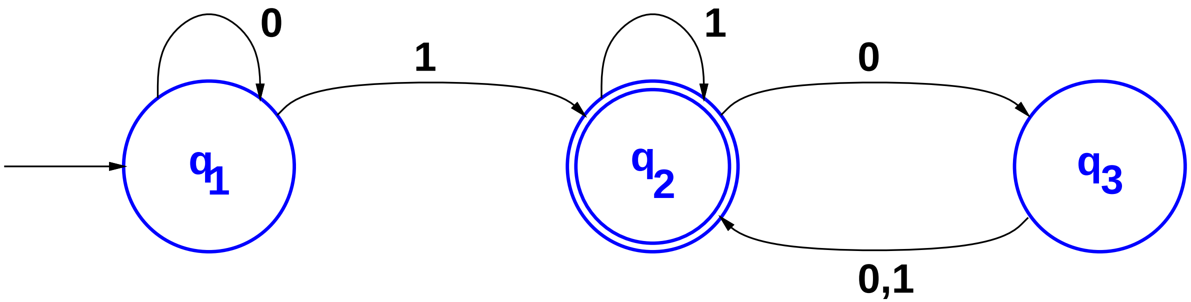
Back to M_1



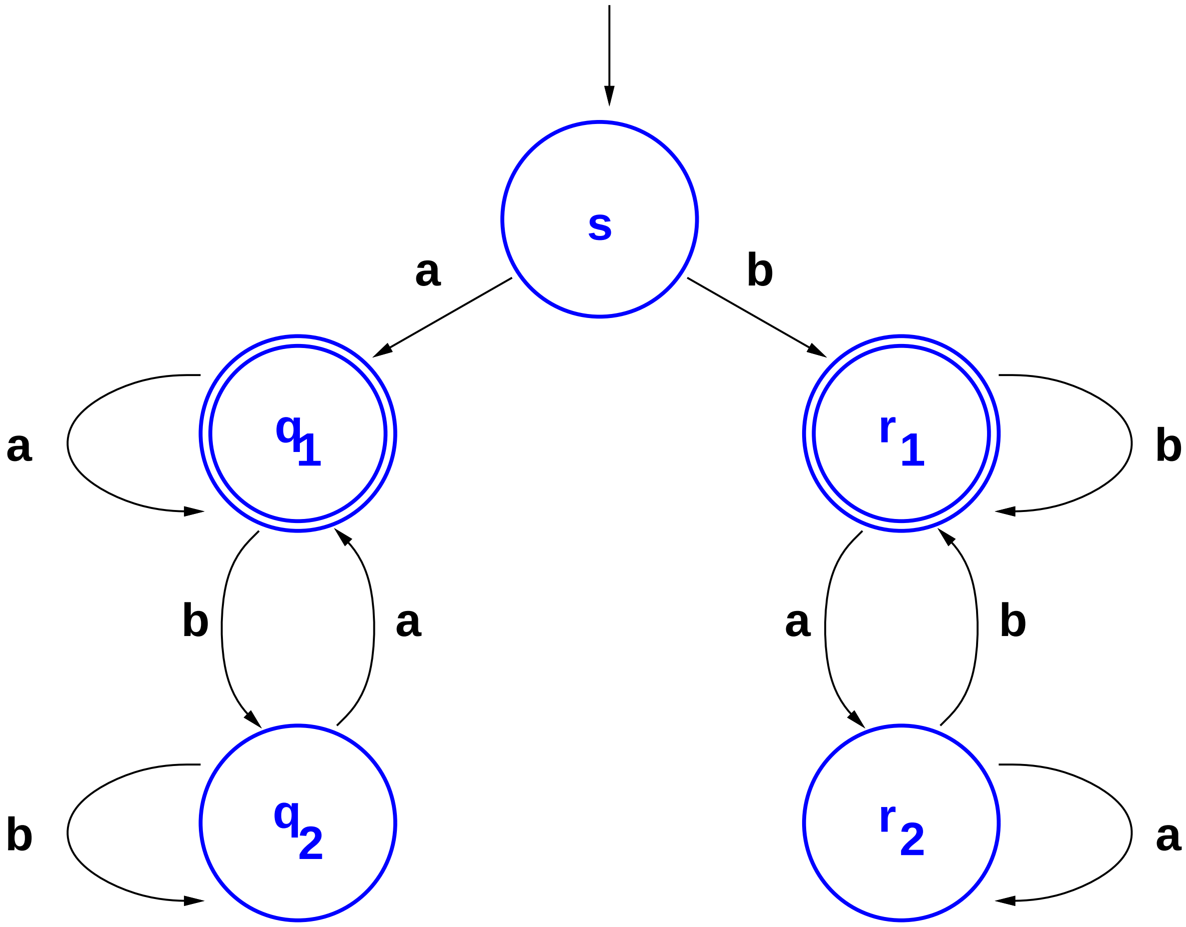
$M_1 = (Q, \Sigma, \delta, q_1, F)$ where

- $Q = \{q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$,
- the transition function δ is

