Hamiltonian Path

Each variable is represented by the following graph:
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Each clause of $\phi$ is a single node.

$c_1$
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Global structure of graph (missing edges)

\( x_1 \)

\( x_2 \)

\( \ldots \)

\( x_k \)
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Center of each diamond has $2k$ nodes, one for each clause.

clause $c_1$  

clause $c_2$
If variable $x_i$ appears in clause $c_j$, add this "detour"
If $\overline{x_i}$ appears in clause $c_j$, add this "detour"
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After adding edges from “diamonds” to clause vertexes, $G$ is complete.

**Claim:** If $\phi$ is satisfiable, then $G$ has a hamiltonian path.

**Strategy:**
- ignore clause nodes for now
- traverse diamonds
If $x_i$ is true in the assignment, then zig-zag.
If $x_i$ is false in the assignment, then zag-zig.
Add clause nodes.

- Each $c_j$ is assigned one true literal.
- For each clause, pick one.

If we select $x_i$ in $c_i$, add “detour”
Hamiltonian Path

Add clause nodes.
- Each $c_j$ is assigned one true literal.
- For each clause, pick one.

If we select $\overline{x_i}$ in $c_i$, add “detour”

This completes one direction of the reduction.
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Claim: If $G$ has a hamiltonian path from $s$ to $t$, then $\phi$ has a satisfying assignment.

Definition: A normal hamiltonian path is one that traverses the diamonds in order.
- if $x_i$ diamond zig-zags, assign true.
- if $x_i$ diamond zags-zig, assign false.
- each clause vertex appears once
- source of detour determines which literal is assigned true.
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Claim: Every hamiltonian path in $G$ is normal.
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- only arrows to $a_2$ from $a_1, a_3, c$
- paths from $a_1$ or $c$ go elsewhere
- path from $a_2$ would leave no exit

Any hamiltonian path is normal, Q.E.D.