Hamiltonian Path

Each variable is represented by the following graph:
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Each clause of $\phi$ is a single node.
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Global structure of graph (missing edges)
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Center of each diamond has $2k$ nodes, one for each clause.
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If variable $x_i$ appears in clause $c_j$, add this "detour"
If $\overline{x_i}$ appears in clause $c_j$, add this “detour”
Hamiltonian Path

After adding edges from “diamonds” to clause vertexes, $G$ is complete.

Claim: If $\phi$ is satisfiable, then $G$ has a Hamiltonian path.

Strategy:
- ignore clause nodes for now
- traverse diamonds
If $x_i$ is true in the assignment, then zig-zag.
If $x_i$ is \textbf{false} in the assignment, then \textbf{zag-zig}.
Hamiltonian Path

Add clause nodes.
- Each $c_j$ is assigned one **true** literal.
- For each clause, pick one.

If we select $x_i$ in $c_i$, add “detour”
Hamiltonian Path

Add clause nodes.
- Each $c_j$ is assigned one true literal.
- For each clause, pick one.

If we select $\overline{x_i}$ in $c_i$, add “detour”

This completes one direction of the reduction.
Hamiltonian Path

Claim: If $G$ has a hamiltonian path from $s$ to $t$, then $\phi$ has a satisfying assignment.

Definition: A normal hamiltonian path is one that traverses the diamonds in order.
- if $x_i$ diamond zig-zags, assign true.
- if $x_i$ diamond zags-zig, assign false.
- each clause vertex appears once
- source of detour determines which literal is assigned true.
Hamiltonian Path

**Claim:** Every Hamiltonian path in $G$ is normal.
Hamiltonian Path

- only arrows to $a_2$ from $a_1, a_3, c$
- paths from $a_1$ or $c$ go elsewhere
- path from $a_2$ would leave no exit

Any hamiltonian path is normal, Q.E.D.