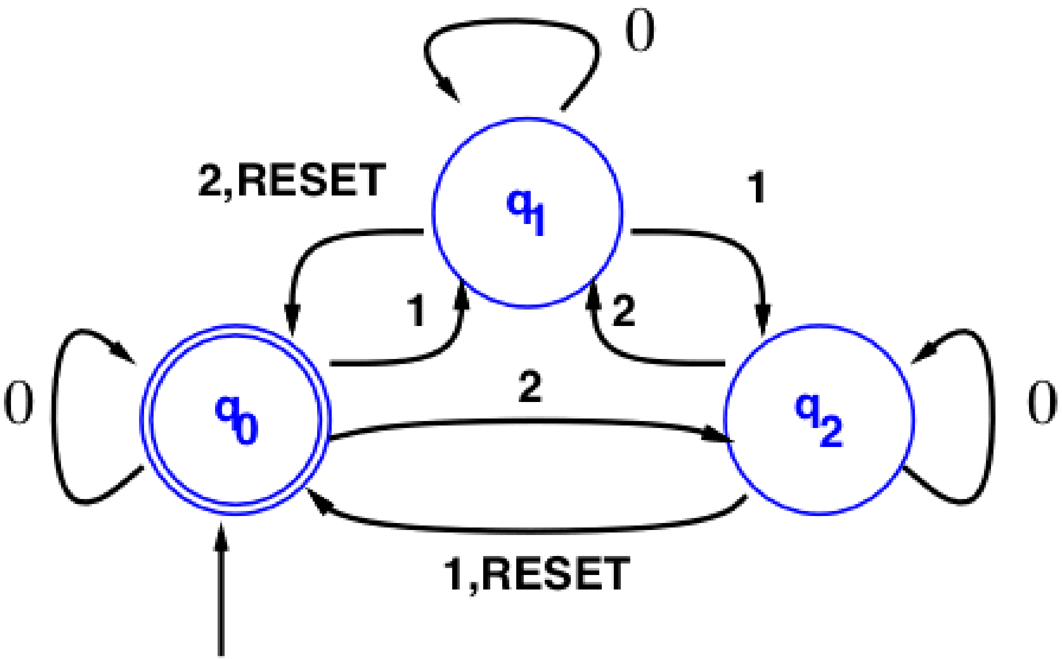
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

- q_1 is the start state, and $F = \{q_2\}$.

Another Example



And Yet Another Example



A Formal Model of Computation

- Let M = $(Q, \Sigma, \delta, q_0, F)$ be a DFA, and
- let w = $w_1 w_2 \cdots w_n$ be a string over Σ.

We say that M accepts w if there is a sequence of states $r_0, \ldots, r_n$ ($r_i \in Q$) such that

- $r_0 = q_0$
- $\delta(r_i, w_{i+1}) = r_{i+1}$, $0 \leq i < n$
- $r_n \in F$

The Regular Operations

Let A and B be languages.

The union operation:

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

The concatenation operation:

$$A \circ B = \{xy | x \in A \text{ and } y \in B\}$$

The star operation:

$$A^* = \{x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A\}$$

The Regular Operations – Examples

Let $A = \{\text{good}, \text{bad}\}$ and $B = \{\text{boy}, \text{girl}\}$.

Union

$$A \cup B = \{\text{good}, \text{bad}, \text{boy}, \text{girl}\}$$

Concatenation

$$A \circ B = \{\text{goodboy}, \text{goodgirl}, \text{badboy}, \text{badgirl}\}$$

Star

$$A^* = \{\varepsilon, \text{good}, \text{bad}, \text{goodgood}, \text{goodbad}, \text{badbad}, \text{badgood}, \ldots\}$$

Claim: Closure Under Union

If A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Approach to Proof:

- some M_1 accepts A_1
- some M_2 accepts A_2
- construct M that accepts $A_1 \cup A_2$.

Attempted Proof Idea:

- first simulate M_1, and
- if M_1 doesn't accept, then simulate M_2.

What's wrong with this?
Fix: Simulate both machines simultaneously.

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Closure Under Union: Correct Proof

- Suppose $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ accepts L_1,
- and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ accepts L_2.

Define M as follows (M will accept $L_1 \cup L_2$):

- $Q = Q_1 \times Q_2$.
- Σ is the same.
- For each $(r_1, r_2) \in Q$ and $a \in \Sigma$, $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$
- $q_0 = (q_1, q_2)$
- $F = \{(r_1, r_2) | r_1 \in F_1$ or $r_2 \in F_2\}$. ♣

(hey, why not choose $F = F_1 \times F_2$?)

What About Concatenation?

Thm: If L_1, L_2 are regular languages, so is $L_1 \circ L_2$.

Example: $L_1 = \{\text{good}, \text{bad}\}$ and $L_2 = \{\text{boy}, \text{girl}\}$.

$$L_1 \circ L_2 = \{\text{goodboy}, \text{goodgirl}, \text{badboy}, \text{badgirl}\}$$

This is much harder to prove.

Idea: Simulate M_1 for a while, then switch to M_2.

Problem: But when do you switch?

This leads us into non-determinism.